Relativity predicts a variable G

F.Lassiaille

frederic.lassiaille@outlook.fr

February 20, 2025

It is demonstrated that relativity predicts a variable G. The proof begins by considering a dimensionless particle in an empty universe. The analysis then extends to two particles, three particles, and an infinite set of particles. This approach enables the calculation of space-time structure for any realistic energy distribution. The proof employs the interchange of limits theorem and ad hoc sequences of energy distributions. With only one particle, the result is a singularity everywhere if the universe is empty outside the particle. These singularities completely disappear with three particles. The calculation is then generalized for any realistic energy distribution, naturally yielding an equation for G. This equation provides a correct approximation for most realistic energy distributions. The fundamental principles underlying Einstein's equation remain valid. However, it is shown that the anthropocentric solar system constant G must be replaced by a variable value, which is weaker in high matter density environments and stronger in low matter density environments. This was called a surrounding effect in previous works [1, 2]. This effect has been shown to resolve current gravitational mysteries in astrophysics and cosmology. Additionally, under a unifying relevant assumption, a solution to the Yang-Mills Millennium problem is also provided.

Contents

1. Introduction

The purpose of this document is to demonstrate that relativity predicts a variable G and to describe a solution to the Millennium Problem [3, 4].

This study builds upon previous works, particularly [2], which showed that the speed of quarks close to the speed of light implies that G is a variable. Here, it is demonstrated that this variability is a theoretical prediction of relativity, independent of the speed of quarks or any other experimental information about energy distributions.

The proof begins by considering a dimensionless particle in an empty universe. The space-time structure is then calculated for several particles up to an infinite set of particles, employing the interchange of limits theorem.

There are several motivations for this study, listed below in order of importance:

1) Solving the Yang-Mills Millennium problem.
2) Addressing today's gravitational mysteries.
3) Searching for a link between energy and the first metric derivative.
4) Resolving the slight caveats of General Relativity (GR) [2].
5) Seeking a theoretical justification for Newton's law.
6) Replacing the anthropocentric solar system value of G with a more general and sound value.
7) Retrieving the information lost in the construction of the stress-energy tensor.
8) Studying the behavior of rest frames.

The concept of rest frames will be further detailed. These frames generalize the traditional rest frames. Exploring this motivation will reveal that Special Relativity (SR) is more than just an algebraic rule of GR; it describes the local space-time deformation caused by energy. This will be the first step in addressing item 3. Item 3 relates to a new chapter of GR that needs to be written and exploited. The most important parts will be addressed in this document.

Initially, only items 2 and 3 were considered. Item 4 encompasses several motivations. Items 1, 3, 5, 6, and 8 will be directly addressed in this document. Item 7 will be indirectly addressed. Item 2 has been addressed in [1]. Item 4 has been addressed in [2], which can be considered a previous version of this document.

It will be proven that relativity predicts a variable G. This variation is driven by the surrounding effect [1], which, in its weaker version, states that gravitational force increases in low matter density environments and decreases in high matter density environments. An equation for G will naturally emerge from this process. Finally, the Yang-Mills Millennium problem will be addressed.

The document is written as a mathematical demonstration. To ease reading, many parts are included in the appendix, and a glossary is provided at the beginning.

2. Glossary

Energy distribution: In this document, energy distributions are assumed to share the same bounded domain. Each distribution is null except for a bounded subset of space-time. They are assumed to be a finite set of dimensionless particles with non-zero masses. No space-time event can contain two different dimensionless particles. The reasons for these restrictions will be explained throughout the document.

Frame in which time elapses the most: This concept comes from the study of the twin paradox, where the twin on Earth experiences time passing faster than the traveling twin. In this document, this concept is used didactically to introduce the concept of GRF.

GRF: Generalized rest frame. This is the central and new concept of the new chapter of relativity developed in this document. It is introduced didactically by the frame in which time elapses the most but is defined as the generalization of the rest frame.

Matter at rest: Matter that is at rest in an inertial frame.

Matter in motion: Matter that is in motion at the speed of light.

Rest frame: A frame in which a particle is at rest. In this document, only sets of dimensionless particles are considered, and only gravitation is taken into account, with other forces either ignored or resulting from gravitation. In this context, a rest frame, with its meaning in relativity, can always be associated with a test particle that is at rest in this frame."

S: The function giving space-time structure from energy distribution.

Test particle: A particle with no mass. In this document, this term refers more precisely to a dimensionless particle with no mass in free fall.

3. Demonstration

3.1. *Principle and contexts of the demonstration*

The principle of the demonstration is based on a physics remark about a dimensionless particle P. For such a particle, two extremely different assumptions can be made about the nature of its energy:

- Assumption (1): Its energy is made of matter at rest. This means there is an inertial frame in which the content of this particle is a solid block of matter at rest.
- Assumption (2): Its energy is made of matter in motion. This means the internal matter of P is always in motion, regardless of the chosen inertial frame. Hence, the speed of this matter is the speed of light. The simplest model is to consider P made of a set of dimensionless bunches of matter in a Brownian distribution, each moving at the speed of light.

The first assumption appears natural and obvious. The second assumption might seem unusual because P is dimensionless and composed of a set of bunches of matter. However, this assumption is as legitimate as the first, where P is dimensionless and composed of a solid block of matter at rest. Therefore, one can state that the second assumption is always true. Hence, one can state that matter is always internally made of infinitely small bunches of matter always moving at the speed of light along infinitely small closed trajectories in a Brownian motion and distribution.

The bunches of matter in the second assumption are unrelated to the particles of particle physics, even though reality in this field has proven to be more compliant with this assumption.

Regardless of the assumption chosen, the result is always a dimensionless particle PP. Thus, the generated space-time structure will be the same. However, as will be seen later, the calculation of the space-time structure will be entirely different.

From this remark, the principle is to calculate this deformation under the second assumption. Let S be the function giving space-time structure from energy distribution. The first constraint for the domain of S is that it is assumed to be the set of energy distributions following the rule of energy conservation. Therefore, for each of them, the following is true:

$$\nabla_\mu T^{\mu 0} = 0 \tag{1}$$

$T^{\mu\nu}$ is the stress energy tensor. ∇_μ is the partial covariant derivative with respect to the μ dimension. This equation is simpler for energy distributions composed of a set of dimensionless particles: it represents the conservation of the number of particles and their energy at rest.

For ease of demonstration, a second constraint is added: the energy distributions are restricted to sets of dimensionless particles. Apart from these two constraints, there are no further constraints for the domain of the S function.

The context described in this paragraph applies only to the calculation of the space-time deformation generated by P under the second assumption. This will be done from paragraph 3.4 to 3.7. Outside of these paragraphs, this context will be withdrawn. The new context will be indicated when needed.

3.2. *Mathematical reminder: interchange of limits theorems*

Norms are equivalent in finite dimensions, hence this is true for the four-dimensional space-time of GR. By other means, spacetime structure is a function of energy distribution. In the present document, the considered energy distributions will be assumed to share the same bounded domain. This will allow the demonstration of the continuity of S. This restriction is allowed since the size of this bounded domain can be huge, for example, it can contain the observable universe.

Under the previous restriction, S is a continuous function using the uniform norm for the two involved spaces. Indeed, if any amount of energy at any spacetime location is decreased to 0, then the effect of this amount of energy on spacetime structure decreases also to 0. This continuity is the result of the conservation of energy principle. The detailed proof is given in Appendix A.

From this continuity of S, the interchange of limits theorem is valid. If any f_n sequence of energy distributions over spacetime tends uniformly to some limit energy distribution, then the following equation arises.

$$S\Big(\underset{n\to+\infty}{lim_u}(f_n)\Big) = \underset{n\to+\infty}{lim_u}\Big(S(f_n)\Big) \tag{2}$$

In equation (2), lim_u means the uniform limit, for energy distributions, and for spacetime structures.

The continuity of the determination of spacetime structure as a function of energy distribution has another interesting consequence. It is possible to imagine thought experiments in which energy is increased or decreased progressively, having a continuous effect on spacetime structure. And this can be done without violation of the conservation of energy principle. Indeed, GR is a mathematical framework which can, and sometimes must be watched independently of reality. Moreover, it is possible in this framework to imagine different universes.

3.3. *A generalized rest frame in relativity*

3.3.1. *Definition and properties*

In relativity, there exist rest frames. This concept can be generalized, and then called a generalized rest frame (GRF). The generalization is done in such a way that the timeline of the generalized frame exists and is a congruent geodesic (part of the same set of geodesics congruence [9]) with respect to the timelines of the rest frames. (A ”congruent geodesic” is the generalization of the concept of ”parallel straight line” in curved spacetime). Moreover, the boost which is associated with the motion of matter in this GRF describes the evolution of this GRF. This is what will be seen in the present paragraph. More details are given in Appendix B.

This new concept can be introduced with a thought experiment, avoiding then any complicated and tedious calculation. It is done in Appendix B. The conclusion is that there exists a GRF in any spacetime event (using the extension of identification which is presented in Appendix B). And for any particle located in a given x event, this GRF exists in x and is transformed by the particle, using the boost which is associated with the four-momentum of the particle. Roughly speaking for the understanding, let’s write that this boost is calculated in the ”old GRF”, that is, the one ”just before the particle”, and that it transforms this old GRF into the new one, that is, the one ”just after the particle”. Another definition is that the old one describes spacetime locally without the existence of the particle, and the new one does it just after the existence of the particle. Moreover, the identification of the new GRF from the old one can be done progressively, using the continuity of the function giving spacetime structure from energy distribution. For this identification, the following scenario is unrealistic but mandatory, and is allowed as a thought experiment, as previously mentioned. The energy of the particle is increased progressively from 0 to its real value.

Also, a rescaling of time and space units occurs after the boost. This will be shown further. But in the present document, it will be written abusively that the local deformation is described by a boost. Indeed, only the context will allow to deduce if a rescaling occurs also, or not. There will be more information about that in Appendix E.

This concept of GRF is central in GR. It is referenced in the literature as the ”rest frame” (please refer to the glossary, paragraph 2). This concept of rest frame is generalized in Appendix B, and then called ”GRF”. It will be associated in the present document with the rule governing its evolution with respect to matter and with the motion of matter. The link between local spacetime deformation and matter will be given by this concept and this rule. This link is local and implies only the first degree of derivation of the metric. It should be possible to induce from that the second degree of derivation, and then compare the result with Einstein's equation. Whatever the result is, a new view of spacetime deformation by energy arises.

This ends what is a reminder about a feature of relativity. But it allows to define this new concept of GRFs which is the starting point of the new chapter of relativity which is partially developed in the present document. In this chapter, the next step will be to study the behavior of these frames with respect to energy.

3.3.2. *Example*

A simple example is a P particle in motion along a straight line in a static universe filled with a constant matter density. Without P, this universe results in a flat Minkowskian universe. The old $R_{0\ system}$ system of GRFs is represented by the same and constant R_0 frame, a frame for which the universe is at rest. This is called the "frame of fixed stars" in old literature. The $R'_{0\ system}$ new system of frames is the real existing one. It results from the existence of P.

This can be refined by assuming a progressive appearance of P in space and time. Then a continuous set of GRFs is constructed progressively, from $R_{0\ system}$ to $R'_{0\ system}$.

For a gravitational wave (GW), the same definition applies: the "old GRF" is the one which would be GRF if the GW was not existing, the "new GRF" takes this GW existence into account.

3.4. *Fundamental assumption*

From now on in the present document, it will be assumed the following assumption.

Assumption (I): a GW propagates at the speed of light.

The relevance of this assumption (I) is well described in the literature [5].

3.5. *Spacetime structure around a particle moving at the speed of light*

Now the aim is to describe the local spacetime deformation of a GW generated by a particle moving at the speed of light. The usual following assumption will be assumed. The P particle is moving at the speed of light along a D straight line in an empty universe previously structured by a flat Minkowskian spacetime.

The spacetime deformations of the GW will be studied at the locations where the GW deformations are the greatest. Being only local, the studied deformation is transforming the time and space axes into new ones. That is, the old GRF is transformed into the new one. Therefore, this local deformation is described by a linear transform.

Whatever the R inertial frame is chosen, this trajectory will always be a straight line and the speed of the particle will always be the speed of light. Moreover, the properties of physics are the same whatever the chosen inertial frame. Therefore, the spacetime structure generated by this particle will always be the same whatever the inertial frame is chosen. It means that in R at any given time the locations in which the deformation is the greatest is always the same space cone centered on D.

Figure (1) shows the GW in two dimensions. In $R(C; ct, x, y, z)$ the P particle moves in the direction of the x axis, and along the D line at the c speed. Let's focus on the GW which is generated by P from the E_C event which is a given C space location pertaining to D, and a given t_C time. This GW propagates from C at the c speed along the space directions of the (y, z) plane which is perpendicular to x and therefore to D, starting from C. Let's choose an E_N spacetime event composed of the N space point and the t_N time in R, such that CN is along the y axis perpendicular to D and E_N receives the GW propagation starting from E_C. During the same $t_N - t_C$ time interval, P moves from C to some C' space point pertaining to D. Also, when P is located in H, which is the middle of C and C', the GW starts another propagation from H in the direction of CN. This propagation is located in M at t_N, such that M is the middle of N and C'. The propagations along CN and HM are done at the c speed. Therefore, this GW propagation is also along the CM line at the $c/\sqrt{2}$ speed. This is the speed of the envelope of the GW along its trajectory. The spatial vector of this propagation speed is normal to the envelope. The envelope is a cone centered on D. This cone is "isosceles". In other words, there is $CN = CC'$ and $HM = HC'$.

Now let's study the deformation generated by the GW. First of all, in C the local deformation which is generated by P is the b_C boost associated with the speed of light in the same direction as P. This is proven in Appendix H.

Along the D line, exactly the same boost propagates itself, at the speed of light. This is proven in Appendix H. Here, since the P trajectory is the same D line, of course, this can't be noticed. But it will be noticed as soon as P will deviate from this D trajectory.

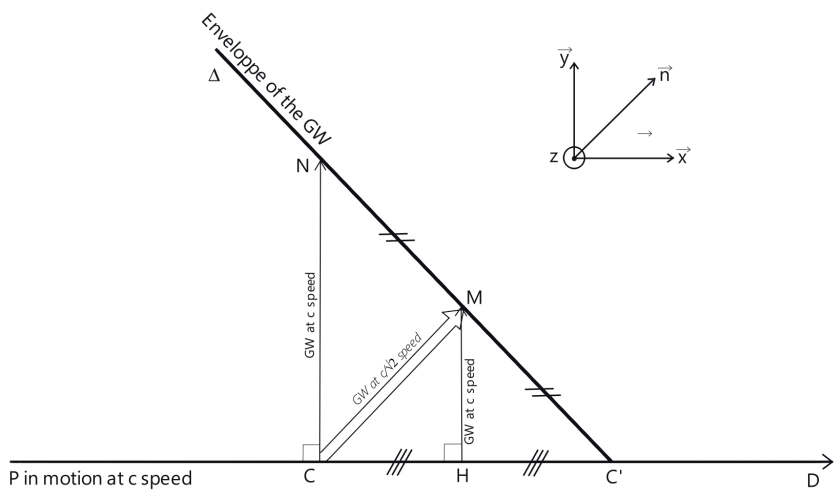

Fig. 1. The GW generated by a P particle moving at the speed of light is shown in two space dimensions. D is the trajectory of P. The spacetime deformation generated by P propagates perpendicularly to D from C to N at the c speed, and also from H to M at the c speed. H is the middle of CC' and M is the middle of NC'. Δ is the envelope of the GW. This envelope propagates at the $c/\sqrt{2}$ speed from C to M.

Outside of D, the local GW deformation is still described by the b_C boost. Indeed, this is the initial deformation in C. As such it is the deformation which is propagated. Also, this is coherent with the propagation speed of the envelope. More details are available in Appendix J and Appendix K.

Let's study the space submanifold at some given time. It is assumed that a slicing of space-time along time is possible. The space-time deformation at any X space location depends on the position of X with respect to the cone of the maximum GW deformation. Outside of the cone (external part of the cone), the space-time structure is unchanged for causality reasons. Indeed, here P has no possible causal link. In other words, it means that here the old and new GRFs are equal. Inside the cone, each event has been reached in the past by the GW generated by P. Indeed, nothing more has ever happened there (no more GW or any effect due to any other matter than P has ever happened there, since there is a vacuum instead of P in the universe). Therefore, here the local space-time deformation is described by b_C.

3.6. *Space-time structure around a particle moving along a circle at the speed of light*

3.6.1. *Context and aim*

The next step is to calculate the space-time deformation generated by a particle at rest, under assumption (2). For modeling this particle, the best model would be a Brownian motion of a huge set of bunches of matter moving at the speed of light. But a simpler model will be used. It's a bunch of matter moving along a circle at the speed of light. The relevance of those models is discussed in Appendix M.

3.6.2. *Space-time deformation generated by the particle moving along a circle*

Therefore, it is assumed that the previous P particle is forced to move along a circle. Of course, using the word "particle" here is an abuse of language. Indeed, this identifies a bunch of matter whose existence is only a hypothesis and this is assumed only for theoretical purposes. But this abuse allows for easier reading.

Then the previous cone transforms into a more complicated geometric figure. But infinitely far from the circle, therefore asymptotically, the envelope of the GW propagation at constant time is a sequence of spheres centered at O, the center of the circle. Asymptotically, these spheres inflate themselves around O at the speed of light. The radiuss of these spheres are separated by the same constant value d, which is the circle's circumference. Now the deformation that is propagated asymptotically is described by a boost associated with the speed of light. This speed is the vector that is normal to the propagation sphere, in the direction that goes out of the sphere. Also, this deformation propagates with this speed. This is proven in Appendix L.

3.6.3. *When the circle's radius tends to 0*

In the mathematical framework of relativity, if the radius of the circle decreases progressively and tends to zero, it means in reality that the circle tends to its limit, which is a dimensionless particle located in O. Then the envelope of the GW transforms itself into the previous sequence of spheres, but also d tends to O. The final result is a sequence of spheres which are infinitely close to each other, all centered on O.

The interchange of limits theorem can be applied to this Dirac distribution of matter, namely here, the dimensionless particle located in O. Let's denote P' as this particle. This can be applied because the circle-like trajectories of microscopic particles represent energy distributions that tend toward this Dirac distribution. If any doubt exists because P is in a rotating motion around O, then it is possible to add another particle sharing the same energy and moving along the same circle-like trajectory in the opposite direction. This would cancel the whole rotating motion without changing the final result, which is the following. Finally, this distribution tends to the Dirac distribution centered in O, when d tends to 0.

$$\underset{n\to+\infty}{lim_u}\big(S_n\big) = m\delta \tag{3}$$

S_n is this energy distribution made of one circle-like trajectory of one bunch of matter like P, in an empty universe, this circle being centered in O. m is the mass of P', which is assumed to be located in O, the center of the δ Dirac distribution. Equation (3) tells that S_n tends to the Dirac distribution of P' since the radius of the S_n circle tends to O.

3.6.4. *Space-time deformation generated by a particle at rest*

Therefore, let's apply the interchange of limit theorem. The space-time structure generated by a 3D Dirac distribution centered in O is the limit of the previously described sequence of spheres, limit when d tends to 0, as written in the following equation.

$$S\big(m\delta\big) = S\left(\underset{n\to+\infty}{lim_u}\big(S_n\big)\right) = \underset{n\to+\infty}{lim_u} S\big(S_n\big) \tag{4}$$

Those limit deformations are described in any M space point by the boost associated with the speed of light which is oriented along OM. Let's write R, a rest frame attached to P' (in which P' is at rest). M belongs to a sphere centered in O which contains all the deformations arriving at the same time in R. Moreover, these spheres are now infinitely close to each other. It means that each space-time event of the universe is modified by this singular boost. This is the space-time structure generated by the 3D Dirac distribution.

Hence the result is radically different from what is told by today's literature. Let's remind that with today's literature the space-time deformation here is the one which corresponds exactly to Newton's law occurring in the solar system. But what has been proven here is that this deformation is a singularity everywhere.

This huge difference will explain why G is not a constant but a variable.

3.6.5. *Avoiding the unknown forces*

P was forced to follow the circle-like trajectory which is not a geodesic. Therefore, there is an unknown force which allows this trajectory not to follow a geodesic. This force is not gravity. Therefore, at first glance, modeling the Dirac distribution under assumption (2) would not be possible in a coherent way and implying only gravitation. But this apparent result is wrong; the final result is quite the opposite, as shown in Appendix M. Interestingly, the result here is a very important result. The result is that in relativity, assumption (1) is impossible and must be replaced.

3.7. *Partial resolution of the issue of Mach's principle*

This resulting space-time structure might be argued to be wrong because it is not realistic. But the correct argument is the opposite. This description appears to be more accurate than the one given in the literature because the distribution was initially assumed to be unrealistic. Indeed, a Dirac distribution of matter is inherently unrealistic since it assumes an empty universe outside of the center of the Dirac distribution. For that reason, only an unrealistic result should be expected.

Moreover, this GR prediction is in perfect agreement with Mach's principle [6]. Let's briefly recall one aspect of the problem with Mach's principle in GR. For example, the issue arises in the case of a static spherically symmetric universe. A particle is located at the center of this spherical symmetry. If ρ is the matter density filling the universe, then one can distinguish two assumptions. The first is $\rho > 0$, and the second is $\rho = 0$. Close to the particle, ρ appears insignificant in both cases. Therefore, in this region, the spacetime deformations will be approximately the same for the two assumptions. However, in the first assumption, it is possible to find an inertial frame, R, at rest with the particle, which is not in rotation with respect to the universe. In R, there are no fictitious forces such as centrifugal forces. But in the second assumption, it is not possible to find such a frame. Assuming that R is at rest with respect to the object is not enough. It is not possible to determine if R remains inertial or not. One cannot say if fictitious forces will appear in R.

The new GR prediction solves this problem. Now, the space-time structure becomes singular everywhere for the second case. An answer is provided: in each frame at rest with the particle, no fictitious force could ever appear, since those singularities would dissolve them completely. Therefore, with this new, correctly unrealistic prediction, GR becomes more Machian.

3.8. *Two particles*

Now let's add another particle, apart from the first one. So there are two particles, at different locations, and the universe outside them is still empty. Everything is still assumed to be static, meaning that the two particles are at rest in some given inertial frame.

There are still singularities, but they are located only along the straight line containing the particles' locations, and only at the points that are not between the particles' locations. They are described by the boost associated with the speed of light in the direction moving away from the particles.

Outside of the particles and those singularities, the space-time structure is determined by the conservation of the GR Lagrangian in vacuum. It is simpler to say that the Ricci tensor is null in vacuum, as shown by the following equation.

$$R^{\nu}_{\mu}(g_{\mu\nu}) = 0 \tag{5}$$

$R^{\nu}_{\mu}()$ is the function giving the Ricci tensor from the metric, and $g_{\mu\nu}$ is the metric. This is a second-degree differential equation. An integration constant is still required at the end of the calculation. Under today's version of GR, the G solar system constant value and the mass of each particle are used for this. Now, the determination of this integration constant remains to be given. Hints and clues for this determination are provided in Appendix D. Although this determination would be an improvement, it is not mandatory for the study of the present document.

Also, asymptotically, the space-time deformation is singular. Indeed, this asymptotic deformation is the same as the previous asymptotic deformation of the distribution with only one particle. And of course, each particle still generates locally a space-time singularity. If the masses of the particles are not equal, a difficult calculation is required. Let's suppose that they are equal. Then, the calculation is simpler since there is perpendicular symmetry with respect to the mediating plane between the particles. This picture is still vastly different from what is written in today's literature.

3.9. *Three or more particles*

Adding a third particle to the scene will dissolve those singularities located along the straight line containing the particles. As usual, it is assumed that the third particle is at rest with respect to the other particles.

With three or more particles, the space-time structure is still determined by equation (5). And under the assumption that the rest mass of the particles is equal, symmetry considerations might help to calculate the space-time structure. For the determination of the integration constant, the same arguments apply as in paragraph 3.8.

3.10. *Space-time structure for any distribution of energy*

The studied distribution of energy is a realistic one: an infinite set of dimensionless particles. Indeed, modeling reality in this way is often a good approximation in astrophysics, from planets to cosmology scales, and in particle physics, because of the sparse nature of matter. Here the wave nature of particles of particle's physics is simply ignored since only gravity is studied at first. Of course, a more general distribution of energy might be studied. Notably, a uniformly continuous distribution of energy could still allow the use of the interchange of limits theorem. But this is outside the scope of the present document.

Hence, let's apply the interchange of limits theorem to this set of Dirac distributions of matter, namely here, the dimensionless particles of the universe. Let's recall that for each P_i particle, where i is the particle's number, i from 0 to infinity, there is a S^i_n sequence of energy distributions. For each i and n, S_n^i is a distribution made of a circle-like trajectory of a virtual microscopic particle moving at the speed of light in an empty universe, this circle being centered on P_i. For each i, i from 0 to infinity, the circle's radius of S_n^i tends to 0, and S_n^i tends to the Dirac distribution of the P_i particle, as n tends to infinity.

$$\underset{n \to +\infty}{lim_u} \left(S_n^i\right) = m_i \delta_i \tag{6}$$

In equation (6), δ_i is the Dirac space distribution centered on the P_i particle, and m_i is the mass of P_i. Although lim_u represents uniform convergence, from the perspective of the set of P_i, this is simple convergence. This simple convergence for each P_i can be transformed into uniform convergence for the entire set of P_i. To achieve this, it suffices to choose, for each n, the same S_n^i circle's radius for any i. For example, the value of $1/(n+2)$ can be chosen for the circle's radius of S_{n+1}^i, in the system of GRFs of space-time structure generated by S_n^i.

The final picture is as follows. The space-time structure is the result of all the microscopic GWs. These microscopic GWs are generated by the virtual microscopic particles moving at the speed of light and representing the real P_i particles in the above study. From now on in this document, these GWs generated by the microscopic virtual particles will be called "virtual gravitational waves" (VGW). Considering only the limit distributions ($n \to +\infty$), the set of S_n^i transforms into the set of $m_i \delta_i$. Thus, each space-time event receives a VGW from each real particle in the universe. This provides a clue for calculating the space-time structure in a different way.

3.11. *Calculating space-time structure with virtual gravitational waves*

3.11.1. *Four-momentum equation*

The same distribution of energy is still assumed. In any x space-time event, the four-momentums of all the GWs propagating in x add themselves. The fundamental reason for this is conservation of energy principle. This is shown in Appendix O.

This is true for the VGWs as shown above, but let's study how any kind of GWs combine themselves when they encounter in x. This will be studied, generally, for any kind of GWs, and then applied for the VGWs.

The resulting equation has been described in [2] and is the following.

$$D^\mu(x) = \Sigma_{n=0}^{\infty} 1_w(x, y_n) f(x, y_n) C^\mu(y_n) \tag{7}$$

Equation (7) shows the calculation of the resulting four-momentum in x. For n from 0 to infinity, each y_n event represents a space-time location in which the P_n particle is possibly propagating a GW in x. The $1_w(x, y_n)$ is equal to 1 if x and y_n events are connected by a null geodesic and if x is located after y_n along this geodesic. It means that the GW generated in y_n is received in x. Considering only the limit distributions of equation (6), $1_w(x, y_n)$ is always equal to 1 and can be supressed from the equations. This will proven further. $f(x, y_n)$ is a scalar positive function. It is assumed to be equal to 1 if y_n is equal to x. This allows to retrieve the rule of local space-time deformation generated by a particle. It expresses the attenuation of the GW energy which is emitted from y_n. $C^\mu(y_n)$ is a four-vector which contains the information of the energy of the GW in y_n. Later on, it will be shown that $C^0(y_n)$ is not the effective energy of the GW, but is proportional to its square root. Nevertheless, in order to avoid a heavy reading, the words "four-momentum" and "energy" will be used respectively for $C^\mu(y_n)$ and $C^0(y_n)$. The context will allow to understand if those are effective energy or contributions to equation (7). And this "contribution" word will mean the $1_w(x, y_n)f(x, y_n)C^\mu(y_n)$ terms which are in the sum of the rhs of equation (7).

Equation (7) can be used for calculating the four-momentum describing the local space-time deformation generated by any of the following objects.

- A single particle located in x,
- a GW propagating in x,
- many GWs encountering in x.

A remark about equation (7) is the question of wether an infinite number can result from this equation or not. For example if an infinite universe is filled with a constant and uniform distribution of particles, then the result is infinite if the f attenuation function decreases less than $1/r^3$. This problem is similar to the Olbers's paradox problem [7]. But a more practical solution can be found here. There is no need to understand the universe expansion and horizon. When translating this equation (7) into a gravitational model such as surrounding [1], a solution is found. Indeed, in surrounding, the fitting of the model with experimental data forces this sum of equation (7) to be translated into a finite value. A practical approach here is simply to ignore the possibility of divergence of this equation, and to fix this issue later on, when working on gravitational models.

3.11.2. *From the four-momentum to the boost*

Let's follow the natural calculations. The distribution of energy of paragraph 3.10 is still assumed. Let's write the resulting four-momentum of equation (7).

$$D^\mu(x) = \gamma \frac{E}{c} \left(1, \frac{v}{c}, 0, 0\right) \tag{8}$$

E and v are respectively the energy at rest of the four-momentum and its speed in a frame. It has been used $\gamma = 1/\sqrt{1 - v^2/c^2}$. This equation (8) has been written in a $R_0(O; ct, x, y, z)$ frame, at rest with the universe, which is an old GRF with respect to the GWs of equation (8). R_0 is such that v is along the Ox line. It is possible to find such a frame. Then, from $D^\mu(x)$ is calculated the local space-time deformation which is generated. This is done [2] by using the boost described in R_0 by the following equation.

$$B^{\mu}_{\nu}(x) = \gamma \begin{pmatrix} 1 & -\frac{v}{c} & 0 & 0 \\ -\frac{v}{c} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{9}$$

This boost is directly deduced from the four-momentum of equation (8).

3.11.3. *From the boost to the metric*

Now it is possible to derive the space-time metric from $B^{\mu}_{\nu}(x)$.

The distribution of energy of paragraph 3.10 is still assumed, but now the universe is filled with a constant matter density. It means, notably, that the grid of particles has cells which are small enough for allowing such an approximation.

Let's write R_0, a frame at rest with the universe. Since the universe is filled with a constant matter density, space-time structure is flat Minkowskian everywhere and R_0 represents a whole system of frames such that R_0 is the GRF everywhere. Now let's assume that a P particle is added to the scene, located in O at $t = 0$, at rest in R_0. Let's write x the event composed by a given M space point, $M \neq O$ at $t = 0$ in R_0. R_0 can be assumed to be the GRF in x, before adding P to the scene. Let's call R'_0 the GRF after adding P to the scene. Therefore, R_0 is the "old GRF", and R'_0 is the "new GRF", with respect to the existence of P and in the M space location. Let's write x' the first event in M when R_0 has been transformed into R'_0, along the time of R_0. R'_0 is obtained by transforming R_0, in x, using the $B^{\mu}_{\nu}(x)$ boost.

From R_0 to R'_0 it can also be generated a successive continuous sequence of GRFs, starting with R_0 and ending with R'_0. For that it suffices to add slowly the P particle energy from 0 to its real value. (Let's remind that avoiding the conservation of energy principle is allowed in a thought experiment).

Then, it is required to rescale the lengths of the "boosted" time and space axis. The boosted time and space axis are the time and space axis which have been modified by the boost, in their states after the boost. The rescaling is done in such a way that the resulting time line described successively by those successive infinitesimal steps is a geodesic. This is detailed by the following equations, relating X'^{ν} the coordinates after the boost, to X^{μ} the coordinates in R_0, and then relating $X"^{\rho}$ the final rescaled coordinates in R'_0 to X'^{ν}.

$$X'^{\nu}(x') = B^{\nu}_{\mu}(x)X^{\mu}(x) \tag{10}$$

$$X"^{\rho}(x') = S^{\rho}_{\nu}(x')X'^{\nu}(x) \tag{11}$$

$$g_{\alpha\beta}(x) = B^{\rho}_{\alpha}(x)S^{\mu}_{\rho}(x')g_{\mu\nu}(x')S^{\nu}_{\kappa}(x')B^{\kappa}_{\beta}(x) \tag{12}$$

$S^\mu_\rho(x')$ is a symetric linear map which has the ability of being diagonalized in R'_0. Its value is determined by the constraint above (the time line of the set of successive GRFs must be a geodesic). Equations (10) (11) and (12) show how $g_{\mu\nu}(x')$ the new metric is deduced from $g_{\alpha\beta}(x)$ the old one, due to the action of $B^\nu_\mu(x)$, which results from the $D^\mu(x)$ added energy of P. Of course equation (12) can be inverted, using the inverse mixed tensors $(B^{-1})^\beta_\kappa(x)$ and $(S^{-1})^\rho_\mu(x')$ of, respectively, $B^\nu_\mu(x)$ and $S^\mu_\rho(x')$. It results the following equation.

$$g_{\mu\nu}(x') = (S^{-1})^\rho_\mu(x')(B^{-1})^\alpha_\rho(x)g_{\alpha\beta}(x)(B^{-1})^\beta_\kappa(x)(S^{-1})^\kappa_\nu(x') \tag{13}$$

Equations (10) (11) and (12) have been obtained by studying the spherically symmetric static case. In the Schwarzschild metric, a P_1 test particle (a "test particle" is defined in the glossary, paragraph 2), being at rest when located infinitely far from the center of the symmetry, follows a time line which is transformed by those equations [8]. Those equations are still valid in a more general case. This is proven in Appendix P. However in the scope of the present document, only the particular spherically symmetric static case is required.

It is already known that $g_{\mu\nu}(x)$ is a diagonal matrix in the R_0 frame and that $g_{\mu\nu}(x')$ is a diagonal matrix in the R'_0 frame. Using equation (8), since the direction of the boost is along the x space axis, only time and x space dimensions are modified by the metric evolution. If the $(S^{-1})^\rho_\nu(x')$ rescaling is written with an α time rescaling and with a β space rescaling, then, using equation (13) and $\alpha\beta = 1$ usual convention, the resulting metric shows $g_{00}(x') = \alpha^2$ and $g_{11}(x') = -\alpha^{-2}$. This allows to check and understand the involved mechanism of equations (10) (11) and (12).

As a conclusion, space-time structure is calculated first by calculating a one and only four-momentum in x which contains the information about the local deformation in x generated by P. From it the final local space-time deformation is determined. This determination is deducing the speed associated with this four-momentum, and then the boost associated with this speed. This boost describes the final space-time deformation occuring locally in x. Taking equation (7) into acount, and the VGWs of paragraph 3.10, the final local space-time deformation in x is described by the boost which is associated with the four-velocity which is the barycentric operation of all the four-velocity of the VGWs propagating in x. This barycentric operation uses the total energy of each VGW as its weight. The attenuation of the propagation of the VGWs follows the rule of Ricci tensor being null in vacuum. From this boost is calculated the metric. This is done in each space-time event. Therefore, the global space-time structure is calculated. The question of the stability of this self-induced mechanism arises. But if the universe is static, with this mechanism, space-time structure converges into a stable structure. Indeed if the universe is static, then a thought experiment can be done in which the energy distribution is constructed by adding progressively the particles one after the other. Also each of them can be added progressively, their energy at rest being increased progressively. Each of those successive energy distributions are static. As usual in GR, space-time structure for each of them can be described with the system of Riemann normal coordinates [9], in which space-time appears virtually flat Minkowskian. In this system, in any x space-time event, the sum of the momentum of the VGWs occuring in x is null. This is valid also for the limit of these energy distributions and the limit of those system of frames.

3.11.4. *An equation of G*

The context and notations of paragraph 3.11.3 are used. The $R_0(O; ct, x, y, z)$ frame is still used, the P particle is located in O, the x event is any possible space-time event. For trying the construction of an equation of G, now let's assume, also, the following.

Assumption (i): Newton's law is valid. But G may differ from its solar system value.

This is based on experimental data. Newtons' law must be supposed to be valid almost in solar system because this law is validated with high accuracy, at least in solar system [12]. And there are theoretical arguments for this law to stay valid out of solar system, though being used with a different value of G. This will be studied in paragraph 5.

Assumption (ii): the energy of a GW propagating spherically evolves always following the same attenuation function (function of the initial starting energy, and of the propagation distance), regardless of its location and starting energy.

Assumption (iii): in equation (7) the sum of the energy of the contributions generated by P is far weaker than the sum of the energy of the other contributions.

This assumption will be confirmed with surrounding, where a sphere with a 15 kpc radius, which is used for calculating the surrounding value, is fitted to experimental data. The energy which is located in this sphere corresponds to the time component of the rhs of equation (7), that is, the sum of the energy of the contributions of this equation.

Assumption (iiii): the contributions of equation (7) can be replaced by their asymptotic values without modifying consistently the result.

This assumption can be valid if the particles of the universe are isolated enough from each other. This might be realistic because matter is known to be extremely sparse in the universe, whatever the scale is, from particle physics scale to cosmological scale. Assumptions (i) (ii) (iii) and (iiii) are easier to accept asymptotically.

The calculations are presented in Appendix Q. They are based on assumptions (i) (ii) (iii) and (iiii), equations (7), equation (8), and the geodesic equation for Newton's law which is reminded in Appendix Q. They result in the following equation of G.

$$G \simeq \frac{c^4}{2\left(\Sigma_{n=0}^{\infty}\sqrt{\frac{E(y_n)}{\|x-y_n\|_3}}\right)^2} \tag{14}$$

In equation (14); $E(y_n) = C^0(y_n)c$ is the total energy of the particle located in y_n. This equation is valid only for VGWs. In Appendix Q it is shown that this equation is a good approximation under (i) (ii) (iii) and (iiii) assumptions.

The possible divergence of equation (7) might appear worse in equation (14) than in equation (7). Indeed, the $1/\sqrt{(r)}$ evolution of the contributions of equation (14) shows potentially a quick divergence of the result of this equation. Nevertheless it must be noticed that the resulting gravitational force will obey to the $1/r^2$ rule. Indeed, equation (14) has been formed from that rule. Therefore the final possible divergence is the Olbers's paradox divergence. And it is easy to modify equation (14), inserting a cut-off value of the contributions, for example resulting in the following equations.

$$\begin{aligned} \Phi_{cut}\big(a, b\big) = \\ b \leqslant R_{cut} : \quad \sqrt{\frac{a}{b}} \\ b > R_{cut} : \quad 0 \end{aligned} \tag{15}$$

$$G \simeq \frac{c^4}{2\Bigg(\Sigma_{n=0}^{\infty}\Phi_{cut}\Big(E(y_n), \|x - y_n\|_3\Big)\Bigg)^2} \tag{16}$$

Here R_{cut} is the maximum GW propagation distance. The $15\ kpc$ value is suggested by surrounding [1].

This cut-off value does not alter much the qualification of "asymptotic" in the previous reasoning and in the calculations of Appendix Q. Indeed in the surrounding gravitational model, the $R_{cut} = 15\ kpc$ value is fitted to experimental data. It is a distance which would require an attracting object like a galaxy in order to contradict this "assymptotic" qualifier. And this equation (16) can still be improved. For example it is possible to replace the simple 0 cut-off value by a slow decrease of the contributions of equation (14).

4. Predicted surrounding effect

The above study shows that relativity predicts a variable G. And this variation given by equation (14) follows the rule untitled "surrounding" in [1]. Of course equation (14) has been constructed under the (i), (ii), (iii) and (iiii) assumptions.

But equation (7) shows already this surrounding effect, without any added assumption. Let's show this by rewriting it, shifting the total energy from left to right, and isolating the resulting speed.

$$\frac{v}{c} = \frac{f(x, y_0)C^0(y_0)}{\Sigma_{n=0}^{\infty} f(x, y_n)C^0(y_n)} \tag{17}$$

The context and notations of paragraph 3.11.4 are assumed. The universe is still assumed to be at rest and filled with a constant matter density, except for only one P particle at rest with the universe and spatially located in O. y_0 is an event spatially located in O, and $C^{\mu}(y_0)$ is the four-momentum of the unique VGW generated by P. The $1_w(x, y_0)$ and $1_w(x, y_n)$ terms disappeared because now only VGWs are considered. Indeed, there is always a unique VGW which is propagated by each particle located in y_n and which is received by any given x event. Equation (17) derives directly from equation (7), and shows that the space velocity of the resulting four-velocity is inversely proportional to the denominator, which increases with the energy surrounding the x location.

It can be noticed also that the denominator of the rhs of this equation is a sum of positive scalars calculated in an isotropic manner. It induces naturally to translate this equation

(17) into a gravitational model, replacing this value by the energy of the surroundings of the location where the gravitational force is exerted. The result is that the surrounding gravitational model or a gravitational model close to it is predicted by relativity. Therefore the so-called gravitational anomalies of today might be no anomalies at all, but regular predictions of relativity.

The modified gravity theories of today must comply with the MOND [13] model predictions, for the greatest part of experimental data. This has been proven by decades of work in astrophysics. This compliance is naturally acomplished by the surrounding model. Indeed, when acceleration is low, MOND increases it. But when acceleration is low, most of the time it means that the surrounding energy and matter are low, also, and then the surrounding effect increases acceleration too.

5. Revisiting Newton's law

A revisiting of Einstein equation is naturally required by the previous study. This equation is nothing more than the most direct translation of Newton's law from non relativistic physics into relativity. Therefore, the first step is to revisit Newton's law. The Poisson's formulation of Newton's law starts the construction of Einstein equation, not only because it's about Newton's law.

First of all, in vacuum, this formulation expresses the following important non relativistic principle.

(i) The flow of the acceleration vector field is constant in vacuum.

It results $div(a) = 0$ in vacuum, where a is the acceleration vector field. This is not something new. But now it has been shown that space-time structure in vacuum can be calculated by considering only the VGWs generated by theoretical particles moving at the speed of light. This gives a new insight about this (i) principle: it corresponds to the conservation of flow of GW energy in vacuum. Hence another argument for the Poisson's formulation of Newton's law is given.

Secondly, matter plays the role of a source for this field: matter is a source of any interaction force. And this is not only true for gravitation. Indeed, the force which attracts a given A particle to another given B particle is acting on A in the direction of B. This direction is tangent to the geodesic relying A to B. At the contrary, vacuum does not generate any force. Therefore the F force vector field has a divergence which is a function of ρ, matter density. Let's write Φ this function. It results the following equation.

$$div(F) = \Phi(\rho) \tag{18}$$

Then, let's write the fundamental principle of dynamics.

$$F = ma \tag{19}$$

F is the force generated on A by ρ. m is the mass of A located in a M space location, and a is the acceleration generated in M by ρ.

From equations (18) and (19) the following equation arises.

$$mdiv(a) = \Phi(\rho) \tag{20}$$

Now the reasoning becomes only valid for gravitation. The weak equivalence principle (WEP) states that a is independent of m for a fixed ρ. It results the following equation.

$$div(a) = f(\rho) \tag{21}$$

f is of course a function which is deduced from Φ. Newton's law in its Poisson's formulation is almost retrieved by the previous theoretical considerations. The divergence of the acceleration vector field should be a function of energy, such that for a null energy there is a null divergence. This gives also the following equation.

$$f(0) = 0 \tag{22}$$

In equations (21) and (22), nothing is told about neither the sign of $f(\rho)$ nor the exact feature of the f function. And the question of a possible proportionality of $f(\rho)$ with ρ is related with conservation of energy and the principle of action and reaction. Let's show this. Equation (21) implies for a P particle in an empty universe the following equation.

$$a = \frac{f(\rho)V}{4\pi x^2} \tag{23}$$

In equation (23), a is the acceleration in any given M location, such as $MO = x$, O being the location of a P particle generating this acceleration. ρ and V are respectively the matter density and the volume of P. It was supposed that ρ is constant in P. Then applying the fundamental principle of dynamics, the following equation arises.

$$F = \frac{m' f(\rho)V}{4\pi x^2} \tag{24}$$

Here, F is the force attracting a P' particle located in M, by P. m' is the P' mass. But the principle of action and reaction implies that this equation is invariant by P and P' permutation. The following equation arises, where m is the mass of P.

$$m' f(\rho)V = mf(\rho')V' \tag{25}$$

This being true for V and V' being constant and for any value of m, m', ρ, and ρ', it implies that $f(\rho)V$ is proportional to m . A better demonstration of that would consider the total energy E of P and P', in place of the principle of action and reaction. This energy is the integral of the forces along space distances. Then it is the invariance of E by the P and P' permutation which would be used. This proportionality would be written the following way.

$$f(\rho) = K\rho \tag{26}$$

Here K is of course the unknown coefficient of this proportionality. Using equation (26) in equation (24), it yields the following equation.

$$F = \frac{Kmm'}{4\pi x^2} \tag{27}$$

But now no theoretical argument can be given here for calculating the $K/(4\pi)$ constant of equation (27). Historically Newton's law has been constructed based on experimental data more than theoretical considerations. To say the least, the G determination was done completely based on experimental data.

But an indirect theoretical argument can be given. Everything was done here under the assumption that a complete vacuum exists outside of P and P'. If the vacuum is not perfect outside of the particles, the reasoning above becomes wrong. The energy surrounding the particles must be taken into account. First of all, matter density of the universe outside of P and P' generates also a divergence by applying equation (21). This added divergence modifies the final result given by equation (24). Secondly, the principle of action and reaction might not be true in its simplest formulation. It is easier to understand that the energy version of the demonstration is wrong. Indeed, rigorously speaking, the total energy of P and P' must be replaced by the total energy of the universe. Indeed, energy exchanges might exist between the particles and their environment. A more practical version would be to approximate the energy of the universe to P and P' energies plus the energy of the surroundings of the particles, to some given extent suitable for a correct approximation.

The whole result of those theoretical arguments is Newton's law. But these arguments tend to prefer a variable G more than a constant G value, this variation depending of the energy surrounding P and P' particles.

6. Revisiting Einstein equation

In the more general relativistic regime the same reasoning might be done, replacing the divergence of equation (21) by Einstein tensor, and matter density by stress-energy tensor. Then the reasoning might give Einstein equation. But this work is above the scope of the present document. Nevertheless, since Einstein equation is the most direct formulation of Newton's law in the context of relativity, the reasoning above done in the non relativistic regime applies indirectly to Einstein equation.

To say the least, what appears still seriously doubtful is the statement that $K/(4\pi)$ is a universal constant, in equation (27). At the contrary, the above discussion shows that one would expect matter density of the universe to play a role in the determination of this constant. A more practical formulation of that would be that the energy of the surroundings of P and P' would play a role in this determination. This was true for Newton's law, and is therefore true for Einstein equation, since it is the most direct translation of Newton's law into relativity.

The $\rho = 0$ particular case implies $div(a) = 0$, from equations (21) and (22), but results also directly from the (i) principle. And the present study shows absolutely no need to modify its relativistic formulation given by equation (5). At the contrary, this equation allows to complete the new construction of space-time structure done in the present document. This equation was used notably, above, in the study about two particles in an empty universe.

By other means, the construction of Einstein equation from Newton's law is extremely simple. It is the simplest way to proceed. Inserting a multiplicative tensor between the stress-energy tensor and Einstein tensor is something natural which were rarely done in the literature. This mutiplicative tensor can be only a function of energy: what else? Now the present document shows that this is exactly the correct translation of Newton's law into relativity. The result would be surrounding, or a gravitational model close to it. A classical way to proceed would be to calculate everything with such a X^{μ}_{ν} hypothetic multiplicative tensor and then compare the predictions to experimental data. Very probably, it would show that X^{μ}_{ν} must be proportional to the surrounding energy of the location where the force is exerted. Indeed, the surrounding gravitational model indicates strongly that this is exactly what would happen.

Also, today during the construction of the GR Lagrangian for matter, the G anthropocentric solar system constant value is forced without any theoretical argument. This is doubtful. At the contrary, the GR Lagrangian for vacuum is the simple and well sounded scalar curvature. This is another argument for applying it in the present study (it is equivalent to equation (5)).

7. Conclusion about G

Apart from the previous demonstrations, it is coherent to adopt a physics view point and add physics arguments in favour of a variable G following the rule of the surrouding effect. They will show that the previous demonstrations do not come from nowhere, but are motivated by strong arguments in physics. They are the following.

1) Mach's principle.
2) Correct theoretical construction of Newton's law.
3) Sophisticating the construction of Einstein equation.
4) Loss of information in the construction of the stress-energy tensor.
5) Implicit assumption of GR.

Items 2) and 4) were listed as motivations in the introduction. Item 1) is part of the motivation 4) of the introduction. Those items were transformed into arguments by the present study. Items 1) 2) and 3) have been described above. Item 4) was used implicitly in the present study. Indeed, there were no such loss of information in the descriptions of the energy distributions of the demonstrations of the present document. This item, as well as item 5), is described in [2]. Item 5) is related to the remark that the quarks are moving at a speed which is close to the speed of light.

Therefore, forgetting one instant the previous demonstrations, from a physics point of view alone the following statement is valid. Outside of solar system, it is much more relevant to use equation (14), or its translation with surrounding, than its solar system value for the determination of G. To say the least, a variable G following the rule of the surrounding effect is much more relevant.

8. Yang-Mills Millennium problem

8.1. *General statement*

The remark done in [2] about the Yang-Mills Millennium problem is still valid. Moreover, this remark is conspicuously reinforced by the study of the present document. Let's remind briefly this remark. It starts by assuming the following.

Assumption (A): unification of the four forces is driven by gravitation.

It is not indicated how this unification takes place. But it can be imagined that each particle of particle physics is constructed by some, internal, amount of energy in motion, the different trajectories of those amount of energy giving different behavior of the three other forces.

Under this unifying assumption, the Yang-Mills Millennium problem finds a solution. Indeed this assumption awake the full relativity in the context of particle physics: now not only SR, but also GR underlines all the forces. Therefore each of four forces is driven by the surrounding effect. And this effect modifies enormously the interactions between the particles.

8.2. *Modified predictions and observations*

The first kind of particle physics observations are particles interactions. The present study modifies the physics predictions in the case of triple nuclear collisions [14]. Indeed those collisions would be predicted to behave in a completely different manner, because of the surrounding effect. But they are almost impossible to realize. The GR modification of the present study predicts a different behaviour when the targetting particle is unchanged but when the target of the interaction is modified. When the target energy decreases, the surrounding effect increases and the scattering is expected to be wider. For example, the Ay puzzle [15, 16] might be enlightened.

The second kind of particle physics observations is cosmic rays. But here the present study will not modify the observed predictions.

The third kind of particle physics observations is static configurations of particles. Let's focus on the most stable group of particles, the atom. The electromagnetic force is now predicted to be much weaker than what is calculated by the simple $1/r^2$ rule, when the electron is part of an atom, than when it is alone. For the nucleus, its most simple form is the hadron. Let's discard hadrons made of two quarks because of their life time. The confinement of quarks is noticeable in hadrons of three quarks. This is very much explained by the present study. The remaining question is why an electron can exist alone, while a quark cannot. The present study gives a simple argument here. An answer is to be found in the difference of magnitude between the electromagnetic force and the strong force, when they are exerted in their confinement state, that is, when such groups are particles are formed. When a hadron is formed, then the strong force is far stronger than the electromagnetic force between an electron and a proton inside of an atom. But when an electron or a quark is left alone, then the surrounding effect increases the interaction force with respect to its value when the particle were not alone. Therefore, for an electron it is increasing a relatively weak force. But for a quark, it means increasing a force which was already strong. Therefore it can be understood why an electron can exist alone, while a quark cannot. It must be added that in this description the strong force can't be assumed to be a very short range force. In paragraph 8.3 it will be shown that this assumption might be wrong.

For the strong force a modification of the present study involves three body interactions. Any group of three particles closed to each other would experience low values of the strong force between them, because the surrounding effect would be strong, due to their close proximity to one another. But any group of two particles, or any group of three particles having one of them far enough from the two others, would experience stronger values of the force, because the surrounding effect would be weak. This can explain why hadrons with two quarks have a very short life-time, while a hadron with three quarks is stable. And it explains why an isolated quark is unobserved.

Therefore this allows quark confinement for long duration only when they are close to each other by groups of three. It results a solution of the Millennium problem [3, 4].

8.3. *Other observations in static configurations*

Nuclear saturation [17, 18] is the observation that the volume of a baryon does not vary with the number of baryons inside of the same nucleus. In today's literature this mechanism is understood as suggesting a very short range for the strong force. But there are the two other possible explanations for it, which follow.

1) The strength of the strong force repulsion allows this volume to stay constant when N varies,
2) The strong force is driven by a surrounding effect.

The first explanation numbered 1) would mean that the repulsion of the strong force would be far greater than its corresponding attraction, resulting probably in strange predictions. Moreover as it will be seen in paragraph 8.5 under assumption (A) each force attenuates following the $1/r^2$ rule. Hence the repulsive strong force follows this rule and the first explanation is wrong.

The present document gives arguments in favor of the second explanation. Indeed the surrounding effect explains this mechanism in the same usual simple manner. The strong force potential, exerted on one baryon, by the other baryons of the nucleus, is substracted by its exact value, resulting in a constant potential. This is the same usual suppressing effect predicted by surrounding [1], which happens for the cosmological version of Einstein equation. It happens also for the bullet cluster. The calculation is presented in Appendix R.

This explains why the volume of the nucleus is proportionnal to the number of baryons: the strong force potential of the other baryons stays the same whatever is the number of baryons.

8.4. *Toward a strong force model*

In particle physics the same issue arises as in gravitation, which were leading to the construction of the surrounding gravitational model. Indeed, the equations of G are not practical here also. Moreover, the exact way in which the three forces of particle physics are constructed from gravitation is unknown. Therefore it is required to model those three other forces. The most important work is about the strong force, since it is the one which appears the most promizing one.

For this the same modeling would be required as the one which were used in the construction of surrounding. This construction were using an homographic function using different values of matter density. Such a recipe might be used in particle physics too. Then fitting the four parameters of this homographic function would be required. It must be done by comparing the predicted results with experimental data. It remains to be done.

8.5. *Relevance of the fundamental assumption*

Let's try to discuss the validity of assumption (A). It is more than an assumption. Indeed, the following argument can be done.

Argument (*): acceleration generated by gravitation is explained by space-time curvature. It is a simple and elegant rule. It is tempting to apply it to each force.

The only remark which would forbid it is that the ratio σ/m is not constant but depends of the type of particle, where σ is the charge of the particle with respect to the considered force and m is its mass. But this remark can be discussed under assumption (A). Indeed, under this assumption this dependence of the WEP with σ can be explained by the possible mechanism which were described previously: the particular behavior of the gravitational force depends of the particular internal structure of each particle.

Another argument is the following. Under assumption (A), the WEP is applied for the four forces. Nevertheless, it can be assumed to stay valid for any fixed type of particle. Then the reasoning of paragraph 5 remains valid for any fixed type of particle. For example, in electrostatic, if σ and m are respectivly the charge and the mass of the electron, then σ/m is known for the electron, and the principle can be extended. Therefore the reasoning of paragraph 5 remains valid for this fixed type of particle. Now equation (27) is yielded for all the forces, having of course a different K value depending of the considered force. The result is that under assumption (A), each force follows the rule of the $1/r^2$ attenuation.

The argument (*) comes from relativity and gravitation. It might appear difficult to find such a convincing hint starting from particle physics. For example it might be difficult to find such a convincing argument with the following assumption.

Assumption (B): unification of the four forces is driven by one of the three forces of particle physics.

Under the (B) assumption, the strong force might be the better candidate. Indeed, for example in figure 1 of [19], it is located on an extreme location as compared to the others. The wave nature of matter proven in particle's physics appears to be an argument here. But this might argue also for (A) assumption since one can interpret them as being related to space-time waves. Unfortunately, no convincing unification might be available under this

(B) assumption. Nevertheless assumptions (A) and (B) appear better than the following one.

Assumption (C): no unification of the four forces exists.

Indeed, there is the apparent energy convergence at Planck scale [20] which contradicts it. But assumptions (A) and (B) must be compared with the following one, which might be done by today's physics.

Assumption (D): unification of the four forces is driven by an effect in which no force plays a leading role.

It might be this (D) assumption which is implicitly assumed todays [21, 22]. But the main arguments for it are symmetry considerations. Their relevance must be compared with the relevance of argument (*).

Also an experimental information in favor of assumption (A) is given by eclipses anomalies. Indeed, under this assumption, strong deviations of the gravitational signal of the sun, by the moon, might be expected during solar eclipses [10, 11].

9. Discussion

A new determination of space-time by energy is presented. It uses the usual GR concept of rest frame, which is generalized and then called GRF in the present document. A property of those frames has been dismissed in the literature. This property is that the evolution of those frames is driven by energy. And this particular behavior is proven to result in equation (7). This equation allows to determine space-time structure from energy and deals with the first degree of derivation of the metric. From there the second degree of derivation is calculated and then compared to Einstein equation. Hence this GR feature relates energy to the first degree of derivation of space-time structure, it is a new chapter of GR to be exploited. Its starting part has been developed here.

It results a more complicated relativity, more Machian than before, in which gravitation follows the rule of a surrounding effect. In its weaker formulation this effect is the following. Increasing the energy of the surroundings of the location, where the gravitational force is exerted, results in decreasing this force with respect to Newton's law. It is proven that G is not a constant, and that its variation is driven by this effect. An equation of G is given, which is a good approximation under four assumptions, which might be valid most of the time in gravitation. For most of the usual energy distributions Einstein equation can be used. In particular it can be used for energy distributions of a small set of sparse objects, (for example two objects), when matter density stays constant over space and time apart from those objects. But this new value of G must be used. For calculating this value, formally equations (14) and (16) exist, but they are not practical since the attenuation function used in these equations remains to be calculated. And its value depends of the energy distribution. However, it remains much better to use Einstein equation with a G value driven by the surrounding effect than its constant anthropocentric solar system value. Since these equations of G are not easy to use, the surrounding gravitational model has been constructed [1].

When the complexity of the energy distribution does not allow to use Einstein equation, then a method has been presented, allowing to calculate space-time structure from equation (7) and from any kind of energy distribution made of dimensionless particles. But, even though it might be calculated numerically, this method is complicated.

That's why a general principle might be searched for, allowing to replace completely Einstein equation. Another Lagrangian might be constructed, based on this new chapter of relativity.

Nevertheless, the compatibility of surrounding [1] with experimental data, so far, along with the demonstration of the present document, shows the following. Each so-called gravitational anomaly might be simply a prediction of relativity. Of course the work is huge until one can replace "might be" by "is" in the previous sentence. Indeed, the surrounding gravitational model appears to solve many of those mysteries in a straightforward way. But it remains several mysteries to address. And this model will have to be tuned. The fixed radius value used for calculating the surrounding value might be replaced by a value varying slowly with the galaxy size. This modification would allow the model to explain Tully-Ficher law and the speed profiles of the small galaxies. Also, the brutal rectangle window used for calculating the surrounding value must be replaced by a smooth window, soon or later. A need for that appears for conforming surrounding to the wide binaries problem [23–25]. Also a shielding mechanism has been proven to appear in the study of the NGC 3310 galaxy, which might allow to replace the α constant of the model, by a more relevant term in the equation of motion. Also, possible regressions might arise, in which this G variation might induce wrong predictions in front of some given experimental data. This work remains to be done also.

But nevertheless a big step is done in gravitation. It has been demonstrated that using Einstein equation with the solar system value is only valid close enough from the sun. (To give an idea, roughly, a location less than 10 parsec from the sun allows a less than $0,1\%$ deviation from Newton's law). It has been demonstrated that far enough, this way, from the sun, using the G value given by surrounding is much more relevant than using solar system value.

In particle physics, this surrounding effect arises under the relevant unifying assumption that the four forces are different aspects of the same and unique force of gravitation. This assumption is relevant for several theoretical reasons which were detailed in the present document. The exact law of motion remains to be determined. The same process which were used for the construction of surrounding might be used, in which a homographic function is fitted to experimental data. Nevertheless, applying a surrounding effect in nuclear physics, three puzzles are either explained or solved. The Ay puzzle and the proton radius puzzle find convincing explanations. The nuclear saturation puzzle is solved. This solution shows that the strong force can still follows a $1/r^2$ law, law which can be argued to be valid from theoretical considerations. This allows to get rid of the wired assumption about its supposedly extremely short range. A simple surrounding effect applied at nucleus scale allows to solve completely this saturation observation.

It is the same effect ruling at the same scale, which predicts that any isolated quark would receive a strong force much greater than the one received inside of the nucleus. The solution is given to the Yang-Mills Millennium problem.

Nevertheless, a significant step has been made in gravitation. It has been demonstrated that using Einstein's equation with the solar system value of G is only valid within a certain proximity to the sun. To give an idea, roughly, a location less than 10 parsecs from the sun allows for less than $0,1\%$ deviation from Newton's law. It has been shown that far enough from the sun, using the G value determined by the surrounding value is much more relevant than using the solar system value.

In particle physics, this surrounding effect emerges under the relevant unifying assumption that the four fundamental forces are different aspects of the same unique force of gravitation. This assumption is supported by several theoretical reasons detailed in the present document. The exact law of motion remains to be determined. The same process used for constructing the surrounding effect might be applied, where a homographic function is fitted to experimental data.

Applying this surrounding effect in nuclear physics resolves three puzzles. The Ay puzzle and the proton radius puzzle find convincing explanations. The nuclear saturation puzzle is solved. This solution shows that the strong force can still follow a $1/r^2$ law, which can be argued to be valid from theoretical considerations. This allows us to discard the peculiar assumption about its supposedly extremely short range. A simple surrounding effect applied at the nucleus scale completely solves this saturation observation.

It is the same effect operating at the same scale that predicts any isolated quark would experience a much stronger force than when inside the nucleus. This provides a solution to the Yang-Mills Millennium problem."

Appendix

Appendix A. Continuity of the S function

The S function gives space-time structure from energy distribution. S starts from the set of energy distributions which is a linear space of infinite dimension. It reaches another linear space of infinite dimension which is the space of space-time structures. The latter can be defined using the matrixes of the metric in some given system of coordinates. Then, this space is the linear space of the distributions over space-time of 4×4 matrixes. This definition depends of the choice of the system of coordinates. Those two linear spaces are equipped with the uniform norm. Let's remind that the considered energy distributions are considered sharing the same bounded domain.

With the uniform norm, and for this restricted set of energy distributions, the claim is that S is continuous.

For proving this, let's prove that, if any amount of energy at any space-time location is decreased to 0, then the effect of this amount of energy on space-time structure decreases also to 0. For this demonstration, let's assume that this is wrong (assumption "w"). Then it exists an f_n sequence of energy distributions tending to 0 and such that their deformations is greater than some given non null value. This can be written the following way.

$$\begin{gathered} \underset{n \to +\infty}{lim_u}\,(f_n) = 0 \\ \forall\, n\; in\; \mathbb{N},\; \|S(E + f_n) - S(E)\| \geqslant M \end{gathered} \tag{A.1}$$

In those equations, E is the energy distribution of the universe, from which the f_n energy are added. $\|.\|$ is the uniform norm on the set of space-time structures. M is some strictly positive real number. Therefore the involved physics is such that an infinitely small amount of energy provokes a non null modification of space-time structure. Let's remind that this word "small" means that the maximum density of energy which is added is small (uniform continuity), and also that the whole quantity of added energy is "small" (the considered energy distributions share the same bounded domain). Since this added amount of energy is infinitely small, then it is possible to imagine a thought experiment in which this energy is added instantaneously. But the result on space-time structure is noticeable. This generates gravitational waves. If the universe is empty then no contradiction arises. But this would be unrealistic. If the universe is not empty, then those gravitational waves provoke the motion of matter in the universe. But with this infinitely small and sudden "addition" of energy of course nothing happens since the added amount is infinitely small. Therefore a sudden modification of energy distribution happens without anything to provoke it. This violates the conservation of energy principle. Therefore assumption "w" was wrong and the claim is proven.

Then, writing this continuity property with the limit of sequences, equation (2) is given.

Appendix B. Generalized rest frame

The concept of generalized rest frame can be introduced with a thought experiment. This thought experiment is simply imagining the energy at rest of a P particle increasing progressively, and at the same time the whole energy of the universe decreasing. At the end of the experiment P and the universe have their roles permuted. Now P contains the energy of the previous universe, and the universe contains the energy of the previous particle. The first result is that the frame in which time elapses the most is no longer the frame attached to the universe. Now this frame is the frame attached to P. It means that the space-time structure is now the symmetrical result of a permutation of those two frames. It means also that during the experiment, the space-time structure has been modified progressively from the first state to the final one. And this operation has allowed to revert the time dilatation. For example, if this was a twin paradox configuration, at the end of his brother's journey, the older twin would become the youngest after the thought experiment. Therefore this space-time modification is simply described by the boost transporting one frame into another. It can be noticed that this reasoning is using the well established supposition that GR is coherent.

Now the need of naming the frames appears. Let's call Ru a frame attached to the universe. It can be supposed that the universe is filled with a constant, homogenous distribution of matter, therefore this matter is supposed to be at rest in Ru. Let's call Rp a frame attached to P. The result of the thought experiment is that P generates locally a space-time deformation which is described by the boost from Ru to Rp. Of course, this deformation is local to P but the more energy at rest of P, the more this deformation is valid around P. A "more valid deformation" means that the space-time deformation exists significatively over a larger space-time domain.

The space-time deformation appearing in the experiment is described by the boost which allows to transform progressively this frame from R_p to R_u. This frame remains a rest frame during the whole process, even though it might be no longer the frame in which time elapses the most. Indeed, this frame is a frame in which P is at rest. The result is that it is possible to extend this identification of the frame in which time elapses the most to any space-time event in which there exists matter. And this identification can be extended even further to events in which vacuum prevails, by interpolation between those events in which there is matter.

This interpolation is done by the following procedure.

1) Let's chose a P dimensionless particle, located in a M point of space, let's write R its rest frame.
2) In M, the time line of R is naturally created, let's write it L, such as M pertains to L, and such as for any N point of L the time vector of R is tangent to L,
3) For each such L let's write G_c the geodesics congruence [9] such that L pertains to G_c.
4) For each L' geodesic of G_c, there exists a unique R' frame such as L' is the time line. By definition, this R' frame is a generalized rest frame (GRF). And the set of those R' is the set of GRFs created around M.

The first point above (the P dimensionless particle) takes into account the entire energy distribution of the universe. Indeed, only energy distributions which are set of dimensionless particles are considered in the present document.

Rigorously a geodesic congruence is not the entire universe (there are possible intersections of such "parallel" geodesics). But with the energy distributions considered in the present document (set of dimensionless particles), it is the entire universe. Indeed, otherwise there would exist space-time event in which two different dimensionless particles would exist (hence they would mix themselves). And an implicit property of energy distributions is that this can't happen. This is why this restriction has been added in the glossary to the definition of energy distribution. Of course this is not a strictly rigorous reasoning since there can still exist time lines encountering in vacuum, even though the rest frames time lines are not. But this would require a strongly deformed space-time structure between rest frame time lines. And this might be argued to be unrealistic.

How is created a geodesic congruence? Of course this is done from the metric. And, outside of events in which there is matter, therefore in vacuum, the metric is given by the rule of the Lagrangian in vacuum. Therefore the constraints allowing the determination of this metric are the following.

1) Each time line of a particle is a geodesic.
2) In vacuum the Ricci scalar is null.

After the definition of a GRF, let's underline that its behavior is also a key point of the chapter of relativity which is explored in the present document. This behavior has been introduced at the beginning of the present appendix. It will be also illustrated throughout the present document when using those GRFs.

Let's also remark that the local space-time deformation around a dimensionless P particle is inflexible. It means that it remains the same whatever is the energy distribution outside of P. What makes this inflexibility is the fact that it is local, thanks to the fact that P is assumed to be dimensionless but having a non null energy. Notably, whatever are the surroundings of P, the local deformation generated by P remains the same. This remains true if P is no longer a dimensionless particle, but gets a high enough matter density. This is a realistic modelization since matter is known to be extremely condensed. And it is true also for the virtual microscopic particles which are used in the thought experiments of the present document.

Appendix C. Rescaling after the boost

Equations (10) (11) and (12) show that a rescaling of ct time and x space axis is required, after execution of the $B^{\mu}_{\nu}(x)$ boost. If no rescaling occurs, then it means that the local deformation generated by P is wholly described by a boost. Then it is simply the usual change of coordinates obeying to SR rule, and no deformation is noticed. Therefore a rescaling occurs, and this one is coherent with time dilation between R and R'. Therefore from equations (8) (10) (11) and (12), if α and β are respectively the s rescaling of ct and x, there is $\alpha^2 = \beta^{-2} = 1 - v^2/c^2$.

Moreover, the calculation of this rescaling can be done in the more general but usual configuration of a flat Minkowskian space-time everywhere into which an ϵ energy distribution is added in a bounded space-time domain. The old and new GRFs refer to this ϵ bounded energy. The space-time deformation generated by ϵ is described by the successive local deformations of the test particles being at rest when located infinitely far from ϵ, along their trajectories. Those calculations are described in (Appendix P). Once again, the SR time dilatation factor is the time component of this rescaling.

Appendix D. Integration constant

This appendix is about the integration constant of the space-time structure calculation for the energy distribution of two particles in an empty universe.

This constant is possibly given by the following information. There are singularities along the straight line containing the particles's locations, but only on those points which are not between the particles's locations. They are described by the boost associated with the speed of light in the direction moving away from the particles.

The rigorous way to proceed is to calculate space-time structure using symmetry consideration and equation (5). Then, the asymptotic values of the deformation should allow to calculate the integration constant.

Let's remind that this procedure is already done for the energy distribution with one particle in an empty universe. Symmetry consideration produces the Schwarzschild metric. Equation (5) gives the information that the metric is of the shape $g_{00}(x) = 1 - M/r$. $g_{00}(x)$ is the time-time component of the metric in the x space-time event, r is the spatial distance from x to the particle which is located at the center of symmetry, and M is the unknown integration contant. Then the asymptotic value of the local space-time deformation given by paragraph 3.6 allows to calculate $M = \infty$.

Therefore, it should be possible to calculate this constant for the energy distribution of two particles in an empty universe.

Appendix E. Particle moving at a speed below the speed of light: local deformation

Let's consider P, a particle moving uniformly at the v speed, $v < c$, along the D space line in a flat Minkowskian space-time. The $R(O; ct, x, y, z)$ frame is chosen such as O is contained by D and Ox is in the direction of the P motion. The local deformation around P is first described by the b_v boost associated with the v speed, as shown before.

But equations (10) (11) and (12) show that a rescaling of ct time and x space axis is required. If no rescaling occurs, then it means that the deformation generated by P is wholly described by a boost. Then it is simply the usual change of coordinates obeying to SR rule, and no deformation is noticed. Therefore a rescaling occurs, and this one is coherent with time dilation between R and R'. Therefore from equations (8) (10) (11) and (12), if α and β are respectively the s rescaling of ct and x, there is $\alpha^2 = \beta^{-2} = 1 - v^2/c^2$.

Hence the complete deformation is described by $b_v s$. This results from equations (10) (11) and (12).

Appendix F. Particle moving at the speed of light: local deformation

Now $v = c$ is assumed. Let's prove that the deformation in C is the b_C boost associated with the speed of light along the D line in the direction of the GW propagation.

Starting from Appendix E, taking the limit when v tends to c, the result is obtained. As usual the interchange of limits theorem is used. The simple convergence is enough for it to work since the studied deformation is only local.

Appendix G. Relation between the boost and the speed of a gravitational wave

Let's study the GW generated by a given P particle in motion with respect to an R old GRF. First of all, the space-time deformation generated by a GW is described by a linear map, because it's a local deformation. But it transforms, at first, an inertial frame into another inertial frame. Therefore the Minkowskian distance is invariant and the linear map is an isometry. Time inversions, space rotations, and space symetries are discarded since they are not realistic. It remains only boosts. Then the results is composed with time and space positive rescaling.

Indeed, no rescaling would mean no space-time deformation. This boost is written in the old R GRF before the existence of the GW. Then for local symmetry considerations, the V speed associated with the boost in R is in the direction of the GW propagation. Let's start by the study of the boost part of this linear map.

It will be assumed that the speed of a GW is a given v positive value with respect to the R frame. What is the relation between v and the V speed? It might be guessed that $V = v$.

This demonstration is easier to understand in a contradiction way. Let's assume the result is wrong. Therefore, it is assumed that the GW speed is v and that the boost of its deformation is V such as $V \neq v$.

Then it is considered a P' particle "surfing" on the GW. That is, P' is always located in close vicinity to a moving M point, where the GW deformation is maximum, and which is in motion with the GW propagation. Let's write R' a frame which is attached to this M point. The local deformation generated by P' is independent of its energy. Indeed, the determination of the new and old GRFs before and after the existence of P are given by SR. There are more details about that at the end of Appendix B. For the same reason, the local deformation generated by P' is also independent of the GW energy. And it is the same for the GW: its generated deformation is independent of its energy and of the P' energy. Therefore, in R', P' generates locally a deformation which is noticed because $V \neq v$. If P' energy is low enough, then the P' local deformation transforms more globally into the GW deformation which is a null deformation in R'. In other words, in R', locally it appears a flat Minkowskian space-time, and also a smaller space-time deformation locally to P'. In R', locally to P' a boost is noticed, associated with a w speed which is not null (w is the relativistic substraction of v by W). But there is the following rule: with respect to a frame, a particle modifies locally space-time structure with the boost which is associated with its speed. And the reverse is true: with respect to a frame, a particle modifying space-time structure with the boost associated with the w speed is in motion with the w speed. Applying this rule here, it results that P' is in motion in R' at the w speed which is not null. A contradiction arises. This proves the above claim.

Therefore the boost part of the GW local deformation is the one which is associated with its v propagation speed. Now, the rescaling is still unknown. But if $v = c$ which is of course well established, then the associated boost is degenerated and any further rescaling would produce no change on the final result.

Appendix H. Particle moving at the speed of light: propagation of the deformation along the trajectory

Let's deduce the following, from the result of Appendix F.

The deformation of P in C propagates along the D line in the same direction as the motion of P and is described by the b_C boost which is associated with the speed of light in the same direction.

This can be noticed as soon as P deviates from this D line trajectory. First of all, the GW generated by P propagates along the D line for symmetry reasons. The propagation speed is the speed of light as assumed by assumption (I). And the propagated local deformation is described by b_C: the demonstration is the same as in Appendix G. Or it is possible to use Appendix G assuming $v < c$ and then considering the limit $v = c$.

Appendix I. Simple description of the local space-time deformation

Let's describe the local space-time deformation generated by a particle or a GW, using the most simple possible way. For that the Poincaré-Einstein synchronization [26] will be used. The Minguzzi synchronization will not be required since the transitivity of the procedure is not mandatory here. This procedure is used here for the construction of a $R'(C, ct', x')$ inertial frame in motion at the v speed with respect to another, let's write it $R(O, ct, x)$. No Lorentz transform nor boost is needed for that, only Eintein synchronization rule and geometry in two dimensions are used. Figure (2) represents this construction. After this construction x' is the symmetric of ct' through the orthogonal symmetry with respect to the light trajectory in the x increasing direction. Indeed, the circle passing by O and M, and having C as its center contains also N because the OMN triangle is rectangle in N. Hence $CM = CN$. Then if H is the middle of M and N then CH is parallel to ON because of

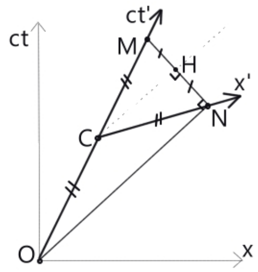

Fig. 2. Two inertial frames are represented in two dimensions, $R(O; ct, x)$, and $R'(C; ct', x')$. The latter is constructed with respect to the first using Einstein synchronisation rule. ON and NM together represent the trajectory of a light beam emitted from O in the direction of x increasing, and bouncing back in N. C is the middle of OM. The x' axis is supported by CN following the synchronisation rule. H is the middle of MN. A simple geometry reasoning in two dimension shows that x' is the symetric of ct' through the orthogonal symmetry with respect to CN, which is parallel to ON.

Thales theorem applied in the OMN triangle. And of course CN is the direction of the light trajectory in the x increasing direction.

Moreover, there is a natural coherence of the calculation of the space axis in four dimensions, using the synchronization procedure. This is seen by using different light beam trajectories. For example the procedure can be executed with a light beam in the x decreasing direction, and then after the turning point, in the x increasing direction. The result is of course the same x' axis. But also using a trajectory along the y, or z, direction will show that along those space axis no deformation is noticed, which is the expected result. Any other light beam back and forth trajectory in space will show a deformed axis which is coherent also, since the procedure yields linear results.

The casual link is the following. The starting point is that the motion of R' with respect to R is v, that is, C is moving at the v speed with respect to R. From this information, then the ct' axis of R' can be drawn with respect to R. Then the synchronization rule determines the location of the x' axis. Even if mathematically there is an equivalence, and ct' can be retrieved from x', nevertheless for physics the casual link is univoque.

A remark can be added about the rescaling studied in Appendix C. The boost transforming R into R' is deduced, therefore implicitly identified by the direction of the new ct' time axis with respect to R. The new GRF and the new metric valid after insertion of the particle or the new GW are deduced from this ct' axis only, without the requirement of the details of the boost itself. Even the usual rescaling of the new time and space units can be done based on the ct' axis only.

Appendix J. Uniformly moving particle: propagation of the deformation everywhere

Let's argue the following.

Assumption (j): The space-time local deformation described by the b_C boost and generated by the P particle moving at thes speed of light in the x direction, generates also a GW which propagates at the speed of light along the directions which are perpendicular to x.

This is much more than an assumption. Indeed, the local deformation which generates this GW is described by this b_C boost. Hence it is more than an assumption that this is exactly what is propagated by this GW. And if D is the name of the P straight line trajectory, then the H space point of D which is the closest to any given M point is such that MH is perpendicular to D. Therefore the propagation of the GW perpendicularly to D is more than an assumption.

Moreover, let's show in the present appendix that this is coherent with the fact that the envelope of the GW propagates globally at the $c/\sqrt{2}$ speed along the CM direction. This will be an added argument for the (j) assumption.

Let's start by studying the motion of the GW because this is the starting point as shown in Appendix I. The GW propagates at the speed of light, as stated by assumption (I) of paragraph 3.4. This propagation at the c speed is done along the directions which are perpendicular to the P trajectory, as stated by assumption (j) above. Therefore this propagation is done from C to N at the c speed. Let's study the Δ half line containing N and C' and starting in C' in the direction of N. The events of Δ at the t_N time are the propagations of the events of the locations of P along the CC' segment. It means that Δ is the envelope of the GW along the (x, y) plane. It is noticed that this Δ envelope propagates along the CN direction at the $c/\sqrt{2}$ speed.

From that motion is deduced the new time axis, let's write it ct", following once again the casual link presented in Appendix I. This ct" time axis in M is located in the (ct, n) plane, where n is the axis supported by the CM direction. Because of the $c/\sqrt{2}$ motion of the GW, the equation of the trajectory of the GW along the n axis is the following.

$$r = (c/\sqrt{2})t \tag{J.1}$$

In equation (J.1), r is the coordinate along the n direction starting from M, with respect to R. It can be written $ct = \sqrt{2}r$. Therefore the β angle between ct" and the (x, y) plane is given by the following equations.

$$tan(\beta) = \sqrt{2} \tag{J.2}$$

$$cos(\beta) = \frac{1}{\sqrt{3}} \tag{J.3}$$

Now let's show that this result is coherent with the local space-time deformation generated by P and its local propagation along the x axis. At t_N, therefore along Δ, the ct time axis has been rotating around the (y, z) plane with a $\pi/4$ angle in the direction of x, resulting in the ct' axis. Indeed this is the effect of the b_C boost along the space points of Δ: the b_C boost transforms R into the $R'(ct', x', y, z)$ frame in such a way. There is $ct' = x'$. This creates a Π plane containing all these lines starting from the points of Δ in the direction of ct'. In other words, Π is the new plane of time after the action of the b_C boosts executed along Δ at $t = t_N$ in R. Let's calculate the α angle between Π, with respect to R, and the (x, y) plane. It is also the angle between the n direction and Π, therefore the angle between CM and Π. The calculation follows.

$$cos(\alpha) = \frac{\overrightarrow{CM}.\overrightarrow{CM'}}{|CM\|_3|CM'\|_3} = \frac{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2}{\sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2}\sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2 + L^2}} = \frac{1}{\sqrt{3}} \tag{J.4}$$

L is the $CN = CC'$ distance. M' is the space-time point being the projection of M on P' along the z direction. P' is the plane which is parallel to Π and which contains C. Equation (J.4) is coherent with Equation (J.3). The result is the following.

$$\alpha = \beta. \tag{J.5}$$

It means that the GW propagation along its envelope creates a space-time deformation which is coherent with the propagation of the b_C boost perpendicularly to the P trajectory.

It must be noticed also that the Π plane makes also a $\pi/4$ angle with respect to the (x, y) plane, but in the direction of y. In other words, the intersection of Π with the (y, z) plane is a line which makes such an angle with respect to the (x, y) plane. And this is coherent with the local GW propagation along the y axis at the speed of light.

The final result is a coherence between the following GW features.

- Locally the GW propagates the b_C boost at the speed of light, along the P trajectory,
- It propagates also locally this boost at the speed of light, perpendicularly to the P trajectory,
- Globally the envelope of the GW propagates at the $c/\sqrt{2}$ speed.

Then, still following the casual link presented in Appendix I, the new space axis of the GW deformation is deduced from the location of the new time axis. This new n" space axis is part of the (ct, n) plane because the GW global motion is along the n axis. n" is the symmetric of ct" with respect of the line supported by the $\vec{ct} + \vec{n}$ vector, $\vec{ct}$ and $\vec{n}$ being respectively the unit vectors of the ct and n axis. Of course the same coherence arises for the new space axis along the n, x and y directions, the same coherence as the one which were shown above for the new time axis.

The conclusion of the present appendix is that under assumptions (I) and (j), a coherent and symmetrical picture of this GW is given. Since assumption (j) was not proven, the picture is not proven to be correct. Hopefully, the full picture is not required in the demonstration of the present document. What is required is the result of Appendix H, that is, the fact that the b_C boost propagates itself along x and x increasing.

Appendix K. Final picture for a particle moving at the speed of light

This results from the previous appendix. The final picture for a particle moving at the speed of light is that in R the deformation occurring in C at the E_C event propagates globally along the Cn space direction, at the $c/\sqrt{2}$ speed. This deformation is described by the boost associated with the c speed in the x direction, which propagates locally along the Cx and direction, and along the Cy directions. But along the Cn direction, this deformation is described by the boost associated with the $c/\sqrt{2}$ speed. This picture is Lorentz invariant.

Appendix L. Asymptotic space-time deformation around a particle moving at the speed of light along a circle

The asymptotic local space-time deformation generated by a particle moving at the speed of light along a circle is deduced from Appendix K. The GW propagates along the direction which is perpendicular to the particle's trajectory. But asymptotically this direction is also the line starting by the location of the particle. And from Appendix K this is done at the c speed: that's the propagation of the b_C boost along D, which was described in paragraph 3.5. It is also the propagation of the envelope which is now the sequence of spheres (the sequence of spheres described in paragraph 3.6, which are centered in O, the center of the circle). Asymptotically those spheres inflate themselves around O at the speed of light. And now this envelope propagates at the c speed. Hence the boost which describe this local deformation is associated to the c motion vector which is normal to these spheres.

Appendix M. Avoiding the unknown forces

P was forced to follow the circle-like trajectory which is not a geodesic. Therefore there is an unknown force which allows this trajectory not to follow a geodesic. This force is not gravity. Therefore, at first glance, modeling the Dirac distribution under assumption (2) would not be possible in a coherent way and implying only gravitation. But this apparent result is wrong, the final result is quite the opposite, as it will be seen further.

In order to avoid such added forces, the sequence of energy distributions converging to the Dirac distribution will be modified. Those energy distributions will respect the following rules.

- a set of particles moving at the speed of light,
- this set is distributed in a homogeneous manner around O, following the spherical symmetry around O,
- at each M point different from O, an infinite set of such particles exist, each of them moving in a perpendicular way with respect to OM. It means that the c speed of each particle is perpendicular to OM.
- the distribution of those c vector is homogeneous in M.
- the space-time structure which is generated by this energy distribution is assumed to allow this distribution to remain static.

In order ensure the latter item above, the calculation will be done at the end of the present study, using the new equation replacing Einstein equation. Therefore what is required is to calculate that such a distribution exists under the new to come equation. In patucular the latter item must be fulfilled. This will ensure the validity of the final result. This is done in Appendix N.

Under assumption (1), such a distribution does not exist. Moreover, under this assumption, there can't be any static spherically symmetric distribution without singularity and without added forces. Indeed, for a static spherically symmetric distribution, the space-time structure would generate a radial acceleration toward O. matter would be accelerated toward O. Any infinitesimal bunch of matter located in M, different from O, would be accelerated along and in the direction of the MO vector.

But a static distribution is mandatory. Indeed, let's remind that the procedure is to model the final Dirac distribution of matter, by studying a sequence of energy distributions, each of them containing no singularity, and which converges to this Dirac distribution. And the Dirac Distribution is static because the aim is to model the gravity of a dimensionless particle at rest in some inertial frame. Therefore this sequence must be made of static distributions. At the opposite, under assumption (2), those static distributions might be possible because the geodesic trajectories are not radial but circles around O.

Therefore the result is exactly opposite to what was claimed at the beginning of the present paragraph. It is modeling the Dirac distribution under assumption (1) which is not possible, not the one under assumption (2). Under assumption (1) the Dirac distributions, used for representing the gravity of particles, are not possible.

Interestingly, the result here is a very important result. The result is that in relativity, assumption (1) is impossible and must be replaced. For this replacement, assumption (2) might be possible.

Appendix N. Existence of a static spherically symmetric distribution

To be written.

Appendix O. The gravitational wave four-momentums add themselves

Let's suppose that n GWs are propagating and encountering themselves in a given x space-time event. Let's write, for i from 0 to n, GW_i the GW having the i indice. For each i, it can be created a P_i particle such that its momentum counteracts exactly the effect of GW_i in x. It means that the P_i speed is the speed of light, and that its total energy in any R frame is the energy of GW_i in R. It means also that the direction of the P_i motion is opposite to the direction of the GW_i motion. The result is that P_i and GW_i together generate no local space-time deformation. Therefore the overall effect of all these P_i particles is such that it cancels the space-time deformation generated by all the GW_i. But the sum of all the P_i particles is a big compound object, let's call it P. So the space-time deformation of all the GW_i is exactly conteracting the space-time deformation generated by P. But conservation of energy principle states that the P four-momentum is the sum of the P_i four-momentums. Therefore the four-momentum of the space-time deformation of the whole set of GW_i is the sum of the GW_i four-momentums.

$$\begin{aligned} F_m\big(\{GW_i, i=0..n\}\big) &= -F_m(P) \\ &= -\Sigma_{i=0}^{n} F_m(P_i) \\ &= -\Sigma_{i=0}^{n}\big(-F_m(GW_i)\big) \\ &= \Sigma_{i=0}^{n} F_m(GW_i) \end{aligned} \tag{O.1}$$

$F_m()$ is a function giving the four-momentum of a particle or a GW. This is the end of the proof.

Appendix P. The free falling particle in the Schwarzschild metric

Let's study, in the Schwarzschild metric, a P_1 test particle, being at rest when located infinitely far from the P particle which is located in the center of the symmetry. Let's show that the local space-time deformation generated successively by P_1 along its trajectory is the same as the space-time deformation generated by P (that is, described by the Schwarzschild metric itself). The same context and notations as in paragraph 3.11.3 are used. Notably the R_0 frame is attached to P.

First of all, the P_1 null mass is replaced by an infinitely small mass. Once again, it means that the calcultions are done in an easier context, and that the final step will use the interchange of limits theorem in order to get the same result for the initial context.

In the R_1 frame which is attached to P_1, let's study another P_2 particle which always stays infinitely close to P_1 and which has an energy at rest far weaker than the P_1 rest energy. P_2 follows the same trajectory as P_1. And this can be understood as only the result of the space-time deformation of P_1. Indeed, P_2 energy is far weaker than P_1 energy, and P_2 is always close to P_1. It must be noticed that the space-time gravitational deformation generated by P is without effect locally to P_1, completely replaced by the space-time deformation of P_1. The P_2 free fall ignores it completely.

Therefore the P_1 deformation of space-time is what forces P_2 to follow the same trajectory as P_1. The R_1 frame attached to P_1 is a GRF because it's a rest frame. It's a rest frame from the start, when P_1 is at rest infinitely far from P. But the P_1 deformation is described by the b_1 boost associated with its motion in R_0. Now let's decrease P_1 energy to 0, progressively, without modifying the P_2 energy (it does not mean that the conservation of energy principle is broken). This P_1 energy decrease will not change the P_2 trajectory. This is because P_1 energy (hence P_2 energy, too) has been assumed to be weak enough, in order to give that result. Indeed, since their energies are always assumed to be weak enough, P_1 and P_2 are always in free fall. Hence their trajectories remain the same whatever are their energies, as far as they are weak enough for allowing the free fall. Noteably it is true for P_2.

The result is that the local space-time deformation generated by P_1 motion in R_0, along its trajectory, is the same as the gravitational space-time deformation generated by P along this trajectory. Hence b_1 describes the gravitational local deformation generated by P, along the P_1 trajectory. And the rescaling after the execution of b_1 is such that the time unit is decreased by $\sqrt{1 - v^2/c^2}$, the proper time usual decrease of the free fall as already known in relativity.

When the mass of P_1 and P_2 tend to 0, the local deformation they generate stay the same as well as of course the deformation of P. Hence the equality of those deformations remain valid. Therefore the previous result applies also to P_1 being a test particle, being at rest when located infinitely far from P.

This ends the demonstration.

This demonstration can be applied to a flat Minkowskian space-time everywhere in which an ϵ energy distribution is added in a bounded space-time domain. The old and new GRFs refer now to this ϵ bounded energy. Therefore in the above demonstration, the only modification is that P is replaced by ϵ. Then the demonstration can be re-executed without change for those generalized energy distributions. Once again, the result is the following. The space-time deformation generated by ϵ is described by the successive local deformations of P_1 along its trajectory. And the SR time dilatation factor is the time component of this rescaling.

Appendix Q. An equation of G: calculations

In this appendix an equation of G is tried. Assumptions (i) (ii) (iii) and (iiii) are assumed. The same context and notations as in paragraph 3.11.3 are used. Let's remind properties of the Schwarzschild metric. The first equation is the following.

$$g_{00}(x') = 1 - \frac{R}{r} \tag{Q.1}$$

Of course $g_{00}(x')$ is the time-time component of the metric in the x' event, R is the Schwarzschild radius of P, r is the distance between P and the x' event. But the following one is another important equation valid in this metric, which relies the metric to the free fall speed.

$$g_{00}(x') = 1 - \frac{v^2}{c^2} \tag{Q.2}$$

v is the speed of the usual P_1 free falling test particle, located in x', which were located initially infinitely far from P. The validity of equation (Q.2) comes from assumption (i). Indeed, equations (Q.1) and (Q.2) are valid together because Newton's law is assumed to be valid. Equation (Q.4), which comes further, can be used to confirm that statement. From equation (8) and (Q.2), using $e = D^1(x)/\big(D^0(x) - D^1(x)\big)$, it results the following equation.

$$g_{00}(x') = \frac{1+2e}{(1+e)^2} \tag{Q.3}$$

By other means, the geodesic equation of P_1 is the following [8].

$$\frac{\partial^2 r}{\partial \tau^2} = -\frac{c^2}{2}\frac{\partial g_{00}}{\partial r} \tag{Q.4}$$

Here τ is the proper time of P_1. Replacing g_{00} by its value given by equation (Q.3), equation (Q.4) becomes the following one.

$$\frac{\partial^2 r}{\partial \tau^2} = c^2 \frac{e}{(1+e)^3}\frac{\partial e}{\partial r} \tag{Q.5}$$

Then, once again, assumption (i) is used, Newton's law is valid. The asymptotic formulation of equation (Q.5) is yielded, relying Newton's law with the asymptotic value of the rhs of equation (Q.5).

$$-\frac{M_0 G}{r^2} \simeq c^2 e \frac{\partial e}{\partial r} \tag{Q.6}$$

Here M_0 is the mass of P. Assumption (iii) has been used: the contribution of P in equation (7) is far weaker than the sum of the other contributions. Therefore, e is asymptotically equal to 0 in front of 1. The solution of this differential equation (Q.6), gives the following asymptotic value of e.

$$e \simeq \sqrt{\frac{R}{r}} \tag{Q.7}$$

From that is deduced $D^1(x)/D^0(x) = e/(1+e) \simeq \sqrt{R/r}$. It results the following equation.

$$\frac{1_w(x,y_0)f(x,y_0)C^0(y_0)}{\Sigma_{n=0}^{\infty}1_w(x,y_n)f(x,y_n)C^0(y_n)} \simeq \sqrt{\frac{R}{r}} \tag{Q.8}$$

It has been supposed that the y_0 virtual particle is the only one pertaining to P. It was used $C^0(y_0) = C^1(y_0)$, which reflects the fact that $C^\mu(y_0)$ is a null four-momentum.

Under (iii) assumption, the denominator of equation (Q.8) is constant, that is, independant of the x location. Therefore this equation shows that the $1_w(x,y_0)\ f(x,y_0)\ C^0(y_0)$ contribution is proportional to $1/\sqrt{r}$. Under the (ii) assumption, it can be deduced that the $1_w(x,y_n)f(x,y_n)C^0(y_n)$ contributions are proportional to $1/\sqrt{\|x-y_n\|_3}$. $\|x-y_n\|_3$ is the space length calculated in a covariant way along the y_n to x geodesic and observed in R_0.

Also x, and therefore r being constant, from equation (Q.8) the $1_w(x,y_0)\ f(x,y_0)$ $C^0(y_0)$ contribution is not proportional to $C^0(y_0)$ but proportional to $\sqrt{R}$, therefore to $\sqrt{M_0}$ and to $\sqrt{C^0(y_0)}$. This is in contradiction with the hypothesis at the construction of equation (7). Here it is noticed a proportionality to the square root of an energy. The answer to this apparent contradiction is that the final picture is coherent. Therefore one can replace each $1_w(x,y_n)\ f(x,y_n)C^0(y_n)$ contribution of equation (Q.8) by $1_w(x,y_n)$ $\sqrt{C^0(y_n)/\|x-y_n\|_3}$. Now, from $f(x,y_0)\ C^0(y_0)$ being proportional to $\sqrt{C^0(y_0)/r}$ and assumption (ii), the result is the following.

$$\frac{1_w(x,y_n)\sqrt{\frac{C^0(y_0)}{r}}}{\Sigma_{n=0}^{\infty}1_w(x,y_n)\sqrt{\frac{C^0(y_n)}{\|x-y_n\|_3}}} \simeq \sqrt{\frac{R}{r}} \tag{Q.9}$$

It can be used $R = 2M_0G/c^2$ and $C^0(y_0) = M_0c$, in order to transform equation (Q.9) into the following one.

$$G \simeq \frac{c^4}{2\left(\Sigma_{n=0}^{\infty}1_w(x,y_n)\sqrt{\frac{E(y_n)}{\|x-y_n\|_3}}\right)^2} \tag{Q.10}$$

$E(y_n) = C^0(y_n)c$ is the total energy of the particle located in y_n. Now it is possible to use this result with VGWs. It means that the energy distributions of the P_i are replaced by the limits of the S_n^i distributions, as shown by equation (6). The result is equation (14), which is equation (Q.10), without the $1_w(x,y_n)$ terms. Indeed, in any given x and y_n possible events of space-time, it is possible to find a m and a S_m^n distribution centered on y_n, such as the GW which is generated by the virtual particle of S_m^n, propagates in x. Moreover, this S_m^n distribution can be chosen with a circle's radius as weak as we want (remember that those S_m^n distributions are circle like trajectories of virtual particles moving at the speed of light). In other words, after the limits of equation (6) are done, each x space-time event is reached by the VGW of each P_n particle.

Appendix R. Nuclear saturation

Let's write N the number of baryons in the nucleus, and β a given baryon, inside of the studied nucleus. β is supposed to have number 0 in the numerotation of the baryons. The interaction force is proportional to the energy of the attractor. This is particularily true under (A) assumption of paragraph 8.1. Let's write below this proportionality of the $F(y_n)$ force exerted by the baryon numbered n, and located in y_n, on β.

$$F(y_n) = K\ G\ C^0(y_n)\ c \tag{R.1}$$

K is the proportionality coefficient. Its dependence with interacton distance is neglected. From equations (R.1) and (14), the Π pressure exerted on β by the other baryons is the following.

$$\Pi = \frac{Kc^4 X}{2S} \tag{R.2}$$

$$X = \frac{N-1}{\left(\Sigma_{n=1}^{N-1} \frac{1}{\sqrt{\|x-y_n\|_3}}\right)^2} \tag{R.3}$$

In equation (R.2), S is a given external surface of the baryon. For example it can be the surface of the minimum possible sphere surrounding the baryon. On S is exerted the $F(y_n)$ force of equation (R.1). It has been assumed that all the baryons of the nucleus share the same $C(y_0)c$ energy. Equation (R.2) shows that Π varies in the same manner as X when N varies. But X does not vary very much as shown by equation (R.3). If the dependence of K with interaction distance was taken into account, the result would be the same. This result is that Π does not vary much with N, the number of baryons. More precisely, this variation appears negligible as compared to what it would be if G was a constant in equation (R.1).

References

1. F. Lassiaille, "Surrounding Matter Theory," *EPJ Web of Conf* **182**, 03006 (2018)
2. F. Lassiaille, "Relativity in Motion: Short Version", *Nuclear Theory* Vol. **39** eds. M. Gaidarov, N. Minkov, Heron Press, Sofia (2022)
3. A. Jaffe and E. Witten, *Quantum Yang-Mills Theory* (Clay Math Institute)
4. L. D. Faddeev, "Mass in Quantum Yang-Mills Theory (Comment on a Clay Millennium Problem)" In: Benedicks M., Jones P.W., Smirnov S., Winckler B. (eds) *Perspectives in Analysis. Mathematical Physics Studies*, vol **27** Springer, Berlin, Heidelberg (2005).(arXiv:0911.1013 [math-ph])

5. BP Abbott, R Abbott, TD Abbott, F Acernese, K Ackley, C Adams, et al., "Gravitational waves and gamma-rays from a binary neutron star merger: GW 170817 and GRB 170817A" *The Astrophysical Journal Letters*, **848** (2): L13. (2017)
6. D. J. Raine, "Mach's Principle in general relativity", *MNRAS* **171** (3): 507–528 (1975)
7. A. Unsöld, B. Baschek, "The New Cosmos: An Introduction to Astronomy and Astrophysics", *Physics and astronomy online, Springer* p. 485 (2001)
8. F. Lassiaille, "Gravitational Model of the Three Elements Theory: Mathematical Explanations," *J. Mod. Phys.* **4** No. 7, pp. 1027-1035 (2013)
9. S. Carroll, *Space time and geometry, An introduction to General Relativity* (San Francisco: Addison-Wesley, 2004).
10. XS. Yang, QS. Wang, "Gravity Anomaly During the Mohe Total Solar Eclipse and New Constraint on Gravitational Shielding Parameter", *Astrophysics and Space Science*, **282** 245–253 (2002)
11. A. F. Pugach, D. Olenici, "Observations of Correlated Behavior of Two Light Torsion Balances and a Paraconical Pendulum in Separate Locations during the Solar Eclipse of January 26th, 2009", *Advances in Astronomy*, **2012** Article ID 263818, 6 pages, (2012)
12. E. V. Pitjeva, "Determination of the Value of the Heliocentric Gravitational Constant (GM0) from Modern Observations of Planets and Spacecraft" *J. Phys. Chem.* Ref. Data 44 031210 (2015)
13. M. Milgrom, "A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis", *ApJ* , **270** 365 (1983)
14. O.V. Vitiuk, V.M. Pugatch, K.A. Bugaev et al, "Triple high energy nuclear and hadron collisions - a new method to study QCD phase diagram at high baryonic densities". Eur. Phys. J. **A 58** 169 (2022)
15. D. Hüber and J. L. Friar, "The Ay puzzle and the nuclear force" *Phys. Rev. C*, **58** 674 (1998)
16. D. R. Entem, R. Machleidt, and H. Witała "Chiral NN model and Ay puzzle" *Phys. Rev. C*, **65** 064005 (2002)
17. L. Coraggio et al, "Nuclear-matter equation of state with consistent two- and three-body perturbative chiral interactions" *Phys. Rev. C*, **89** 044321 (2014)
18. F. Sammarruca et al, "Toward order-by-order calculations of the nuclear and neutron matter equations of state in chiral effective field theory" *Phys. Rev. C*, **91** 054311 (2015)
19. W. Juven, "Unified model beyond grand unification", *Physical Review D.*, **103** (10): 105024 (2021)
20. F. Wilczek, "The Future of Particle Physics as a Natural Science" *International Journal of Modern Physics A.* **13** (6): 863–886 (1998)
21. H. Georgi and S. L. Glashow, "Unity of All Elementary Particle Forces", *Phys. Rev. Lett.*, **32** 438–441 (1974)
22. H. Fritzsch and P. Minkowski, "Unified Interactions of Leptons and Hadrons", *Annals Phys.*, **93** 193–266 (1975)
23. I. Banik et al., "Strong constraints on the gravitational law from Gaia DR3 wide binaries", *MNRAS*, **527** 4573 (2023)
24. K.-H. Chae, "Breakdown of the Newton–Einstein Standard Gravity at Low Acceleration in Internal Dynamics of Wide Binary Stars", *ApJ*, **952** 128 (2023)
25. X. Hernandez, "Internal kinematics of Gaia DR3 wide binaries: anomalous behaviour in the low acceleration regime", *MNRAS*, **525** 1401 (2023)
26. E. Minguzzi, "The Poincaré-Einstein synchronization: historical aspects and new developments" *J. Phys.: Conf. Ser.* **306** 012059