Relativity predicts a variable G

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It is demonstrated that relativity predicts a variable G. The proof begins by considering a dimensionless particle in an empty universe. The analysis then extends to two particles, three particles, and an infinite set of particles. This approach enables the calculation of spacetime structure for any realistic energy distribution. The proof employs the interchange of limits theorem and ad hoc sequences of energy distributions. With only one particle, the result is a singularity everywhere if the universe is empty outside the particle. These singularities completely disappear with three particles. The calculation is then generalized for any realistic energy distribution, naturally yielding an equation for G. This equation provides a correct approximation for most realistic energy distributions. The fundamental principles underlying Einstein's equation remain valid. However, it is shown that the anthropocentric solar system constant G must be replaced by a variable value, which is weaker in high matter density environments and stronger in low matter density environments. This was called a surrounding effect in previous works [1, 2]. This effect has been shown to resolve current gravitational mysteries in astrophysics and cosmology. Additionally, under a unifying relevant assumption, several puzzles in nuclear physics are either explained or solved, and a solution to the Yang-Mills Millennium problem is also provided.

Contents

1. Introduction

The purpose of this document is to demonstrate that relativity predicts a variable G and to describe a solution to the Millennium Problem [3, 4].

This study builds upon previous works, particularly [2], which showed that a particular assumption implies that G is a variable. Here, it is demonstrated that this variability is a theoretical prediction of relativity, independent of the speed of quarks or any other experimental information about energy distributions.

The proof begins by considering a dimensionless particle in an empty universe. The spacetime structure is then calculated for several particles up to an infinite set of particles, employing the interchange of limits theorem.

There are several motivations for this study, listed below in order of importance:

1) Solving the Yang-Mills Millennium problem.
2) Addressing today's gravitational mysteries.
3) Searching for a link between energy and the first metric derivative.
4) Resolving the slight caveats of General Relativity (GR) [2].
5) Seeking a theoretical justification for Newton's law.
6) Replacing the anthropocentric solar system value of G with a more general and sound value.
7) Retrieving the information lost in the construction of the stress-energy tensor.
8) Studying the behavior of rest frames.

The concept of rest frames will be further detailed. These frames generalize the traditional rest frames. Exploring this motivation will reveal that Special Relativity (SR) is more than just an algebraic rule of GR; it describes the local spacetime deformation caused by energy. This will be the first step in addressing item 3. Item 3 relates to a new chapter of GR that needs to be written and exploited. The most important parts will be addressed in this document.

Initially, only items 2 and 3 were considered. Item 4 encompasses several motivations. Items 1, 3, 5, 6, and 8 will be directly addressed in this document. Item 7 will be indirectly addressed. Item 2 has been addressed in [1]. Item 4 has been addressed in [2], which can be considered a previous version of this document.

It will be proven that relativity predicts a variable G. This variation is driven by the surrounding effect [1], which, in its weaker version, states that gravitational force increases in low matter density environments and decreases in high matter density environments. An equation for G will naturally emerge from this process. Finally, the Yang-Mills Millennium problem will be addressed.

The document is written as a mathematical demonstration. To ease reading, many parts are included in the appendix, and a glossary is provided at the beginning.

2. Glossary

Energy distribution: In this document, energy distributions are assumed to share the same bounded domain. Each distribution is null except for a bounded subset of spacetime. They are assumed to be a finite set of dimensionless particles with non-zero masses. No space-time event can contain two different dimensionless particles. The reasons for these restrictions will be explained throughout the document.

Frame in which time elapses the most: This concept comes from the study of the twin paradox, where the twin on Earth experiences time passing faster than the traveling twin. In this document, this concept is used didactically to introduce the concept of GRF.

GRF: Generalized rest frame. This is the central and new concept of the new chapter of relativity developed in this document. It is introduced didactically by the frame in which time elapses the most but is defined as the generalization of the rest frame.

Matter at rest: Matter that is at rest in an inertial frame.

Matter in motion: Matter that is in motion at the speed of light.

Rest frame: A frame in which a particle is at rest. In this document, only sets of dimensionless particles are considered, and only gravitation is taken into account, with other forces either ignored or resulting from gravitation. In this context, a rest frame, with its meaning in relativity, can always be associated with a test particle that is at rest in this frame."

S: The function giving spacetime structure from energy distribution.

Test particle: An idealized particle with negligible mass, in free fall, that does not affect the space-time geometry. In this document, test particles are also considered to be dimensionless.

3. Demonstration

3.1. *Principle and contexts of the demonstration*

The principle of the demonstration is based on a physics remark about a dimensionless particle P. For such a particle, two extremely different assumptions can be made about the nature of its energy:

- Assumption (1): Its energy is made of matter at rest. This means there is an inertial frame in which the content of this particle is a solid block of matter at rest.
- Assumption (2): Its energy is made of matter in motion. This means the internal matter of P is always in motion, regardless of the chosen inertial frame. Hence, the speed of this matter is the speed of light. The simplest model is to consider P made of a set of dimensionless bunches of matter in a Brownian distribution, each moving at the speed of light.

The first assumption appears natural and obvious. The second assumption might seem unusual because P is dimensionless and composed of a set of bunches of matter. However, this assumption is as legitimate as the first, where P is dimensionless and composed of a solid block of matter at rest. Therefore, one can state that the second assumption is always true. Hence, one can state that matter is always internally made of infinitely small bunches of matter always moving at the speed of light along infinitely small closed trajectories in a Brownian motion and distribution.

The bunches of matter in the second assumption are unrelated to the particles of particle physics, even though reality in this field has proven to be more compliant with this assumption.

Regardless of the assumption chosen, the result is always a dimensionless particle P. Thus, the generated spacetime structure will be the same. However, as will be seen later, the calculation of the spacetime structure will be entirely different.

From this remark, the principle is to calculate this deformation under the second assumption. Let S be the function giving spacetime structure from energy distribution. The first constraint for the domain of S is that it is assumed to be the set of energy distributions following the rule of energy conservation. Therefore, for each of them, the following is true:

$$\nabla_\mu T^{\mu 0} = 0 \tag{1}$$

$T^{\mu\nu}$ is the stress energy tensor. ∇_μ is the partial covariant derivative with respect to the μ dimension. This equation is simpler for energy distributions composed of a set of dimensionless particles: it represents the conservation of the number of particles and their energy at rest.

For ease of demonstration, a second constraint is added: the energy distributions are restricted to sets of dimensionless particles. Apart from these two constraints, there are no further constraints for the domain of the S function.

The context described in this paragraph applies only to the calculation of the spacetime deformation generated by P under the second assumption. This will be done from paragraph 3.4 to 3.7. Outside of these paragraphs, this context will be withdrawn. The new context will be indicated when needed.

3.2. *Mathematical reminder: interchange of limits theorems*

Norms are equivalent in finite dimensions, hence this is true for the four-dimensional spacetime of GR. By other means, spacetime structure is a function of energy distribution. In the present document, the considered energy distributions will be assumed to share the same bounded domain. This will allow the demonstration of the continuity of S. This restriction is allowed since the size of this bounded domain can be huge, for example, it can contain the observable universe.

Under the previous restriction, S is a continuous function using the uniform norm for the two involved spaces. Indeed, if any amount of energy at any spacetime location is decreased to 0, then the effect of this amount of energy on spacetime structure decreases also to 0. This continuity is the result of the conservation of energy principle. The detailed proof is given in Appendix A.

From this continuity of S, the interchange of limits theorem is valid. If any f_n sequence of energy distributions over spacetime tends uniformly to some limit energy distribution, then the following equation arises.

$$S\Big(\underset{n\to+\infty}{lim_u}(f_n)\Big) = \underset{n\to+\infty}{lim_u}\Big(S(f_n)\Big) \tag{2}$$

In equation (2), lim_u means the uniform limit, for energy distributions, and for spacetime structures.

The continuity of the determination of spacetime structure as a function of energy distribution has another interesting consequence. It is possible to imagine thought experiments in which energy is increased or decreased progressively, having a continuous effect on spacetime structure. And this can be done without violation of the conservation of energy principle. Indeed, GR is a mathematical framework which can, and sometimes must be watched independently of reality. Moreover, it is possible in this framework to imagine different universes.

3.3. *A generalized rest frame in relativity*

3.3.1. *Definition*

In relativity, rest frames exist (see glossary, paragraph 2). This concept can be generalized and is then referred to as a generalized rest frame (GRF). This generalization is achieved by evaluating the metric between rest frames. In vacuum, GRFs can be constructed by iteratively applying a process involving small displacements. This construction is detailed in Appendix B.

3.3.2. *Old and new GRFs*

Thus, for any particle located at a given event x, a GRF exists at x. Let us denote the "old GRF" as the GRF "just before the particle," which then transforms into the "new GRF," i.e., the one "just after the particle." Alternatively, the old GRF locally describes spacetime without the presence of the particle, while the new GRF describes it just after the particle's appearance. In other words, the identification of the new GRF from the old one can be carried out progressively, using the continuity of the function that relates the spacetime structure to the energy distribution. To illustrate this, consider the following scenario — not realistic, but valid as a thought experiment as previously stated: the energy of the particle is increased gradually from 0 to its actual value.

3.3.3. *Behavior*

The nature of these GRFs depends on the existing energy distribution. However, transitioning from one energy distribution to another by introducing a dimensionless particle can be analyzed. Suppose a dimensionless particle is inserted into the universe. The old GRF refers to the frame before this insertion, and the new GRF refers to the frame after it. The boost associated with the motion of this new matter in the old GRF describes the evolution from the old GRF to the new one and allows the calculation of the new GRF. More details and demonstrations are provided in Appendix B.

3.3.4. *Rescaling*

Additionally, a rescaling of time and space units occurs after the boost. This will be discussed in more detail later. However, in the present document, we will, somewhat abusively, refer to the local deformation simply as a boost. The context will determine whether a rescaling also occurs. Further discussion on this can be found in Appendix B and Appendix C.

3.3.5. *Conclusion*

This GRF concept and its behavior are local and involve only the first derivative of the metric. From this, the second derivative can be inferred, enabling a comparison with Einstein's equation.

3.3.6. *Examples*

Consider a simple example: a particle P moving along a straight line in a static universe filled with uniform matter density. In the absence of P, this universe is flat and Minkowskian. The old system of GRFs, denoted $R_{0\ system}$, corresponds to a constant R_0 frame in which the universe is at rest. This is traditionally referred to as the "frame of fixed stars." The new system, $R'_{0\ system}$, is the one that actually exists and results from the presence of P.

This scenario can be refined by assuming that P appears progressively in space and time. In this case, a continuous set of GRFs is constructed progressively from $R_{0\ system}$ to $R'_{0\ system}$.

In this example, even if P has a finite size, the result remains the same. The spacetime deformation caused by P — that is, by its very existence — is locally described by the boost associated with the velocity of P in its old GRF. This boost yields the new GRF from the old one.

For a GW, the same definition applies: the "old GRF" is the one that would exist if the GW and the matter generating it were absent. The "new GRF" accounts for their presence.

3.4. ***Fundamental assumption***

From now on in the present document, it will be assumed the following assumption.

Assumption (I): a GW propagates at the speed of light.

The relevance of this assumption (I) is well described in the literature [5]. This is for the experimental data. Also a theoretical argument is made by the usual study of a GW using the linearized Einstein equation.

3.5. ***Spacetime structure around a particle moving at the speed of light***

Now the aim is to describe the local spacetime deformation of a GW generated by a particle moving at the speed of light. The usual following assumption will be assumed. The P particle is moving at the speed of light along a D straight line in an empty universe previously structured by a flat Minkowskian spacetime.

The spacetime deformations of the GW will be studied at the locations where the GW deformations are the greatest. Being only local, the studied deformation is transforming the time and space axes into new ones. That is, the old GRF is transformed into the new one. Therefore, this local deformation is described by a linear transform.

Whatever the R inertial frame is chosen, this trajectory will always be a straight line and the speed of the particle will always be the speed of light. Moreover, the properties of physics are the same whatever the chosen inertial frame. Therefore, the spacetime structure generated by this particle will always be the same whatever the inertial frame is chosen. It means that in R at any given time the locations in which the deformation is the greatest is always the same space cone centered on D.

Figure (1) shows the GW in two dimensions. In $R(C; ct, x, y, z)$ the P particle moves in the direction of the x axis, and along the D line at the c speed. Let's focus on the GW which is generated by P from the E_C event which is a given C space location pertaining to D, and a given t_C time. This GW propagates from C at the c speed along the space directions of the (y, z) plane which is perpendicular to x and therefore to D, starting from C. Let's choose an E_N spacetime event composed of the N space point and the t_N time in R, such that CN is along the y axis perpendicular to D and E_N receives the GW propagation starting from E_C. During the same $t_N - t_C$ time interval, P moves from C to some C' space point pertaining to D. Also, when P is located in H, which is the middle of C and C', the GW starts another propagation from H in the direction of CN. This propagation is located in M at t_N, such that M is the middle of N and C'. The propagations along CN and HM

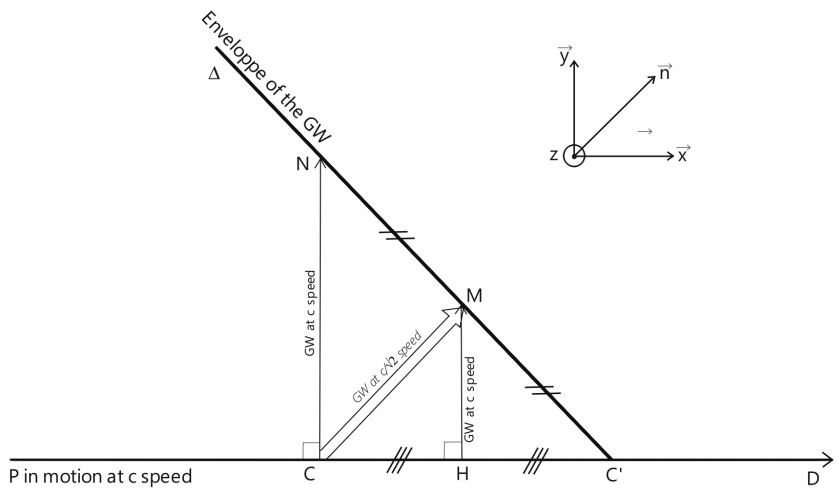

Fig. 1. The GW generated by a P particle moving at the speed of light is shown in two space dimensions. D is the trajectory of P. The spacetime deformation generated by P propagates perpendicularly to D from C to N at the c speed, and also from H to M at the c speed. H is the middle of CC' and M is the middle of NC'. Δ is the envelope of the GW. This envelope propagates at the $c/\sqrt{2}$ speed from C to M.

are done at the c speed. Therefore, this GW propagation is also along the CM line at the $c/\sqrt{2}$ speed. This is the speed of the envelope of the GW along its trajectory. The spatial vector of this propagation speed is normal to the envelope. The envelope is a cone centered on D. This cone is "isosceles". In other words, there is $CN = CC'$ and $HM = HC'$.

Now let's study the deformation generated by the GW. First of all, in C the local deformation which is generated by P is the b_C boost associated with the speed of light in the same direction as P. This is proven in Appendix E.

Along the D line, exactly the same boost propagates itself, at the speed of light. This is proven in Appendix F. Here, since the P trajectory is the same D line, of course, this can't be noticed. But it will be noticed as soon as P will deviate from this D trajectory.

Outside of D, the local GW deformation is still described by the b_C boost. Indeed, this is the initial deformation in C. As such it is the deformation which is propagated. Also, this is coherent with the propagation speed of the envelope. More details are available in Appendix I and Appendix J.

Let's study the space submanifold at some given time. It is assumed that a slicing of spacetime along time is possible. The spacetime deformation at any X space location depends on the position of X with respect to the cone of the maximum GW deformation. Outside of the cone (external part of the cone), the spacetime structure is unchanged for causality reasons. Indeed, here P has no possible causal link. In other words, it means that here the old and new GRFs are equal. Inside the cone, each event has been reached in the past by the GW generated by P. Indeed, nothing more has ever happened there (no more GW or any effect due to any other matter than P has ever happened there, since there is a vacuum instead of P in the universe). Therefore, here the local spacetime deformation is described by b_C.

3.6. *Spacetime structure around a particle moving along a circle at the speed of light*

3.6.1. *Context and aim*

The next step is to calculate the spacetime deformation generated by a particle at rest, under assumption (2). For modeling this particle, the best model would be a Brownian motion of a huge set of bunches of matter moving at the speed of light. But a simpler model will be used. It's a bunch of matter moving along a circle at the speed of light. The relevance of those models is discussed in Appendix L.

3.6.2. *Spacetime deformation generated by the particle moving along a circle*

Therefore, it is assumed that the previous P particle is forced to move along a circle. Of course, using the word "particle" here is an abuse of language. Indeed, this identifies a bunch of matter whose existence is only a hypothesis and this is assumed only for theoretical purposes. But this abuse allows for easier reading.

Then the previous cone transforms into a more complicated geometric figure. But infinitely far from the circle, therefore asymptotically, the envelope of the GW propagation at constant time is a sequence of spheres centered at O, the center of the circle. Asymptotically, these spheres inflate themselves around O at the speed of light. The radiuss of these spheres are separated by the same constant value d, which is the circle's circumference. Now the deformation that is propagated asymptotically is described by a boost associated with the speed of light. This speed is the vector that is normal to the propagation sphere, in the direction that goes out of the sphere. Also, this deformation propagates with this speed. This is proven in Appendix K.

3.6.3. *When the circle's radius tends to 0*

In the mathematical framework of relativity, if the radius of the circle decreases progressively and tends to zero, it means in reality that the circle tends to its limit, which is a dimensionless particle located in O. Then the envelope of the GW transforms itself into the previous sequence of spheres, but also d tends to O. The final result is a sequence of spheres which are infinitely close to each other, all centered on O.

The interchange of limits theorem can be applied to this Dirac distribution of matter, namely here, the dimensionless particle located in O. Let's denote P' as this particle. This can be applied because the circle-like trajectories of microscopic bunches of matter represent energy distributions that tend toward this Dirac distribution. If any doubt exists because P is in a rotating motion around O, then it is possible to add another bunch of matter sharing the same energy and moving along the same circle-like trajectory in the opposite direction. This would cancel the whole rotating motion without changing the final result, which is the following. Finally, this distribution tends to the Dirac distribution centered in O, when d tends to 0.

$$\underset{n \to +\infty}{lim_u} \big(S_n\big) = m\delta \tag{3}$$

S_n is this energy distribution made of one circle-like trajectory of one bunch of matter like P, in an empty universe, this circle being centered in O. m is the mass of P', which is assumed to be located in O, the center of the δ Dirac distribution. Equation (3) tells that S_n tends to the Dirac distribution of P' since the radius of the S_n circle tends to O.

3.6.4. *Spacetime deformation generated by a particle at rest*

Therefore, let's apply the interchange of limit theorem. The spacetime structure generated by a 3D Dirac distribution centered in O is the limit of the previously described sequence of spheres, limit when d tends to 0, as written in the following equation.

$$S\big(m\delta\big) = S\Bigg(\underset{n \to +\infty}{lim_u} \big(S_n\big)\Bigg) = \underset{n \to +\infty}{lim_u} S\big(S_n\big) \tag{4}$$

Those limit deformations are described in any M space point by the boost associated with the speed of light which is oriented along OM. Let's write R, a rest frame attached to P' (in which P' is at rest). M belongs to a sphere centered in O which contains all the deformations arriving at the same time in R. Moreover, these spheres are now infinitely close to each other. It means that each spacetime event of the universe is modified by this singular boost. This is the spacetime structure generated by the 3D Dirac distribution.

Hence the result is radically different from what is told by today's literature. Let's remind that with today's literature the spacetime deformation here is the one which corresponds exactly to Newton's law occurring in the solar system. But what has been proven here is that this deformation is a singularity everywhere.

This huge difference will explain why G is not a constant but a variable.

3.6.5. *Avoiding the unknown forces*

P was forced to follow the circle-like trajectory which is not a geodesic. Therefore, there is an unknown force which allows this trajectory not to follow a geodesic. This force is not gravity. Therefore, at first glance, modeling the Dirac distribution under assumption (2) would not be possible in a coherent way and implying only gravitation. But this apparent result is wrong; the final result is quite the opposite, as shown in Appendix L. Interestingly, the result here is a very important result. The result is that in relativity, assumption (1) is impossible and must be replaced.

3.7. *Partial resolution of the issue of Mach's principle*

This resulting spacetime structure might be argued to be wrong because it is not realistic. But the correct argument is the opposite. This description appears to be more accurate than the one given in the literature because the distribution was initially assumed to be unrealistic. Indeed, a Dirac distribution of matter is inherently unrealistic since it assumes an empty universe outside of the center of the Dirac distribution. For that reason, only an unrealistic result should be expected.

Moreover, this GR prediction is in perfect agreement with Mach's principle [6]. Let's briefly recall one aspect of the problem with Mach's principle in GR. For example, the issue arises in the case of a static spherically symmetric universe. A particle is located at the center of this spherical symmetry. If ρ is the matter density filling the universe, then one can distinguish two assumptions. The first is $\rho > 0$, and the second is $\rho = 0$. Close to the particle, ρ appears insignificant in both cases. Therefore, in this region, the spacetime deformations will be approximately the same for the two assumptions. However, in the first assumption, it is possible to find an inertial frame, R, at rest with the particle, which is not in rotation with respect to the universe. In R, there are no fictitious forces such as centrifugal forces. But in the second assumption, it is not possible to find such a frame. Assuming that R is at rest with respect to the object is not enough. It is not possible to determine if R remains inertial or not. One cannot say if fictitious forces will appear in R.

The new GR prediction solves this problem. Now, the spacetime structure becomes singular everywhere for the second case. An answer is provided: in each frame at rest with the particle, no fictitious force could ever appear, since those singularities would dissolve them completely. Therefore, with this new, correctly unrealistic prediction, GR becomes more Machian.

3.8. *Two particles*

Now let's add another particle, apart from the first one. So there are two particles, at different locations, and the universe outside them is still empty. Everything is still assumed to be static, meaning that the two particles are at rest in some given inertial frame.

There are still singularities, but they are located only along the straight line containing the particles' locations, and only at the points that are not between the particles' locations. They are described by the boost associated with the speed of light in the direction moving away from the particles.

Outside of the particles and those singularities, the spacetime structure is determined by the conservation of the GR Lagrangian in vacuum. It is simpler to say that the Ricci tensor is null in vacuum, as shown by the following equation.

$$R_\mu^\nu(g_{\mu\nu}) = 0 \tag{5}$$

$R_\mu^\nu()$ is the function giving the Ricci tensor from the metric, and $g_{\mu\nu}$ is the metric. This is a second-degree differential equation. An integration constant is still required at the end of the calculation. Under today's version of GR, the G solar system constant value and the mass of each particle are used for this. Now, the determination of this integration constant remains to be given. Hints and clues for this determination are provided in Appendix D. Although this determination would be an improvement, it is not mandatory for the study of the present document.

Also, asymptotically, the spacetime deformation is singular. Indeed, this asymptotic deformation is the same as the previous asymptotic deformation of the distribution with only one particle. And of course, each particle still generates locally a spacetime singularity. If the masses of the particles are not equal, a difficult calculation is required. Let's suppose that they are equal. Then, the calculation is simpler since there is perpendicular symmetry with respect to the mediating plane between the particles. This picture is still vastly different from what is written in today's literature.

3.9. *Three or more particles*

Adding a third particle to the scene will dissolve those singularities located along the straight line containing the particles. As usual, it is assumed that the third particle is at rest with respect to the other particles.

With three or more particles, the spacetime structure is still determined by equation (5). And under the assumption that the rest mass of the particles is equal, symmetry considerations might help to calculate the spacetime structure. For the determination of the integration constant, the same arguments apply as in paragraph 3.8.

3.10. *Spacetime structure for any distribution of energy*

The studied distribution of energy is a realistic one: an infinite set of dimensionless particles. Indeed, modeling reality in this way is often a good approximation in astrophysics, from planets to cosmology scales, and in particle physics, because of the sparse nature of matter. Here the wave nature of particles of particle's physics is simply ignored since only gravity is studied at first. Of course, a more general distribution of energy might be studied. Notably, a uniformly continuous distribution of energy could still allow the use of the interchange of limits theorem. But this is outside the scope of the present document.

Hence, let's apply the interchange of limits theorem to this set of Dirac distributions of matter, namely here, the dimensionless particles of the universe. Let's recall that for each P_i particle, where i is the particle's number, i from 0 to infinity, there is a S_n^i sequence of energy distributions. For each i and n, S_n^i is a distribution made of a circle-like trajectory of a virtual microscopic bunch of matter moving at the speed of light in an empty universe, this circle being centered on P_i. For each i, i from 0 to infinity, the circle's radius of S_n^i tends to 0, and S_n^i tends to the Dirac distribution of the P_i particle, as n tends to infinity.

$$\underset{n\to+\infty}{lim_u}\left(S_n^i\right) = m_i\delta_i \tag{6}$$

In equation (6), δ_i is the Dirac space distribution centered on the P_i particle, and m_i is the mass of P_i. Although lim_u represents uniform convergence, from the perspective of the set of P_i, this is simple convergence. This simple convergence for each P_i can be transformed into uniform convergence for the entire set of P_i. To achieve this, it suffices to choose, for each n, the same S_n^i circle's radius for any i. For example, the value of $1/(n+2)$ can be chosen for the circle's radius of S_{n+1}^i, in the system of GRFs of spacetime structure generated by S_n^i.

$$S\left(\Sigma_{i=1}^N m_i\delta_i\right) = S\Big(\Sigma_{i=1}^N \underset{n\to+\infty}{lim_u}\left(S_n^i\right)\Big) = \underset{n\to+\infty}{lim_u} S\Big(\left(\Sigma_{i=1}^N (S_n^i)\right)\Big) \tag{7}$$

Equation (6) has been used, as well as the interchange of limits theorem. N is the number of particles in the universe. Of course an infinite number of such particles is not required in the study. The finite N number of the visible particles of the universe is enough. Hence we are left with the spacetime structure given by the final term $\underset{n\to+\infty}{lim_u} S\Big(\left(\Sigma_{i=1}^N (S_n^i)\right)\Big)$. Each S_n^i amount of matter is moving at the speed of light. Asymptotically it has been shown that each of them propagates a GW which is described locally by the boost associated with the motion of the enveloppe of this GW, which propagates at the speed of light.

Furthermore, for each particle, asymptotically the gravitational effect of the particle is infinitely greater that the effects of the other particles. Therefore asympotically close to the particle, the arising context is exactly the same as studied in paragraph 3.6.4. Consequently, each of the previous result applies: the shape of the GW enveloppe is a sphere and its ray increases at the speed of light. And the GW spacetime deformation is described by the boost associated with the speed of light of the ray of this enveloppe. Asymptotically when the ray of the circle of the trajectory of the bunch of matter tends to zero, and also asymptotically close to the resulting particle, the whole spacetime receives the GW. In other words, in the infinitely small solid sphere centered in the location of the particle, every space location receives the GW, at any time.

Now far from the particle, this description remains valid. Indeed, the previous demonstration still holds if the old spacetime structure (the spacetime structure before the particle) is not assumed to be flat Minkowskian. The reasoning was working because of the vacuum around the particle. Whatever is the nature of old GRFs of the partticle, it is possible to consider the system of frames in which the GW propagates along geodesics which are straight lines containing the location of the particle. By the way the old GRFs is such a system of frames. Even if the Riemann tensor does not describe a Minkowskian flat spacetime, those straight lines implies that the enveloppe is a sphere in this system of frames, and then the previous reasoning remains correct. Hence all the previous results hold, asymptotically, the GW propagates at the speed of light, is described by the boost associated with this speed, and each event of space-time receives the GW.

Hence asymptotically in each x event of spacetime which is located outside of the particles, the GW of those particles encounter themselves. This is been proven in Appendix N. This encounter results in a local spacetime deformation which is described by the sum of the boosts of the GW of the particles.

$$S\big(\Sigma_{i=1}^{N} m_i \delta_i\big) = \{B\Big(\Sigma_{i=1}^{N} 1_w(x, y_i) f(x, y_i) C^{\mu}(y_i)\Big),\ x\ in\ U\} \tag{8}$$

Equation (8) describes exactly that. B is a function giving the boost which is associated to a four-momentum, from this four-momentum. y_i is the location of the P_i particle. $C^{\mu}(y_i)$ is the radical boost describing the spacetime local deformation of the GW generated by the virtual particle modeling the P_i particle. x is the generic event describing the set in the rhs of this equation (8). The $1_w(x, y_i)$ and $f(x, y_i)$ terms are related to the propagation of this deformation and will be given further during the explanation of the next equation, which is similar.

Therefore the set of those boosts over spacetime identifies the new spacetime structure resulting by the existence of the P_i particles.

The final picture is as follows. The spacetime structure is the result of all the microscopic GWs. These microscopic GWs are generated by the virtual microscopic bunches of matter moving at the speed of light and representing the real P_i particles in the above study. From now on in this document, these GWs generated by the microscopic virtual bunches of matter will be called "virtual gravitational waves" (VGW). Considering only the limit distributions ($n \to +\infty$), the set of S_n^i transforms into the set of $m_i \delta_i$. Thus, each spacetime event receives a VGW from each real particle in the universe. This provides a clue for calculating the spacetime structure in a different way.

3.11. *Calculating spacetime structure with virtual gravitational waves*

3.11.1. *Four-momentum equation*

The same distribution of energy is still assumed. In any x spacetime event, the four-momentums of all the GWs propagating in x add themselves. The fundamental reason for this is conservation of energy principle. This is shown in Appendix N.

This is true for the VGWs as shown above, but let's study how any kind of GWs combine themselves when they encounter in x. This will be studied, generally, for any kind of GWs, and then applied for the VGWs.

The resulting equation has been described in [2] and is the following.

$$D^\mu(x) = \Sigma_{n=0}^{\infty} 1_w(x, y_n) f(x, y_n) C^\mu(y_n) \tag{9}$$

Equation (9) shows the calculation of the resulting four-momentum in x. For n from 0 to infinity, each y_n event represents a spacetime location in which the P_n particle is possibly propagating a GW in x. The $1_w(x, y_n)$ is equal to 1 if x and y_n events are connected by a null geodesic and if x is located after y_n along this geodesic. It means that the GW generated in y_n is received in x. Considering only the limit distributions of equation (6), $1_w(x, y_n)$ is always equal to 1 and can be supressed from the equations. This will proven further. $f(x, y_n)$ is a scalar positive function. It is assumed to be equal to 1 if y_n is equal to x. This allows to retrieve the rule of local spacetime deformation generated by a particle. It expresses the attenuation of the GW energy which is emitted from y_n. $C^\mu(y_n)$ is a four-vector which contains the information of the energy of the GW in y_n. Later on, it will be shown that $C^0(y_n)$ is not the effective energy of the GW, but is proportional to its square root. Nevertheless, in order to avoid a heavy reading, the words "four-momentum" and "energy" will be used respectively for $C^\mu(y_n)$ and $C^0(y_n)$. The context will allow to understand if those are effective energy or contributions to equation (9). And this "contribution" word will mean the $1_w(x, y_n) f(x, y_n) C^\mu(y_n)$ terms which are in the sum of the rhs of equation (9).

Equation (9) can be used for calculating the four-momentum describing the local spacetime deformation generated by any of the following objects.

- A single particle located in x,
- a GW propagating in x,
- many GWs encountering in x.

A remark about equation (9) is the question of wether an infinite number can result from this equation or not. For example if an infinite universe is filled with a constant and uniform distribution of particles, then the result is infinite if the f attenuation function decreases less than $1/r^3$. This problem is similar to the Olbers's paradox problem [7]. But a more practical solution can be found here. There is no need to understand the universe expansion and horizon. When translating this equation (9) into a gravitational model such as surrounding [1], a solution is found. Indeed, in surrounding, the fitting of the model with experimental data forces this sum of equation (9) to be translated into a finite value. A practical approach here is simply to ignore the possibility of divergence of this equation, and to fix this issue later on, when working on gravitational models.

Other remarks can be made regarding the structure of this equation. Although it resembles an energy conservation law, it also acts as a barycentric equation, relating the final four-velocity vector of spacetime at x to the GW four-velocity vectors. The weights in this barycentric operation are given by the total energy of each contribution. This resolves the initial coherence problem of spacetime deformation: a P bunch of matter deforms spacetime globally according to its motion, but if it is composed of A and B components moving differently, then the spacetime deformations due to A and B differ from the one generated by P. The associative property of the barycentric operator solves this inconsistency. This also addresses the question raised by the existence of Willer rotations $[8, 9]$ in the composition of boosts. Furthermore, the equation can be viewed as a Green's function integral representation involving four-vectors. However, the underlying mechanism is non-linear. In addition, the attenuation function f is unknown — yet this apparent obstacle can be overcome. Indeed, in the most common realistic distributions of energy, this function is approximately the same for each y_n event, allowing for both reasoning and simple calculations.

3.11.2. *From the four-momentum to the boost*

Let's follow the natural calculations. The distribution of energy of paragraph 3.10 is still assumed. Let's write the resulting four-momentum of equation (9).

$$D^\mu(x) = \gamma \frac{E}{c} \left(1, \frac{v}{c}, 0, 0\right) \tag{10}$$

E and v are respectively the energy at rest of the four-momentum and its speed in a frame. It has been used $\gamma = 1/\sqrt{1 - v^2/c^2}$. This equation (10) has been written in a $R_0(O; ct, x, y, z)$ frame, at rest with the universe, which is an old GRF with respect to the GWs of equation (10). R_0 is such that v is along the Ox line. It is possible to find such a frame. Then, from $D^\mu(x)$ is calculated the local spacetime deformation which is generated. This is done [2] by using the boost described in R_0 by the following equation.

$$B^\mu_\nu(x) = \gamma \begin{pmatrix} 1 & -\frac{v}{c} & 0 & 0 \\ -\frac{v}{c} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{11}$$

This boost is directly deduced from the four-momentum of equation (10).

3.11.3. *From the boost to the metric*

Now it is possible to derive the spacetime metric from $B^\mu_\nu(x)$.

The distribution of energy of paragraph 3.10 is still assumed, but now the universe is filled with a constant matter density. It means, notably, that the grid of particles has cells which are small enough for allowing such an approximation.

Let's write R_0, a frame at rest with the universe. Since the universe is filled with a constant matter density, spacetime structure is flat Minkowskian everywhere and R_0 represents a whole system of frames such that R_0 is the GRF everywhere. Now let's assume that a P particle is added to the scene, located in O at $t = 0$, at rest in R_0. Let's write x the event composed by a given M space point, $M \neq O$ at $t = 0$ in R_0. R_0 can be assumed to be the

GRF in x, before adding P to the scene. Let's call R'_0 the GRF after adding P to the scene. Therefore, R_0 is the "old GRF", and R'_0 is the "new GRF", with respect to the existence of P and in the M space location. Let's write x' the first event in M when R_0 has been transformed into R'_0, along the time of R_0. R'_0 is obtained by transforming R_0, in x, using the $B^\mu_\nu(x)$ boost.

From R_0 to R'_0 it can also be generated a successive continuous sequence of GRFs, starting with R_0 and ending with R'_0. For that it suffices to add slowly the P particle energy from 0 to its real value. (Let's remind that avoiding the conservation of energy principle is allowed in a thought experiment).

Then, it is required to rescale the lengths of the "boosted" time and space axis. The boosted time and space axis are the time and space axis which have been modified by the boost, in their states after the boost. The rescaling is done in such a way that the resulting time line described successively by those successive infinitesimal steps is a geodesic. This is detailed by the following equations, relating X'^ν the coordinates after the boost, to X^μ the coordinates in R_0, and then relating $X"^\rho$ the final rescaled coordinates in R'_0 to X'^ν.

$$X'^\nu(x') = B^\nu_\mu(x)X^\mu(x) \tag{12}$$

$$X"^\rho(x') = S^\rho_\nu(x')X'^\nu(x) \tag{13}$$

$$g_{\alpha\beta}(x) = B^\rho_\alpha(x)S^\mu_\rho(x')g_{\mu\nu}(x')S^\nu_\kappa(x')B^\kappa_\beta(x) \tag{14}$$

$S^\mu_\rho(x')$ is a symetric linear map which has the ability of being diagonalized in R'_0. Its value is determined by the constraint above (the time line of the set of successive GRFs must be a geodesic). Equations (12) (13) and (14) show how $g_{\mu\nu}(x')$ the new metric is deduced from $g_{\alpha\beta}(x)$ the old one, due to the action of $B^\nu_\mu(x)$, which results from the $D^\mu(x)$ added energy of P. Of course equation (14) can be inverted, using the inverse mixed tensors $(B^{-1})^\beta_\kappa(x)$ and $(S^{-1})^\rho_\mu(x')$ of, respectively, $B^\nu_\mu(x)$ and $S^\mu_\rho(x')$. It results the following equation.

$$g_{\mu\nu}(x') = (S^{-1})^\rho_\mu(x')(B^{-1})^\alpha_\rho(x)g_{\alpha\beta}(x)(B^{-1})^\beta_\kappa(x)(S^{-1})^\kappa_\nu(x') \tag{15}$$

Equations (12) (13) and (14) have been obtained by studying the spherically symmetric static case. In the Schwarzschild metric, a P_1 test particle (a "test particle" is defined in the glossary, paragraph 2), being at rest when located infinitely far from the center of the symmetry, follows a time line which is transformed by those equations [10]. Those equations are still valid in a more general case. This is proven in Appendix F.3. However in the scope of the present document, only the particular spherically symmetric static case is required.

It is already known that $g_{\mu\nu}(x)$ is a diagonal matrix in the R_0 frame and that $g_{\mu\nu}(x')$ is a diagonal matrix in the R'_0 frame. Using equation (10), since the direction of the boost is along the x space axis, only time and x space dimensions are modified by the metric evolution. If the $(S^{-1})^{\rho}_{\nu}(x')$ rescaling is written with an α time rescaling and with a β space rescaling, then, using equation (15) and $\alpha\beta = 1$ usual convention, the resulting metric shows $g_{00}(x') = \alpha^2$ and $g_{11}(x') = -\alpha^{-2}$. This allows to check and understand the involved mechanism of equations (12) (13) and (14).

As a conclusion, spacetime structure is calculated first by calculating a one and only four-momentum in x which contains the information about the local deformation in x generated by P. From it the final local spacetime deformation is determined. This determination is deducing the speed associated with this four-momentum, and then the boost associated with this speed. This boost describes the final spacetime deformation occuring locally in x. Taking equation (9) into acount, and the VGWs of paragraph 3.10, the final local spacetime deformation in x is described by the boost which is associated with the four-velocity which is the barycentric operation of all the four-velocity of the VGWs propagating in x. This barycentric operation uses the total energy of each VGW as its weight. The attenuation of the propagation of the VGWs follows the rule of Ricci tensor being null in vacuum. From this boost is calculated the metric. This is done in each spacetime event. Therefore, the global spacetime structure is calculated. The question of the stability of this self-induced mechanism arises. But if the universe is static, with this mechanism, spacetime structure converges into a stable structure. Indeed if the universe is static, then a thought experiment can be done in which the energy distribution is constructed by adding progressively the particles one after the other. Also each of them can be added progressively, their energy at rest being increased progressively. Each of those successive energy distributions are static. As usual in GR, spacetime structure for each of them can be described with the system of Riemann normal coordinates [11], in which spacetime appears virtually flat Minkowskian. In this system, in any x spacetime event, the sum of the momentum of the VGWs occuring in x is null. This is valid also for the limit of these energy distributions and the limit of those system of frames.

3.11.4. *An equation of G*

The context and notations of paragraph 3.11.3 are used. The $R_0(O; ct, x, y, z)$ frame is still used, the P particle is located in O, the x event is any possible spacetime event. For trying the construction of an equation of G, now let's assume, also, the following.

Assumption (i): Newton's law is valid. But G may differ from its solar system value.

This is based on experimental data. Newtons' law must be supposed to be valid almost in solar system because this law is validated with high accuracy, at least in solar system [14]. And there are theoretical arguments for this law to stay valid out of solar system, though being used with a different value of G. This will be studied in paragraph 5.

Assumption (ii): the energy of a GW propagating spherically evolves always following the same attenuation function (function of the initial starting energy, and of the propagation distance), regardless of its location and starting energy.

Assumption (iii): in equation (9) the sum of the energy of the contributions generated by P is far weaker than the sum of the energy of the other contributions.

This assumption will be confirmed with surrounding, where a sphere with a 15 kpc

radius, which is used for calculating the surrounding value, is fitted to experimental data. The energy which is located in this sphere corresponds to the time component of the rhs of equation (9), that is, the sum of the energy of the contributions of this equation.

Assumption (iiii): the contributions of equation (9) can be replaced by their asymptotic values without modifying consistently the result.

This assumption can be valid if the particles of the universe are isolated enough from each other. This might be realistic because matter is known to be extremely sparse in the universe, whatever the scale is, from particle physics scale to cosmological scale. Assumptions (i) (ii) (iii) and (iiii) are easier to accept asymptotically.

The calculations are presented in Appendix O. They are based on assumptions (i) (ii) (iii) and (iiii), equations (9), equation (10), and the geodesic equation for Newton's law which is reminded in Appendix O. They result in the following equation of G.

$$G \simeq \frac{c^4}{2\left(\Sigma_{n=0}^{\infty}\sqrt{\frac{E(y_n)}{\|x-y_n\|_3}}\right)^2} \tag{16}$$

In equation (16); $E(y_n) = C^0(y_n)c$ is the total energy of the particle located in y_n. This equation is valid only for VGWs. In Appendix O it is shown that this equation is a good approximation under (i) (ii) (iii) and (iiii) assumptions.

The possible divergence of equation (9) might appear worse in equation (16) than in equation (9). Indeed, the $1/\sqrt(r)$ evolution of the contributions of equation (16) shows potentially a quick divergence of the result of this equation. Nevertheless it must be noticed that the resulting gravitational force will obey to the $1/r^2$ rule. Indeed, equation (16) has been formed from that rule. Therefore the final possible divergence is the Olbers's paradox divergence. And it is easy to modify equation (16), inserting a cut-off value of the contributions, for example resulting in the following equations.

$$\begin{aligned} \Phi_{cut}\big(a, b\big) &= \\ b \leqslant R_{cut} :& \quad \sqrt{\frac{a}{b}} \\ b > R_{cut} :& \quad 0 \end{aligned} \tag{17}$$

$$G \simeq \frac{c^4}{2\left(\Sigma_{n=0}^{\infty}\Phi_{cut}\Big(E(y_n), \|x-y_n\|_3\Big)\right)^2} \tag{18}$$

Here R_{cut} is the maximum GW propagation distance. The 15 kpc value is suggested by surrounding [1].

This cut-off value does not alter much the qualification of "asymptotic" in the previous reasoning and in the calculations of Appendix O. Indeed in the surrounding gravitational model, the $R_{cut} = 15\ kpc$ value is fitted to experimental data. It is a distance which would require an attracting object like a galaxy in order to contradict this "assymptotic" qualifier.

And this equation (18) can still be improved. For example it is possible to replace the simple 0 cut-off value by a slow decrease of the contributions of equation (16).

4. Predicted surrounding effect

The above study shows that relativity predicts a variable G. And this variation given by equation (16) follows the rule untitled "surrounding" in [1]. Of course equation (16) has been constructed under the (i), (ii), (iii) and (iiii) assumptions.

But equation (9) shows already this surrounding effect, without any added assumption. Let's show this by rewriting it, shifting the total energy from left to right, and isolating the resulting speed.

$$\frac{v}{c} = \frac{f(x, y_0)C^0(y_0)}{\Sigma_{n=0}^{\infty} f(x, y_n)C^0(y_n)} \tag{19}$$

The context and notations of paragraph 3.11.4 are assumed. The universe is still assumed to be at rest and filled with a constant matter density, except for only one P particle at rest with the universe and spatially located in O. y_0 is an event spatially located in O, and $C^\mu(y_0)$ is the four-momentum of the unique VGW generated by P. The $1_w(x, y_0)$ and $1_w(x, y_n)$ terms disappeared because now only VGWs are considered. Indeed, there is always a unique VGW which is propagated by each particle located in y_n and which is received by any given x event. Equation (19) derives directly from equation (9), and shows that the space velocity of the resulting four-velocity is inversely proportional to the denominator, which increases with the energy surrounding the x location.

It can be noticed also that the denominator of the rhs of this equation is a sum of positive scalars calculated in an isotropic manner. It induces naturally to translate this equation (19) into a gravitational model, replacing this value by the energy of the surroundings of the location where the gravitational force is exerted. The result is that the surrounding gravitational model or a gravitational model close to it is predicted by relativity. Therefore the so-called gravitational anomalies of today might be no anomalies at all, but regular predictions of relativity.

The modified gravity theories of today must comply with the MOND [15] model predictions, for the greatest part of experimental data. This has been proven by decades of work in astrophysics. This compliance is naturally acomplished by the surrounding model. Indeed, when acceleration is low, MOND increases it. But when acceleration is low, most of the time it means that the surrounding energy and matter are low, also, and then the surrounding effect increases acceleration too.

5. Revisiting Newton's law

A revisiting of Einstein equation is naturally required by the previous study. This equation is nothing more than the most direct translation of Newton's law from non relativistic physics into relativity. Therefore, the first step is to revisit Newton's law. The Poisson's formulation of Newton's law starts the construction of Einstein equation, not only because it's about Newton's law.

First of all, in vacuum, this formulation expresses the following important non relativistic principle.

(i) The flow of the acceleration vector field is constant in vacuum.

It results $div(a) = 0$ in vacuum, where a is the acceleration vector field. This is not something new. But now it has been shown that spacetime structure in vacuum can be calculated by considering only the VGWs generated by theoretical particles moving at the speed of light. This gives a new insight about this (i) principle: it corresponds to the conservation of flow of GW energy in vacuum. Hence another argument for the Poisson's formulation of Newton's law is given.

Secondly, matter plays the role of a source for this field: matter is a source of any interaction force. And this is not only true for gravitation. Indeed, the force which attracts a given A particle to another given B particle is acting on A in the direction of B. This direction is tangent to the geodesic relying A to B. At the contrary, vacuum does not generate any force. Therefore the F force vector field has a divergence which is a function of ρ, matter density. Let's write Φ this function. It results the following equation.

$$div(F) = \Phi(\rho) \tag{20}$$

Then, let's write the fundamental principle of dynamics.

$$F = ma \tag{21}$$

F is the force generated on A by ρ. m is the mass of A located in a M space location, and a is the acceleration generated in M by ρ.

From equations (20) and (21) the following equation arises.

$$mdiv(a) = \Phi(\rho) \tag{22}$$

Now the reasoning becomes only valid for gravitation. The weak equivalence principle (WEP) states that a is independent of m for a fixed ρ. It results the following equation.

$$div(a) = f(\rho) \tag{23}$$

f is of course a function which is deduced from Φ. Newton's law in its Poisson's formulation is almost retrieved by the previous theoretical considerations. The divergence of the acceleration vector field should be a function of energy, such that for a null energy there is a null divergence. This gives also the following equation.

$$f(0) = 0 \tag{24}$$

In equations (23) and (24), nothing is told about neither the sign of $f(\rho)$ nor the exact feature of the f function. And the question of a possible proportionality of $f(\rho)$ with ρ is related with conservation of energy and the principle of action and reaction. Let's show this. Equation (23) implies for a P particle in an empty universe the following equation.

$$a = \frac{f(\rho)V}{4\pi x^2} \tag{25}$$

In equation (25), a is the acceleration in any given M location, such as $MO = x$, O being the location of a P particle generating this acceleration. ρ and V are respectively the matter density and the volume of P. It was supposed that ρ is constant in P. Then applying the fundamental principle of dynamics, the following equation arises.

$$F = \frac{m' f(\rho)V}{4\pi x^2} \tag{26}$$

Here, F is the force attracting a P' particle located in M, by P. m' is the P' mass. But the principle of action and reaction implies that this equation is invariant by P and P' permutation. The following equation arises, where m is the mass of P.

$$m' f(\rho)V = m f(\rho')V' \tag{27}$$

This being true for V and V' being constant and for any value of m, m', ρ, and ρ', it implies that $f(\rho)V$ is proportional to m . A better demonstration of that would consider the total energy E of P and P', in place of the principle of action and reaction. This energy is the integral of the forces along space distances. Then it is the invariance of E by the P and P' permutation which would be used. This proportionality would be written the following way.

$$f(\rho) = K\rho \tag{28}$$

Here K is of course the unknown coefficient of this proportionality. Using equation (28) in equation (26), it yields the following equation.

$$F = \frac{Kmm'}{4\pi x^2} \tag{29}$$

But now no theoretical argument can be given here for calculating the $K/(4\pi)$ constant of equation (29). Historically Newton's law has been constructed based on experimental data more than theoretical considerations. To say the least, the G determination was done completely based on experimental data.

But an indirect theoretical argument can be given. Everything was done here under the assumption that a complete vacuum exists outside of P and P'. If the vacuum is not perfect outside of the particles, the reasoning above becomes wrong. The energy surrounding the particles must be taken into account. First of all, matter density of the universe outside of P and P' generates also a divergence by applying equation (23). This added divergence modifies the final result given by equation (26). Secondly, the principle of action and reaction might not be true in its simplest formulation. It is easier to understand that the energy version of the demonstration is wrong. Indeed, rigorously speaking, the total energy of P

and P' must be replaced by the total energy of the universe. Indeed, energy exchanges might exist between the particles and their environment. A more practical version would be to approximate the energy of the universe to P and P' energies plus the energy of the surroundings of the particles, to some given extent suitable for a correct approximation.

The whole result of those theoretical arguments is Newton's law. But these arguments tend to prefer a variable G more than a constant G value, this variation depending of the energy surrounding P and P' particles.

6. Revisiting Einstein equation

In the more general relativistic regime the same reasoning might be done, replacing the divergence of equation (23) by Einstein tensor, and matter density by stress-energy tensor. Then the reasoning might give Einstein equation. But this work is above the scope of the present document. Nevertheless, since Einstein equation is the most direct formulation of Newton's law in the context of relativity, the reasoning above done in the non relativistic regime applies indirectly to Einstein equation.

To say the least, what appears still seriously doubtful is the statement that $K/(4\pi)$ is a universal constant, in equation (29). At the contrary, the above discussion shows that one would expect matter density of the universe to play a role in the determination of this constant. A more practical formulation of that would be that the energy of the surroundings of P and P' would play a role in this determination. This was true for Newton's law, and is therefore true for Einstein equation, since it is the most direct translation of Newton's law into relativity.

The $\rho = 0$ particular case implies $div(a) = 0$, from equations (23) and (24), but results also directly from the (i) principle. And the present study shows absolutely no need to modify its relativistic formulation given by equation (5). At the contrary, this equation allows to complete the new construction of spacetime structure done in the present document. This equation was used notably, above, in the study about two particles in an empty universe.

By other means, the construction of Einstein equation from Newton's law is extremely simple. It is the simplest way to proceed. Inserting a multiplicative tensor between the stress-energy tensor and Einstein tensor is something natural which were rarely done in the literature. This mutiplicative tensor can be only a function of energy: what else? Now the present document shows that this is exactly the correct translation of Newton's law into relativity. The result would be surrounding, or a gravitational model close to it. A classical way to proceed would be to calculate everything with such a X^{μ}_{ν} hypothetic multiplicative tensor and then compare the predictions to experimental data. Very probably, it would show that X^{μ}_{ν} must be proportional to the surrounding energy of the location where the force is exerted. Indeed, the surrounding gravitational model indicates strongly that this is exactly what would happen.

Also, today during the construction of the GR Lagrangian for matter, the G anthropocentric solar system constant value is forced without any theoretical argument. This is doubtful. At the contrary, the GR Lagrangian for vacuum is the simple and well sounded scalar curvature. This is another argument for applying it in the present study (it is equivalent to equation (5)).

7. Conclusion about G

Apart from the previous demonstrations, it is coherent to adopt a physics view point and add physics arguments in favour of a variable G following the rule of the surrouding effect. They will show that the previous demonstrations do not come from nowhere, but are motivated by strong arguments in physics. They are the following.

1) Mach's principle.
2) Correct theoretical construction of Newton's law.
3) Sophisticating the construction of Einstein equation.
4) Loss of information in the construction of the stress-energy tensor.
5) Implicit assumption of GR.

Items 2) and 4) were listed as motivations in the introduction. Item 1) is part of the motivation 4) of the introduction. Those items were transformed into arguments by the present study. Items 1) 2) and 3) have been described above. Item 4) was used implicitly in the present study. Indeed, there were no such loss of information in the descriptions of the energy distributions of the demonstrations of the present document. This item, as well as item 5), is described in [2]. Item 5) is related to the remark that the quarks are moving at a speed which is close to the speed of light.

Therefore, forgetting one instant the previous demonstrations, from a physics point of view alone the following statement is valid. Outside of solar system, it is much more relevant to use equation (16), or its translation with surrounding, than its solar system value for the determination of G. To say the least, a variable G following the rule of the surrounding effect is much more relevant.

8. Yang-Mills Millennium problem

8.1. *General statement*

The remark done in [2] about the Yang-Mills Millennium problem is still valid. Moreover, this remark is conspicuously reinforced by the study of the present document. Let's remind briefly this remark. It starts by assuming the following.

Assumption (A): unification of the four forces is driven by gravitation.

It is not indicated how this unification takes place. But it can be imagined that each particle of particle physics is constructed by some, internal, amount of energy in motion, the different trajectories of those amount of energy giving different behavior of the three other forces.

Under this unifying assumption, the Yang-Mills Millennium problem finds a solution. Indeed this assumption awake the full relativity in the context of particle physics: now not only SR, but also GR underlines all the forces. Therefore each of four forces is driven by the surrounding effect. And this effect modifies enormously the interactions between the particles.

8.2. *Modified predictions and observations*

The first kind of particle physics observations are particles interactions. The present study modifies the physics predictions in the case of triple nuclear collisions [16]. Indeed those collisions would be predicted to behave in a completely different manner, because of the surrounding effect. But they are almost impossible to realize. The GR modification of the present study predicts a different behaviour when the targetting particle is unchanged but when the target of the interaction is modified. When the target energy decreases, the surrounding effect increases and the scattering is expected to be wider. For example, the Ay puzzle [17, 18] might be enlightened.

The second kind of particle physics observations is cosmic rays. But here the present study will not modify the observed predictions.

The third kind of particle physics observations is static configurations of particles. Let's focus on the most stable group of particles, the atom. The electromagnetic force is now predicted to be much weaker than what is calculated by the simple $1/r^2$ rule, when the electron is part of an atom, than when it is alone. For the nucleus, its most simple form is the hadron. Let's discard hadrons made of two quarks because of their life time. The confinement of quarks is noticeable in hadrons of three quarks. This is very much explained by the present study. The remaining question is why an electron can exist alone, while a quark cannot. The present study gives a simple argument here. An answer is to be found in the difference of magnitude between the electromagnetic force and the strong force, when they are exerted in their confinement state, that is, when such groups are particles are formed. When a hadron is formed, then the strong force is far stronger than the electromagnetic force between an electron and a proton inside of an atom. But when an electron or a quark is left alone, then the surrounding effect increases the interaction force with respect to its value when the particle were not alone. Therefore, for an electron it is increasing a relatively weak force. But for a quark, it means increasing a force which was already strong. Therefore it can be understood why an electron can exist alone, while a quark cannot. It must be added that in this description the strong force can't be assumed to be a very short range force. In paragraph 8.3 it will be shown that this assumption might be wrong.

For the strong force a modification of the present study involves three body interactions. Any group of three particles closed to each other would experience low values of the strong force between them, because the surrounding effect would be strong, due to their close proximity to one another. But any group of two particles, or any group of three particles having one of them far enough from the two others, would experience stronger values of the force, because the surrounding effect would be weak. This can explain why hadrons with two quarks have a very short life-time, while a hadron with three quarks is stable. And it explains why an isolated quark is unobserved.

Therefore this allows quark confinement for long duration only when they are close to each other by groups of three. It results a solution of the Millennium problem [3, 4].

8.3. *Other observations in static configurations*

Nuclear saturation [19,20] is the observation that the volume of a baryon does not vary with the number of baryons inside of the same nucleus. In today's literature this mechanism is understood as suggesting a very short range for the strong force. But there are the two other possible explanations for it, which follow.

1) The strength of the strong force repulsion allows this volume to stay constant when N varies,
2) The strong force is driven by a surrounding effect.

The first explanation numbered 1) would mean that the repulsion of the strong force would be far greater than its corresponding attraction, resulting probably in strange predictions. Moreover as it will be seen in paragraph 8.5 under assumption (A) each force attenuates following the $1/r^2$ rule. Hence the repulsive strong force follows this rule and the first explanation is wrong.

The present document gives arguments in favor of the second explanation. Indeed the surrounding effect explains this mechanism in the same usual simple manner. The strong force potential, exerted on one baryon, by the other baryons of the nucleus, is substracted by its exact value, resulting in a constant potential. This is the same usual suppressing effect predicted by surrounding [1], which happens for the cosmological version of Einstein equation. It happens also for the bullet cluster. The calculation is presented in Appendix P.

This explains why the volume of the nucleus is proportional to the number of baryons: the strong force potential of the other baryons stays the same whatever is the number of baryons.

8.4. *Toward a strong force model*

In particle physics the same issue arises as in gravitation, which were leading to the construction of the surrounding gravitational model. Indeed, the equations of G are not practical here also. Moreover, the exact way in which the three forces of particle physics are constructed from gravitation is unknown. Therefore it is required to model those three other forces. The most important work is about the strong force, since it is the one which appears the most promizing one.

For this the same modeling would be required as the one which were used in the construction of surrounding. This construction were using an homographic function using different values of matter density. Such a recipe might be used in particle physics too. Then fitting the four parameters of this homographic function would be required. It must be done by comparing the predicted results with experimental data. It remains to be done.

8.5. *Relevance of the fundamental assumption*

Let's try to discuss the validity of assumption (A). It is more than an assumption. Indeed, the following argument can be done.

Argument (*): acceleration generated by gravitation is explained by spacetime curvature. It is a simple and elegant rule. It is tempting to apply it to each force.

The only remark which would forbid it is that the ratio σ/m is not constant but depends of the type of particle, where σ is the charge of the particle with respect to the considered force and m is its mass. But this remark can be discussed under assumption (A). Indeed, under this assumption this dependence of the WEP with σ can be explained by the possible mechanism which were described previously: the particular behavior of the gravitational force depends of the particular internal structure of each particle.

Another argument is the following. Under assumption (A), the WEP is applied for the four forces. Nevertheless, it can be assumed to stay valid for any fixed type of particle. Then the reasoning of paragraph 5 remains valid for any fixed type of particle. For example, in electrostatic, if σ and m are respectivly the charge and the mass of the electron, then σ/m is known for the electron, and the principle can be extended. Therefore the reasoning of paragraph 5 remains valid for this fixed type of particle. Now equation (29) is yielded for all the forces, having of course a different K value depending of the considered force. The result is that under assumption (A), each force follows the rule of the $1/r^2$ attenuation.

The argument (*) comes from relativity and gravitation. It might appear difficult to find such a convincing hint starting from particle physics. For example it might be difficult to find such a convincing argument with the following assumption.

Assumption (B): unification of the four forces is driven by one of the three forces of particle physics.

Under the (B) assumption, the strong force might be the better candidate. Indeed, for example in figure 1 of [21], it is located on an extreme location as compared to the others. The wave nature of matter proven in particle's physics appears to be an argument here. But this might argue also for (A) assumption since one can interpret them as being related to spacetime waves. Unfortunately, no convincing unification might be available under this (B) assumption. Nevertheless assumptions (A) and (B) appear better than the following one.

Assumption (C): no unification of the four forces exists.

Indeed, there is the apparent energy convergence at Planck scale [22] which contradicts it. But assumptions (A) and (B) must be compared with the following one, which might be done by today's physics.

Assumption (D): unification of the four forces is driven by an effect in which no force plays a leading role.

It might be this (D) assumption which is implicitly assumed todays [23, 24]. But the main arguments for it are symmetry considerations. Their relevance must be compared with the relevance of argument (*).

Also an experimental information in favor of assumption (A) is given by eclipses anomalies. Indeed, under this assumption, strong deviations of the gravitational signal of the sun, by the moon, might be expected during solar eclipses [12, 13].

9. Discussion

A new determination of spacetime by energy is presented. It uses the usual GR concept of rest frame, which is generalized and then called GRF in the present document. A property of those frames has been dismissed in the literature. This property is that the evolution of those frames is driven by energy. And this particular behavior is proven to result in equation (9). This equation allows to determine spacetime structure from energy and deals with the first degree of derivation of the metric. From there the second degree of derivation is calculated and then compared to Einstein equation. Hence this GR feature relates energy to the first degree of derivation of spacetime structure, it is a new chapter of GR to be exploited. Its starting part has been developed here.

It results a more complicated relativity, more Machian than before, in which gravitation follows the rule of a surrounding effect. In its weaker formulation this effect is the following. Increasing the energy of the surroundings of the location, where the gravitational force is exerted, results in decreasing this force with respect to Newton's law. It is proven that G is not a constant, and that its variation is driven by this effect. An equation of G is given, which is a good approximation under four assumptions, which might be valid most of the time in gravitation. For most of the usual energy distributions Einstein equation can be used. In particular it can be used for energy distributions of a small set of sparse objects, (for example two objects), when matter density stays constant over space and time apart from those objects. But this new value of G must be used. For calculating this value, formally equations (16) and (18) exist, but they are not practical since the attenuation function used in these equations remains to be calculated. And its value depends of the energy distribution. However, it remains much better to use Einstein equation with a G value driven by the surrounding effect than its constant anthropocentric solar system value. Since these equations of G are not easy to use, the surrounding gravitational model has been constructed [1].

When the complexity of the energy distribution does not allow to use Einstein equation, then a method has been presented, allowing to calculate spacetime structure from equation (7) and from any kind of energy distribution made of dimensionless particles. But, even though it might be calculated numerically, this method is complicated.

That's why a general principle might be searched for, allowing to replace completely Einstein equation. Another Lagrangian might be constructed, based on this new chapter of relativity.

However, the compatibility of surrounding [1] with experimental data, so far, along with the demonstration of the present document, shows the following. Each so-called gravitational anomaly might be simply a prediction of relativity. Of course the work is huge until one can replace "might be" by "is" in the previous sentence. Indeed, it remains several mysteries to address. And this model will have to be tuned. More details are given in Appendix Q.

Nevertheless, a significant step has been made in gravitation. It has been shown that far enough from the sun, using the G value determined by the surrounding value is much more relevant than using the solar system value.

In particle physics, this surrounding effect emerges under the relevant unifying assumption that the four fundamental forces are different aspects of the same unique force of gravitation. This assumption is supported by several theoretical reasons detailed in the present document. The exact law of motion remains to be determined. The same process used for constructing the surrounding effect might be applied, where a homographic function is fitted to experimental data.

Applying this surrounding effect in nuclear physics resolves three puzzles. The Ay puzzle and the proton radius puzzle find convincing explanations. The nuclear saturation puzzle is solved. This solution shows that the strong force can still follow a $1/r^2$ law, which can be argued to be valid from theoretical considerations. This allows us to discard the peculiar assumption about its supposedly extremely short range. A simple surrounding effect applied at the nucleus scale completely solves this saturation observation.

It is the same effect operating at the same scale that predicts any isolated quark would experience a much stronger force than when inside the nucleus. This provides a solution to the Yang-Mills Millennium problem.

Appendix

Appendix A. Continuity of the S function

The S function gives spacetime structure from energy distribution. S starts from the set of energy distributions, which is a linear space of infinite dimension. It reaches another linear space of infinite dimension, which is the space of spacetime structures. The latter can be defined using the matrices of the metric in some given system of coordinates. Then, this space is the linear space of the distributions over spacetime of 4×4 matrices. This definition depends on the choice of the system of coordinates. Both linear spaces are equipped with the uniform norm. Let's remind that the considered energy distributions are assumed to share the same bounded domain.

With the uniform norm, and for this restricted set of energy distributions, the claim is that S is continuous.

To prove this, let's show that if any amount of energy at any spacetime location is decreased to 0, then the effect of this amount of energy on spacetime structure also decreases to 0. For this demonstration, let's assume that this is wrong (assumption "w"). Then there exists an f_n sequence of energy distributions tending to 0 and such that their deformations are greater than some given non-null value. This can be written as follows:

$$\begin{aligned} \lim_{n \to +\infty} {}_u \left(f_n\right) = 0 \\ \forall\, n \; in \; \mathbb{N},\ \|S(E + f_n) - S(E)\| \geqslant M \end{aligned} \tag{A.1}$$

In these equations, E is the energy distribution of the universe, to which the f_n energy is added. $\|.\|$ is the uniform norm on the set of spacetime structures. M is some strictly positive real number. Therefore, the involved physics is such that an infinitely small amount of energy provokes a non-null modification of spacetime structure. Let's remind that this word "small" means that the maximum density of energy which is added is small (uniform continuity), and also that the whole quantity of added energy is "small" (the considered energy distributions share the same bounded domain). Since this added amount of energy is infinitely small, it is possible to imagine a thought experiment in which this energy is added instantaneously. But the result on spacetime structure is noticeable. This generates gravitational waves. If the universe is empty, then no contradiction arises. But this would be unrealistic. If the universe is not empty, then those gravitational waves cause the motion of matter in the universe. But with this infinitely small and sudden "addition" of energy, of course, nothing happens since the added amount is infinitely small. Therefore, a sudden modification of energy distribution happens without anything to provoke it. This violates the conservation of energy principle. Therefore, assumption "w" was wrong, and the claim is proven.

Then, writing this continuity property with the limit of sequences, equation (2) is given.

Appendix B. Generalized rest frame

Appendix B.1. *Definition of a GRF*

The construction of GRFs of the present appendix is done for an energy distribution following the principle of energy conservation and following the principle stating that a free fall particle follows the trajectory of a geodesic.

In relativity, rest frames exist. This concept can be generalized into what is called a generalized rest frame (GRF). Let us now perform this generalization. The metric is evaluated between rest frames using the GR Lagrangian in vacuum. Based on this, GRFs can be defined in the following way.

A rest frame R is chosen. This means there is a particle P at rest in R, where P is a dimensionless particle with nonzero mass. A displacement is then considered: that is, a spacetime event E is chosen such that P is located at E, and $E' = E + dl$ is any other event infinitesimally close to E, where dl is an infinitesimal displacement. From E', a test particle P' is considered, meaning that P' is at rest relative to R. However, since P' is not exactly at E, its "potential" rest frame is not R. The term "potential" is used because P' has no mass. Therefore, the rest frame of P' is another GRF passing through E'. By repeating this process infinitely, a GRF can be constructed at every spacetime event in vacuum.

It may be proven that this generalization ensures the existence of a time line for the generalized frame, which is a congruent geodesic (part of the same congruence of geodesics [11]) with respect to the time lines of the rest frames. (A "congruent geodesic" is the generalization of the concept of a "parallel straight line" in curved spacetime.) However, such a demonstration is not necessary for the present document.

Appendix B.2. *Fundamental behavior of a rest frame*

The fundamental behavior of a generalized rest frame can be illustrated through a thought experiment. Consider gradually increasing the rest energy of a particle P, while simultaneously decreasing the total energy of the universe. At the end of the experiment, P and the universe have exchanged roles: P now contains the energy that was previously in the universe, and the universe contains the energy that was previously in the particle.

The first result is that the frame in which time elapses the most is no longer the one attached to the universe. That frame is now attached to P. This implies that the spacetime structure is now the symmetric result of a frame exchange between the two. It also means that the spacetime structure has been gradually modified from the initial state to the final one. This process effectively reverses time dilation. For example, in a twin paradox configuration, at the end of the traveling twin's journey, the older twin would become the younger after this thought experiment. Therefore, the spacetime modification is described simply by the boost that transforms one frame into the other. This reasoning is based on the well-established assumption that general relativity is internally consistent.

At this point, it becomes useful to assign names to the frames. Let R_u be a frame attached to the universe. Assume the universe is filled with a constant, homogeneous distribution of matter; hence, this matter is considered to be at rest in R_u. Let R_p be a frame attached to P. The result of the thought experiment is that P locally generates a spacetime deformation described by the boost from R_u to R_p. Of course, this deformation is local to P, but the greater the rest energy of P, the more valid this deformation becomes around P. A "more valid deformation" means that the spacetime deformation has significant influence over a larger spacetime region.

The spacetime deformation arising in the experiment is described by the boost that progressively transforms the frame from R_u to R_p. This frame remains a rest frame throughout the process, even if it is no longer the frame in which time elapses the most. It is, nonetheless, the frame in which P is at rest.

This implies that the "absolute" spatial lines used in explaining the twin paradox have been exchanged between R_u and R_p. Accordingly, their time lines have also been exchanged (and the paradox can likewise be interpreted using absolute time lines). Furthermore, it follows that only the relative values of the respective energies determine which frame experiences the fastest passage of time. Consequently, any dimensionless particle P with nonzero mass deforms spacetime locally in such a way that, sufficiently close to P, its deformation dominates over that of any other energy source. Thus, in the vicinity of P, the absolute space and time lines are the space and time axes of P's rest frame.

The result is as follows: for any particle located at a given event x, an old GRF exists at x and is transformed into a new GRF by the presence of the particle, using the boost associated with the particle's four-momentum. Equation 11 describes the formulation of this boost in the old GRF, as derived from the expression of the four-momentum in the old GRF, given by equation 10.

Appendix B.3. *Inflexibility of the local spacetime deformation*

Let us also note that the local spacetime deformation around a dimensionless particle P is inflexible.

This means that the associated boost remains the same regardless of the energy distribution outside P. In other words, while increasing the energy of P may make the deformation more valid over a larger region, it does not alter the deformation in the immediate vicinity of P. Increasing or decreasing the energy of P has no effect on the local deformation of the spacetime structure generated by P. This deformation is always described by the same corresponding boost.

What causes this inflexibility is the fact that the deformation is local. Particle P is dimensionless but possesses nonzero energy, so its energy distribution is modeled by a Dirac delta function. Since S is a continuous function, the deformation introduced by P is also described by a Dirac delta multiplied by this boost. Specifically, regardless of the surroundings of P, the local deformation it generates remains the same.

Practically speaking, this remains approximately true even if P is not strictly dimensionless, provided it has a sufficiently high matter density. This is a realistic model, given the known extreme density of matter. The same applies to the virtual microscopic bunches of matter considered in the thought experiments of this document.

Appendix B.4. *Conclusion*

The conclusions of this appendix are as follows:

1) For each spacetime event, a frame called a GRF has been constructed.
2) This GRF concept generalizes the notion of a rest frame.
3) Each particle P located at a given spacetime event x is associated with an old GRF and a new GRF. The old GRF is defined in x under the assumption that P is absent; the new GRF is defined in x under the assumption that P is present.
4) At x, the new GRF is obtained from the old GRF by applying the boost associated with the motion of P relative to the old GRF.

Appendix C. Rescaling after the boost

Let's consider P, a particle moving uniformly at speed v, where $v < c$, along the D space line in a flat Minkowskian spacetime. The $R(O; ct, x, y, z)$ frame is chosen such that O lies on D, and Ox is in the direction of P's motion. The local deformation around P is first described by the b_v boost associated with the v speed, as shown previously.

But equations (12), (13), and (14) show that a rescaling of ct time and x space axis is required. If no rescaling occurs, it means that the deformation generated by P is entirely described by a boost. In that case, it is simply the usual change of coordinates following SR rules, and no deformation is observed. Therefore, a rescaling occurs, and this is consistent with time dilation between R and R'. From equations (10), (12), (13), and (14), if α and β are the s rescalings of ct and x, respectively, we have $\alpha^2 = \beta^{-2} = 1 - v^2/c^2$.

Hence, the complete deformation is described by $b_v s$. This results from equations (12), (13), and (14).

This is true for the Schwarzschild metric and for a test particle at rest infinitely far from the center of symmetry. But the calculation of this rescaling can be done for any energy distribution, with respect to any R_0 frame, at any t given time in R_0. For this calculation a test particle being at rest with respect to R_0 at t is used for calculating the boost. And then the rescaling is the time dilatation from the time of R_0 to the proper time of the particle.

Appendix D. Integration constant

This appendix deals with the integration constant in the spacetime structure calculation for the energy distribution of two particles in an empty universe.

This constant can potentially be determined from the following information. Singularities occur along the straight line containing the particles' locations, but only at those points that are not between the particles' locations. These are described by the boost associated with the speed of light in the direction moving away from the particles.

The rigorous approach is to calculate the spacetime structure using symmetry considerations and equation (5). Then, the asymptotic values of the deformation should allow the calculation of the integration constant.

Let's recall that this procedure has already been done for the energy distribution with one particle in an empty universe. Symmetry considerations produce the Schwarzschild metric. Equation (5) provides the information that the metric takes the form $g_{00}(x) = 1 - M/r$. Here, $g_{00}(x)$ is the time-time component of the metric at the x spacetime event, r is the spatial distance from x to the particle located at the center of symmetry, and M is the unknown integration constant. Then, the asymptotic value of the local spacetime deformation, as given by paragraph 3.6, allows the calculation of $M = \infty$.

Therefore, it should be possible to calculate this constant for the energy distribution of two particles in an empty universe.

Appendix E. Particle moving at the speed of light: local deformation

Now, assume $v = c$. Let's prove that the deformation in C is the b_C boost associated with the speed of light along the D line in the direction of the GW propagation.

Starting from the fundamental behavior of a rest frame which is described in Appendix B, taking the limit as v approaches c, the result is obtained. As usual, the interchange of limits theorem is used. The simple convergence is sufficient for it to work since the studied deformation is only local.

Appendix F. Particle moving at the speed of light: propagation of the deformation along the trajectory

Appendix F.1. *Aim*

The main aim of the present appendix is to study simply the GW generated by a given particle P in motion at the speed of light. This study is "simple" because it is restricted to the trajectory of the particle and also to the geodesics which share the same tangent as the P trajectory (if this trajectory deviates from being a geodesic at some point).

The complete description of this GW is done in Appendix I, but no demonstration is really given for this description, only clues. That's why here the aim is to have a rigorous demonstration of the nature of the GW along those particular geodesics. This will be enough for the whole demonstration of the present document.

The study will start by the description of important properties of test particles.

Appendix F.2. *Context and notations*

A given E energy distribution is in the universe. Let's denote $GRF(E)$ the system of GRFs associated to E. In other words, $GRF(E)$ is the set of the new GRFs after insertion of E. An e spacetime event is chosen. Let's denote, for any such e, $GRF(E, e)$ the GRF which is a part of the $GRF(E)$ system, and which is located in e.

Then a Q test particle is considered, and added to E. Q is such that it is moving at the V speed with respect to $R_0 = GRF(E, e)$ at e. Hence after the insertion of Q, the total energy distribution of the universe is $E + E_Q$, where E_Q is the Dirac distribution of Q. Let's always denote E_M the Dirac distribution of any given M dimensionless particle. Let's denote $v(t)$ the speed of Q with respect to R_0 at any t time.

Appendix F.3. *Deformation generated by a test particle at rest in R_0*

First, let's assume $V = 0$. Therefore Q is assumed to be at rest with respect to R_0 at e.

In e, a geodesic, let's call it $g(e)$, has been constructed at the construction of the $GRF(E)$ system. $g(e)$ is congruent to the nearby particle trajectories which are geodesics. This has been done at the construction of the $GRF(E, e)$ system, in Appendix B. Let's remind that this construction is done for an energy distribution following the principle of energy conservation and following the principle stating that a free fall particle follows the trajectory of a geodesic. $g(e)$ as well as the Q trajectory are geodesics. But they share the same tangent in e because $V = 0$. Hence those geodesics are equal. Therefore, the speed of Q, with respect locally to each frame of the $GRF(E)$ system of frames, remains equal to 0.

Therefore, along $g(e)$, the new GRF, $GRF(E + E_Q)$, is equal to its old, that is, $GRF(E)$. But by other means, along the Q trajectory, $GRF(E + E_Q)$ is given by transforming $R_0 = GRF(E, e)$ by the boost associated with $v(t)$ in R_0 at any t. Therefore since $GRF(E) = GRF(E + E_Q)$, it is also $GRF(E)$ which is given by transforming $R_0 = GRF(E, e)$ by the boost associated with $v(t)$ in R_0 at any t. Therefore, a test particle allows to calculate the spacetime structure generated by E, along its trajectory.

Therefore a test particle being at rest with respect to $GRF(E)$ at a given e event allows to calculate spacetime structure along its trajectory.

The final result is that the test particles being at rest with respect to $GRF(E)$ at any given event allow to calculate spacetime structure everywhere. This is an important result which will be used directly throughout the document.

Appendix F.4. *Deformation generated by a test particle moving at V speed in R_0*

Second, $V = 0$ is no longer assumed. The Q test particle being in free fall, its motion with respect to the local GRF which is part of $GRF(E)$, the system of old GRFs, is always equal to V. This is ensured, for instance, by conservation of energy of motion. Therefore, along Q's trajectory, $GRF(E + E_Q)$, is given by transforming $GRF(E)$ by the boost associated with V with respect to $GRF(E)$ at any t time.

Therefore $GRF(E)$, is given by transforming R_0 by the boost associated with the relativistic substraction of $v(t)$ by V in R_0 at any t.

This allows to calculate spacetime structure in another way in which V is not null. This way might appear wired because it is simpler to choose R_0 such that $V = 0$ at t_0, which is always possible. But this wired result helps for the following paragraph.

Appendix F.5. *Deformation generated by a particle moving at the speed of light along its trajectory*

Now let's add another particle, P. Let's assume that P is moving at the speed of light along a geodesic before a given e spacetime event, at time t in a given R frame, and that after t, P deviates from this geodesic. Let's call g this geodesic. This deviation may result from the existence of some unknown force. The question about the existence of such a force is discussed in Appendix L. The aim here is only to study the deformation generated by the GW produced by P.

g is a null geodesic before e therefore g is a allways a null geodesic. Let's add another particle, Q. Q is a test particle "surfing" on the GW generated by P along g. It means that Q is located in e, infinitely close to P, and in e the two particles share the same speed of light. Then after e, P and Q get apart from each other.

Since Q is in free fall and since it moves at the speed of light in e with respect to $GRF(E)$, its speed remains the speed of light after e, with respect to $GRF(E)$, along g.

Therefore, after e, $GRF(E + E_P + E_Q)$ is given by transforming $GRF(E)$, by the boost associated with this particular speed of light. A first demonstration is given by considering that this is the rule valid for any test particle not moving at the speed of light. It remains valid for a particle moving at the speed of light this can be proven by taking the limit for a test particle moving at the $V < c$ speed, limit when V tend to c, using also the interchange of limit theorem. Another demonstration uses the Doppler effect. It is that if the deformation generated by this particle was not equal to this one then it would mean that its energy would become null. Indeed with respect to the system of $GRF(E + E_P + E_Q)$, in e, Q

Q is at rest with respect to $GRF(E + E_P)$ at e.

Therefore Q which is a test particle, which remains at rest with respect, locally, to $GRF(E + E_P)$, along g.

Therefore, along g, $GRF(E + E_P) = GRF(E + E_P + E_Q)$ (the transforming boost is associated to the null speed).

Therefore, along g, $GRF(E + E_P)$ is given by transforming $GRF(E)$ by the boost associated with the speed of light, the speed of Q.

But $GRF(E + E_P)$ is the system of new GRFs given by the existence of E and P. Therefore this system describes the GW generated by P.

Appendix F.6. *Conclusion*

Therefore the GW propagates at the speed of light along g and the local deformation which is propagated is the radical deformation described by the boost which is associated with the speed of light of this GW.

Using the notations of Figure (1), the final picture is the following. The deformation of P in C propagates along the D line in the same direction as the motion of P and is described by the b_C boost associated with the speed of light in the same direction. This can't be really observed when P moves along D, because the deformation generated by P can't be distinguished from the deformation generated by its GW propagating in the same direction. But this can be observed as soon as P deviates from this D line trajectory. The propagation speed is the speed of light (this has been demonstrated without using assumption (I).) The important result here is that this propagated local deformation is described by b_C.

Appendix F.7. *Context and notations*

A given E energy distribution is in the universe. Let's denote $GRF(E)$ the system of GRFs associated to E. In other words, $GRF(E)$ is the set of the new GRFs after insertion of E. A e spacetime event is chosen. Let's denote, for any such e, $GRF(E, e)$ the GRF which is a part of the $GRF(E)$ system, and which is located in e.

Then a P test particle is considered, and added to E. P is such that it is moving at the c speed with respect to $R_0 = GRF(E, e)$ at e. Hence after the insertion of P, the total energy distribution of the universe is $E + E_P$, where E_P is the Dirac distribution of P.

Appendix F.8. *Deviation of P from its geodesic*

SO P is following a null geodesic, let's denote it g. Now let's assume that at a given e event of spacetime, P deviates from g. This deviation may result from the existence of some unknown force. The question about the existence of such a force is discussed in Appendix L. The aim here is only to study the deformation generated by the GW produced by P.

Appendix F.9. *Surfing particle*

Let's add another particle, Q. Q is a test particle "surfing" on the GW generated by P along g. It means that Q is located in e, infinitely close to P, and in e the two particles share the same speed of light. Then after e, P and Q get apart from each other.

Since Q is in free fall at the speed of light in e, its speed remains the speed of light after e. This results from conservation of energy with respect to any inertial frame. Also the speed of Q remains the speed of light with respect to the $GRF(E + E_P)$ system of frames.

Hence the spacetime deformation generated by Q remains null. Otherwise its energy would become null. With respect to the $GRF(E + E_P)$ system of frames, this deformation is null.

Therefore, after e, $GRF(E + E_P + E_Q)$ is given by transforming $GRF(E)$, by the boost associated with this particular speed of light. Q is at rest with respect to $GRF(E + E_P)$ at e. Therefore Q is a test particle which remains at rest with respect, locally, to $GRF(E + E_P)$, along g. Therefore, along g, $GRF(E + E_P) = GRF(E + E_P + E_Q)$ (the transforming boost is associated to the null speed).

Therefore, along g, $GRF(E + E_P)$ is given by transforming $GRF(E)$ by the boost associated with the speed of light, the speed of Q. But $GRF(E + E_P)$ is the system of new GRFs given by the existence of E and P therefore this system describes the GW generated by P, it's the new GRFs of P and of its generated GW.

Appendix G. Simple description of the local spacetime deformation

Let's describe the local spacetime deformation generated by a particle or a GW, using the simplest possible approach. For that, the Poincaré-Einstein synchronization [28] will be used. The Minguzzi synchronization is not strictly required since the transitivity of the procedure is not mandatory here. This procedure is used here for the construction of an inertial frame $R'(C, ct', x')$, moving at speed v with respect to another frame, denoted $R(O, ct, x)$. No Lorentz transform or boost is needed for that, only the Einstein synchronization rule and geometry in two dimensions are used. Figure (2) represents this construction. After this construction, x' is the symmetric of ct' through the orthogonal symmetry with respect to the light trajectory in the increasing x direction. Indeed, the circle passing through O and M, and having C as its center, also contains N because the OMN triangle is a right triangle at N. Hence, $CM = CN$. Then, if H is the midpoint of M and N, then CH is parallel to ON by Thales' theorem applied to the OMN triangle. And, of course, CN is the direction of the light trajectory in the increasing x direction.

Moreover, there is a natural coherence in the calculation of the space axis in four dimensions using the synchronization procedure. This is demonstrated by using different light beam trajectories. For example, the procedure can be executed with a light beam in the decreasing x direction, and then, after the turning point, in the increasing x direction. The result is, of course, the same x' axis. But also, using a trajectory along the y or z directions will show that along these space axes no deformation is noticed, which is the expected result. Any other light beam back-and-forth trajectory in space will show a deformed axis, which is also coherent since the procedure yields linear results.

The causal link is as follows. The starting point is that the motion of R' with respect to R is v, i.e., C is moving at speed v with respect to R. From this information, the ct' axis of R' can be drawn with respect to R. Then, the synchronization rule determines the location of the x' axis. Even if mathematically there is an equivalence, and ct' can be retrieved from x', nevertheless, for physics, the causal link is unambiguous.

A remark can be added about the rescaling studied in Appendix C. The boost transforming R into R' is deduced and, therefore, implicitly identified by the direction of the new ct' time axis with respect to R. The new GRF and the new metric valid after the insertion of the particle or the new GW are deduced from this ct' axis only, without the need for the details of the boost itself. Even the usual rescaling of the new time and space units can be done based on the ct' axis alone.

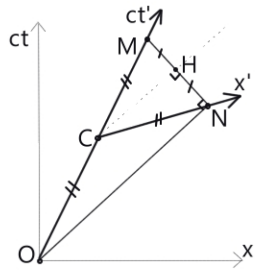

Fig. 2. Two inertial frames are represented in two dimensions, $R(O; ct, x)$, and $R'(C; ct', x')$. The latter is constructed with respect to the first using Einstein's synchronization rule. ON and NM together represent the trajectory of a light beam emitted from O in the direction of increasing x, and bouncing back at N. C is the midpoint of OM. The x' axis is supported by CN following the synchronization rule. H is the midpoint of MN. A simple geometric reasoning in two dimensions shows that x' is symmetric to ct' through the orthogonal symmetry with respect to CH, which is parallel to ON.

Appendix H. Particle moving at the speed of light: propagation of the deformation everywhere

This paragraph gives different demonstrations of the nature of the spacetime deformation generated by a particle moving at the speed of light. The first demonstration use the Schwarzschild metric and the second demonstration is based on a postulate of scalability.

Appendix H.1. *Schwartzchild metric from a relativistic frame*

In this demonstration, a P particle is considered in vacuum. Therefore the spacetime deformation generated by P is described by the following metric.

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \qquad \text{(H.1)}$$

This equation is given in a $R'(O'; ct', x', y', z')$ frame attached to P. O' is the space location of P, ct' is the time dimension, and x', y', z' are the space dimensions. Now let's consider another frame, $R(O; ct, x, y, z)$, in motion with respect to R' at the speed $-c$ along the x' axis of R' and also in the direction of x decreasing with respect to R. It means that with respect to R, P is moving at the speed of light along x.

Let's calculate the new formulation of the metric in R. For this, the motion of R with respect to R' will first be assumed to be equal to $-v$, with $0 < v < c$.

The new formulation of ds in R is the following.

$$\begin{aligned} ds^2 = dr^2 + \left(\frac{u}{x} - d\alpha\right)\left(-\frac{u}{x}dx + d\alpha + 2du\right) \\ r = \sqrt{x^2 + y^2 + z^2} \\ u = \sqrt{y^2 + z^2} \\ d\alpha = \frac{ctdx - xcdt}{ct - x} \end{aligned} \qquad \text{(H.2)}$$

Let's take the limit of this value when v tends to c. The result is the following.

$$\begin{aligned} ds^2 &\simeq -\, x \, dY \, d\alpha \\ Y &= ct - x \end{aligned} \qquad \text{(H.3)}$$

Singularities appear in this formulation. They are located along the $x = ct$ variety, which corresponds to the $x = ct$ planes in space at each constant t. The P trajectory is in this variety. And along this trajectory, the local spacetime deformation generated by P locally around itself is described by the b_C boost. This has been shown in Appendix E.

Therefore, in equation H.3, the singularity corresponds to this local deformation described by the b_C boost. Hence this local deforamtion described by this boost exist also over the whole variety.

The result is that the GW generated by P is described by the same b_C boost and propagate along the $x = ct$ planes.

As usual, this result might appear wired. But P is alone is an empty universe, and that is a unrealistic assumption. Only an unrealistic assumption must be expected. Another remark helping to understand that is that here the total energy of P has been increased as with respect to R, for $v = c$, it is infinity.

Appendix H.2. *Assumption of scalability*

In this demonstration, the same configuration is used. The P particle is moving at the speed of light in the direction of the x axis, x increasing, with respect to the $R(O; ct, x, y, z)$ frame.

The energy distribution is invariant by the perpendicular symmetry with respect to the Ox axis. As a result, the spacetime deformation is also invariant by this symmetry. We are left to describe the GW generated by P by a ϕ function, giving the location of the maximum local deformation of the GW. At any t time in R for each M space location where this deformation is maximum, if x, y, z are the space coordinates of M, then there is $h = \phi(x)$ where $h = \sqrt{y^2 + z^2}$.

The question of the meaning and the existence of this maximum gets the following answer. Its meaning is what has been described in Appendix B. At each spacetime event there is an old GRF and a new one, with respect to the existence of P. This "local deformation" is the boost transforming the old GRF into the new one. The maximum value of such a boost relates to the norm of the speed which is associated to this boost. This value is bounded between 0 and c. Therefore on a compact such a bounded domain around P this value gets a maximum. The question of the unicity of this maximum remains. But unicity is not mandatory has it will be seen further.

There is $0 = \phi(x_P)$ at any time if x_P is the x location of P at t. Getting more information about ϕ would be a work but is not mandatory.

Let's consider two space points M and M' at a given t time in R, and such that they are points of maximum value of the GW, ans also such that M' is in the space plane containing M and the Ox straight line. The (M, M') straight line containing M and M' gets an equation of the form $h = ax + b$, using the same notation as above.

Let's also consider another $R'(O'; x', y', z')$ frame in motion with respect to $R(O; x, y, z)$, at the v speed in the direction of x, being positive when x increases. The corresponding $h' = a'x' + b'$ equation of (M, M') can be calculated, with obvious notations for h', a', b', x'. After some calculation, the following equations result.

$$\text{.............} \tag{H.4}$$

The result is that a is transformed into a' in such a way that the limit value of a' for $v = -c$ is $-\infty$ if $a < 0$ and $+\infty$ if $a > 0$. But from the assumption of scalability of the configuration, there must be $a' = a$ for any value of v. It results, either $a = 0$, or $a = +\infty$ or $a = -\infty$. The GW is a cone centered on P trajectory but it is a "plane" cone. The GW is the Oyz plane.

Appendix I. Particle moving at the speed of light: final picture of the deformation everywhere

Appendix I.1. *Demonstration of the GW structure*

Let's study the envelope of the GW generated by a particle moving at the speed of light along the D line. The notations of Figure (1) are used. Apart from P, the universe is assumed to be empty. This envelope is a cone, its axis is the D line. Indeed, the energy distribution is symmetric with respect to the orthogonal symmetry around D. Hence the spacetime structure is also symmetric with respect to the same symmetry. When P moves from C to C', this is done at the speed of light with respect to any given inertial R frame. During the same delay of time in R, the generated GW moves from C to N. This is the definition of N, along with CN being perpendicular to D. Lets take an R' inertial frame in motion with respect to R along CN. If CN is different from

Appendix I.2. *Details about the resulting GW*

Figure (1) will illustrate the events described in the present appendix. Let's argue the following.

Assumption (j): The spacetime local deformation described by the b_C boost, generated by the particle P moving at the speed of light in the x direction, also generates a GW that propagates at the speed of light along the directions perpendicular to x.

This is much more than an assumption. Indeed, the local deformation that generates this GW is described by the b_C boost. This is proven in Appendix E. Hence, it is more than an assumption that this is exactly what is propagated by this GW. If D represents the straight-line trajectory of P, then the point H on D, closest to any given point M, is such that MH is perpendicular to D. Therefore, the propagation of the GW perpendicularly to D is more than an assumption.

Moreover, let's show in the present appendix that this is coherent with the fact that the envelope of the GW propagates globally at the speed $c/\sqrt{2}$ along the CM direction. This will be an additional argument for assumption (j).

Let's start by studying the motion of the GW because this is the starting point, as shown in Appendix G. The GW propagates at the speed of light, as stated by assumption (I) of paragraph 3.4. This propagation at speed c occurs along directions perpendicular to the P trajectory, as stated by assumption (j) above. Therefore, this propagation occurs from C to N at speed c. Let's study the Δ half-line containing N and C', starting at C' and going in the direction of N. The events along Δ at time t_N correspond to the propagation of events at the locations of P along the CC' segment. This means that Δ is the envelope of the GW along the (x, y) plane. It is noted that this Δ envelope propagates along the CN direction at speed $c/\sqrt{2}$.

From this motion, the new time axis is deduced, let's call it ct'', starting the causal link presented in Appendix G. This ct'' time axis in M is located in the (ct, n) plane, where n is the axis aligned with the CM direction. Due to the $c/\sqrt{2}$ motion of the GW, the equation of the trajectory of the GW along the n axis is as follows.

$$r = \frac{c}{\sqrt{2}} t \tag{I.1}$$

In equation (I.1), r is the coordinate along the n direction starting from M, with respect to R. It can be written as $ct = \sqrt{2}r$. Therefore, the angle β between ct'' and the (x, y) plane is given by the following equations.

$$\tan(\beta) = \sqrt{2} \tag{I.2}$$

$$\cos(\beta) = \frac{1}{\sqrt{3}} \tag{I.3}$$

Now let's show that this result is coherent with the local spacetime deformation generated by P and its local propagation along the x axis. At time t_N, along Δ, the ct time axis has been rotated around the (y, z) plane by an angle of $\pi/4$ in the direction of x, resulting in the ct' axis. Indeed, this is the effect of the b_C boost along the space points of Δ: the b_C boost transforms R into the $R'(ct', x', y, z)$ frame in this way. There is $ct' = x'$. This creates a Π plane containing all these lines starting from the points of Δ in the direction of ct'. In other words, Π is the new plane of time after the action of the b_C boost executed along Δ at $t = t_N$ in R. Let's calculate the angle α between Π and the (x, y) plane, with respect to R. This is also the angle between the n direction and Π, and therefore the angle between CM and Π. The calculation follows.

$$\cos(\alpha) = \frac{\overrightarrow{CM} \cdot \overrightarrow{CM'}}{|CM|_3 |CM'|_3} = \frac{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2}{\sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2}\sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2 + L^2}} = \frac{1}{\sqrt{3}} \tag{I.4}$$

L is the $CN = CC'$ distance. Let's denote Π' as the plane parallel to Π and containing C. M' is the spacetime point that is the projection of M onto Π' along the ct direction. Equation (I.4) is coherent with equation (I.3). The result is the following.

$$\alpha = \beta \tag{I.5}$$

This means that the GW propagation along its envelope creates a spacetime deformation that is coherent with the propagation of the b_C boost perpendicularly to the P trajectory. It should also be noted that the Π plane makes a $\pi/4$ angle with respect to the (x, y) plane, but in the direction of y. In other words, the intersection of Π with the (y, ct) plane is a line that makes such an angle with respect to the (x, y) plane. This is coherent with the local GW propagation along the y axis at the speed of light.

The final result is a coherence between the following GW features:

1) Locally, the GW propagates the b_C boost at the speed of light along the P trajectory,

2) It also propagates this boost at the speed of light, perpendicularly to the P trajectory. This describes the entire GW spacetime deformation (in the whole space at a given time).
3) The GW propagation along any space direction perpendicular to the P trajectory occurs at the speed of light. This is a spacetime deformation also described by the boost associated with the speed of light in this direction. It describes exactly the same entire GW spacetime deformation as described above.
4) Globally, the envelope of the GW propagates at the speed $c/\sqrt{2}$. The boost associated with this speed and direction also describes exactly the same entire GW spacetime deformation as described above.

Then, still following the causal link presented in Appendix G, the new space axis of the GW deformation is deduced from the location of the new time axis. This new n'' space axis is part of the (ct, n) plane because the GW global motion is along the n axis. n'' is the symmetric of ct'' with respect to the line supported by the $\vec{ct} + \vec{n}$ vector, where $\vec{ct}$ and $\vec{n}$ are the unit vectors of the ct and n axes, respectively. Of course, the same coherence arises for the new space axes along the n, x, and y directions, that is, the same coherence as shown above for the new time axis (items 1 to 4).

The conclusion of this appendix is that under assumptions (I) and (j), a coherent and symmetrical picture of this GW is presented. Since assumption (j) has not been proven, the picture cannot be considered definitively correct. Hopefully, the full picture is not required for the demonstration in the current document. What is required is the result from Appendix F, namely, the fact that the b_C boost propagates itself along x and the increasing x direction.

Appendix J. Final picture for a particle moving at the speed of light

This follows from Appendix I. The final picture for a particle moving at the speed of light is that, in R, the deformation occurring at event E_C in C propagates globally along the Cn spatial direction, at a speed of $c/\sqrt{2}$. This deformation is described by the boost associated with the speed of light in the x direction, which propagates locally at the speed of light along the Cx direction and also at the speed of light along the Cy direction. However, along the Cn direction, the deformation is described by the boost associated with the $c/\sqrt{2}$ speed. This picture is Lorentz invariant.

Appendix K. Asymptotic spacetime deformation around a particle moving at the speed of light along a circle

The asymptotic local spacetime deformation generated by a particle moving at the speed of light along a circle is deduced from Appendix J. The GW propagates in the direction perpendicular to the particle's trajectory. This is what is assumed by assumption (j) of Appendix I, and this assumption provides a coherent picture, as described in this appendix.

Moreover, asymptotically, this direction is also the line which is tangent to the particle's trajectory. In Appendix F, it has been proven that this propagation happens at the speed of light, and that this GW is the propagation of the b_C boost along D.

Therefore, it has been demonstrated the present starting paragraph. Asymptotically, the envelope is represented by a sequence of spheres (the sequence of spheres described in section 3.6), all centered on O, the center of the circle. Asymptotically, these spheres expand around O at the speed of light, and thus this envelope propagates at the speed of light. The boost which describes the local deformation of the GW on those spheres is associated with the speed of light which is normal to these spheres, in the direction that goes out of the spheres.

Appendix L. Avoiding the Unknown Forces

P was forced to follow a circular trajectory, which is not a geodesic. Therefore, there is an unknown force that prevents this trajectory from following a geodesic. This force is not gravity. At first glance, this suggests that modeling the Dirac distribution under assumption (2) would not be possible in a coherent way which would only involve gravitation. However, this apparent result is incorrect. The final result is actually quite the opposite, as will be shown.

Let us assume that P is at rest in a given inertial frame R, at the point O in space. The goal is to model P using a set of energy distributions that converge to it.

Let us start by reformulating the constraint above, which is indeed necessary for modeling a dimensionless particle P. Another constraint is added, and the result is the following two conditions, which must be satisfied by each energy distribution in the sequence converging to P:

- The energy distribution has a non-zero size.
- The spacetime structure generated by the energy distribution allows it to remain static.

As usual, the definition of the word "static" is that, as time passes, the energy distribution remains unchanged with respect to R, but matter may change. This means that matter can still be in motion in R, but in such a way that the matter density remains the same at any given space point.

The first constraint simply prevents trivial sequences. The second constraint is mandatory because the goal is to model the final Dirac distribution of matter for P by studying a sequence of energy distributions that converge to this Dirac distribution. The Dirac distribution is static because the aim is to model the gravity of this dimensionless particle P, which is at rest in R. Strictly speaking, this constraint may not be strictly mandatory. Indeed, one could imagine a cumbersome sequence of complex energy distributions, each of which is non-static, but still avoids any force other than gravity, allowing their existence, and ultimately converging to the energy distribution of P. However, the goal here is simply to examine the remark made at the beginning of the present appendix regarding the required unknown force for the static distribution of a circular trajectory. Moreover, the ultimate goal here in the present document is to construct a static sequence of energy distributions converging to the energy distribution of P without the need of any other force than gravity and following the rule given by assumption (2).

However, under assumption (1), such a distribution does not exist. Indeed, for a static distribution with a non-zero size, the spacetime structure would generate a radial acceleration toward some space point I, located somewhere within this energy distribution. For example, if the distribution is symmetric with respect to O, then this point is $I = O$. Therefore, matter would be accelerated toward I. Any infinitesimal chunk of matter located at M, different from I, would be accelerated along and in the direction of the MI vector. The result is that this is not a static energy distribution.

On the other hand, under assumption (2), such static distributions may be possible because geodesic trajectories can be circles around O. It is more difficult to prove that assumption (2) does not allow such a sequence of static distributions converging to P. And it remains challenging to prove that such a sequence exists under this assumption. In Appendix M, this problem will be studied. In this appendix, it will be shown that, under assumption (2), there exists a static energy distribution that is extremely close to the one of P (it is important to recall that this is not possible under assumption (1), as shown above).

Therefore, the result is exactly the opposite of what was claimed at the beginning of the present paragraph. It is straightforwardly modeling the Dirac distribution under assumption (1), which is, obviously, not possible. Under assumption (2), no such obvious contradiction arises, and some arguments suggest that it might be possible.

Interestingly, this result is very important. It shows that in relativity, assumption (1) is, to say the least, not straightforward. For this replacement, assumption (2) might be a viable alternative.

Appendix M. Existence of a static spherically symmetric distribution

Appendix M.1. *Existence of a sequence of static distributions converging to* P

Instead of proving the existence of such a sequence, let us list some specifications for such energy distributions that might converge to P in a static manner.

- A set of particles moving at the speed of light,
- This set is distributed homogeneously around O, following the spherical symmetry around O,
- At each point M different from O, an infinite set of such particles exists, each moving perpendicularly with respect to OM. This means that the v speed of light of each particle is perpendicular to OM,
- The distribution of these v vectors is homogeneous in M.

There are many energy distributions that satisfy these constraints. The goal is to find a sequence of such distributions converging to P, while also fulfilling the following constraint.

- The spacetime structure generated by each energy distribution must allow it to remain static.

This work is still in progress. However, the following example will show that assumption (2) is better than assumption (1) for modeling P.

Appendix M.2. ***Existence of a static distribution close to P under assumption (2)***

Let us try to roughly calculate the radius of the circle in the trajectory of a particle moving at the speed of light, in such a way that the resulting energy distribution would remain static. This radius is roughly the r Schwarzschild radius of the mass M of the particle at rest which is being modeled.

$$r = \frac{2MG}{c^2} \tag{M.1}$$

Let us calculate this using the gravitational constant G given by Surrounding. (Using the usual gravitational value valid in the solar system would yield a very similar result.)

$$G = G_0 \frac{\alpha_0 \rho_0 + \rho_{u0}}{\rho_{15kpc} + \rho_u} \tag{M.2}$$

G_0 is the gravitational constant valid in the solar system. ρ_{15kpc} is the matter density in the sphere centered on the measurement location, with a $15\ kpc$ radius. ρ_{u0} is the universe's matter density today, and ρ_u is the universe's matter density at the time of measurement. The result is the following:

$$r \approx 10^{-18} \text{ Planck Length} \tag{M.3}$$

We used $G_0 = 6.67 \times 10^{-11}$ m^{3}kg^{-1}s^{-2}, $\alpha_0 = 1.6 \times 10^{-5}$, $\rho_0 = 0.98 \times 10^{-21}$ kg m^{-3}, $\rho_{15} = \rho_{u0} = \rho_u = 0.05\ \rho_c$ with $\rho_c = 8.5 \times 10^{-27}$ kg m^{-3}, and $PlanckLength = 10^{-35}$ m. For the mass M, we used the proton mass of $M = 1.67 \times 10^{-27}$ kg.

The result shows that the following modeling of a proton is possible:

- Modeling a proton as being made of an internal bunch of matter moving at the speed of light along a circular trajectory,
- Using the gravitational constant given by Surrounding in the particular situation of a proton located outside any galaxy at the current time,
- In a static manner, that is, without collapse,
- Without the help of any force except gravitation.

Of course, it has not been demonstrated that the resulting spacetime structure allows this energy distribution to remain static. Nevertheless, since the size of the circle is extremely small, this energy distribution is close to another one, which consists of a circle of matter rotating at the speed of light around its center and in the plane of the circle. This energy distribution generates a symmetric spacetime structure with respect to the straight line passing through the center of the circle and perpendicular to the plane of the circle. Also there is a symmetry with respect to the plane of the circle. At the locations of the circle, the attraction of matter towards O is exactly the one that allows the orbital motion along the circle at the speed of light because the distance to O is r, the Schwarzschild radius. If any doubt exists about the fact that this orbital distance is r, then it is possible to use this orbital distance as the radius of the circle. This orbital distance will be close to r.

Also, this model does not account for the fact that, in reality, a proton is made of quarks. Nevertheless, this modeling is not far from reality. The goal here is purely theoretical: it models a dimensionless particle P at rest in R. The mass of P has simply been assumed to be equal to the mass of a proton.

Therefore, under the above conditions, it is realistic and possible to model a particle with the mass of a proton, using only static distributions, and under assumption (2). This result must be compared with the fact that such a modeling is not possible under assumption (1).

Appendix N. The gravitational wave four-momenta add themselves

Let's suppose that n gravitational waves (GWs) are propagating and encountering each other at a given spacetime event x. Let GW_i represent the i-th gravitational wave, for i ranging from 0 to n. For each i, a P_i particle can be created such that its momentum exactly counteracts the effect of GW_i at x. This means that the P_i speed is the speed of light and that its total energy in any reference frame R is the energy of GW_i in R. It also implies that the direction of P_i's motion is opposite to that of GW_i. The result is that P_i and GW_i together generate no local spacetime deformation. Therefore, the overall effect of all these P_i particles is such that they cancel the spacetime deformation generated by all the GW_i. But the sum of all the P_i particles forms a large compound object, which we will call P. Thus, the spacetime deformation of all the GW_i is exactly counteracted by the spacetime deformation generated by P. According to the principle of conservation of energy, the four-momentum of P is the sum of the four-momenta of all the P_i. Therefore, the four-momentum of the spacetime deformation of the entire set of GW_i is the sum of the four-momenta of the GW_i.

$$\begin{aligned} F_m\big(\{GW_i, i = 0..n\}\big) &= -F_m(P) \\ &= -\sum_{i=0}^{n} F_m(P_i) \\ &= -\sum_{i=0}^{n} \big(- F_m(GW_i)\big) \\ &= \sum_{i=0}^{n} F_m(GW_i) \end{aligned} \tag{N.1}$$

$F_m()$ is a function that gives the four-momentum of a particle or a GW. This concludes the proof.

Appendix O. An equation of *G*: calculations

In this appendix, an equation for G is derived. Assumptions (i), (ii), (iii), and (iiii) are made. The same context and notations as in paragraph 3.11.3 are used. Let's remind the properties of the Schwarzschild metric. The first equation is as follows.

$$g_{00}(x') = 1 - \frac{R}{r} \tag{O.1}$$

Of course, $g_{00}(x')$ is the time-time component of the metric at the x' event, R is the Schwarzschild radius of P, and r is the distance between P and the x' event. The following is another important equation valid in this metric, which relates the metric to the free-fall speed.

$$g_{00}(x') = 1 - \frac{v^2}{c^2} \tag{O.2}$$

Here, v is the speed of the usual P_1 free-falling test particle, located at x', which was initially located infinitely far from P. The validity of equation (O.2) comes from assumption (i). Indeed, equations (O.1) and (O.2) are valid together because Newton's law is assumed to be valid. Equation (O.4), which follows, can be used to confirm this statement. From equations (10) and (O.2), using $e = D^1(x)/\big(D^0(x) - D^1(x)\big)$, the following equation is obtained.

$$g_{00}(x') = \frac{1 + 2e}{(1+e)^2} \tag{O.3}$$

Furthermore, the geodesic equation of P_1 is as follows [10].

$$\frac{\partial^2 r}{\partial \tau^2} = -\frac{c^2}{2}\frac{\partial g_{00}}{\partial r} \tag{O.4}$$

Here, τ is the proper time of P_1. Substituting g_{00} with its value given by equation (O.3), equation (O.4) becomes the following.

$$\frac{\partial^2 r}{\partial \tau^2} = c^2 \frac{e}{(1+e)^3}\frac{\partial e}{\partial r} \tag{O.5}$$

Then, once again, assumption (i) is used, and Newton's law is assumed valid. The asymptotic formulation of equation (O.5) is derived, relying on Newton's law with the asymptotic value of the right-hand side of equation (O.5).

$$-\frac{M_0 G}{r^2} \simeq c^2 e \frac{\partial e}{\partial r} \tag{O.6}$$

Here, M_0 is the mass of P. Assumption (iii) has been used: the contribution of P in equation (9) is far weaker than the sum of the other contributions. Therefore, asymptotically $D^1(x) << D^0(x)$, and asymptotically e is equal to 0 compared to 1. The solution of this differential equation (O.6) gives the following asymptotic value for e.

$$e \simeq \sqrt{\frac{R}{r}} \tag{O.7}$$

From this, it follows that $D^1(x)/D^0(x) = e/(1+e) \simeq \sqrt{R/r}$. This results in the following equation.

$$\frac{1_w(x,y_0)f(x,y_0)C^0(y_0)}{\Sigma_{n=0}^{\infty}1_w(x,y_n)f(x,y_n)C^0(y_n)} \simeq \sqrt{\frac{R}{r}} \tag{O.8}$$

It is assumed that the y_0 virtual particle is the only one associated with P. It was used $C^0(y_0) = C^1(y_0)$, which reflects the fact that $C^\mu(y_0)$ is a null four-momentum. Under assumption (iii), the denominator of equation (O.8) is constant, that is, independent of the x location. Therefore, this equation shows that the $1_w(x,y_0)\ f(x,y_0)\ C^0(y_0)$ contribution is proportional to $1/\sqrt{r}$. Under assumption (ii), it can be deduced that the $1_w(x,y_n)f(x,y_n)C^0(y_n)$ contributions are proportional to $1/\sqrt{\|x-y_n\|_3}$. $\|x-y_n\|_3$ is the spatial length calculated covariantly along the geodesic from y_n to x and observed in R_0.

Also, assuming r constant, and varying $C^0(y_0)$, from equation (O.8), the $1_w(x,y_0)$ $f(x,y_0)\ C^0(y_0)$ contribution is not proportional to $C^0(y_0)$ but proportional to $\sqrt{R}$, and therefore to $\sqrt{M_0}$ and $\sqrt{C^0(y_0)}$. This contradicts the hypothesis made during the construction of equation (9). Here, a proportionality to the square root of an energy is observed. The answer to this apparent contradiction is that the final picture is coherent. Therefore, one can replace each $1_w(x,y_n)\ f(x,y_n)C^0(y_n)$ contribution in equation (O.8) by $1_w(x,y_n)$ $\sqrt{C^0(y_n)/\|x-y_n\|_3}$. Now, from $f(x,y_0)\ C^0(y_0)$ being proportional to $\sqrt{C^0(y_0)/r}$ and assumption (ii), the result is the following.

$$\frac{1_w(x,y_n)\sqrt{\frac{C^0(y_0)}{r}}}{\Sigma_{n=0}^{\infty}1_w(x,y_n)\sqrt{\frac{C^0(y_n)}{\|x-y_n\|_3}}} \simeq \sqrt{\frac{R}{r}} \tag{O.9}$$

We can use $R = 2M_0G/c^2$ and $C^0(y_0) = M_0c$, in order to transform equation (O.9) into the following one.

$$G \simeq \frac{c^4}{2\left(\Sigma_{n=0}^{\infty}1_w(x,y_n)\sqrt{\frac{E(y_n)}{\|x-y_n\|_3}}\right)^2} \tag{O.10}$$

Here, $E(y_n) = C^0(y_n)c$ is the total energy of the particle located at y_n. Now, it is possible to use this result with VGWs. This means that the energy distributions of the P_i are replaced by the limits of the S_n^i distributions, as shown by equation (6). The result is equation (16), which is equation (O.10), without the $1_w(x,y_n)$ terms. Indeed, in any given x and y_n spacetime events, it is possible to find a m and a S_m^n distribution centered on y_n, such that the gravitational wave (GW) generated by the virtual particle of S_m^n propagates in x. Moreover, this S_m^n distribution can be chosen with a circle's radius as small as desired (remember that those S_m^n distributions are circular-like trajectories of virtual particles moving at the speed of light). In other words, after the limits of equation (6) are taken, each x spacetime event is reached by the VGW of each P_n particle.

Appendix P. Nuclear saturation

Let N be the number of baryons in the nucleus, and let β be a given baryon inside the studied nucleus. β is assumed to have the number 0 in the numbering of the baryons. The interaction force is proportional to the energy of the attractor. This is particularly true under assumption (A) from paragraph 8.1. Let us write the proportionality of the $F(y_n)$ force exerted by the baryon numbered n, located at y_n, on β.

$$F(y_n) = K\ G\ C^0(y_n)\ c \tag{P.1}$$

K is the proportionality constant. Its dependence on the interaction distance is neglected. From equations (P.1) and (16), the Π pressure exerted on β by the other baryons is given by the following equation.

$$\Pi = \frac{Kc^4 X}{2S} \tag{P.2}$$

$$X = \frac{N-1}{\left(\Sigma_{n=1}^{N-1} \frac{1}{\sqrt{\|x-y_n\|_3}}\right)^2} \tag{P.3}$$

In equation (P.2), S is a given external surface of the baryon. For example, it can be the surface of the minimum possible sphere surrounding the baryon. The $F(y_n)$ force from equation (P.1) is exerted on S. It has been assumed that all the baryons in the nucleus share the same $C(y_0)c$ energy. Equation (P.2) shows that Π varies in the same manner as X when N changes. However, X does not vary much, as shown by equation (P.3). If the dependence of K on the interaction distance were taken into account, the result would be the same. The result is that Π does not vary much with N, the number of baryons. More precisely, this variation is negligible compared to what it would be if G were a constant in equation (P.1).

Appendix Q. Surrounding: next developments

The surrounding gravitational model appears to solve many of the gravitational mysteries of today in a straightforward way. But it remains several mysteries to address. And this model will have to be tuned. The fixed radius value used for calculating the surrounding value might be replaced by a value varying slowly with the galaxy size. This modification would allow the model to explain Tully-Ficher law and the speed profiles of the small galaxies. Also, the brutal rectangle window used for calculating the surrounding value must be replaced by a smooth window, soon or later. A need for that appears for conforming surrounding to the wide binaries problem [25–27]. Also a shielding mechanism has been proven to appear in the study of the NGC 3310 galaxy, which might allow to replace the α constant of the model, by a more relevant term in the equation of motion. Also, possible regressions might arise, in which this G variation might induce wrong predictions in front of some given experimental data. This work remains to be done also.

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