Resources Policy 77 (2022) 102753

Contents lists available at ScienceDirect

Resources Policy

journal homepage: www.elsevier.com/locate/resourpol

# An economic framework for producing critical minerals as joint products

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ARTICLE INFO

*JEL classification:*
Q3
D2

*Keywords:*
Ore
Returns to scale
Geological input
Mineral recovery
Mineral policy
USGS
Rare earth elements

ABSTRACT

The topic of economics of critical minerals production has received little attention in the economics literature. This study presents a two-stage optimization model to frame the economics of critical minerals. In the first-stage, firms minimize cost to choose input levels, including the extraction of a common ore to produce a critical mineral with or after the production of a main mineral. We examine the impact of geological, cost, and technology parameters on the level of input use and the decision to further process for critical minerals. We characterize the marginal cost of producing critical minerals, which is used in the second stage to determine production decisions. Results suggest that (1) production of critical minerals could be expanded by investing in technically efficient technologies and technologies with increasing returns to scale, (2) prescriptive mandates requiring firms to process a given percentage of geological input to recover critical minerals would have unintended consequences such as increasing the marginal cost of producing main minerals, and (3) the supply elasticity of critical minerals depends on returns to scale of production.

## 1. Introduction

Non-fuel mineral commodities, produced from mines, are essential inputs for several downstream industries that make up a nation's economy, such as steel, aerospace, electronics, and renewable energy generation (Calvo and Valero, 2022). According to reports by the United States Geological Survey (2020b), the United States continues to significantly rely on foreign sources for raw and processed non-fuel mineral materials. For example, in 2020, the US imported more than 50% of the nation's consumption of 46 non-fuel minerals and 100% of the nation's consumption of another 17 minerals.

In the US, a sub-set of 50 non-fuel minerals have been deemed as *critical* based on a screening methodology that measures and assesses supply risk (United States Geological Survey, 2022). Nassar et al. (2020) and Nassar and Fortier (2021) estimate the supply risk of mineral commodities as a function of disruption potential (production concentration in few countries), trade exposure (high net imports as a percent of consumption), and vulnerability to changing market conditions such as increase in costs relative to prices. These minerals are deemed critical because they are "essential to the economic and national security of the United States", they have a "supply chain vulnerable to disruption, and serve an essential function in the manufacturing of a product, the absence of which would have significant consequences for the economy or national security". (Federal Register, 2017). Most critical minerals are difficult to substitute since they have specialized applications in key sectors such as the technology and energy sectors.

The US has low production (mine exploration, processing, refining, and secondary production from scrap) and high reliance on imports for a majority of the 50 critical minerals (Federal Register, 2017; Center, 2020). Production levels have not kept pace with increasing and new sources of demand which rely on critical minerals as a major input. This has generated concerns on the risk of supply disruption and the consequences this would have on the nation's economy and security. Addressing these concerns would require finding new deposits by using advanced technologies, investing in alternative methods of increasing the supply of critical minerals (e.g., processing mine wastes, etc.), and investing in technologies/processes that increase the efficient use and recovery of critical minerals as well as their extraction from multiple sources (Bonvillian, 2021; House, 2021). Such actions are expected to reduce the cost of critical minerals production, improve cost advantages over global competitors, and lead to increased production capacity in the US.

Like the US, other countries like Canada, Australia, and Japan have also identified certain non-fuel mineral commodities as critical and are developing strategies to encourage local production and reduce import reliance (Coulomb et al., 2015). For example, in Japan criticality of metals is measured by increase in demand, production

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https://doi.org/10.1016/j.resourpol.2022.102753
Received 6 July 2021; Received in revised form 30 April 2022; Accepted 3 May 2022
Available online 23 May 2022

concentrations in few countries, and recycling difficulty (Hatayama and Tahara, 2015). Australia's critical minerals strategy focuses on promoting investment in processing, providing incentives that lower costs and increase comparative advantage, and developing infrastructure that supports the critical minerals sector (Australian Government Department of Industry, 2021; Whittle et al., 2020).

Most critical minerals are produced as joint products after or with another mineral processing or recovery. The study by Nassar and Fortier (2021) finds that at least 23 of the 50 critical minerals in the US are predominately produced as joint products.[1] A joint production system allows mining companies to use a single ore deposit as an input (or raw material) to produce multiple mineral goods, the primary of which is the *main mineral*. A further processing of the ore deposit is required to produce one or more *critical minerals* (Jordan, 2018). For example, gallium is a critical mineral that can be recovered by processing bauxite and zinc ores; cobalt can be recovered from copper or other metal deposits; germanium can be produced from zinc concentrates; and tellurium can be recovered from copper refining (Goldfarb et al., 2017; Schulz et al., 2017; United States Geological Survey, 2020a,b). The joint production system generally includes geological, market, and technology parameters, which are essential considerations in framing the economic problem of critical minerals production. However, the literature lacks studies that examine the impact of such parameters on production decisions. So far the literature on critical minerals has focused on creating metrics to measure criticality (Coulomb et al., 2015; Nassar et al., 2020; McCullough and Nassar, 2017; Hayes and McCullough, 2018; Helbig et al., 2016; Hatayama and Tahara, 2015, 2018; Nassar and Fortier, 2021), framing the geopolitics of critical minerals production (Kalantzakos, 2020), addressing issues related to inequality and justice (Heffron, 2020), and examining the impact of critical minerals shortage on the development of renewable energy sources (Viebahn et al., 2015; McLellan et al., 2016). Without a comprehensive framework modeling relationships between incentives and constraints, governments might propose sub-optimal policies that do not result in the intended outcomes or create unintended consequences.

This study develops an economic model that can be used to frame the current economic problem with critical minerals production in the US (and other countries). In particular, the model in this work is useful for evaluating extraction of critical minerals that are produced after or during a main mineral extraction process using a joint production technology.

First, we present a generic joint production model where miners or firms make decisions in two stages. In the first stage, firms minimize costs and choose the optimal demand for inputs including extraction of the common input or ore used in the joint production. Firms also decide whether to further process for critical minerals or not by choosing the percent of geologically usable and available ore to process. We examine the impact of geological, cost, and technology parameters on the level of input use and the decision to further process for critical minerals. Second, given the conditional demand for inputs and resulting cost functions, we model the production decision of firms by maximizing profits in the second stage. Equilibrium conditions are examined under several scenarios to understand the role of geological, market, and technological parameters on critical minerals production. Market data from a sample of 17 critical minerals are used to illustrate model implementations by deriving threshold levels of average ore processing costs that allow for the production of each mineral under different assumptions.

Section 2 presents the building blocks of the model by using general functional forms and presenting the conceptual framework for the joint production system. Section 3 characterizes solutions for input demand and the cost function. Section 4 characterizes the production decision and illustrates model implementation using market data on critical minerals. Section 5 presents a discussion of questions for future work that can be used to evaluate potential policy options and Section 6 presents a concluding remark.

[1] Although some studies use the term *byproducts*, we use the term *joint products* because the ore needs to be intentionally processed further to produce critical minerals rather than critical minerals being incidentally produced.

## 2. Building blocks of the model

In this section, we present a generic model used to examine the production of two minerals, the main mineral and the critical mineral. We model the decision-making of firms by considering their production technology and cost of production. Firms decide to further process and produce critical minerals or not. Firms also decide how much inputs to use, and the level of production of the main and critical minerals.

While we acknowledge that, in many instances, mines produce more than just two minerals, we make this simplifying assumption to facilitate modeling. We believe there is no significant loss of generality because, even in cases where there are more than two minerals, decisions to exploit each critical mineral is made relative to the main mineral. We present a generic market demand for the main and critical minerals and the market structure features competition among several firms.

### 2.1. Production functions and costs

Mine owners or firms extract and process an ore deposit (subscript $o$) which contains the main (subscript $m$) and the critical (subscript $b$) mineral. The total quantity of ore extracted from the ground is represented by $q_o$ and this is the common input used in the production of the two mineral goods (main and critical). For example, copper porphyry deposits are used to recover tellurium during the production of copper (Goldfarb et al., 2017). Since countries like the US are well-endowed with several natural resources, the model assumes no short-run resource constraints for the mined ore (Schulz et al., 2017; Australian Government Department of Industry, 2021; Coulomb et al., 2015). The main mineral is processed and then sold in the main mineral's market. If firms decide to further process the ore, they can recover critical minerals and sell them in the critical minerals market.

We introduce a parameter $0 \leq \theta_1 \leq 1$ to represent the natural concentration rate of the main mineral found in the ore such that a total of $q_o\theta_1$ units of the ore are geologically available for the production of the main mineral. When $\theta_1$ approaches to one, the ore is fully endowed with the main mineral. The production function of the main mineral is represented by $q_m = f(\theta_1 q_o, z)$ where $z$ is a numeraire input used in the production and refining processes. For example, $z$ can be the amount of energy required for processing the main mineral.

After or during the extraction of the main mineral from the ore, the ore concentrate can further be processed to produce a critical mineral, the natural concentration rate of which is represented by a percentage $0 \leq \theta_2 \leq 1$. Typically critical minerals have a much lower concentration ratios compared to the main mineral. For example, copper porphyry deposits typically contain copper in concentrations of 0.2–1.5% while tellurium concentrations go up to 300 ppm (Moats et al., 2021). Thus, a total of $q_o\theta_2$ units of the ore are available to be used as an input for critical minerals production — for the context of critical minerals recovery we refer to this as geologically *usable and available ore*. However, not all firms decide to further process the ore to produce critical minerals so we use $0 \leq e \leq 1$ to capture the percentage of this geologically available input a given firm processes to produce the critical mineral. When $e = 1$ it means the firm chooses to use all available geological resources to produce the critical mineral and when $e = 0$ it means the firm decides not to produce the critical mineral at all.

Processing the ore for critical minerals has both costs and benefits and the firm weighs costs and benefits to decide the optimal rate of $e$. The variable $e$ also captures the rate of joint production of critical

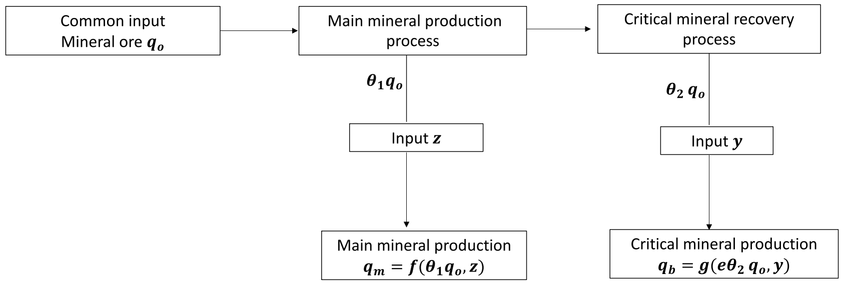


**Fig. 1.** A simplified joint production model.

minerals. Thus, the production function for the critical mineral can be presented as $q_b = g(e\theta_2 q_o, y)$ where $y$ is a numeraire input used for critical minerals production (e.g., processing reagents specifically used to extract critical minerals or energy consumed for extra flotation stage used to concentrate critical minerals). While the amount $q_o\theta_2$ represents the geologically usable and available input, the level $eq_o\theta_2$ represents the exact amount used in the production process. Fig. 1 presents a generic joint production model adopted from Fisher (2011).

The economic cost of production is made up of the cost of ore extraction, cost of input purchase, the cost of processing inputs and fixed costs. The price of the numeraire inputs, $z$ and $y$, are both assumed to be equal to one and $F$ represents fixed costs. First each firm mines and extracts the ore from the ground and prepares the ore for the production process (e.g., separating waste from usable input) where cost is a function of the quantity of ore mined, $C_0(q_o)$. This cost is also assumed to capture any user costs of mining. Second, the cost of processing the ore for the main mineral is represented by $C_1(q_o\theta_1)$ where cost is increasing in $q_o\theta_1$. Third, the cost of separating/recovering the critical mineral in the joint production is $C_2(eq_o\theta_2)$ where $e = 0$ leads to $C_2 = 0$. All costs are assumed to be increasing in the level of input use. Thus, the total cost of a representative firm is given as:

$$TC(q_o, z, y, e) = C_0(q_o) + C_1(q_o\theta_1) + C_2(eq_o\theta_2) + z + y + F \tag{1}$$

Each firm decides the level of input use ($z$, $y$), the quantity of ore to extract ($q_o$) and its rate of reliance on the joint production technology ($e$). Thus, $e$, $q_o$, $z$ and $y$ are choice variables of the model.

### 2.2. Demand for minerals

We consider the global market for minerals where there are $n \geq 1$ firms. The market structure is determined by a combination of factors including number of firms, product differentiation, and barriers to entry (Gocht et al., 1988). We model a representative firm $i$ which faces the world demand for minerals.

The inverse global demand function for the main mineral is presented by $p_{m,i} = p_m(\sum q_{m,i})$ where $i = 1, 2, \ldots, n$, and $dp_{m,i}/dq_{m,i}$ represents the own-effect whereas $dp_{m,i}/dq_{m,j}$ represents the cross-effect. $p_{m,i}$ is the price of the main mineral produced by firm $i$ and the term $\sum q_{m,i}$ represents total global production of the main mineral. Main minerals are produced in several countries and demanded in several industries such as the demand for copper as an input in manufactured goods and in the construction industry.

Similarly, we present a general inverse demand function for the critical mineral given by $p_{b,i} = p_b(\sum q_{b,i})$ where $dp_{b,i}/dq_{b,i}$ represents the own-effect and $dp_{b,i}/dq_{b,j}$ is the cross-effect. $p_{b,i}$ is the price of the critical mineral produced by firm $i$. Critical minerals have several applications in industries such as renewable energy, electronics, and infrastructure (Goldfarb et al., 2017).

The demand functions can be used to examine different market structures where $n = 1$ is a monopoly miner in the world market and a sufficiently large $n$ represents a competitive global market. With $dp_{m,i}/dq_{m,j} \neq 0$ and $dp_{b,i}/dq_{b,j} \neq 0$ one could also examine product differentiation across firms.

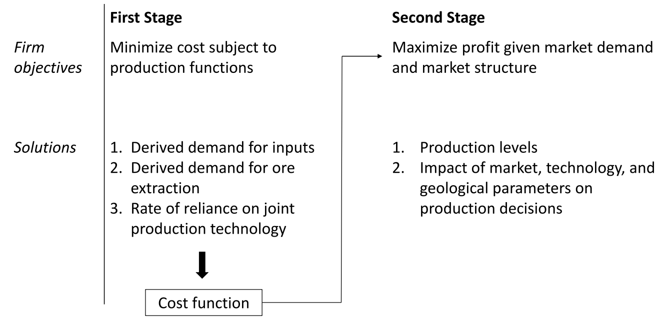


**Fig. 2.** Firm objectives and decision-making.

### 2.3. Firm objectives and constraints

We characterize and solve solutions for the choice variables by using two stages as presented in Fig. 2. In the first stage, each firm determines the conditional or derived demand for inputs ($z, y, q_o$) and determines the rate of use of the joint production technology ($e$). This is done by minimizing cost subject to the production functions. More formally, minimize $TC(q_o, z, y, e)$ subject to $q_m = f(.)$ and $q_b = g(.)$ to obtain solutions for, $e^*$, $q_o^*$, $z^*$, and $y^*$, as functions of $q_b$ and $q_m$.

For a representative firm, the Lagrangian equation for the cost minimization problem is given as follows:

$$L(q_o, z, y, e, \lambda, \mu) = TC(q_o, z, y, e) + \lambda[q_m - f(.)] + \mu[q_b - g(.)] \tag{2}$$

The optimal solutions are achieved when the following conditions hold:

$$\begin{aligned} 1 - \lambda\frac{df(.)}{dz} &= 0 \\ 1 - \mu\frac{dg(.)}{dy} &= 0 \\ \frac{dTC(.)}{dq_o} - \lambda\frac{df(.)}{dq_o} - \mu\frac{dg(.)}{dq_o} &= 0 \\ \frac{dTC(.)}{de} - \mu\frac{dg(.)}{de} &= 0 \\ q_m &= f(.) \\ q_b &= g(.) \end{aligned} \tag{3}$$

The Lagrangian multipliers, $\lambda$ and $\mu$, indicate the marginal cost of producing the main and critical minerals respectively (Baxley and Moorhouse, 1984). Solutions from this problem are inserted into Eq. (1) to derive the total cost function, $TC(q_m, q_b)$.

In the second stage, each firm $i$ maximizes profit to determine the level of production, $q_m$ and $q_b$, where profit is defined as revenue minus cost:

$$\pi_i = p_{m,i}q_{m,i} + p_{b,i}q_{b,i} - TC(q_{m,i}, q_{b,i}) \tag{4}$$

Given the global inverse demand functions, the following first-order conditions of the profit function are used to find each firm's production level given market demand and competition:

$$\frac{\partial \pi_i}{\partial q_{m,i}} = p_{m,i} - \frac{dp_{m,i}}{dq_{m,i}} q_{m,i} - \frac{dTC(.)}{dq_{m,i}} = 0$$
$$\frac{\partial \pi_i}{\partial q_{b,i}} = p_{b,i} - \frac{dp_{b,i}}{dq_{b,i}} q_{b,i} - \frac{dTC(.)}{dq_{b,i}} = 0 \quad (5)$$

By solving solutions for Eq. (5) each firm determines the production level of the main and critical minerals. The model is general enough to examine the strategic reaction of firms to the production level of other firms as well as examine the impact of shocks (e.g., technology improvement and regulatory changes) on production levels.

### 2.4. Simplifying assumptions

To solve the system of equations, derive closed form solutions, and perform comparative static exercises, three assumptions are made: a Cobb–Douglas production function, linear costs, and competitive markets. Each of these are discussed below.

We derive solutions for the conditional input demands by using a Cobb–Douglas production function (Rybak, 2019):

$$q_m = k_1(\theta_1 q_o)^{r_1} z^{r_2} \quad (6)$$

$$q_b = k_2(e\theta_2 q_o)^{t_1} y^{t_2} \quad (7)$$

The parameters $k_1, k_2 > 0$ represent the technical efficiency of the production of main and critical minerals, respectively.

**Assumption 2.1.** Main mineral production exhibits increasing returns to scale with $r_1 = r_2 = 1$. Returns to scale for critical minerals production is determined by $t_1 + t_2$.

We assume that main mineral production exhibits increasing returns to scale because it is computationally easier to calculate and derive comparative statics. Increasing returns to scale has been documented for several mines across the global, such as coal mining (Boyd, 1987), gold mining (Gajigo and Dhaou, 2015) and others (Roman and Daneshmend, 2000). For critical minerals production, the returns to scale is determined by the parameters $t_1$ and $t_2$ where $t_1 + t_2 = 1$ represents constant returns to scale, $t_1 + t_2 > 1$ increasing returns to scale, and $t_1 + t_2 < 1$ decreasing returns to scale. The parameter $t_1$ measures the rate at which $q_b$ can be increased with a one percent increase in the exact amount of usable ore the firm decides to use, $q_o\theta_2 e$. $t_2$ measures the rate at which $q_b$ can be increased with a one percent increase in input $y$.

Our second assumption is that all costs are positive constants to facilitate the derivation of solutions and interpretation of results. The average cost of extraction and preparing ore for production is given by $c_0$ dollars per unit of $q_o$. The average cost of processing the usable and available ore for the main mineral is $c_1$ dollars for each unit of ore used in main mineral production. The average cost of separating/recovering the critical mineral is $c_2$ dollars per unit of ore the firm decides to use. Thus, the total cost of a representative firm is given as $TC(q_o, z, y, e) = c_0 q_o + q_o[\theta_1 c_1 + c_2 e\theta_2] + z + y + F$.

Third, we assume that mineral markets are competitive and prices are determined in the world market where world quantity supplied is equal to quantity demanded. Each firm is a price-taker where the price of the main mineral is $p_m$ and the price of the critical mineral is $p_b$. Since we are interested in the production (or lack of) of critical minerals we assume that firms already produce the main mineral. For example, the US produces copper using primary and secondary refinery; and in 2019 the US exported 330 thousand metric tons of copper ores and concentrates and 140 thousand metric tons of refined copper (United States Geological Survey, 2020a,b). However, the US did not have any primary refinery production for critical minerals, such as gallium and tellurium, which are often processed after copper is refined as the main mineral. We consider a simple decision-rule where firms produce a critical mineral only when the average cost of processing ore for critical minerals is low enough that marginal costs do not exceed the price of the critical mineral, $p_b$. Mines will close and exploration for critical minerals will stop when average costs are high enough that prices fall below marginal costs (Henckens et al., 2016).

## 3. Demand for inputs and cost

We present solutions for the conditional input demand functions and the optimal rate of use of the joint production technology to recover critical minerals ($e$) by solving for the systems in Eq. (3):

$$q_o^* = [\frac{q_m}{k_1\theta_1(c_0 + \theta_1 c_1)}]^{1/2}$$
$$z^* = [\frac{q_m(c_0 + \theta_1 c_1)}{k_1\theta_1}]^{1/2}$$
$$y^* = [\frac{q_b}{k_2}(\frac{c_2 t_2}{t_1})^{t_1}]^{1/(t_1+t_2)} \quad (8)$$
$$e^* = [\frac{q_b}{k_2}]^{1/(t_1+t_2)}[\frac{c_2 t_2}{t_1}]^{-t_2/(t_1+t_2)}[\frac{k_1\theta_1(c_0 + c_1\theta_1)}{q_m}]^{1/2}[\frac{1}{\theta_2}]$$

The demand for inputs $z$ and $y$ ultimately depends on the demand for the main and critical minerals respectively, where $dz^*/dq_m > 0$ and $dy^*/dq_b > 0$. Demand for ore extraction depends on demand for the main mineral where $dq_o^*/dq_m > 0$ but not on demand for the critical mineral, $dq_o^*/dq_b = 0$. This suggests that demand for the main mineral is the main factor affecting the decision to extract the ore and how much. This is because once a given amount of ore is extracted the entire ore is available for critical mineral extraction/recovery, without affecting the production level of the main mineral. Thus, there is no additional need to extract more ore to produce more critical minerals.

**Lemma 3.1.** *Optimal demand for ore extraction is positively affected by demand for the main mineral, $dq_o^*/dq_m > 0$. Demand for the critical mineral does not affect the extraction of the common input (ore), $dq_o^*/dq_b = 0$.*

The cost parameters ($c_0$, $c_1$), production technical efficiency index ($k_1$), and the natural concentration rate ($\theta_1$) affect the optimal demand for ore extraction negatively. When the technical efficiency of production is high, the firm is able to use a lower quantity of usable ore as an input ($\theta_1 q_o$); and when the cost of extracting and processing the ore is low, firms can afford to extract more ore deposit. In addition, a higher concentration ratio $\theta_1$ reduces the need for mining higher quantities of the ore to extract a given quantity of main mineral.

The optimal percentage of use of geologically available ore for the production of critical minerals ($e^*$) is affected by all cost parameters ($c_0$, $c_1$, and $c_2$), by both the natural mineral availability indicators ($\theta_1$ and $\theta_2$), and by the technical efficiency index of both production functions ($k_1$ and $k_2$). The optimal $e^*$ positively depends on the demand for critical minerals and negatively on the demand for the main mineral. When demand for $q_b$ increases, the firm needs to increase $e$ to minimize costs for a given $q_o$. When demand for $q_m$ increases, the firm needs to reduce $e$ to minimize costs for a given $q_o$ (i.e., the firm needs to focus on meeting the increased demand for the main mineral).

An increase in the average cost of processing ore for critical minerals, $c_2$, reduces $e^*$. An increase in average cost of extracting and processing the main mineral, $c_0 + c_1\theta_1$, reduces the amount of ore extracted ($q_o$) and this in turn increases $e^*$ to minimize costs. An increase in $\theta_1$ increases $e^*$ because with a higher $\theta_1$ firms will extract lower quantities of the ore ($dq_o/d\theta_1 < 0$) and so they need to increase $e$ to minimize costs.

An increase in $\theta_2$ decreases $e^*$ because it is naturally easier to recover a given unit of critical mineral. A higher technical efficiency for critical mineral production lowers the need to use a higher percentage of usable and available ore for critical mineral recovery, $de^*/dk_2 < 0$. Finally, if the technical efficiency of producing the main mineral is high, the firm can afford to increase $e^*$, where $de^*/dk_1 > 0$.

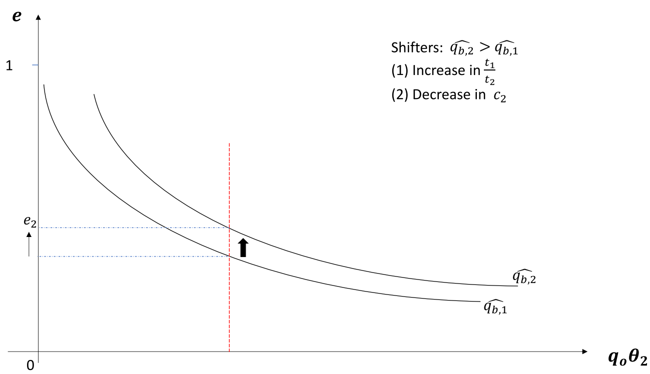


**Fig. 3.** The relationship between $q_o\theta_2$ and $e$.

**Proposition 3.2.** *Let the amount of geologically usable and available ore for critical minerals recovery be $q_o\theta_2$. The cost-minimizing percent to be used for critical minerals production, that is $e^*$, (i) increases with demand for critical mineral ($q_b$), natural concentration rate of main mineral $\theta_1$, technical efficiency of producing main mineral $k_1$, and the cost of ore extraction and main mineral production $c_0 + c_1\theta_1$, (ii) declines with the demand for main mineral $q_m$, natural concentration rate of critical mineral $\theta_2$, technical efficiency of producing critical mineral $k_2$, and the average cost of processing ore for critical mineral recovery $c_2$.*

### 3.1. Geological input and cost-minimizing choice

To further examine the relationship between usable and available ore input ($q_o\theta_2$) and the rate at which the firm decides to further process this ore for critical minerals production ($e$) we use solutions from the first-stage problem to map out the relationship between these two variables. To do this, we solve two of the first-order conditions from the Lagrangian equation, that is $dL(.)/dy = 0$ and $dL(.)/de = 0$, which yields the following relationship:

$$(\theta_2 q_o)e = \frac{yt_1}{c_2 t_2} \tag{9}$$

Eq. (9) implies that to produce a given level of critical mineral, say $\hat{q_{b,1}}$, the firm can use a lower amount of usable and available ore, $\theta_2 q_o$, together with a higher percentage $e$ or a higher $\theta_2 q_o$ together with a smaller percentage $e$. This gives rise to the negative relationship between $e$ and $\theta_2 q_o$ for a given $q_b$. Fig. 3 presents this negative relationship and also illustrates two factors that cause a shift of the curve. A higher $t_1/t_2$ ratio yields a higher $e$ for a given $\theta_2 q_o$ and a given $y$ (therefore a higher $q_b$ such that $\hat{q_{b,1}} < \hat{q_{b,2}}$) because of the relatively higher output elasticity of critical mineral production for a percent change in usable and available ore. Similarly, for a given $\theta_2 q_o$ and $y$ a firm can produce a higher level of critical minerals by increasing $e$ when $c_2$ declines.

**Proposition 3.3.** *For a given amount of usable and available ore for critical minerals recovery $q_o\theta_2$ and a given amount of input $y$, firms increase $e$ to produce more critical minerals ($q_b$) when $c_2$ declines and/or when $t_1/t_2$ increases.*

The amount $q_o\theta_2$ can be considered as a geological input (naturally usable and available for critical minerals production), where as $e$ is the share of this geological input which is cost-minimizing to further process. Each unit of geological input corresponds to a given cost-minimizing $e$ as presented by the curve in Fig. 3. For a given $q_o\theta_2$ an $e$ above or below the curve is not cost-minimizing to process and leads to higher cost.

### 3.2. Derivation of marginal costs

The characterization of marginal costs is important because firms make production decisions by comparing the marginal cost of production to prices. When all inputs are optimally chosen and when $e$ is optimal as given in Eq. (8), total cost is calculated as a function of outputs, $TC(q_m, q_b)$. We calculate two marginal costs, one with respect to $q_m$ and the other with respect to $q_b$ as follows:

$$MC_m = \frac{dTC}{dq_m} = \sqrt{\frac{c_0 + \theta_1 c_1}{k_1\theta_1 q_m}} \tag{10}$$

$$MC_b = \frac{dTC}{dq_b} = q_b^{(1-t_1-t_2)/(t_1+t_2)}[(\frac{c_2}{t_1})^{t_1}(\frac{1}{t_2})^{t_2}\frac{1}{k_2}]^{1/(t_1+t_2)} \tag{11}$$

Eq. (10) implies that marginal cost of producing an additional unit of the main mineral is affected by, (1) the natural concentration rate $\theta_1$ where a higher concentration rate has a cost-reducing effect ($dMC_m/d\theta_1 < 0$), (2) cost parameters where a higher average cost of extracting and processing increases marginal cost ($dMC_m/dc_1 > 0, dMC_m/dc_0 > 0$), and (3) the technical efficiency of production where higher technical efficiency reduces marginal cost ($dMC_m/dk_1 < 0$).

The marginal cost of producing an additional unit of critical mineral is affected by returns to scale of production, the technical efficiency of the production function ($dMC_b/dk_2 < 0$), and the average cost of processing ore for critical minerals ($dMC_b/dc_2 > 0$). The natural concentration rate $\theta_2$ does not affect the marginal cost of production because $e$ has been optimally chosen for the given $\theta_2$. This means that $\theta_2$ has no net effect on marginal cost of producing critical minerals as long as the firm performs the first-stage cost-minimization problem before maximizing profits.

Suppose $e$ is given and not optimally chosen when the common input or ore is extracted, then marginal costs are calculated as follows:

$$MC_m = \sqrt{\frac{c_0 + \theta_1 c_1}{k_1\theta_1 q_m}} + \frac{c_2\theta_2 e}{2\sqrt{q_m k_1\theta_1(c_0 + \theta_1 c_1)}} \tag{12}$$

$$MC_b = [\frac{t_2}{t_1 + t_2}]q_b^{(1-t_1-t_2)/(t_1+t_2)}[(\frac{c_2}{t_1})^{t_1}(\frac{1}{t_2})^{t_2}\frac{1}{k_2}]^{1/(t_1+t_2)} \tag{13}$$

When $e$ is given, a higher $e$ increases the marginal cost of producing the main mineral. This suggests that the production of the two minerals is not synergistic when $e$ is not optimally chosen. The production of more critical minerals by increasing $e$ increases the firm's marginal cost of producing the main mineral. In addition, a higher $\theta_2$ leads to a higher marginal cost of producing the main mineral which suggests that the joint production does not benefit firms unless they choose $e$ optimally.

When $e$ is exogenously given, the production of critical minerals interferes with the production of the main mineral by raising the entire $MC_m$ curve, and hence requiring a higher main mineral price ($p_m$) to maximize profits or minimize losses. Furthermore, with $e > 0$, an increasing level of $e$ raises marginal cost $MC_m$ and signals to the firm to reduce production of the main mineral.

**Proposition 3.4.** *The natural concentration rate of critical minerals, $\theta_2$, does not have a net effect on marginal costs as long as $e$ is optimally chosen.*

**Proposition 3.5.** *When $e$ is exogenously given it increases the marginal cost of producing the main mineral.*

When $e$ is given, marginal cost of producing critical minerals is scaled down by a factor $t_2/(t_1+t_2) < 1$ and this will scale up production decisions without altering general comparative static results. Since marginal cost is now lower by the given scale, it is more likely that a given price induces firms to produce critical minerals. The reason is because the decrease in cost is passed on to the production of the main mineral when $e$ is exogenously set. For example, prescribing a given $e$ (e.g., through mandates) could affect main mineral production unless it aligns with individual firms' first-stage cost-minimization problem.

This result suggests that regulations that attempt to set the level of ore mining firms should process for critical minerals, $e$, could have unintended consequences. Assuming that mining firms are able to optimally set $e$, government regulation that mandates $e$ is likely to set $e$ at a level that is not optimal. The results of this work shows that setting $e$ sub-optimally will interfere with the production of the main mineral and increase the marginal costs of the main mineral. Thus, this work shows that government mandates to compel mining firms to increase $e$ is not sound policy. Similarly, Tilton et al. (2018) argue against the use of production quotas, standards, and prescriptive technology (e.g., output goals, requiring a particular technology or processing method, requiring recycling, requiring the use of renewable resources, etc.) to increase the production of critical minerals.

## 4. Production decision in competitive markets

In this section we focus on the decision to produce critical minerals. We determine combinations of parameters (average costs $c_2$, output elasticity $t_1$ and $t_2$, returns to scale $t_1 + t_2$, and technical efficiency $k_2$) that yield a production decision in competitive markets. These parameters affect the marginal cost of producing critical minerals and hence are expected to affect production decisions. For instance, a higher $c_2$ increases marginal cost of critical minerals, a higher $k_2$ reduces it, and the effect of $t_1/t_2$ depends on the ratio (e.g., greater or less than one) and returns to scale.

We are interested in identifying scenarios that lead to lower/higher production of critical minerals. We do this by solving the profit maximizing or loss-minimizing problem (Eq. (5)) when the market price is given. Solutions are derived by setting prices equal to marginal costs for both minerals. We consider a local firm's (e.g., a miner in the US) production decisions by comparing its marginal costs to the given market price. It is important to differentiate between the economic problem of critical minerals production at the local versus global level because when local miners are not producing the mineral, several other miners elsewhere may still be in operation. The model in this study does not address strategic responses in the global context.

### 4.1. Constant returns to scale

First we consider constant returns to scale where $t_1 + t_2 = 1$. Fig. 4 presents the results. With $p_{b,1}$ and $t_1 = t_2$, a firm produces critical minerals only if $c_2 \leq c_2^*$, that is if the average cost of processing ore to recover critical minerals is less or equal to $c_2^*$. If $k_2$ increases then $c_2^*$ also increases, making it more likely to have the production of critical minerals. With $p_{b,1}$ if $t_1 > t_2$ then a much lower $c_2$ ($c_2 < c_2^*$) is required for production to take place making it less likely to produce. Alternatively when $t_1 > t_2$, at a given $c_2$ the price should be higher than $p_{b,1}$ to induce production of critical minerals.

For constant returns to scale, the level of marginal cost is higher with a higher $t_1/t_2$ ratio for the range $c_2 > \hat{c_2}$. Fig. 4 shows that a higher $t_1/t_2$ leads to a higher level of marginal cost for a given $c_2 > \hat{c_2}$ requiring a high price for profit-maximization or loss minimization. Even if a higher $t_1/t_2$ ratio with constant returns to scale allows for more volumes to be produced (by increasing $e$) at a lower overall cost, the level of marginal cost will be raised for the range $c_2 > c_2^*$ and a higher $c_2$ exacerbates this.

For the range $c_2 < \hat{c_2}$ marginal cost is lower with $t_1/t_2 > 1$ than with $t_1 = t_2$ (cost reducing effect of output elasticity kicks in). In this scenario, if price is sufficiently low then firms with $t_1/t_2 > 1$ are more likely to produce critical minerals than firms with $t_1/t_2 = 1$.

Fig. 4 serves as a decision-making tool for critical minerals production which exhibits a constant returns to scale. If firms have information about the cost parameter, $c_2$, and production parameters, $t_1/t_2$ and $k_2$, they can determine whether they should produce or not given market prices. Firms can also predict how high prices should be, how efficient production should be, or how low costs should be for them to decide to produce.

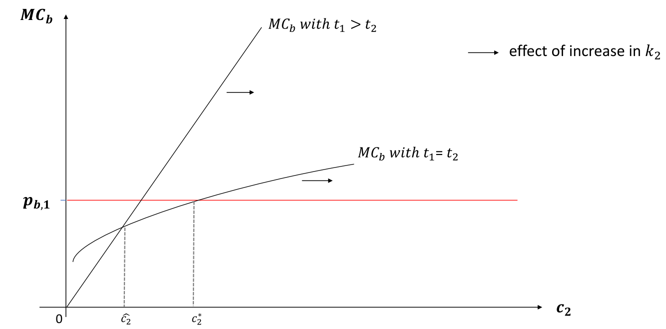


**Fig. 4.** Price, marginal cost, and $c_2$ with constant returns to scale.

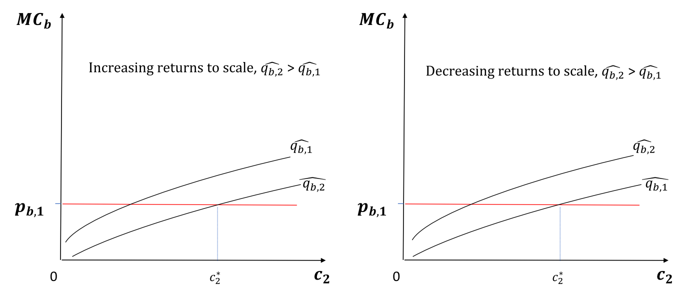


**Fig. 5.** Price, marginal cost, and $c_2$ with increasing and decreasing returns to scale.

### 4.2. Decreasing and increasing returns to scale

Fig. 5 illustrates that for a given $c_2$, when there is increasing returns to scale, firms need to increase production to reduce marginal costs, and with decreasing returns to scale firms need to reduce production to reduce marginal costs. With increasing returns, if $c_2 = c_2^*$ and given the market price, the firm should produce the higher quantity, $q_{\hat{b},2}$, to maximize profits or minimize losses. If the firm produces the lower quantity $q_{\hat{b},1}$ marginal costs would be higher requiring a higher price for profit maximization or loss minimization. With decreasing returns, if $c_2 = c_2^*$ and given the market price, the firm should produce the lower quantity, $q_{\hat{b},1}$, to maximize profits or minimize loss. If the firm produces the higher quantity $q_{\hat{b},2}$ marginal costs would be higher requiring a higher price for profit maximization or loss minimization.

**Proposition 4.1.** *Consider any type of returns to scale and a global price of $p_{b,1}$ for critical minerals. Firms decide to produce critical minerals when $c_2 \leq c_2^*$.*

Given the current low level of domestic production of most critical minerals in the US (United States Geological Survey, 2020a,b), technologies used for critical minerals recovery and processing are less likely to exhibit increasing returns to scale, and less likely to have a higher $k_2$. Furthermore, the low level of production of critical minerals can be explained by a relatively high $c_2$ where $c_2 > c_2^*$. In fact, lead mines in the US, for instance, choose to ship their concentrate overseas rather than refine lead in the US and possibly extract critical minerals such as bismuth, tellurium, and germanium.

### 4.3. Critical mineral supply and supply elasticity

We consider the case of decreasing and constant returns to scale in this sub-section ($t_1 + t_2 \leq 1$) to understand the responsiveness of local firms to a change in the world price. The profit maximizing (or loss minimizing) condition holds when price equals to marginal cost:

$$q_b = [\frac{p_b}{\Psi}]^{(t_1+t_2)/(1-t_1-t_2)} \quad (14)$$

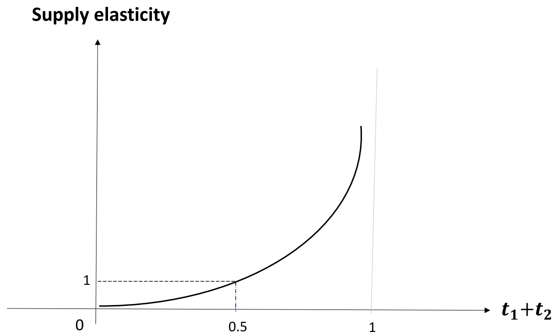


**Fig. 6.** Price elasticity of supply of critical minerals.

We define $\Psi = [(\frac{c_2}{t_1})^{t_1}(\frac{1}{t_2})^{t_2}\frac{1}{k_2}]^{1/(t_1+t_2)} > 0$. With constant returns to scale $MC_b = \Psi$ and a local firm's decision is to produce critical minerals as long as $p_b \geq \Psi$. With decreasing returns to scale, Eq. (14) provides a local firm's supply of critical minerals .

For the given firm, the price elasticity of supply of critical minerals is equal to $(t_1 + t_2)/(1 - t_1 - t_2)$. This shows that supply elasticity depends on the output elasticities ($t_1$ and $t_2$) and neither on natural concentration rates nor cost parameters. With constant returns to scale, a local firm is perfectly responsive to a change in the market price - a very small decline in world price is enough to induce a large reduction in production. This implies that with constant returns to scale, firms are vulnerable to the price dynamics in the world market, and could halt production when world prices fall by a small amount.

Fig. 6 shows that as the technology gets closer to constant returns to scale, elasticity increases approaching to a perfectly elastic supply. There is a range of $t_1+t_2 > 0.5$ ($t_1+t_2 < 0.5$) where elasticity is greater (less) than one where the firm is very (not) responsive to a change in the market price. While Tilton et al. (2018) argue that short-run supply is less elastic, Fig. 6 illustrates that elasticity depends on output elasticities and returns to scale. For example, with decreasing returns to scale supply may be less responsive to price.

**Proposition 4.2.** *Suppose the production of critical minerals exhibits constant returns to scale, then an individual firm has a perfectly elastic supply for critical minerals production. Suppose the production function exhibits decreasing returns to scale, then the price elasticity of supply depends on output elasticities, $t_1$ and $t_2$.*

### *4.4. Model implementation*

Many of the mineral deposits in the US have the potential to produce critical minerals that are on the United States Geological Survey's critical list (Orris and Grauch, 2002; Burger et al., 2018; Mercer et al., 2020). For example, some lead deposits can produce cobalt (Hitzman et al., 2017), some zinc deposits can produce germanium and gallium (Bonnet et al., 2017), and some copper deposits can produce tellurium (Moats et al., 2021; Calvo and Valero, 2022). Rather than focusing on all the 50 critical minerals, which in most cases are difficult to compare due to differences in production and recovery processes, we focus on critical minerals that are predominately produced as joint products. Nassar and Fortier (2021) present a list of 23 such minerals and we were able to find market data for 17.

Market data are obtained from the United States Geological Survey Mineral Commodities Summary (United States Geological Survey, 2020a). Table 1 indicates that the 17 critical minerals are heterogeneous with respect to import volumes, market prices, and the US's net import reliance rate. For example, the net import reliance rate for gallium and tellurium is among the highest while net import rates for palladium and zirconium are lower.

In Table 1 we present values for the maximum $c_2$ that allows a firm to produce a given critical mineral, i.e., $c_2^*$ measured in dollars per unit of processed ore. We do this by setting the given price equal to marginal cost and solving for $c_2$ under several scenarios. Any value of $c_2$ lower than the calculated amount makes critical minerals production feasible because it leads to prices being at least as high as marginal costs. We consider a *benchmark scenario* with constant returns to scale where $t_1 = t_2 = 0.5$ and $k_2 = 1$ and estimate values for the maximum value of $c_2$ that allows production. Next, we consider four alternative scenarios and calculate changes in $c_2$ against the benchmark scenario.

The four alternative scenarios are: (i) constant returns to scale with $t_1 = t_2 = 0.5$ and $k_2 = 2$, which we refer to as a complete or 100% *improvement in technology*, (ii) constant returns to scale with $t_1 = 0.9, t_2 = 0.1, k_2 = 1$ which we refer to as an 80% *improvement in output elasticity* with respect to usable and available ore, (iii) decreasing returns to scale where $t_1 = t_2 = 0.25, k_2 = 1$, and (iv) increasing returns to scale where $t_1 = t_2 = 0.6, k_2 = 1$. For increasing and decreasing returns to scale we consider $q_b$ for each mineral to be equal to 1% of total imports. Thus, we assume that a given local firm is considering to produce 1% of total imports.

To illustrate how to interpret results in Table 1 we consider tellurium. According to Anderson (2017) the production of tellurium is directly affected by the production of the main mineral from which it is derived — copper. In 2021, the average price of tellurium was $56 per kilogram (kg), and the US imported 12 thousand kg. The US is a net importer of tellurium with very low primary production (United States Geological Survey, 2017). We find that production is feasible for the benchmark case only when $c_2 < \$784$ per kg of ore, that is the average cost of processing each kg of usable and available ore should be below $784 for each kg of ore processed. The scenario that increases the likelihood of tellurium production is a technology that exhibits increasing returns and improvement in technical efficiency ($k_2$).

**Constant returns to scale**: For the benchmark case of constant returns to scale, minerals with a higher price have a higher $c_2$ suggesting that a high market price allows production of high-cost minerals. With a 100% improvement in technology, the range of $c_2$ increases by 300% for all minerals, allowing even more high-cost minerals to be produced.

With the given *improvement in output elasticity* the range of $c_2$ that allows production shrinks for all minerals. This result illustrates the impact of a higher $t_1/t_2$ on increasing marginal costs and discouraging production. Even if improvements in output elasticity with respect to usable and available input allows for a higher level of production, by increasing $e$ for given level of inputs ($y$ and $\theta_2 q_o$), (see Fig. 4 where $c_2^* > \hat{c}_2$) it leads to an increase in the level of marginal costs when there is constant returns to scale. Thus, firms will not produce unless the price increases to compensate for the high marginal cost.

**Increasing returns to scale**: When we consider the case where a given local firm is considering to produce 1% of the nation's gross imports, the estimates for $c_2$ are higher than those with the benchmark scenario. Higher values of $c_2$ indicate that more high-cost minerals can be produced with increasing returns to scale. Increasing returns allows firms to enjoy economies of scale by producing larger quantities and for each mineral each firm needs to produce at least 1% of total imports for production to be more likely with increasing returns to scale than constant returns to scale.

With increasing returns to scale, more firms could find it profitable to produce more quantities because the $c_2$ should be significantly high for production to halt. This suggests that right now, the US may not yet have increasing returns to scale; and that even with constant returns to scale the $k_2$ may be too low to allow for profitable production.

**Decreasing returns to scale**: Decreasing returns to scale implies that firms would have to produce less to lower marginal costs. Our estimates for $c_2$ are all close to 100% lower than the benchmark scenario suggesting that due to decreasing returns to scale the probability of production will decline for each mineral, because the cost parameter $c_2$ needs to be very low and in most cases close to zero.

**Table 1**
Sample of critical minerals produced as joint products.

| Critical minerals | Import reliance (%) | Average price ($/kg) | Imports for consumption (thousand kg) | $c_2^*$ for benchmark scenario ($/kg ore) | Change in $c_2^*$ with improved technology | Change in $c_2^*$ with improved output elasticity |
|---|---|---|---|---|---|---|
| Antimony | 81 | $5.89 | 20631 | $8.66 | 300% | -94% |
| Arsenic | 100 | $0.98 | 10500 | $0.24 | 300% | -68% |
| Bismuth | 93 | $6.00 | 1650 | $8.99 | 300% | -94% |
| Cadmium | < 50 | $2.29 | 375 | $1.31 | 300% | -85% |
| Cobalt | 76 | $32.98 | 9740 | $271.94 | 300% | -99% |
| Gallium | 100 | $379.50 | 182.46 | $36,005 | 300% | -100% |
| Germanium | > 50 | $885 | 30.3 | $195,806 | 300% | -100% |
| Hafnium | n.a | $750 | 16 | $140,625 | 300% | -100% |
| Indium | 100 | $395 | 115 | $39,006 | 300% | -100% |
| Iridium | n.a. | $52,518.57 | 1.62 | $689,549,944 | 300% | -100% |
| Palladium | 35 | $70,901 | 76.4 | $1,256,740,675 | 300% | -100% |
| Rhodium | n.a. | $360,251 | 20.7 | $32,445,203,738 | 300% | -100% |
| Ruthenium | n.a | $8,739 | 13.9 | $19,094,878 | 300% | -100% |
| Tellurium | > 95 | $56 | 12 | $784.00 | 300% | -99% |
| Vanadium | 99 | $14.77 | 3463 | $54.55 | 300% | -97% |
| Yttrium | 100 | $34.00 | 650 | $289.00 | 300% | -99% |
| Zirconium | < 25 | $1.38 | 17932 | $0.48 | 300% | -77% |

Overall the results in this sub-section suggest that decreasing returns to scale combined with lower technical efficiency of technologies used for critical mineral recovery may contribute to the low production and high import reliance in the US. In addition, the average cost of processing ore for critical minerals ($c_2$) may be relatively high in the US compared to what allows for profit maximization or loss minimization.

## 5. Policy implications

### 5.1. Current policy approaches

Since 2011, the US mineral policy has focused on building the knowledge infrastructure to strengthen domestic production and establish a framework to define and identify critical minerals. In December 2017, Executive Order 13,817 tasked the Department of Interior with publishing a list of critical minerals which led to the development of a federal strategy towards critical minerals.

In 2021, the United States Department of Energy, as the key agency in developing the national federal strategy on critical minerals, published its strategy on how to support the domestic critical minerals supply chain. The goal of the strategy is to develop the domestic supply chain and encourage the private sector to produce and process more critical minerals. This is to be achieved through a series of actions and initiatives aimed at increasing the productive capacity of US critical minerals production through supporting the development of primary, secondary, and unconventional sources of critical minerals (United States Department of Energy, 2021).

One of the policy actions is to support research and development for novel technologies and processes that increase the efficient use of critical minerals, recover more critical minerals, and substitute them with other materials or commodities. The initiative also includes finding new deposits using advanced mapping technologies or alternative methods of increasing the supply of critical minerals such as recycling and/or processing mine wastes. Furthermore, opening up federal lands for exploration and reducing the permitting framework, is a proposed policy action which is expected to speed up the production of critical minerals by making more exploration land available, and reducing regulatory burdens, respectively (United States Department of Energy, 2021).

### 5.2. Considerations for future research

In this study, we show that prescriptive mandates requiring firms to process ore for critical minerals recovery could interfere with main mineral production by raising marginal costs. In addition, the findings of this study suggests that the economic problem of critical minerals production could be addressed by investing in technology improvements to increase $k_2$ and investing in technologies that exhibit increasing returns.

However, now, the question is, would these new technological developments happen without policy support and without government intervention? Tilton et al. (2018) argue that public policies should not be used to sponsor research and development (R&D) or promote new processing technologies beyond what is signaled by market forces. This is because with short-run shortages, prices will rise thereby incentivizing (1) miners to produce more or invest more in explorations, (2) industries to lower demand (e.g., by discovering cheaper substitutes or switching to new processes that use fewer critical mineral inputs), (3) stockpiling, and (4) undertaking R&D as an insurance against future market shocks (Tilton et al., 2018). This means, miners will invest in new technology by comparing their costs and benefits and markets create the incentive to invest in new technologies. Thus, Tilton et al. (2018) recommend policies that address market failure problems such as supporting education as a public good, addressing information asymmetry problems by supporting collection and dissemination of data, and reducing negative externalities from pollution.

We argue that more research is needed to examine and measure the extent of market inefficiencies and welfare losses created by supporting policies. For example, studies can examine under what conditions policy-induced welfare losses outweigh the economic value achieved by reducing potential supply risks and vulnerabilities, even by a marginal amount. This is because even when the probability of supply disruption is low, the expected costs if it occurs may, under some circumstances, be higher than short-run welfare losses caused by policy intervention. Research is needed to identify and quantify (any) net benefits of policy intervention such as decreasing the supply risk, reducing the probability of disruption of production activities, reducing the extent of perceived or real vulnerabilities, reducing the criticality of the 50 minerals, establishing a domestic source of supply, etc., all of which could potentially be economically valuable ‘services’. More research is needed to measure any potential policy benefits and compare these to policy-induced social welfare losses. In addition, before ruling out policy intervention, studies should be conducted to understand society's willingness to pay to avoid some of the stated risks and disruptions or reduce their likelihood of occurrence.

Moreover, some countries (e.g., Australia, Canada) are currently actively devising strategic policies and approaches to enhance their competitive advantage (Trade, 2019; Commonwealth of Australia, 2022; Government of Ontario, 2022) while others have direct subsidizing policies to support both production and R&D (Yi et al., 2021; Andersson, 2020). Thus, there may be some room for considering and evaluating strategic policy in response to what other countries are doing. Finally, given the uncertainties around when comparative advantages will be organically achieved for US miners, and given the uncertainties in demand among the downstream industries (e.g., mining firms are not sure how much cobalt or lithium the battery market will need because battery makers can always revise the battery chemistry to reduce demand if the price of one or the other mineral input becomes too high) more studies are needed to examine the upstream and downstream market dynamics in order to identify whether and how policy actions can be used to cost-effectively harness market forces to address the economic problem of critical minerals production in the US (Fattahi, 2017; McNulty and Jowitt, 2021).

## 6. Conclusion

The topic of economics of critical minerals production has received little attention in the economics literature. This study contributes to the economics of critical minerals production with foundations in microeconomic theory that can be translated to policy implications. The study presents a generic joint production model where miners or firms make decisions in two stages. In the first stage, firms minimize costs to choose the level of inputs including extraction of an ore used to produce the main mineral and possibly a critical mineral, if firms decide to further process the ore. We examine the impact of geological, cost, and technology parameters on the level of input use and the decision to further process for critical minerals. We find that the optimal demand for ore extraction is positively affected by demand for the main mineral, but demand for critical minerals does not affect the decision to extract the common input.

Furthermore we find that firms choose to use a higher percent of geologically usable and available ore for critical minerals production when (i) demand for critical minerals, the natural concentration rate of main mineral, $\theta_1$, the technical efficiency of producing the main mineral, $k_1$, and/or the cost of ore extraction and main mineral production, $c_0 + c_1\theta_1$, increase, and (ii) the demand for main mineral, natural concentration rate of critical mineral, $\theta_2$, technical efficiency of producing critical mineral, $k_2$, and the cost of processing ore for critical mineral recovery, $c_2$, decline. As part of the first-stage solutions, we also find that firms can produce more critical minerals when either the cost of processing ore declines or the relative output elasticity of the geologically usable and available ore improves. Using derived input demand, we show that the natural concentration rate of a mineral affects its marginal cost only for the main mineral but not the critical mineral.

In the second stage, we model the production decision of firms by maximizing profits to examine the role of geological, market, and technology parameters on critical minerals production under several scenarios. Our results show that firms are more likely to produce critical minerals when the joint production technology is more technically efficient and when average cost of processing the input ore for critical mineral recovery is low. The natural concentration rate of critical minerals does not affect marginal costs and hence does not influence production decisions. Market data from 17 critical minerals are used to derive maximum thresholds of average ore processing costs that allow for the production of critical minerals.

This study is not without limitations. One limitation of the model is that it only considers marginal cost where the assumption is that firms produce critical minerals if price is higher than the marginal cost. In reality, because there is limited capital available to mining firms, they spend their money where they will get the highest returns. So to actually invest in R&D (beyond exploration), they will have to believe that prices will be high enough to yield higher rate of returns than other mining commodities. Another limitation is the assumption of competitive mineral markets. While for some mineral commodities, local markets are less competitive due to barriers to entry and/or concentration of market power, the global market may be more competitive. In addition, most mineral markets are subject to environmental and safety regulations that increase the cost of production (Wilkerson, 2010). Such regulations and environmental standards may be more stringent in some countries such as the US and Canada compared to others, further lowering cost advantages.

## CRediT authorship contribution statement

**Mahelet G. Fikru:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Resources, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. **Kwame Awuah-Offei:** Conceptualization, Formal analysis, Investigation, Methodology, Resources, Software, Validation, Writing – original draft, Writing – review & editing.

## Data availability

Data will be made available on request.

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