**Constraint Satisfaction Problems (CSPs)**

A Constraint Satisfaction Problem (CSP) is a mathematical problem where we need to find a set of values for a given set of variables that satisfy a set of constraints. These constraints define the relationships between the variables.

**Components of a CSP**

* **Variables:** A finite set of variables, each of which can take a value from a finite domain.
* **Domains:** A finite set of possible values for each variable.
* **Constraints:** A set of restrictions that specify which combinations of values are allowed for the variables.

**Types of Constraints**

* **Unary Constraints:** Constraints involving a single variable.
* **Binary Constraints:** Constraints involving two variables.
* **Higher-order Constraints:** Constraints involving more than two variables.

**Solving CSPs**

There are several techniques for solving CSPs:

* **Backtracking:** A systematic search algorithm that explores the search space by trying different values for the variables.
* **Forward Checking:** A technique that propagates constraints to reduce the domain of unassigned variables during backtracking.
* **Arc Consistency:** A technique that ensures that for every pair of variables, there is at least one value in the domain of each variable that satisfies their constraint.
* **Local Consistency:** A more general form of consistency that ensures that there is a consistent assignment for a subset of variables.
* **Heuristics:** Techniques that can help to guide the search process, such as choosing variables with the smallest domain or constraints that are most restrictive.

**Applications of CSPs**

CSPs have a wide range of applications, including:

* **Artificial Intelligence:** Planning, scheduling, and reasoning.
* **Computer Vision:** Image analysis and scene interpretation.
* **Natural Language Processing:** Parsing and semantic analysis.
* **Robotics:** Motion planning and task coordination.
* **Engineering Design:** Circuit layout and VLSI design.

**Example: Map Colouring**

A classic example of a CSP is the map colouring problem. Given a map with countries and their borders, the goal is to assign a color to each country such that no two adjacent countries have the same color. This problem can be formulated as a CSP where the variables are the countries, the domains are the available colours, and the constraints ensure that adjacent countries have different colours.