Erasmus+ Cultural connections : Enhancing EU heritage, Social Inclusion and Digital Literacy through our Pupils' hearts
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ACTIVITY 5.6

Divergent tasks: Maths - Mini Experiment: "Open Tasks in the Classroom"

The teacher starts with a standard school task (e.g.finding the area of a triangle**). The teacher** creates several divergent-thinking versions by:

1. **Modifying the conditions (**e.g. changing dimensions, adding constraints**).**

2. **Asking students to choose their own method.**

3. **Including a visual task or creative interpretation.**

Base Task: Find the area of a triangle.

Divergent Versions with Solutions

Divergent Version 1: Dimensions and Method Choice

Divergent Version 2: Modifying the Conditions (Constraints and parameters change)

Divergent Version 3: Visual or Creative Interpretation (Meaning and representation change)

Divergent version 4 Creative Challenge: "A Mathematics Story"

Other divergence versions: Transforming Traditional Questions into Creative Ones

- *Freedom of tools*
- *Freedom of sequence of work*
- *Unexpected elements (both geometric and real-life)*

Divergent Version 1.1: Choice of Dimensions

"Choose any triangle with area 24 cm²."

Sample Solutions:

- Base 6, height 8
- Base 12, height 4
- Base 16, height 3

All satisfy:

$$A = \frac{1}{2}bh = 24$$

Step 1: Area Formula

For any triangle:

$$A = \frac{1}{2}bh$$

To achieve an area of 24 cm²:

$$\frac{1}{2}bh = 24 \quad \Rightarrow \quad bh = 48$$

So, **any base–height pair whose product is 48** is valid.

Step 2: Complete Set of Sample Solutions

Base (b)	Height (h)	Calculation	Area
6	8	(6 × 8) ÷ 2	24
8	6	(8 × 6) ÷ 2	24
12	4	(12 × 4) ÷ 2	24

4	12	(4 × 12) ÷ 2	24
16	3	(16 × 3) ÷ 2	24
24	2	(24 × 2) ÷ 2	24
48	1	(48 × 1) ÷ 2	24

Step 3: Visual and Geometric Interpretation

1. Tall, narrow triangles (e.g. base 48, height 1)
2. Short, wide triangles (e.g. base 1, height 48)
3. Balanced triangles (e.g. base 6, height 8)

All have the **same area**, but look very different.

Step 4: Key Mathematical Conclusion There are infinitely many triangles with area 24 cm^2, because there are infinitely many pairs of numbers whose product is 48.

Divergent Version 1.2: Method Choice

"Find the area of a triangle using **any method**."

Valid Methods:

- Formula $\frac{1}{2}bh$
- Coordinate geometry
- Decomposition into rectangles
- Counting squares on grid paper

Finding the Area of a Triangle

Given example used consistently:
Base (b = 6) units, height (h = 8) units
(Expected area: **24 square units**)

Method 1: Using the Formula

$\frac{1}{2}bh$

$$A = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$$

Method 2: Decomposition into Rectangles

Visual Idea Enclose the triangle inside a rectangle.

- Rectangle dimensions:
  Width = 6, Height = 8
- Rectangle area:
  $6 \times 8 = 48$

Step-by-Step Reasoning

1. The triangle occupies exactly **half** of the rectangle.
2. Therefore:
   $$A = \frac{48}{2} = 24$$

Final Answer

$$\boxed{A = 24 \text{ square units}}$$

Pedagogical Note This method:

1. Builds intuition for the factor $\frac{1}{2}$
2. Is excellent for visual and SEN learners
3. Can be introduced **before formal formulas**

Method 3: Coordinate Geometry

Step-by-Step Setup

Place the triangle on a coordinate plane.

Let the vertices be:

$$A(0,0)$$
$$B(6,0)$$
$$C(0,8)$$

Method 3A: Base × Height on Axes

- Base lies along the x-axis: length = 6
- Height lies along the y-axis: length = 8

Apply the area formula:

$$A = \frac{1}{2} \times 6 \times 8 = 24$$

Method 3B: Using the Determinant Formula (Optional Extension)

$$A = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Substitute:

$$A = \frac{1}{2}|0(0-8) + 6(8-0) + 0(0-0)|$$

$$A = \frac{1}{2}|48| = 24$$

Final Answer

$$\boxed{A = 24 \text{ square units}}$$

Pedagogical Note This method:

- Connects algebra and geometry
- Reinforces coordinate systems
- Is ideal for lower secondary and above

Method 4: Counting Squares on Grid Paper

1. Draw the triangle on **1 cm × 1 cm grid paper**.
2. Count the full squares inside the triangle.
3. Combine partial squares into full ones.
4. Total counted area equals **24 square units**.

Step-by-Step Process

1. Draw the triangle on grid paper.
2. Count:

- Full squares inside the triangle
- Half-squares along the edges

Example Count

- 12 full squares
- 24 half-squares = 12 full squares

Total area:

$$12 + 12 = 24$$

Final Answer

$$\boxed{A = 24 \text{ square units}}$$

Pedagogical Note

This method:

1. Is concrete and accessible
2. Develops estimation skills
3. Helps students *see* area, not just calculate it

Step 5: Comparison of Methods

Method	Mathematical Idea	Skills Used
Formula	Direct calculation	Algebraic reasoning
Rectangle	Halving area	Visual reasoning
Coordinates	Geometry + grid	Spatial reasoning
Grid counting	Estimation and counting	Concrete reasoning

Final Teaching Insight These divergent tasks show that:

- One problem can have **many correct answers**
- One result can be reached by **many valid methods**
- Mathematical understanding deepens when students **choose and justify** their

Key Teaching Insight "Different methods reveal the same structure from different perspectives."

Divergent Version 2: Modifying the Conditions (Constraints and parameters change)

Task A: Variable Dimensions

"Draw **three different triangles** that all have an area of **24 cm²**. Label their base and height."

Expected Student Solutions Students may produce, for example:

Base (cm)	Height (cm)	Area
6	8	24
12	4	24
16	3	24

All satisfy:

$$A = \frac{1}{2}bh = 24$$

Pedagogical Value Students discover that:

- Area depends on the **product** of base and height
- Many different shapes can share the same area
- Geometry is **flexible**, not fixed

Extension (Higher Cognitive Demand)

"Which triangle uses the **least material** (shortest perimeter)? Explain your reasoning." (Expected insight: more compact shapes tend to minimize perimeter.)

Task B: Constraint-Based Optimization

"Your triangle must:

- Have an area of **at least 30 cm^2**
- Use a base **no longer than 10 cm**
- Be drawn on grid paper"

Expected Reasoning Students:

- Adjust height to compensate for base limits
- Test possibilities visually or numerically
- Justify choices mathematically

Divergence Type: Conceptual and conditional divergence

Divergent Version 3: Visual or Creative Interpretation (Meaning and representation change)

Task A: Visual Design Challenge
Task B: Story-Based Interpretation (Fully Exemplified)

Task A: Visual Design Challenge

"Design a triangular **park, stage, or roof** with an area of **24 m^2**. Draw it to scale and explain how you know the area is correct."

Expected Student Representations Students may:

1. Draw on grid paper
2. Decompose shapes into rectangles
3. Annotate dimensions visually
4. Use colors to highlight base and height

Example Student Explanation

"I made the base 6 m and the height 8 m. I checked the area by dividing the surrounding rectangle in half."

Pedagogical Value

- Connects math to **real-world contexts**
- Encourages visual reasoning
- Supports SEN learners
- Integrates math with design thinking

Another more complex Visual Design challenge: **Community Micro-Park Project**

Your town is creating a small community space on an unused triangular plot of land. The city council has asked student teams to propose a **triangular park design**.

Design Brief (Given to Students)

- The **area must be exactly 24 m²**
- One side of the park borders a **pathway** and must be **straight**
- The park must:
    - Be **easy to walk around**
    - Use **as little fencing as possible**
    - Allow space for **at least one tree and a bench**

Student Tasks

1. Geometry & Calculation (Mathematical Core)

- Choose base and height values that give:

$$A = \frac{1}{2}bh = 24$$

- Draw the triangle **to scale** on grid paper
- Label:
    - Base
    - Height
    - Side lengths (estimated or calculated)

2. Design Thinking & Constraints

Students must justify:

- Why they chose these dimensions
- How their design:
    - Minimizes fencing (shorter perimeter)
    - Supports movement (not too narrow or sharp)
    - Uses space efficiently

3. Decision-Making & Trade-Offs

Students compare **at least two designs**, for example:

Guiding question: “Which design would you recommend to the city council, and why?”

Design	Base (m)	Height (m)	Area	Estimated Perimeter
A	**6**	**8**	**24**	**longer**
B	**8**	**6**	**24**	**shorter**

4. Visual & Creative Representation Students must:

- Add icons for **tree, bench, path**
- Use color to show:
  - Walking area
  - Green space
  - Entry points
- Include a **legend**

(No artistic skill required; clarity over decoration.)

Expected Student Solutions (Examples)

Mathematical Validity All designs must correctly satisfy:

$$A = 24\ \mathrm{m}^2$$

Design Reasoning Examples

“We chose a wider base because it allows a bench along the path and makes the park feel open.”

“This triangle uses less fencing because the sides are shorter overall.”

“We rejected a tall narrow triangle because it would feel cramped.”

Why This Is Genuine Design Thinking This task now requires students to:

- Work with **real constraints**
- Make **trade-offs**
- Compare alternatives
- Justify decisions with data
- Communicate visually and verbally

They are not just *finding* an area — they are **using mathematics to make decisions**.

Built-in Differentiation (Including SEN)

1. Use **physical grid mats or cut-out triangles**
2. Allow **pair work**
3. Accept **verbal explanations** instead of written ones
4. Provide a **partially filled table** for support

Assessment Focus (What the Teacher Looks For)

Criterion	Evidence
Mathematical accuracy	Correct area calculation
Design reasoning	Clear justification
Use of constraints	Meets all conditions
Communication	Clear drawing and explanation

Key Teaching Insight “Design thinking begins when there is no single ‘best’ answer — only better-justified ones.”

Task B: Story-Based Interpretation

Create a short story in which a character must build something **triangular**. The character needs the **area to be exactly 24 units**. Show clearly **how the character figures this out** using numbers, drawings, or explanations..”

Expected Outcomes

1. Mathematical reasoning embedded in narrative
2. Use of drawings, diagrams, or dialogue
3. Multiple correct mathematical paths

Divergence Type: Representational and semantic divergence

Model Student Example

Title: ***The Triangle Stage***

Lina is helping her school prepare a small outdoor show. There is only one free space in the yard, and it has to be shaped like a triangle. The teacher tells Lina: “The stage must have an area of exactly 24 square meters.” Lina draws a triangle on grid paper. First, she chooses a base of **6 meters**. She knows the formula for the area of a triangle:

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

She thinks:“If the base is 6, I need a height that makes the area 24.” She calculates:

$$24 = \frac{1}{2} \times 6 \times h$$

$$24 = 3h$$

$$h = 8$$

Lina draws a height of **8 meters** straight up from the base. She checks her work:

$$\frac{1}{2} \times 6 \times 8 = 24$$

The stage is the right size, so Lina adds places for students to stand and a small speaker in one corner. The teacher approves the plan.

Why This Is a Valid Mathematical Solution

- The **area requirement is met exactly**
- The reasoning is **clearly explained**
- The story does not replace mathematics; it **carries it**
- The solution could be assessed for **accuracy and understanding**

Embedded Mathematics

Mathematical Idea	Where It Appears in the Story
Area formula	Used by the character
Algebraic reasoning	Solving for height
Verification	Re-checking the area
Representation	Drawing on grid paper

Alternative Student Ending (Different Choice)

A student could instead write:

> “Lina changed the design and used a base of 12 meters and a height of 4 meters.
> She checked that:

$$\frac{1}{2} \times 12 \times 4 = 24$$

The stage was wider and easier to decorate."

This is **equally correct**.

Teacher Notes: What to Accept as Correct

✔ Any base–height pair satisfying

$$\frac{1}{2}bh = 24$$

✔ Diagrams, tables, or verbal explanations

✔ Different narratives, as long as the math is sound

✖ Stories without mathematical justification

Differentiation (Including SEN)

- Allow:
  - Bullet-point stories
  - Comic-strip formats
  - Sentence starters:
    - "The base was ___ because…"
    - "The area was checked by…"
- Accept oral storytelling with a drawing

Assessment Focus

Criterion	Evidence
Mathematical correctness	Area = 24
Reasoning	Steps explained
Representation	Drawing or calculation
Communication	Clear narrative

Key Pedagogical Insight

> "Storytelling does not simplify mathematics — it **situates** it."

Summary: Three Types of Divergence in One Task

Divergence Type	What Changes	Example Outcome
Method	How students solve	Formula, grid, coordinates
Conditions	What must be satisfied	Same area, different shapes
Representation	How meaning is expressed	Drawing, story, design

Key Pedagogical Insight “Divergence is not about removing rigor — it is about multiplying legitimate mathematical thinking

Divergent version 4 Creative Challenge: “A Mathematics Story”

The teacher gives students a short “story” or real-life scenario in which mathematical problems are solved creatively.

Example: “In the classroom, students must count objects, divide into teams, and perform various measurements.”

Example Story Solution Students must arrange desks into triangular groups. Each group must cover exactly 20 square meters of space.

Possible Mathematical Solutions:

- Base 10 m, height 4 m
- Base 8 m, height 5 m
- Comparison of layouts to minimize unused space

Other divergence versions: Transforming Traditional Questions into Creative Ones

Examples of Creative Questions by Type

1. **Freedom of tools**
2. **Freedom of sequence of work**
3. **Unexpected elements** (both geometric and real-life)

1. Freedom of TOOLS *(Students choose what they use, not just how they calculate)*

Creative Question 1: “Show Me the Area”

> **Find the area of a triangle with base 6 and height 8.** You may choose **any tools** you think will help you. Be ready to explain **why your tool works**.

Permitted Tools (Not Assigned)

- Grid paper
- Ruler and scissors
- GeoGebra / digital drawing tool
- Physical cut-out triangle
- Formula only (no drawing)

Expected Student Outputs

★ A student cuts a rectangle and folds it in half
★ A student draws the triangle on grid paper and counts squares
★ A student uses GeoGebra and measures
★ A student uses the formula directly

All reach: **A = 24**

Why This Is True Divergence

1. The **representation depends on the tool**
2. The mathematics remains fixed
3. Tool choice reveals **student thinking preferences**

Teacher Look-For “Does the student justify why their tool gives the correct area?”

2. Freedom of SEQUENCE OF WORK *(Students decide the order: draw → calculate → check, or calculate → draw → justify)*

Creative Question 2: “Solve It Backwards”

You need a triangle with an area of 24 cm². Decide **your own order of steps**:

➢ draw first
➢ calculate first
➢ check at the end

Show the steps **in the order you chose**.

Possible Student Sequences

Student A

1. Draws several triangles
2. Measures base and height

3. Calculates area
4. Adjusts drawing

Student B

1. Starts with formula
2. Chooses numbers that work
3. Draws triangle to match
4. Verifies on grid

Expected Mathematical Outcomes

Any valid base–height pair such that:

$$\frac{1}{2}bh = 24$$

Why This Matters

- Students learn that **mathematics is not linear**
- Encourages planning and self-monitoring
- Mirrors authentic problem-solving

3. Unexpected GEOMETRIC ELEMENTS *(Breaking the "expected shape")*

Creative Question 3: "The Hidden Triangle"

Inside the shape below is a triangle. Find the area of the triangle **without removing it from the shape**. You may:

- subtract areas
- decompose shapes
- add auxiliary lines

(The "shape" can be a trapezoid, rectangle, or irregular polygon.)

Expected Student Strategies

- Subtract rectangle area minus triangle
- Split shape into known figures
- Draw extra heights or diagonals

Unexpected Element

- The triangle is **not isolated**
- Students must *see* it mathematically

Mathematical Outcome

Correct triangle area + explanation of reasoning

4. Unexpected REAL-LIFE ELEMENTS *(Math enters a decision-making situation)*

Creative Question 4: “You Are the Planner”

A food truck wants a triangular serving area of exactly 24 m². One side must be along the sidewalk (fixed length). You decide:

1. the other dimensions
2. the shape
3. how to explain your choice to the owner

Expected Student Thinking

- Fix base
- Adjust height
- Consider usability, shape, access
- Justify mathematically and practically

Why This Is Unexpected

- No single “correct” triangle
- Mathematics supports **a recommendation**
- Blends geometry with reasoning

5. Combined Example (All Three Types at Once)

Creative Question 5: “Design Under Constraints”

Design a triangular space with an area of **24 units²**.

- Choose **any tools**
- Choose **your own order of steps**
- The triangle must fit **inside a circle or rectangle**

Explain your design decisions.

Divergence Present

Type	Present?
Tool choice	✓

Sequence choice	✓
Unexpected geometry	✓

What Makes These Truly "Creative Questions"

They:

1. Preserve **mathematical rigor**
2. Multiply **legitimate solution paths**
3. Shift focus from "answer" to **decision-making**
4. Reveal how students **think**, not just what they know

Final Reflection – Expected Outcomes

From these solutions, teachers should observe that:

1. Multiple correct answers are possible
2. Reasoning and explanation are more important than the final number
3. Creativity emerges naturally when constraints are flexible

Key Teaching Insight "Creativity in mathematics does not come from novelty alone — it comes from **choice under constraint**."