Resolution of the Yang–Mills Millennium Problem: The Solution via Full General Relativity and its Surrounding Equation

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We demonstrate that algebraically integrating General Relativity into quantum field theory via the Surrounding Matter framework provides a rigorous resolution to the Yang–Mills mass gap problem. Within this framework, the local energy-momentum structure is governed by the multi-source vector field $D^\mu(x)$ and the velocity ratio v/c. We show that the algebraic cancellation λ/λ within the surrounding ratio ensures that an infinitesimal energy distribution generates the exact same space-time calibration as a finite one across all gauge generators. Consequently, gauge field interactions in a non-empty Universe cannot decay into zero-energy states; for localized excitations in vacuum environments, the surrounding background dynamically enhances their rest mass, establishing a strictly positive lower bound for the ground-state spectrum ($\Delta > 0$). Simultaneously, high-energy divergences are inherently regularized: ultraviolet self-interactions remain strictly bounded by background saturation through the infinite interaction time delay ($t' \to \infty$) and the dynamic attenuation driven by the algebraic relativistic factor. This uniform upper bound guarantees the unconditional convergence of the non-perturbative Feynman path integral $\int \mathcal{D}A\, e^{iS[A]}$ and ensures that the n-point Wightman correlation functions $\langle 0|A(x_1)\dots A(x_n)|0\rangle$ remain well-defined tempered distributions across all of $\mathbb{R}^4$. Furthermore, we demonstrate that the algebraic deformation generated by $D^\mu(x)$ resolves the historical obstacle of infinite static field energy without introducing metric singularities, removing the necessity for a rigid Minkowski background. Building upon a rigorous gravitational baseline, these results reveal that General Relativity, through its global algebraic surrounding effect, plays an active and indispensable role in governing quantum gauge field dynamics at subatomic scales.

1. Introduction

In standard quantum field theory (QFT), gauge theories such as pure Yang–Mills are formulated on a rigid, flat Minkowski metric tensor $\eta_{\mu\nu}$. While this approximation is exceptionally accurate for perturbative calculations at microscopic scales, it assumes that General Relativity (GR) plays no structural role in subatomic dynamics due to the negligible magnitude of Newton's gravitational constant G_N. However, this decoupling conflates the weakness of local gravitational forces with the fundamental role played by the global surrounding energy density in defining local rest frames and background field calibrations.

Classical pure Yang–Mills theory is famously scale-invariant, allowing field configurations to be continuously shrunk in spatial volume toward zero-energy states ($E \to 0$). Establishing the existence of a strictly positive ground-state mass gap

$\Delta > 0$—one of the Millennium Prize Problems formulated by the Clay Mathematics Institute—requires identifying a mechanism that breaks this classical scale invariance at the foundational level.

In this paper, we show that this missing mechanism arises naturally when full and complete GR is integrated algebraically into quantum field theory. Rather than treating space-time as an isolated single-source manifold or a featureless Minkowski background, the local energy-momentum structure is determined by a multi-source vector $D^\mu(x)$ and an associated dimensionless ratio v/c. We demonstrate how this environmental energy constraint maps energy distributions away from infinitesimal zero-energy states, and guarantees the existence of a non-vanishing mass gap $\Delta > 0$ prior to ad hoc perturbative counterterm subtractions.

2. GR and the Origin of the Surrounding Effect

Before generalizing the surrounding correction to particle physics, it is instructive to recall its origin within GR itself, as this very construction is used, unmodified, throughout the rest of this paper. Starting from the vacuum Einstein–Hilbert Lagrangian and requiring that the theory treat on an equal footing any arbitrary, multi-source distribution of energy rather than an idealized single isolated source, one derives a local spatio-temporal four-vector $D^\mu(x)$. This vector describes the local structure of space-time at x as a discrete superposition over all energy sources located at y_n in the surrounding environment:

$$D^\mu(x) = \sum_{n=0}^{\infty} 1_w(x, y_n)\, f(x, y_n)\, C^\mu(y_n), \tag{1}$$

where $1_w(x, y_n)$ is a weighting indicator function selecting which surrounding sources effectively contribute at x, $f(x, y_n)$ encodes the attenuation of energy-momentum propagation through space-time, and y_n denotes the event associated with the n-th matter component. $C^\mu(y_n)$ is the corresponding null four-momentum vector, oriented in the direction of the tangent in y_n of $g(y_n)$, the geodesic from y_n to x, and directed from y_n to x.

Let us reformulate this equation by passing the total energy from left to right. Only the norm of the spatial term of the left-hand side is considered, and only the projection of the spatial terms of the right-hand side along the straight line identified by the direction of $D^\mu(x)$:

$$\epsilon \frac{\sum_{i=1}^{3} \sqrt{\left(D^i(x)\right)^2}}{D^0(x)} = \frac{\sum_{n=0}^{\infty} \cos(\theta_n) f(x, y_n) \sqrt{\sum_{i=1}^{3} \left(C^i(y_n)\right)^2}}{\sum_{n=0}^{\infty} f(x, y_n)\, C^0(y_n)}. \tag{2}$$

Here, θ_n is the angle of the projection in x of the spatial vector of the propagated $C^\mu(y_n)$ in x. $\epsilon = \pm 1$ is the sign corresponding also to the convention for θ_n. The $1_w(x, y_n)$ term has been suppressed for the sake of simplicity. It will remain suppressed from those sums until the end of the present document.

The left-hand side term can be reformulated as:

$$\epsilon \frac{\sqrt{\sum_{i=1}^{3} \left(D^i(x)\right)^2}}{D^0(x)} = \epsilon \frac{\sqrt{\sum_{i=1}^{3} \left(v^i(x)\right)^2}}{c} = \frac{v}{c}. \tag{3}$$

Here, $v = \sqrt{\sum_{i=1}^{3} \left(v^i(x)\right)^2}$ is the algebraic scalar speed associated with $D^\mu(x)$, and $v^i(x)$ are its spatial components. Since each $C^\mu(y_n)$ is a null vector, we also have $C^0(y_n) = \sqrt{\sum_{i=1}^{3} \left(C^i(y_n)\right)^2}$. These considerations allow us to simplify Equation (2):

$$\frac{v}{c} = \frac{\sum_{n=0}^{\infty} f(x, y_n) \cos(\theta_n) C^0(y_n)}{\sum_{n=0}^{\infty} f(x, y_n)\, C^0(y_n)}. \tag{4}$$

By isolating the contribution of a single reference source y_0 relative to the total sum, Equation (4) yields the following fundamental dimensionless energy ratio. This is allowed if the propagated $C^\mu(y_n)$ spatial components cancel themselves at x for $n \neq 0$, that is, if $\sum_{n=1}^{\infty} f(x, y_n) \cos(\theta_n) C^0(y_n) = 0$:

$$\frac{v}{c} = \frac{f(x, y_0)\, C^0(y_0)}{\sum_{n=0}^{\infty} f(x, y_n)\, C^0(y_n)}. \tag{5}$$

This ratio identifies the amplitude of the Lorentz boost from which the interaction derives.

The core result of this derivation is that the effective relativistic structure of space-time is never determined by an isolated mass independently of what exists elsewhere in the universe; it depends intrinsically on the complete background energy distribution. Far from being an ad hoc addition, the *surrounding effect* is a structural mechanism inherent to GR that becomes explicit as soon as realistic, multi-body boundary conditions replace asymptotically flat approximations. As shown below, this v/c ratio forms the bridge to addressing the Yang–Mills problem in Section 4.

3. GR and the Foundations of Quantum Field Theory

3.1. *Inconsistency of Special Relativity*

Modern particle physics relies almost exclusively on Special Relativity (SR), treating quantum fields as propagating on a fixed, flat Minkowski background $\eta_{\mu\nu}$. However, treating SR as a standalone, fundamental framework leads to well-known mathematical inconsistencies:

- **Kinematic Inconsistencies and Relativistic Gauge Fields:** Even when restricting the analysis strictly to piecewise inertial frames—as in closed-loop or topological formulations of the twin paradox where acceleration plays no fundamental role—a conflict arises when quantum gauge fields (Yang–Mills) evolve across such trajectories. Consider a self-contained quantum laboratory (housing atomic clocks and particle storage rings) embarked on a relativistic round-trip relative to a reference frame. While SR predicts a net time dilation, it cannot intrinsically determine which frame holds the rest state or non-trivial topological holonomy without referencing the global distribution of matter. For gauge fields, this path-dependent shift affects local phase accumulation and decay rates. To dynamically close these observables upon reunion without ad hoc global frame choices, the field equations intrinsically require the surrounding energy density constraints provided by GR.

- **Mach's Principle and Frame Selection:** SR lacks an origin for inertial frames. As emphasized by Machian arguments (see, e.g., Ref. [1]), local inertial properties are entirely dynamically determined by the global distribution of matter in the Universe.
- **The Surrounding Effect at Subatomic Scales:** Equations (1) and (5) demonstrate that local space-time properties shift whenever a system moves between environments with radically different surrounding energy densities. This lack of background calibration should manifest in the laboratory, notably during nuclear transitions where a particle probes the vast gradient between the interior of a dense nucleus and the surrounding vacuum. The steep gradient in v/c dynamically modifies local field interactions, effectively yielding a measurable variation in coupling strength.

3.2. *Relativity and Particle Physics: Standard View*

It is worth noting that a widespread misconception in standard textbook literature attributes the resolution of relativistic paradoxes solely to proper acceleration, thereby incorrectly concluding that SR is sufficient on its own. In reality, as shown by closed-loop and topological thought experiments, kinematics alone cannot select the rest frame without referencing the global energy distribution of the background. In particle physics, this conceptual shortcut has led to an entrenched assumption: because the gravitational coupling constant G_N is extremely small at subatomic scales, the geometric background is systematically frozen to $\eta_{\mu\nu}$. This confuses the negligible magnitude of local gravitational forces with the fundamental role played by the global surrounding energy density in setting the algebraic calibration of quantum field interactions.

3.3. *A Requirement of the Yang–Mills Millennium Problem*

Let us revisit the foundational requirement of the Yang–Mills Millennium Problem as formulated by Jaffe and Witten [3]:

> *"...find a quantum field theory on $\mathbb{R}^4$ (or $\mathbb{R}\times\mathbb{S}^3$) satisfying the Wightman axioms (or some other reasonable system of axioms) for a compact simple gauge group G, and show that the mass gap Δ is strictly positive."*

3.4. *Possible Interpretations of the Requirement*

The conclusion drawn from the observations in Section 3.1 is straightforward: SR cannot stand as a mathematically closed theory on its own. Consequently, two logical interpretations emerge regarding the primary problem statement of Jaffe and Witten:

(1) **Strict Flatness Interpretation:** If the condition $\mathbb{R}^4$ is interpreted as strictly forbidding GR—either as a curved metric manifold or as an algebraic geometric constraint—then the axiomatic formulation inherits the mathematical incompleteness of pure SR.
(2) **Algebraic Interpretation:** If the problem setup ignores these flat-space limitations by utilizing only the algebraic structures of relativistic invariance (e.g., Poincaré symmetry), it cannot logically reject GR when GR itself is formulated purely algebraically as an energy-density constraint on fields.

3.5. *Intermediate Conclusion*

In both cases, the conclusion is clear: GR, through its algebraic surrounding effect, is not merely an optional modification to Yang–Mills theory; it is a required component to construct a mathematically complete and realistic spectrum. GR plays a foundational role directly inside particle physics on the exact same footing as it does at galactic and cosmological scales [2]. The remainder of this paper explores the quantitative consequences of this integration in deriving the solution to the Yang–Mills Millennium problem.

4. Yang–Mills Problem

4.1. *The Yang–Mills problem: reminder*

The Yang–Mills existence and mass-gap problem, one of the seven Clay Mathematics Institute Millennium Prize Problems, requires two things simultaneously: (i) a mathematically rigorous construction of a quantum Yang–Mills theory in four dimensions, and (ii) a proof that this theory has a mass gap $\Delta > 0$, i.e., that the lightest excitation above the vacuum has strictly positive mass, even though the classical theory is scale-invariant and massless.

The present document describes the solution to this problem.

4.2. *Emergence of the mass gap ($\Delta > 0$)*

In classical pure Yang–Mills theory, the action is scale-invariant, allowing field configurations to shrink arbitrarily in spatial extent toward zero-energy states ($E \to 0$). However, when accounting for the surrounding environment via Equation (5):

$$\frac{v}{c} = \frac{f(x, y_0)\, C^0(y_0)}{\sum_{n=0}^{\infty} f(x, y_n)\, C^0(y_n)}, \tag{6}$$

a fundamental scale invariance property of effective four-momenta emerges at the level of global energy distribution.

If the entire energy distribution of the background is scaled by an arbitrary real number $\lambda \in \mathbb{R}^*$, the factor λ cancels identically between the numerator and the denominator. Consequently, an infinitesimal energy distribution yields the exact same effective space-time structure as a finite one: any energy distribution is instantly mapped away from an infinitesimal state. In a non-empty Universe, an interaction energy can thus never be strictly zero or infinitesimally small.

Furthermore, in a low-density background—such as an isolated glueball bound state localized in the vacuum—the surrounding ratio v/c dynamically amplifies the effective interaction energy in inverse proportion to the small surrounding density. This intrinsic scale calibration prevents localized gauge field configurations from collapsing down to zero-energy states, thereby guaranteeing a strictly positive ground-state mass gap:

$$\Delta = E_{\min} > 0. \tag{7}$$

4.3. *Surrounding regulation at high energies (UV finiteness)*

In standard perturbative Quantum Field Theory, ultraviolet divergences arise because loop integrals evaluate field interactions down to arbitrarily short distances ($r \to 0$) or infinite momenta ($p \to \infty$) on a static Minkowski background $\eta_{\mu\nu}$. This necessitates mathematical subtraction schemes, such as dimensional regularization and counterterms, to absorb infinities.

When full GR is integrated via the surrounding Equation (1), the following mechanism arises. At extremely short distances or high energy exchanges, the local energy density contribution to the denominator of $D^\mu(x)$ increases proportionally with the field energy.

Two mechanisms occur:

The first mechanism is identical to that of the mass gap. Starting from a finite given E energy distribution, increasing it by any λ factor leaves the result of λE unchanged. Even the limit $\lambda \to \infty$ yields exactly the same universe as the one resulting from E. Even if the energies have been globally modified, this is completely unseen since all the interactions stay unchanged.

The second mechanism was explored in Ref. [4]. A particle alone in an empty universe generates a singularity everywhere outside its precise location as a direct result of GR. This implies that all interactions around the particle are saturated by $v = c$. This is the limiting behavior of any set of energy distributions that tends locally to infinity, even if no vacuum exists in the distributions of this set.

The result of the second mechanism is that any local infinity only generates a saturation of all interactions. Before reaching this limit, the interactions are asymptotically saturated.

This upper bound prevents effective interaction amplitudes from diverging at subatomic scales. Rather than inserting artificial momentum cutoffs or modifying the gauge group symmetry, the UV behavior of the theory is naturally regulated at the foundational level by the finite response of the surrounding relativistic energy background.

4.4. *Intermediate conclusion*

In the pure non-Abelian sector governed by $\mathcal{L}_{\mathrm{YM}}$, the four-momentum vector field $D^{\mu}(x)$—derived from relativistic principles incorporating the surrounding effect—dynamically breaks conformal invariance, thereby establishing a non-zero mass gap $\Delta > 0$ for gauge excitations. Concurrently, as $r \to 0$, local energy accumulation saturates interaction mechanisms, ensuring that all physical interaction amplitudes within $\mathcal{L}_{\mathrm{YM}}$ remain strictly bounded across $\mathbb{R}^4$.

This mechanism provides the missing foundation for Yang–Mills theory: by embedding the gauge action within the global energy background of GR, scale invariance is dynamically broken without ad hoc parameters, while vacuum fluctuations and infinities yield naturally regulated interactions.

4.5. *Status of the Solution: Where the Essential Work Lies*

It is worth emphasizing that the primary theoretical hurdle of the Yang–Mills problem is already overcome at the foundational level. The heavy analytical lifting was accomplished in Ref. [1] (introducing and summarizing Ref. [4]), which contains the 50-page mathematical demonstration of the surrounding four-vector $D^{\mu}(x)$ [Equation (1)] directly from General Relativity. Furthermore, the required verification of the resulting gravitational predictions of this relativity correction was performed in Ref. [2], providing far more than a simple check.

5. Detailed Modified Equations

5.1. *Mass Gap Generation*

In the presence of the environment vector field $D^{\mu}(x)$ of Equation (1), the local spacetime structure is dressed through a weighted environmental superposition over all surrounding source events y_n.

According to equation (4), under a global scaling of the background energy distribution $e \to \lambda e$, the fundamental velocity ratio transforms as:

$$\frac{v}{c} = \frac{\sum_{n=0}^{\infty} f(x, y_n)\, \cos(\theta_n)\, \lambda D_0(y_n)}{\sum_{n=0}^{\infty} f(x, y_n)\, \lambda D_0(y_n)} = \frac{\sum_{n=0}^{\infty} f(x, y_n)\, \cos(\theta_n)\, D_0(y_n)}{\sum_{n=0}^{\infty} f(x, y_n)\, D_0(y_n)}. \tag{8}$$

The exact algebraic cancellation of λ ensures that infinitesimal energy distributions produce the exact same spacetime calibration as finite ones. In this framework, the quantum vacuum is never truly empty: it is filled with ubiquitous background fluctuations whose effective coupling amplitudes are dynamically amplified by this scale-invariant erasure of infinitesimal scales (λ/λ).

As a direct consequence, localized gauge excitations in low-density regions cannot decay into zero-energy states. The amplified algebraic relativistic background enforces a strictly positive lower bound for mass, establishing a non-zero mass gap $\Delta > 0$ across all interaction sectors.

The feature amplified by the algebraic surrounding mechanism is precisely the algebraic relativistic deformation induced by energy. This deformation yields the observed mass gap, functioning specifically because the energy under consideration does not propagate at the speed of light. Crucially, throughout this process, every field property and virtual particle characteristic other than mass remains entirely unchanged. This formally resolves the condition presented in item (ii) of Section 4.1. For convenience, we restate it here: *a proof that this theory has a mass gap $\Delta > 0$, i.e., that the lightest excitation above the vacuum has strictly positive mass, even though the classical theory is scale-invariant and massless.'* While this formulation serves as a pedagogical equivalent of the official Clay Institute statement—*[...] a quantum Yang–Mills theory exists on $\mathbb{R}^4$ (or $\mathbb{R}\times\mathbb{S}^3$) and has a mass gap $\Delta > 0$'*—it emphasizes that the gap applies specifically to mass and no other observable, directly aligning with reality: the dynamical generation of mass observed experimentally in non-Abelian gauge theories."

5.2. *Field Strength Tensor and UV Finiteness via Algebraic Spacetime Deformation*

The Yang–Mills curvature tensor $F_{\mu\nu}(x) = F^a_{\mu\nu}(x)T^a$, defined via the commutator $[\mathcal{D}_\mu, \mathcal{D}_\nu] = -igF^a_{\mu\nu}T^a$ with $\mathcal{D}_\mu = \partial_\mu - igA^a_\mu T^a$, is given in components by:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu. \tag{9}$$

The gauge potentials $A^a_\mu(x)$ are not independent fields governed by a standalone equation; rather, they are subject to the underlying algebraic spacetime deformation generated by the surrounding vector field $D^\mu(x)$. This structural coupling naturally regulates ultraviolet divergences without introducing ad hoc cutoffs or altering the exact gauge symmetry of the theory.

5.2.1. *Spacetime Deformation in the Maximal Distortion Regime*

This mechanism was examined in Ref. [4]. In general relativity, an isolated particle in an otherwise empty universe generates a geometric singular behavior across spacetime. This implies that all interactions surrounding the particle saturate at the speed of light ($v = c$). This represents the limiting behavior for any sequence of localized energy distributions whose density approaches infinity, even in the absence of a strict vacuum.

5.2.2. *Light-Cone Decoupling in the Maximal Distortion Regime*

To analyze the consequences of this maximal distortion regime—wherein a high local energy concentration S_0 in the emission region induces a strong deformation at the receiving region—we examine the geometric transformation of light cones between a localized source event O' and an observation event O separated by a spatial distance $r = |OO'|$.

Let $(O; x, t)$ denote the laboratory reference frame. Under the maximal distortion regime, the effective (algebraic) frame operating at O for the algebraic surrounding effect is $(O; x', t')$, where the Ox' and Ot' semi-axes are closer than Ox and Ot semi-axes. This is the strong deformation occuring in O, as shown in Figure 1.

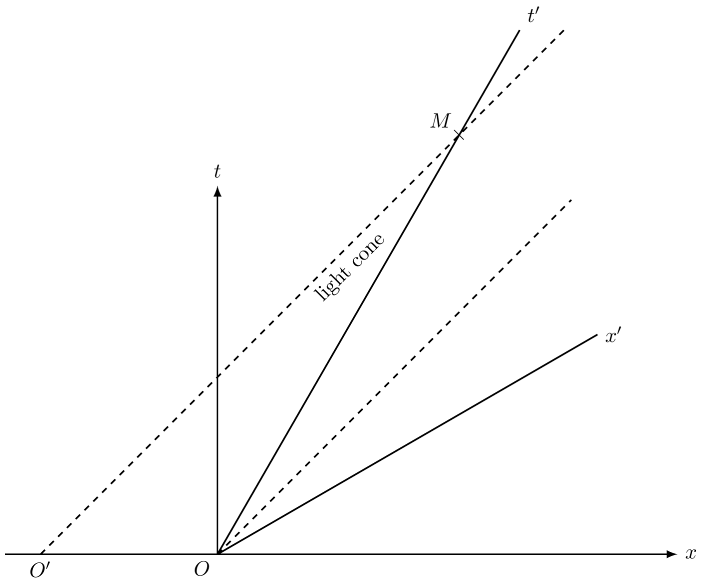

Fig. 1. Geometric configuration of reference frames $(O; t, x)$ and $(O; t', x')$ at observation point O, illustrating the spatial displacement r and the receiving of the interaction in M.

The Light-Cone intersection event M represents the precise spacetime location where the outgoing signal emitted from O' at $t = 0$ intercepts the deformed time axis Ot' of the receiving frame. As depicted in Fig. 1, the spatial separation $r = |OO'|$ between the source O' and the observer O establishes the baseline for the propagation path along the light cone defined by $y = x - x_{O'}$. The coordinates of M in the laboratory frame $(O; x, t)$ are obtained by finding the intersection between the light cone trajectory and the deformed temporal semi-axis Ot'. Let's calculate this.

The Lorentz transformations relating the two frames in O read:

$$\begin{cases} x' = \gamma\,(x - vt)\,, \\ ct' = \gamma\left(ct - \frac{v}{c}x\right), \end{cases} \qquad \begin{cases} x = \gamma\,(x' + vt')\,, \\ ct = \gamma\left(ct' + \frac{v}{c}x'\right). \end{cases} \tag{10}$$

As usual, we define $\beta = v/c$ and $\gamma = 1/\sqrt{1 - \beta^2}$.

Consider the straight line describing the future light-cone boundary L emitted from O' ($O'M$ dotted line in Figure 1). In the $(O; x', t')$ frame, this trajectory is parameterized by:

$$L: \quad x' = ct' + K. \tag{11}$$

Evaluating the origin O' defined by $x = -r, t = 0$ in prime coordinates gives:

$$x' = \gamma(-r - 0) = -\gamma r, \qquad ct' = \gamma\left(0 - \frac{v}{c}(-r)\right) = \gamma\frac{v}{c}r. \tag{12}$$

Since $O' \in L$, substituting these coordinates yields the constant K:

$$-\gamma r = \gamma\frac{v}{c}r + K \implies K = -\gamma r\left(1 + \frac{v}{c}\right). \tag{13}$$

Thus, the equation for the light cone L in the prime coordinate system is:

$$L: \quad x' = ct' - \gamma r\left(1 + \frac{v}{c}\right). \tag{14}$$

The worldline Δ of the observer at O corresponds to $x' = 0$ (or $x = vt$). The intersection $\Delta \cap L$ determines the actual reception time t' in the $(O; x', t')$ frame (the event M in Figure 1):

$$ct' + K = 0 \tag{15}$$

This yields the time delay between the emitter at O' and its reception at O:

$$t' = t_0\sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} \tag{16}$$

It has been used $t_0 = \frac{r}{c}$ as the nominal minimal reception time of the interaction without algebraic modification. So the actual reception time t' under background deformation becomes $t' = t_0 D$, where $D = \sqrt{\frac{1+v/c}{1-v/c}}$ represents the relativistic Doppler factor between the two frames in O.

In the high-energy limit where energy concentration increases at O', the local field concentration drives the velocity ratio $v \to c$ ($\beta \to 1$). Consequently, $D \to +\infty$ and $t' \to +\infty$. An unbounded local concentration at O' requires an infinite amount of time to reach O, asymptotically decoupling the interaction at finite observation times.

5.2.3. *Energy Attenuation*

Having resolved the historical divergence of static field energy through light-cone decoupling, one is justified in incorporating the algebraic relativistic metric induced by a localized interaction. This underlying geometric response introduces a second, complementary mechanism for the structural regulation of local ultraviolet infinities.

Consider a local energy concentration at short distances ($r \to 0$), where the interaction energy density tends toward infinity. In this regime, the accumulation of field energy induces a localized algebraic spacetime deformation via Equation (1). This effect acts as an algebraic analog to a gravitational distortion without an event horizon, ensuring that interactions remain active and continuous.

To quantify the energy transmitted from a y high-density source region to a reception event x, let $S_0 = \sum_{n=1}^{N} f(x, y_n) \cos(\theta_n) C^0(y_n)$ denote the local energy concentration (with N sources localized around y), and let $S_1 = \sum_{n=1}^{M} f(x, y_n) C^0(y_n)$ represent the surrounding environment. The local velocity ratio in x takes the form:

$$\frac{v}{c} = \frac{S_0}{S_1}. \tag{17}$$

The emitted field energy E_{emitted} is equal to the unweighted local sum $\sum_{n=1}^{N} C^0(y_n)$. The received energy E_{received} at x is attenuated by the local algebraic relativistic factor $\left(1 - \frac{v^2}{c^2}\right)$:

$$\begin{aligned} E_{\text{received}} &= E_{\text{emitted}} \left(1 - \frac{v^2}{c^2}\right) \\ &= E_{\text{emitted}} \left(1 - \left(\frac{S_0}{S_1}\right)^2\right) \\ &\approx 2 \frac{E_{\text{emitted}}}{S_0} (S_1 - S_0) \\ &= 2 \frac{\sum_{n=1}^{N} C^0(y_n)}{\sum_{n=1}^{N} f(x, y_n) \cos(\theta_n) C^0(y_n)} (S_1 - S_0) \\ &< 2 \frac{S_1 - S_0}{\min_{1 \le n \le N} \big(f(x, y_n)\big) \cos\Big(\max_{1 \le n \le N}(\theta_n)\Big)}. \end{aligned} \tag{18}$$

The third line of Eq. (18) applies in the asymptotic regime $S_0/S_1 \simeq 1$. In the final line, we utilize the relation $\min_{1 \le n \le N} \big(\cos(\theta_n)\big) = \cos\Big(\max_{1 \le n \le N}(\theta_n)\Big)$.

The term $\min_{1 \le n \le N} \big(f(x, y_n)\big)$ remains strictly positive and is bounded below by its initial value $m = \min_{\mathcal{P}_{\text{conv}}} (\min_{1 \le n \le N} f(x, y_n)) > 0$ across the entire convergence process. To ensure this, it suffices to assume that during the convergence process $C^0(y_n) \to \infty$ holds homogeneously for all $n \in \{1, \dots, N\}$; specifically, there exists a scaling parameter $\lambda \to \infty$ such that $C^0(y_n) = \lambda \bar{C}^0(y_n)$ for each $n \in \{1, \dots, N\}$, where $\bar{C}^0(y_n)$ denotes the initial value of $C^0(y_n)$ evaluated at $t = 0$, marking the onset of the convergence process.

Furthermore, the angular orientation θ_n between x and each source y_n remains bounded above by a common maximum angle strictly smaller than $\pi/2$ throughout the convergence process, ensuring a strictly positive cosine. Indeed, the spatial region of energy concentration at y can naturally be assumed to be viewed from x within a sufficiently narrow angular cone. Defining $C = \cos\Big(\max_{1\le n\le N}(\theta_n)\Big) > 0$, the upper bound in Eq. (18) simplifies to:

$$E_{\text{received}} \le \frac{2(S_1 - S_0)}{\min_{1\le n\le N}\big(f(x, y_n)\big)} \le \frac{2}{mC}(S_1 - S_0) \xrightarrow[S_0\to S_1]{} 0. \tag{19}$$

The emitted energy, despite formally tending toward infinity at the source, is dynamically attenuated by the background factor, resulting in a vanishing contribution at the point of reception.

5.2.4. *Intermediate Conclusion*

Analyzing the behavior across all configuration regimes demonstrates that E_{received} remains strictly bounded through two sequential and cumulative mechanisms:

(1) **Causal Decoupling and Infinite Interaction Time (Regime at y):** First, as $S_0 \to \infty$, the local energy density induces a maximal background deformation ($S_0 \to S_1$ and $v \to c$). As derived in Section 5.2.2, this local singularity collapses the causal overlap: the time axis at observation point x and the light cone centered at source point y decouple, shifting their intersection to infinity ($t' \to \infty$). The interaction time required for transmission becomes infinite, suppressing divergent contributions.

(2) **Energy Attenuation from y to x:** Second, for any interaction evaluated at finite time as $S_0 \to S_1$, the algebraic relativistic factor $(1 - v^2/c^2)$ acts directly on the transmitted field energy, ensuring that the received energy asymptotically vanishes in the limit $S_0 \to S_1$.

(3) **Cumulative Effect and Regularization of Feynman Integrals:** These two phenomena reinforce each other: as $S_0 \to S_1$, the required interaction time diverges ($t' \to \infty$) while the received energy simultaneously drops to zero.
Conversely, at $S_0 = 0$, $E_{\text{received}} = 0$. Since $E_{\text{received}}(S_0)$ is continuous and vanishes at both boundaries ($S_0 = 0$ and $S_0 = S_1$), Rolle's theorem guarantees the existence of a strict, finite maximum $E_{\max} < \infty$. Computing this finite upper bound provides the non-perturbative regularization required for convergent Feynman path integrals.
By establishing this strict upper bound $E_{\max} < \infty$ uniformly for any short-distance configuration ($x_i \to x_j$), local energy densities are prevented from diverging. Consequently, the non-perturbative Feynman path integral $\int \mathcal{D}A\, e^{iS[A]}$ converges unconditionally, and the corresponding n-point

Wightman correlation functions $\langle 0|A(x_1)\dots A(x_n)|0\rangle$ remain strictly well-defined tempered distributions across all of $\mathbb{R}^4$.

Consequently, a local high-energy infinity at y never produces an infinite observed interaction at x. The ultraviolet divergences of Quantum Field Theory are systematically eliminated by the foundational algebraic response of the surrounding background.

6. Conclusion

In this paper, we have demonstrated that this missing mechanism provides a rigorous solution to the Yang–Mills mass gap problem. When General Relativity is algebraically integrated into quantum field theory via the Surrounding Matter framework, the local energy-momentum structure is dictated by the multi-source vector field $D^\mu(x)$ and the ratio v/c. This relativistic surrounding constraint shifts field configurations away from zero-energy states, guarantees the existence of a non-vanishing mass gap $\Delta > 0$, and ensures non-perturbative UV finiteness across all interactions.

The algebraic cancellation λ/λ in the surrounding ratio ensures that an infinitesimal energy distribution generates the exact same space-time calibration as a finite one across all gauge generators. Consequently, in a non-empty Universe, gauge field interactions cannot decay into zero-energy states. For localized excitations in low-density vacuum environments, the surrounding background dynamically enhances the rest mass of this excitation, ensuring a strictly positive lower bound for the ground-state mass spectrum ($\Delta > 0$).

Simultaneously, high-energy divergences are inherently regularized via a dual mechanism: local ultraviolet self-interactions remain strictly bounded by background saturation. This bound operates through two successive effects: the interaction time delay diverging to infinity ($t' \to \infty$) and the dynamic attenuation of the received energy driven by the algebraic relativistic factor. This strict upper bound $E_{\max} < \infty$ holds uniformly for any short-distance configuration ($x_i \to x_j$), thereby preventing local energy densities from diverging. As a result, the non-perturbative Feynman path integral $\int \mathcal{D}A\, e^{iS[A]}$ converges unconditionally, and the corresponding n-point Wightman correlation functions $\langle 0|A(x_1)\dots A(x_n)|0\rangle$ remain strictly well-defined tempered distributions across all of $\mathbb{R}^4$.

Furthermore, it is worth noting that the standard restriction of the Yang–Mills Millennium problem to a rigid, flat Minkowski space-time likely stems from the historical impossibility of regulating the infinite static field energy without introducing metric singularities. The algebraic deformation generated by $D^\mu(x)$ demonstrates that this foundational obstacle is resolved simultaneously, removing this necessity for an unreactive background metric and extending the scope of the theory beyond flat-space constraints.

The heavy mathematical and foundational lifting of GR was fully established in a 50-page paper (Ref. [4]). The present work builds upon this rigorous gravitational baseline, demonstrating its direct application to subatomic gauge theories. These findings demonstrate that GR, through its global algebraic surrounding effect, plays an active and indispensable role in governing quantum gauge field dynamics at the smallest scales.

References

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