Erasmus+ Cultural connections : Enhancing EU heritage, Social Inclusion and Digital Literacy through our Pupils' hearts
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ACTIVITY 7

Divergent tasks: Creative Maths

Other divergence versions

Transforming Traditional Questions into Creative Ones - version 1

Traditional Task “Find the area of a right triangle where a = 6 and b = 8.”

Mathematical Solution

$$A = \frac{1}{2} \cdot 6 \cdot 8 = 24$$

Creative Version (Mini Theater Problem)

Task: Choose side lengths aaa and bbb so the area is maximized while using minimal material.

Sample Solutions

- Larger area achieved by increasing both sides proportionally
- Comparison of multiple designs:
  - a = 6, b = 8 → area 24
  - a = 8, b = 8 → area 32
- Discussion of **optimization trade-offs** (area vs. material length)

Creative Version: Mini Theater Problem — Complete Solution

Problem Restatement

A class is building a **mini theater in the shape of a right triangle**.

- The perpendicular sides have lengths (a) and (b).
- The **area** of the stage should be as **large as possible**.
- The amount of **material used** (interpreted as the total length of the two perpendicular sides) should be **minimal**.

The task is to choose values of (a) and (b) that balance these two goals.

Step 1: Define the Mathematics

Area of the Stage For a right triangle:

$$A = \frac{1}{2}ab$$

Material Used If the material is used only for the two perpendicular sides (Alternative interpretations could include the hypotenuse, but here we follow the simplest classroom assumption.):

$$M = a + b$$

Step 2: Optimization Under a Constraint

To meaningfully compare designs, we assume the **same amount of material is available**.

Let:

$$a + b = k \quad (\text{constant})$$

Then:

$$b = k - a$$

Substitute into the area formula:

$$A(a) = \frac{1}{2}a(k - a)$$

Step 3: Maximize the Area

Expand:

$$A(a) = \frac{1}{2}(ka - a^2)$$

This is a **quadratic function** opening downward, so it has a maximum at its vertex. The vertex occurs when:

$$a = \frac{k}{2}$$

Thus:

$$a = b = \frac{k}{2}$$

Step 4: Interpretation of the Result

For a fixed amount of material, the area of a right triangle is maximized when the two perpendicular sides are equal.

In other words:

$$\boxed{a = b}$$

Step 5: Numerical Comparison (Concrete Examples)

Assume total material:

$$a + b = 14$$

Design 1

- $a = 6, b = 8$
- Area:

$$A = \frac{1}{2} \cdot 6 \cdot 8 = 24$$

Design 2

- $a = 7, b = 7$
- Area:

$$A = \frac{1}{2} \cdot 7 \cdot 7 = 24.5$$

Design 3

- $a = 5, b = 9$
- Area:

$$A = \frac{1}{2} \cdot 5 \cdot 9 = 22.5$$

Conclusion from Examples

The **balanced design** produces the **largest area** with the same amount of material.

Step 6: Final Answer (Student-Friendly Conclusion)

- ➢ To **maximize the stage area** while **using minimal material**, the two sides should be **equal in length**.
- ➢ The optimal mini theater is a **right isosceles triangle**.
- ➢ This demonstrates a key optimization principle: *Balanced dimensions lead to maximum efficiency.*

Below is a **lower-grade adaptation** of the *Mini Theater Problem*, designed for students who have **not studied algebraic optimization**. The focus is on **tables, diagrams, comparison, and visual reasoning**.

Mini Theater Problem (Lower Grades Version)

Problem Story Your class is building a **mini theater** in the shape of a **right triangle**.

1. You have a fixed amount of material for **two sides** of the stage.
2. You want the **stage area to be as large as possible**.
3. You may choose different side lengths, but the **total length must stay the same**.

Step 1: Fixed Material (Concrete Condition)

Tell students: “You have **14 meters of material** for the two sides of the triangle.”

So: $a + b = 14$

Step 2: Draw and Measure (Diagram-Based Reasoning)

Students draw right triangles on **grid paper**.

Example diagrams:

- A triangle with sides **6 and 8**
- A triangle with sides **7 and 7**
- A triangle with sides **5 and 9**

They count squares or use the formula:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

Step 3: Use a Table to Compare Students fill in a table like this:

Side a	Side b	Total (a + b)	Area = (a × b) ÷ 2
5	9	14	22.5
6	8	14	24
7	7	14	24.5
8	6	14	24
9	5	14	22.5

Step 4: Visual Comparison (Class Discussion) Guide students to observe:

1. When one side is **much longer**, the area is **smaller**.
2. When the sides become **more equal**, the area **increases**.
3. The **largest area** appears when the sides are **the same**.

Teacher prompt: “Which triangle looks the most balanced?” “Which one gives us the biggest stage?”

Step 5: Student Conclusion (Guided) Students complete sentences such as:

- “The biggest area happens when the sides are __________.”
- “If we keep the same amount of material, it is better to make the sides more __________.”

Expected answers:

- *equal*
- *balanced*

Step 6: Simple Diagram Explanation

On the board, draw three triangles with the same total side length:

- Tall and narrow

- Medium
- Balanced (isosceles)

Label the areas visually using square counting or shading.

Explain: "Balanced shapes use space better."

Step 7: Extension (Optional)

For fast finishers:

- Try a **different total**, e.g. 12 or 16.
- Predict which pair will give the biggest area before calculating.
- Explain the result using a picture.

Key Learning Outcomes (Lower Grades) Students learn that:

- The same material can make **different shapes**.
- **Comparing results** helps find the best solution.
- Mathematics can help make **real-life decisions**.
- The idea of **optimization** can be understood visually.

Transforming Traditional Questions into Creative Ones - version 2

1. Area of a Triangle

- **Creative Version 1: Design Challenge**
- **Creative Version 2: Visual Comparison**

2. Counting Squares

- **Creative Version 1: Strategy Choice**
- **Creative Version 2: Pattern Extension**

3. Perimeter of a Rectangle **- Creative Version : Garden Design**

4. Fractions

- **Creative Version 1: Visual Representation**
- **Creative Version 2: Story Context**

5. Linear Equations

- **Creative Version 1: Reverse Thinking**
- **Creative Version 2: Multiple Representations**

6. Volume of a Rectangular Prism**: Creative Version: Box Design**

1. Area of a Triangle

Traditional Question “Find the area of a triangle with base 10 cm and height 6 cm.”

Expected Answer:

$$A = \frac{1}{2} \cdot 10 \cdot 6 = 30 \text{ cm}^2$$

Creative Version 1: Design Challenge

Task: “Design a triangular park with an area of **30 m²**. Choose different base and height values. Draw at least two designs and explain which you prefer.”

Expected Solutions Students must satisfy:

$$\frac{1}{2}bh = 30 \Rightarrow bh = 60$$

Valid Examples:

- Base 10 m, height 6 m
- Base 12 m, height 5 m
- Base 15 m, height 4 m

Expected Reasoning

1. Recognition that multiple base–height pairs produce the same area
2. Justification based on shape, space use, or aesthetics

Creative Version 2: Visual Comparison

Task:“Draw three different triangles with the **same area**. Explain how you know their areas are equal.”

Expected Answers

- Drawings with different shapes but same calculated area
- Use of formula, grid counting, or decomposition

Acceptable Explanation:
“All triangles have the same base–height product, so the area is equal.

2. Counting Squares

Traditional Question “Count the number of squares in the diagram.”

Expected Answer: Depends on the diagram (e.g., 16, 20, or 40).

Creative Version 1: Strategy Choice

Task: “Count the squares using **two different methods**.”

Expected Methods

1. Counting small squares only
2. Grouping into larger squares
3. Using symmetry or patterns

Expected Answers

- Same total number reached via different strategies
- Clear explanation of both methods

Creative Version 2: Pattern Extension

Task: “Create a larger diagram and predict how many squares it will have.”

Expected Answers

- A correct count for the new diagram
- A rule such as: “When the grid size doubles, the number of squares increases more than twice.”

3. Perimeter of a Rectangle

Traditional Question

> “Find the perimeter of a rectangle with length 8 cm and width 5 cm.”

Expected Answer:

$$P = 2(8+5) = 26\ \text{cm}$$

Creative Version 1: Garden Design

Task: “You have **26 meters of fencing**. What rectangles can you build? Which gives the **largest area**?”

Expected Solutions

Length	Width	Area
12	1	12
10	3	30
8	5	40
6.5	6.5	42.25

Expected Conclusion The rectangle closest to a square gives the largest area.

4. Fractions

Traditional Question

"Calculate $\frac{3}{4} + \frac{1}{2}$."

Expected Answer:

$$\frac{3}{4} + \frac{1}{2} = \frac{5}{4} = 1\frac{1}{4}$$

Creative Version 1: Visual Representation

Task: "Show the sum using drawings or models."

Expected Solutions

1. Area model
2. Fraction bars
3. Number line

Expected Explanation "The shaded parts together make one whole and one quarter."

Creative Version 2: Story Context

Task: "Create a real-life problem using these fractions and solve it."

Expected Example

"I ate ¾ of a pizza and then ½ more. Altogether I ate 1¼ pizzas."

5. Linear Equations

Traditional Question "Solve (2x + 5 = 17)."

Expected Answer: Algebraic Steps

1. Subtract 5 from both sides:
   $$2x + 5 - 5 = 17 - 5$$
   $$2x = 12$$
2. Divide both sides by 2:
   $$x = 6$$

Creative Version 1: Reverse Thinking

Task: “Create an equation with the solution (x = 6).”

Expected Answers

- (x + 4 = 10)
- (3x = 18)
- (2x + 5 = 17)

Creative Version 2: Multiple Representations

Expected Methods

1. Algebraic steps (see above)
2. Balance model
3. Number line jumps

Expected Outcome All methods correctly lead to (x = 6).

Representation 2: Balance Model (Conceptual / Visual)

Interpretation The equation represents a balanced scale. Whatever is done to one side must also be done to the other.

- Left side: two equal weights (x) plus 5 extra units
- Right side: 17 units

1. Remove 5 units from **both sides**:
    - Left side becomes **two equal weights**
    - Right side becomes **12 units**
2. Divide the remaining 12 units equally between the two (x)'s:
   $$12 \div 2 = 6$$

Result Each (x) weighs 6 units.

Representation 3: Number Line Jumps

1. Start at 17 on the number line.
2. Jump **back 5 units** to represent subtracting 5:
    - You land on 12.
3. Split the remaining distance equally into **2 equal jumps**:
    - Each jump is 6 units.

Conclusion x = 6

Optional Representation 4: Table (Lower Grades / Support)

Step	Value

Start	17
Minus 5	12
Divide by 2	6

Summary Table: One Equation, Multiple Representations

Method	**Key Idea**	**Result**
Algebraic	Inverse operations	(x = 6)
Balance model	Keep both sides equal	(x = 6)
Number line	Undo operations visually	(x = 6)
Table	Step-by-step tracking	(x = 6)

Expected Student Conclusion "No matter which method we use, we always get the same value for (x), so the solution must be correct."

6. Volume of a Rectangular Prism

Traditional Question "Find the volume of a prism with dimensions 4 × 3 × 5."

Expected Answer:

$$V = 60\ \text{cm}^3$$

Creative Version: Box Design Task: "Design a box with volume 60 cm³. Which design uses the least material?"

Expected Solutions

Dimensions	**Volume**	**Surface Area**

1 × 6 × 10	60	152
2 × 3 × 10	60	112
3 × 4 × 5	60	94

Expected Conclusion More balanced dimensions use less material.

Mini Presentation – Expected Outcomes

Each group should present:

1. The original task
2. Their creative version
3. Sample solutions
4. Explanation of why multiple answers are valid