

C H A P T E R

5



Numbers and Operations Review

This chapter reviews key concepts of numbers and operations that you need to know for the SAT. Throughout the chapter are sample questions in the style of SAT questions. Each sample SAT question is followed by an explanation of the correct answer.

► Real Numbers

All numbers on the SAT are real numbers. Real numbers include the following sets:

- **Whole numbers** are also known as counting numbers.
0, 1, 2, 3, 4, 5, 6, . . .
- **Integers** are positive and negative whole numbers and the number zero.
. . . -3, -2, -1, 0, 1, 2, 3 . . .
- **Rational numbers** are all numbers that can be written as fractions, terminating decimals, and repeating decimals. Rational numbers include integers.
 $\frac{3}{4}$ $\frac{2}{1}$ 0.25 0.38658 $0.\overline{666}$
- **Irrational numbers** are numbers that cannot be expressed as terminating or repeating decimals.
 π $\sqrt{2}$ 1.6066951524 . . .

Practice Question

The number -16 belongs in which of the following sets of numbers?

- a. rational numbers only
- b. whole numbers and integers
- c. whole numbers, integers, and rational numbers
- d. integers and rational numbers
- e. integers only

Answer

d. -16 is an integer because it is a negative whole number. It is also a rational number because it can be written as a fraction. All integers are also rational numbers. It is not a whole number because negative numbers are not whole numbers.

► Comparison Symbols

The following table shows the various comparison symbols used on the SAT.

SYMBOL	MEANING	EXAMPLE
=	is equal to	$3 = 3$
≠	is not equal to	$7 \neq 6$
>	is greater than	$5 > 4$
≥	is greater than or equal to	$x \geq 2$ (x can be 2 or any number greater than 2)
<	is less than	$1 < 2$
≤	is less than or equal to	$x \leq 8$ (x can be 8 or any number less than 8)

Practice Question

If $a > 37$, which of the following is a possible value of a ?

- a. -43
- b. -37
- c. 35
- d. 37
- e. 41

Answer

e. $a > 37$ means that a is greater than 37. Only 41 is greater than 37.

You can **combine like terms** by grouping like terms together using mathematical operations:

$$3x + 9x = 12x$$

$$17a - 6a = 11a$$

Practice Question

$$4x^2y + 5y + 7xy + 8x + 9xy + 6y + 3xy^2$$

Which of the following is equal to the expression above?

- a. $4x^2y + 3xy^2 + 16xy + 8x + 11y$
- b. $7x^2y + 16xy + 8x + 11y$
- c. $7x^2y^2 + 16xy + 8x + 11y$
- d. $4x^2y + 3xy^2 + 35xy$
- e. $23x^4y^4 + 8x + 11y$

Answer

- a. Only like terms can be combined in an expression. $7xy$ and $9xy$ are like terms because they share the same variables. They combine to $16xy$. $5y$ and $6y$ are also like terms. They combine to $11y$. $4x^2y$ and $3xy^2$ are not like terms because their variables have different exponents. In one term, the x is squared, and in the other, it's not. Also, in one term, the y is squared and in the other it's not. Variables must have the exact same exponents to be considered like terms.

► Properties of Addition and Multiplication

- **Commutative Property of Addition.** When using addition, the order of the addends does not affect the sum:

$$a + b = b + a$$

$$7 + 3 = 3 + 7$$

- **Commutative Property of Multiplication.** When using multiplication, the order of the factors does not affect the product:

$$a \times b = b \times a$$

$$6 \times 4 = 4 \times 6$$

- **Associative Property of Addition.** When adding three or more addends, the grouping of the addends does not affect the sum.

$$a + (b + c) = (a + b) + c$$

$$4 + (5 + 6) = (4 + 5) + 6$$

- **Associative Property of Multiplication.** When multiplying three or more factors, the grouping of the factors does not affect the product.

$$5(ab) = (5a)b$$

$$(7 \times 8) \times 9 = 7 \times (8 \times 9)$$

- **Distributive Property.** When multiplying a sum (or a difference) by a third number, you can multiply each of the first two numbers by the third number and then add (or subtract) the products.

$$7(a + b) = 7a + 7b$$

$$9(a - b) = 9a - 9b$$

$$3(4 + 5) = 12 + 15$$

$$2(3 - 4) = 6 - 8$$

Practice Question

Which equation illustrates the commutative property of multiplication?

- a. $7(\frac{8}{9} + \frac{3}{10}) = (7 \times \frac{8}{9}) + (7 \times \frac{3}{10})$
- b. $(4.5 \times 0.32) \times 9 = 9 \times (4.5 \times 0.32)$
- c. $12(0.65 \times 9.3) = (12 \times 0.65) \times (12 \times 9.3)$
- d. $(9.04 \times 1.7) \times 2.2 = 9.04 \times (1.7 \times 2.2)$
- e. $5 \times (\frac{3}{7} \times \frac{4}{9}) = (5 \times \frac{3}{7}) \times \frac{4}{9}$

Answer

- b. Answer choices **a** and **c** show the distributive property. Answer choices **d** and **e** show the associative property. Answer choice **b** is correct because it represents that you can change the order of the terms you are multiplying without affecting the product.

Order of Operations

You must follow a specific order when calculating multiple operations:

Parentheses: First, perform all operations within parentheses.

Exponents: Next evaluate exponents.

Multiply/Divide: Then work from left to right in your multiplication and division.

Add/Subtract: Last, work from left to right in your addition and subtraction.

You can remember the correct order using the acronym **PEMDAS** or the mnemonic *Please Excuse My Dear Aunt Sally*.

Example

$$\begin{aligned}
 &8 + 4 \times (3 + 1)^2 \\
 &= 8 + 4 \times (4)^2 && \text{Parentheses} \\
 &= 8 + 4 \times 16 && \text{Exponents} \\
 &= 8 + 64 && \text{Multiplication (and Division)} \\
 &= 72 && \text{Addition (and Subtraction)}
 \end{aligned}$$

Practice Question

$$3 \times (49 - 16) + 5 \times (2 + 3^2) - (6 - 4)^2$$

What is the value of the expression above?

- a. 146
- b. 150
- c. 164
- d. 220
- e. 259

Answer

- b.** Following the order of operations, the expression should be simplified as follows:

$$3 \times (49 - 16) + 5 \times 3(2 + 3^2) - (6 - 4)^2$$

$$3 \times (33) + 5 \times (2 + 9) - (2)^2$$

$$3 \times (33) + 5 \times (11) - 4$$

$$[3 \times (33)] + [5 \times (11)] - 4$$

$$99 + 55 - 4$$

$$= 150$$

► Powers and Roots

Exponents

An **exponent** tells you how many times a number, the **base**, is a factor in the product.

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243 \quad 3 \text{ is the } \textit{base}. 5 \text{ is the } \textit{exponent}.$$

Exponents can also be used with variables. You can substitute for the variables when values are provided.

b^n The “ b ” represents a number that will be a factor to itself “ n ” times.

If $b = 4$ and $n = 3$, then $b^n = 4^3 = 4 \times 4 \times 4 = 64$.

Practice Question

Which of the following is equivalent to 7^8 ?

a. $7 \times 7 \times 7 \times 7 \times 7 \times 7$

b. $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$

c. $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$

d. $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$

e. $7 \times 8 \times 7 \times 8$

Answer

- d.** 7 is the base. 8 is the exponent. Therefore, 7 is multiplied 8 times.

Laws of Exponents

- Any base to the zero power equals 1.

$$(12xy)^0 = 1 \quad 80^0 = 1 \quad 8,345,832^0 = 1$$

- When multiplying identical bases, keep the same base and add the exponents:

$$b^m \times b^n = b^{m+n}$$

Examples

$$9^5 \times 9^6 = 9^{5+6} = 9^{11}$$

$$a^2 \times a^3 \times a^5 = a^{2+3+5} = a^{10}$$

- When dividing identical bases, keep the same base and subtract the exponents:

$$b^m \div b^n = b^{m-n}$$

$$\frac{b^m}{b^n} = b^{m-n}$$

Examples

$$6^5 \div 6^3 = 6^{5-3} = 6^2$$

$$\frac{a^9}{a^4} = a^{9-4} = a^5$$

- If an exponent appears outside of parentheses, multiply any exponents inside the parentheses by the exponent outside the parentheses.

$$(b^m)^n = b^{m \times n}$$

Examples

$$(4^3)^8 = 4^3 \times 8 = 4^{24}$$

$$(j^4 \times k^2)^3 = j^{4 \times 3} \times k^{2 \times 3} = j^{12} \times k^6$$

Practice Question

Which of the following is equivalent to 6^{12} ?

- a. $(6^6)^6$
- b. $6^2 + 6^5 + 6^5$
- c. $6^3 \times 6^2 \times 6^7$
- d. $\frac{18^{15}}{3^3}$
- e. $\frac{6^4}{6^3}$

Answer

- c. Answer choice **a** is incorrect because $(6^6)^6 = 6^{36}$. Answer choice **b** is incorrect because exponents don't combine in addition problems. Answer choice **d** is incorrect because $\frac{bm}{bn} = b^{m-n}$ applies only when the base in the numerator and denominator are the same. Answer choice **e** is incorrect because you must subtract the exponents in a division problem, not multiply them. Answer choice **c** is correct: $6^3 \times 6^2 \times 6^7 = 6^{3+2+7} = 6^{12}$.

► Squares and Square Roots

The **square** of a number is the product of a number and itself. For example, the number 25 is the **square** of the number 5 because $5 \times 5 = 25$. The square of a number is represented by the number raised to a power of 2:

$$a^2 = a \times a \quad 5^2 = 5 \times 5 = 25$$

The **square root** of a number is one of the equal factors whose product is the square. For example, 5 is the square root of the number 25 because $5 \times 5 = 25$. The symbol for square root is $\sqrt{\quad}$. This symbol is called the **radical**. The number inside of the radical is called the **radicand**.

$\sqrt{36} = 6$ because $6^2 = 36$ 36 is the square of 6, so 6 is the square root of 36.

Practice Question

Which of the following is equivalent to $\sqrt{196}$?

- a. 13
- b. 14
- c. 15
- d. 16
- e. 17

Answer

b. $\sqrt{196} = 14$ because $14 \times 14 = 196$.

Perfect Squares

The square root of a number might not be a whole number. For example, there is not a whole number that can be multiplied by itself to equal 8. $\sqrt{8} = 2.8284271 \dots$

A whole number is a **perfect square** if its square root is also a whole number:

- 1 is a perfect square because $\sqrt{1} = 1$
- 4 is a perfect square because $\sqrt{4} = 2$
- 9 is a perfect square because $\sqrt{9} = 3$
- 16 is a perfect square because $\sqrt{16} = 4$
- 25 is a perfect square because $\sqrt{25} = 5$
- 36 is a perfect square because $\sqrt{36} = 6$
- 49 is a perfect square because $\sqrt{49} = 7$

Practice Question

Which of the following is a perfect square?

- a. 72
- b. 78
- c. 80
- d. 81
- e. 88

Answer

d. Answer choices **a**, **b**, **c**, and **e** are incorrect because they are not perfect squares. The square root of a perfect square is a whole number; $\sqrt{72} \approx 8.485$; $\sqrt{78} \approx 8.832$; $\sqrt{80} \approx 8.944$; $\sqrt{88} \approx 9.381$; 81 is a perfect square because $\sqrt{81} = 9$.

Properties of Square Root Radicals

The product of the square roots of two numbers is the same as the square root of their product.

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

$$\sqrt{7} \times \sqrt{3} = \sqrt{7 \times 3} = \sqrt{21}$$

- The quotient of the square roots of two numbers is the square root of the quotient of the two numbers.

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}, \text{ where } b \neq 0$$

$$\frac{\sqrt{24}}{\sqrt{8}} = \sqrt{\frac{24}{8}} = \sqrt{3}$$

- The square of a square root radical is the radicand.

$$(\sqrt{N})^2 = N$$

$$(\sqrt{4})^2 = \sqrt{4} \times \sqrt{4} = \sqrt{16} = 4$$

- When adding or subtracting radicals with the same radicand, add or subtract only the coefficients. Keep the radicand the same.

$$a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b}$$

$$4\sqrt{7} + 6\sqrt{7} = (4 + 6)\sqrt{7} = 10\sqrt{7}$$

- You cannot combine radicals with different radicands using addition or subtraction.

$$\sqrt{a} + \sqrt{b} \neq \sqrt{a + b}$$

$$\sqrt{2} + \sqrt{3} \neq \sqrt{5}$$

- To simplify a square root radical, write the radicand as the product of two factors, with one number being the largest perfect square factor. Then write the radical over each factor and simplify.

$$\sqrt{8} = \sqrt{4} \times \sqrt{2} = 2 \times \sqrt{2} = 2\sqrt{2}$$

$$\sqrt{27} = \sqrt{9} \times \sqrt{3} = 3 \times \sqrt{3} = 3\sqrt{3}$$

Practice Question

Which of the following is equivalent to $2\sqrt{6}$?

a. $2\sqrt{3} \times \sqrt{3}$

b. $\sqrt{24}$

c. $\frac{2\sqrt{9}}{\sqrt{3}}$

d. $2\sqrt{4} + 2\sqrt{2}$

e. $\sqrt{72}$

Answer

- b. Answer choice **a** is incorrect because $2\sqrt{3} \times \sqrt{3} = 2\sqrt{9}$. Answer choice **c** is incorrect because $\frac{2\sqrt{9}}{\sqrt{3}} = 2\sqrt{3}$. Answer choice **d** is incorrect because you cannot combine radicals with different radicands using addition or subtraction. Answer choice **e** is incorrect because $\sqrt{72} = \sqrt{2 \times 36} = 6\sqrt{2}$. Answer choice **b** is correct because $\sqrt{24} = \sqrt{6 \times 4} = 2\sqrt{6}$.

► Negative Exponents

Negative exponents are the opposite of positive exponents. Therefore, because positive exponents tell you how many of the base to *multiply* together, negative exponents tell you how many of the base to *divide*.

$$a^{-n} = \frac{1}{a^n} \quad 3^{-2} = \frac{1}{3^2} = \frac{1}{3 \times 3} = \frac{1}{9} \quad -5^{-3} = -\frac{1}{5^3} = -\frac{1}{5 \times 5 \times 5} = -\frac{1}{125}$$

Practice Question

Which of the following is equivalent to -6^{-4} ?

- a. $-1,296$
- b. $-\frac{6}{1,296}$
- c. $-\frac{1}{1,296}$
- d. $\frac{1}{1,296}$
- e. $1,296$

Answer

c. $-6^{-4} = -\frac{1}{6^4} = -\frac{1}{6 \times 6 \times 6 \times 6} = -\frac{1}{1,296}$

► Rational Exponents

Rational numbers are numbers that can be written as fractions (and decimals and repeating decimals). Similarly, numbers raised to rational exponents are numbers raised to fractional powers:

$$4^{\frac{1}{2}} \quad 25^{\frac{1}{2}} \quad 8^{\frac{1}{3}} \quad 3^{\frac{2}{3}}$$

For a number with a fractional exponent, the numerator of the exponent tells you the power to raise the number to, and the denominator of the exponent tells you the root you take.

$$4^{\frac{1}{2}} = \sqrt{4^1} = \sqrt{4} = 2$$

The numerator is 1, so raise 4 to a power of 1. The denominator is 2, so take the square root.

$$25^{\frac{1}{2}} = \sqrt{25^1} = \sqrt{25} = 5$$

The numerator is 1, so raise 25 to a power of 1. The denominator is 2, so take the square root.

$$8^{\frac{1}{3}} = \sqrt[3]{8^1} = \sqrt[3]{8} = 2$$

The numerator is 1, so raise 8 to a power of 1. The denominator is 3, so take the cube root.

$$3^{\frac{2}{3}} = \sqrt[3]{3^2} = \sqrt[3]{9}$$

The numerator is 2, so raise 3 to a power of 2. The denominator is 3, so take the cube root.

Practice Question

Which of the following is equivalent to $8^{\frac{2}{3}}$?

- a. $\sqrt[3]{4}$
- b. $\sqrt[3]{8}$
- c. $\sqrt[3]{16}$
- d. $\sqrt[3]{64}$
- e. $\sqrt{512}$

Answer

- d. In the exponent of $8^{\frac{2}{3}}$, the numerator is 2, so raise 8 to a power of 2. The denominator is 3, so take the cube root; $\sqrt[3]{8^2} = \sqrt[3]{64}$.

► Divisibility and Factors

Like multiplication, division can be represented in different ways. In the following examples, 3 is the **divisor** and 12 is the **dividend**. The result, 4, is the **quotient**.

$$12 \div 3 = 4 \qquad 3 \overline{)12} = 4 \qquad \frac{12}{3} = 4$$

Practice Question

In which of the following equations is the divisor 15?

- a. $\frac{15}{5} = 3$
- b. $\frac{60}{15} = 4$
- c. $15 \div 3 = 5$
- d. $45 \div 3 = 15$
- e. $10 \overline{)150} = 15$

Answer

- b. The divisor is the number that divides *into* the dividend to find the quotient. In answer choices a and c, 15 is the dividend. In answer choices d and e, 15 is the quotient. Only in answer choice b is 15 the divisor.

Odd and Even Numbers

An **even** number is a number that can be divided by the number 2 to result in a whole number. Even numbers have a 2, 4, 6, 8, or 0 in the ones place.

2 34 86 1,018 6,987,120

Consecutive even numbers differ by two:

2, 4, 6, 8, 10, 12, 14 . . .

An **odd** number cannot be divided evenly by the number 2 to result in a whole number. Odd numbers have a 1, 3, 5, 7, or 9 in the ones place.

1 13 95 2,827 7,820,289

Consecutive odd numbers differ by two:

1, 3, 5, 7, 9, 11, 13 . . .

Even and odd numbers behave consistently when added or multiplied:

$even + even = even$	and	$even \times even = even$
$odd + odd = even$	and	$odd \times odd = odd$
$odd + even = odd$	and	$even \times odd = even$

Practice Question

Which of the following situations must result in an odd number?

- a. even number + even number
- b. odd number \times odd number
- c. odd number + 1
- d. odd number + odd number
- e. $\frac{\text{even number}}{2}$

Answer

b. **a**, **c**, and **d** definitely yield even numbers; **e** could yield either an even or an odd number. The product of two odd numbers (**b**) is an odd number.

Dividing by Zero

Dividing by zero is impossible. Therefore, the denominator of a fraction can never be zero. Remember this fact when working with fractions.

Example

$$\frac{5}{n - 4}$$

We know that $n \neq 4$ because the denominator cannot be 0.

Factors

Factors of a number are whole numbers that, when divided into the original number, result in a quotient that is a whole number.

Example

The factors of 18 are 1, 2, 3, 6, 9, and 18 because these are the only whole numbers that divide evenly into 18.

The **common factors** of two or more numbers are the factors that the numbers have in common. The **greatest common factor** of two or more numbers is the largest of all the common factors. Determining the greatest common factor is useful for reducing fractions.

Examples

The *factors* of 28 are 1, 2, 4, 7, 14, and 28.

The *factors* of 21 are 1, 3, 7, and 21.

The *common factors* of 28 and 21 are therefore 1 and 7 because they are factors of both 28 and 21.

The *greatest common factor* of 28 and 21 is therefore 7. It is the largest factor shared by 28 and 21.

Practice Question

What are the common factors of 48 and 36?

- a. 1, 2, and 3
- b. 1, 2, 3, and 6
- c. 1, 2, 3, 6, and 12
- d. 1, 2, 3, 6, 8, and 12
- e. 1, 2, 3, 4, 6, 8, and 12

Answer

- c. The factors of 48 are 1, 2, 3, 6, 8, 12, 24, and 48. The factors of 36 are 1, 2, 3, 6, 12, 18, and 36. Therefore, their common factors—the factors they share—are 1, 2, 3, 6, and 12.

► Multiples

Any number that can be obtained by multiplying a number x by a whole number is called a **multiple** of x .

Examples

Multiples of x include $1x$, $2x$, $3x$, $4x$, $5x$, $6x$, $7x$, $8x$. . .

Multiples of 5 include 5, 10, 15, 20, 25, 30, 35, 40 . . .

Multiples of 8 include 8, 16, 24, 32, 40, 48, 56, 64 . . .

The **common multiples** of two or more numbers are the multiples that the numbers have in common. The **least common multiple** of two or more numbers is the smallest of all the common multiples. The least common multiple, or LCM, is used when performing various operations with fractions.

Examples

Multiples of 10 include 10, 20, 30, 40, 50, 60, 70, 80, 90 . . .

Multiples of 15 include 15, 30, 45, 60, 75, 90, 105 . . .

Some *common multiples* of 10 and 15 are therefore 30, 60, and 90 because they are multiples of both 10 and 15.

The *least common multiple* of 10 and 15 is therefore 30. It is the smallest of the multiples shared by 10 and 15.

► Prime and Composite Numbers

A positive integer that is greater than the number 1 is either prime or composite, but not both.

- A **prime number** has only itself and the number 1 as factors:
2, 3, 5, 7, 11, 13, 17, 19, 23 . . .
- A **composite** number is a number that has more than two factors:
4, 6, 8, 9, 10, 12, 14, 15, 16 . . .
- The number 1 is neither prime nor composite.

Practice Question

n is a prime number and

$n > 2$

What must be true about n ?

- a. $n = 3$
- b. $n = 4$
- c. n is a negative number
- d. n is an even number
- e. n is an odd number

Answer

- e. All prime numbers greater than 2 are odd. They cannot be even because all even numbers are divisible by at least themselves *and* the number 2, which means they have at least two factors and are therefore composite, not prime. Thus, answer choices **b** and **d** are incorrect. Answer choice **a** is incorrect because, although n could equal 3, it does not necessarily equal 3. Answer choice **c** is incorrect because $n > 2$.

► Prime Factorization

Prime factorization is a process of breaking down factors into prime numbers.

Example

Let's determine the prime factorization of 18.

Begin by writing 18 as the product of two factors:

$$18 = 9 \times 2$$

Next break down those factors into smaller factors:

9 can be written as 3×3 , so $18 = 9 \times 2 = 3 \times 3 \times 2$.

The numbers 3, 3, and 2 are all prime, so we have determined that the prime factorization of 18 is $3 \times 3 \times 2$.

We could have also found the prime factorization of 18 by writing the product of 18 as 3×6 :

6 can be written as 3×2 , so $18 = 6 \times 3 = 3 \times 3 \times 2$.

Thus, the prime factorization of 18 is $3 \times 3 \times 2$.

Note: Whatever the road one takes to the factorization of a number, the answer is always the same.

Practice Question

$2 \times 2 \times 2 \times 5$ is the prime factorization of which number?

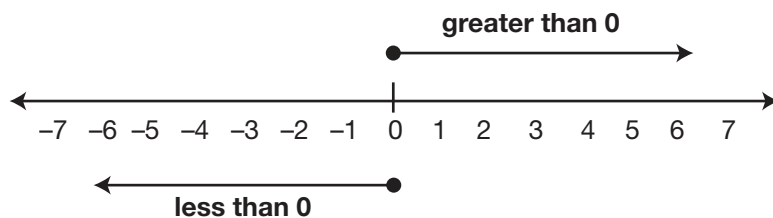
- a. 10
- b. 11
- c. 20
- d. 40
- e. 80

Answer

- d. There are two ways to answer this question. You could find the prime factorization of each answer choice, or you could simply multiply the prime factors together. The second method is faster: $2 \times 2 \times 2 \times 5 = 4 \times 2 \times 5 = 8 \times 5 = 40$.

► Number Lines and Signed Numbers

On a number line, *less than 0* is to the *left* of 0 and *greater than 0* is to the *right* of 0.



Negative numbers are the opposites of positive numbers.

Examples

5 is five to the *right* of zero.

-5 is five to the *left* of zero.

If a number is *less than* another number, it is farther to the left on the number line.

Example

-4 is to the left of -2, so $-4 < -2$.

If a number is *greater than* another number, it is farther to the right on the number line.

Example

3 is to the right of -1, so $3 > -1$.

A positive number is always greater than a negative number. A negative number is always less than a positive number.

Examples

2 is greater than -3,675.

-25,812 is less than 3.

As a shortcut to avoiding confusion when comparing two negative numbers, remember the following rules:

When a and b are positive, if $a > b$, then $-a < -b$.

When a and b are positive, if $a < b$, then $-a > -b$.

Examples

If $8 > 6$, then $-6 > -8$. (8 is to the right of 6 on the number line. Therefore, -8 is to the left of -6 on the number line.)

If $132 < 267$, then $-132 > -267$. (132 is to the left of 267 on the number line. Therefore, -132 is to the right of -267 on the number line.)

Practice Question

Which of the following statements is true?

- a. $-25 > -24$
- b. $-48 > 16$
- c. $14 > 17$
- d. $-22 > 19$
- e. $-37 > -62$

Answer

- e. $-37 > -62$ because -37 is to the right of -62 on the number line.

► Absolute Value

The **absolute value** of a number is the distance the number is from zero on a number line. Absolute value is represented by the symbol $||$. Absolute values are *always* positive or zero.

Examples

$ -1 = 1$	The absolute value of -1 is 1. The distance of -1 from zero on a number line is 1.
$ 1 = 1$	The absolute value of 1 is 1. The distance of 1 from zero on a number line is 1.
$ -23 = 23$	The absolute value of -23 is 23. The distance of -23 from zero on a number line is 23.
$ 23 = 23$	The absolute value of 23 is 23. The distance of 23 from zero on a number line is 23.

The **absolute value** of an expression is the distance the value of the expression is from zero on a number line. Absolute values of expressions are *always* positive or zero.

Examples

$ 3 - 5 = -2 = 2$	The absolute value of $3 - 5$ is 2. The distance of $3 - 5$ from zero on a number line is 2.
$ 5 - 3 = 2 = 2$	The absolute value of $5 - 3$ is 2. The distance of $5 - 3$ from zero on a number line is 2.

Practice Question

$$|x - y| = 5$$

Which values of x and y make the above equation NOT true?

- a. $x = -8$ $y = -3$
- b. $x = 12$ $y = 7$
- c. $x = -20$ $y = -25$
- d. $x = -5$ $y = 10$
- e. $x = -2$ $y = 3$

Answer

- d. Answer choice a: $|(-8) - (-3)| = |(-8) + 3| = |-5| = 5$
 Answer choice b: $|12 - 7| = |5| = 5$
 Answer choice c: $|(-20) - (-25)| = |(-20) + 25| = |5| = 5$
 Answer choice d: $|(-5) - 10| = |-15| = 15$
 Answer choice e: $|(-2) - 3| = |-5| = 5$

Therefore, the values of x and y in answer choice **d** make the equation NOT true.

► Rules for Working with Positive and Negative Integers

Multiplying/Dividing

- When multiplying or dividing two integers, if the signs are the same, the result is positive.

Examples

negative \times positive = negative	$-3 \times 5 = -15$
positive \div positive = positive	$15 \div 5 = 3$
negative \times negative = positive	$-3 \times -5 = 15$
negative \div negative = positive	$-15 \div -5 = 3$

- When multiplying or dividing two integers, if the signs are different, the result is negative:

Examples

positive \times negative = negative	$3 \times -5 = -15$
positive \div negative = negative	$15 \div -5 = -3$

Adding

- When adding two integers with the same sign, the sum has the same sign as the addends.

Examples

positive + positive = positive	$4 + 3 = 7$
negative + negative = negative	$-4 + -3 = -7$

- When adding integers of different signs, follow this two-step process:
 - Subtract the absolute values of the numbers. Be sure to subtract the lesser absolute value from the greater absolute value.
 - Apply the sign of the larger number

Examples

$$-2 + 6$$

First subtract the absolute values of the numbers: $|6| - |-2| = 6 - 2 = 4$

Then apply the sign of the larger number: 6.

The answer is 4.

$$7 + -12$$

First subtract the absolute values of the numbers: $|-12| - |7| = 12 - 7 = 5$

Then apply the sign of the larger number: -12 .

The answer is -5 .

Subtracting

- When subtracting integers, change all subtraction to addition and change the sign of the number being subtracted to its opposite. Then follow the rules for addition.

Examples

$$(+12) - (+15) = (+12) + (-15) = -3$$

$$(-6) - (-9) = (-6) + (+9) = +3$$

Practice Question

Which of the following expressions is equal to -9 ?

- a. $-17 + 12 - (-4) - (-10)$
- b. $13 - (-7) - 36 - (-8)$
- c. $-8 \times (-2) - 14 + (-11)$
- d. $(-10 \times 4) - (-5 \times 5) - 6$
- e. $[-48 \div (-3)] - (28 \div 4)$

Answer

- c. Answer choice a: $-17 + 12 - (-4) - (-10) = 9$
 Answer choice b: $13 - (-7) - 36 - (-8) = -8$
 Answer choice c: $-8 \times (-2) - 14 + (-11) = -9$
 Answer choice d: $(-10 \times 4) - (-5 \times 5) - 6 = -21$
 Answer choice e: $[-48 \div (-3)] - (28 \div 4) = 9$
 Therefore, answer choice c is equal to -9 .

► Decimals

Memorize the order of place value:

3	7	5	9	•	1	6	0	4
T H O U S A N D S	H U N D R E D S	T E N S	O N E S	D E C I M A L P O I N T	T E N T H S	H U N D R E D T H S	T H O U S A N D T H S	T E N T H O U S A N D T H S

The number shown in the place value chart can also be expressed in expanded form:

$$3,759.1604 =$$

$$(3 \times 1,000) + (7 \times 100) + (5 \times 10) + (9 \times 1) + (1 \times 0.1) + (6 \times 0.01) + (0 \times 0.001) + (4 \times 0.0001)$$

Comparing Decimals

When comparing decimals less than one, line up the decimal points and fill in any zeroes needed to have an equal number of digits in each number.

Example

Compare 0.8 and 0.008.

Line up decimal points 0.800

and add zeroes 0.008.

Then ignore the decimal point and ask, which is greater: 800 or 8?

800 is bigger than 8, so 0.8 is greater than 0.008.

Practice Question

Which of the following inequalities is true?

- a. $0.04 < 0.004$
- b. $0.17 < 0.017$
- c. $0.83 < 0.80$
- d. $0.29 < 0.3$
- e. $0.5 < 0.08$

Answer

- d. Answer choice **a**: $0.040 > 0.004$ because $40 > 4$. Therefore, $0.04 > 0.004$. This answer choice is FALSE.
- Answer choice **b**: $0.170 > 0.017$ because $170 > 17$. Therefore, $0.17 > 0.017$. This answer choice is FALSE.
- Answer choice **c**: $0.83 > 0.80$ because $83 > 80$. This answer choice is FALSE.
- Answer choice **d**: $0.29 < 0.30$ because $29 < 30$. Therefore, $0.29 < 0.3$. This answer choice is TRUE.
- Answer choice **e**: $0.50 > 0.08$ because $50 > 8$. Therefore, $0.5 > 0.08$. This answer choice is FALSE.

► Fractions

Multiplying Fractions

To multiply fractions, simply multiply the numerators and the denominators:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \qquad \frac{5}{8} \times \frac{3}{7} = \frac{5 \times 3}{8 \times 7} = \frac{15}{56} \qquad \frac{3}{4} \times \frac{5}{6} = \frac{3 \times 5}{4 \times 6} = \frac{15}{24}$$

Practice Question

Which of the following fractions is equivalent to $\frac{2}{9} \times \frac{3}{5}$?

- a. $\frac{5}{45}$
- b. $\frac{6}{45}$
- c. $\frac{5}{14}$
- d. $\frac{10}{18}$
- e. $\frac{37}{45}$

Answer

b. $\frac{2}{9} \times \frac{3}{5} = \frac{2 \times 3}{9 \times 5} = \frac{6}{45}$

Reciprocals

To find the reciprocal of any fraction, swap its numerator and denominator.

Examples

Fraction: $\frac{1}{4}$ Reciprocal: $\frac{4}{1}$

Fraction: $\frac{5}{6}$ Reciprocal: $\frac{6}{5}$

Fraction: $\frac{7}{2}$ Reciprocal: $\frac{2}{7}$

Fraction: $\frac{x}{y}$ Reciprocal: $\frac{y}{x}$

Dividing Fractions

Dividing a fraction by another fraction is the same as multiplying the first fraction by the **reciprocal** of the second fraction:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c} \qquad \frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8} \qquad \frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{3 \times 6}{4 \times 5} = \frac{18}{20}$$

Adding and Subtracting Fractions with Like Denominators

To add or subtract fractions with like denominators, add or subtract the numerators and leave the denominator as it is:

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \frac{1}{6} + \frac{4}{6} = \frac{1+4}{6} = \frac{5}{6}$$

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c} \qquad \frac{5}{7} - \frac{3}{7} = \frac{5-3}{7} = \frac{2}{7}$$

Adding and Subtracting Fractions with Unlike Denominators

To add or subtract fractions with unlike denominators, find the **Least Common Denominator**, or **LCD**, and convert the unlike denominators into the LCD. The LCD is the smallest number divisible by each of the denominators. For example, the LCD of $\frac{1}{8}$ and $\frac{1}{12}$ is 24 because 24 is the least multiple shared by 8 and 12. Once you know the LCD, convert each fraction to its new form by multiplying both the numerator and denominator by the necessary number to get the LCD, and then add or subtract the new numerators.

Example

$$\frac{1}{8} + \frac{1}{12}$$

$$\frac{1}{8} = 1 \times \frac{3}{8} \times 3 = \frac{3}{24}$$

$$\frac{1}{12} = 1 \times \frac{2}{12} \times 2 = \frac{2}{24}$$

$$\frac{3}{24} + \frac{2}{24} = \frac{5}{24}$$

LCD is 24 because $8 \times 3 = 24$ and $12 \times 2 = 24$.

Convert fraction.

Convert fraction.

Add numerators only.

Example

$$\frac{4}{9} - \frac{1}{6}$$

$$\frac{4}{9} = 4 \times \frac{6}{9} \times 6 = \frac{24}{54}$$

$$\frac{1}{6} = 1 \times \frac{9}{6} \times 9 = \frac{9}{54}$$

$$\frac{24}{54} - \frac{9}{54} = \frac{15}{54} = \frac{5}{18}$$

LCD is 54 because $9 \times 6 = 54$ and $6 \times 9 = 54$.

Convert fraction.

Convert fraction.

Subtract numerators only. Reduce where possible.

Practice Question

Which of the following expressions is equivalent to $\frac{5}{8} \div \frac{3}{4}$?

a. $\frac{1}{3} + \frac{1}{2}$

b. $\frac{3}{4} + \frac{5}{8}$

c. $\frac{1}{3} + \frac{2}{3}$

d. $\frac{4}{12} + \frac{1}{12}$

e. $\frac{1}{6} + \frac{3}{6}$

Answer

a. The expression in the equation is $\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3} = \frac{5 \times 4}{8 \times 3} = \frac{20}{24} = \frac{5}{6}$. So you must evaluate each answer choice to determine which equals $\frac{5}{6}$.

Answer choice **a**: $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$.

Answer choice **b**: $\frac{3}{4} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8} = \frac{11}{8}$.

Answer choice **c**: $\frac{1}{3} + \frac{2}{3} = \frac{3}{3} = \frac{6}{6} = 1$.

Answer choice **d**: $\frac{4}{12} + \frac{1}{12} = \frac{5}{12}$.

Answer choice **e**: $\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$.

Therefore, answer choice **a** is correct.

► Sets

Sets are collections of certain numbers. All of the numbers within a set are called the **members** of the set.

Examples

The set of integers is $\{ \dots -3, -2, -1, 0, 1, 2, 3, \dots \}$.

The set of whole numbers is $\{0, 1, 2, 3, \dots \}$.

Intersections

When you find the elements that two (or more) sets have in common, you are finding the **intersection** of the sets. The symbol for intersection is \cap .

Example

The set of negative integers is $\{ \dots, -4, -3, -2, -1 \}$.

The set of even numbers is $\{ \dots, -4, -2, 0, 2, 4, \dots \}$.

The intersection of the set of negative integers and the set of even numbers is the set of elements (numbers) that the two sets have in common:

$\{ \dots, -8, -6, -4, -2 \}$.

Practice Question

Set X = even numbers between 0 and 10

Set Y = prime numbers between 0 and 10

What is $X \cap Y$?

- a. $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- b. $\{1, 2, 3, 4, 5, 6, 7, 8\}$
- c. $\{2\}$
- d. $\{2, 4, 6, 8\}$
- e. $\{1, 2, 3, 5, 7\}$

Answer

- c. $X \cap Y$ is “the intersection of sets X and Y .” The intersection of two sets is the set of numbers shared by both sets. Set $X = \{2, 4, 6, 8\}$. Set $Y = \{1, 2, 3, 5, 7\}$. Therefore, the intersection is $\{2\}$.

Unions

When you combine the elements of two (or more) sets, you are finding the **union** of the sets. The symbol for union is \cup .

Example

The positive even integers are $\{2, 4, 6, 8, \dots \}$.

The positive odd integers are $\{1, 3, 5, 7, \dots \}$.

If we combine the elements of these two sets, we find the union of these sets:

$\{1, 2, 3, 4, 5, 6, 7, 8, \dots \}$.

Practice Question

Set $P = \{0, \frac{3}{7}, 0.93, 4, 6.98, \frac{27}{2}\}$

Set $Q = \{0.01, 0.15, 1.43, 4\}$

What is $P \cup Q$?

- a. $\{4\}$
- b. $\{\frac{3}{7}, \frac{27}{2}\}$
- c. $\{0, 4\}$
- d. $\{0, 0.01, 0.15, \frac{3}{7}, 0.93, 1.43, 6.98, \frac{27}{2}\}$
- e. $\{0, 0.01, 0.15, \frac{3}{7}, 0.93, 1.43, 4, 6.98, \frac{27}{2}\}$

Answer

- e. $P \cup Q$ is “the union of sets P and Q .” The union of two sets is all the numbers from the two sets combined. Set $P = \{0, \frac{3}{7}, 0.93, 4, 6.98, \frac{27}{2}\}$. Set $Q = \{0.01, 0.15, 1.43, 4\}$. Therefore, the union is $\{0, 0.01, 0.15, \frac{3}{7}, 0.93, 1.43, 4, 6.98, \frac{27}{2}\}$.

Mean, Median, and Mode

To find the average, or **mean**, of a set of numbers, add all of the numbers together and divide by the quantity of numbers in the set.

$$\text{mean} = \frac{\text{sum of numbers in set}}{\text{quantity of numbers in set}}$$

Example

Find the mean of 9, 4, 7, 6, and 4.

$$\frac{9+4+7+6+4}{5} = \frac{30}{5} = 6$$

The denominator is 5 because there are five numbers in the set.

To find the **median** of a set of numbers, arrange the numbers in ascending order and find the middle value.

- If the set contains an odd number of elements, then simply choose the middle value.

Example

Find the median of the number set: 1, 5, 3, 7, 2.

First arrange the set in ascending order: 1, 2, 3, 5, 7.

Then choose the middle value: 3.

The median is 3.

- If the set contains an even number of elements, then average the two middle values.

Example

Find the median of the number set: 1, 5, 3, 7, 2, 8.

First arrange the set in ascending order: 1, 2, 3, 5, 7, 8.

Then choose the middle values: 3 and 5.

Find the average of the numbers 3 and 5: $\frac{3+5}{2} = \frac{8}{2} = 4$.

The median is 4.

The **mode** of a set of numbers is the number that occurs most frequently.

Example

For the number set 1, 2, 5, 3, 4, 2, 3, 6, 3, 7, the number 3 is the mode because it occurs three times. The other numbers occur only once or twice.

Practice Question

If the mode of a set of three numbers is 17, which of the following must be true?

- I. The average is greater than 17.
 - II. The average is odd.
 - III. The median is 17.
- a. none
 - b. I only
 - c. III only
 - d. I and III
 - e. I, II, and III

Answer

- c. If the mode of a set of three numbers is 17, the set is $\{x, 17, 17\}$. Using that information, we can evaluate the three statements:

Statement I: *The average is greater than 17.*

If x is less than 17, then the average of the set will be less than 17. For example, if $x = 2$, then we can find the average:

$$2 + 17 + 17 = 36$$

$$36 \div 3 = 12$$

Therefore, the average would be 12, which is not greater than 17, so number I isn't necessarily true. Statement I is FALSE.

Statement II: *The average is odd.*

Because we don't know the third number of the set, we don't know that the average must be even. As we just learned, if the third number is 2, the average is 12, which is even, so statement II ISN'T NECESSARILY TRUE.

Statement III: *The median is 17.*

We know that the median is 17 because the median is the middle value of the three numbers in the set. If $X > 17$, the median is 17 because the numbers would be ordered: $X, 17, 17$. If $X < 17$, the median is still 17 because the numbers would be ordered: $17, 17, X$. Statement III is TRUE.

Answer: Only statement III is NECESSARILY TRUE.

► Percent

A percent is a ratio that compares a number to 100. For example, $30\% = \frac{30}{100}$.

- To convert a decimal to a percentage, move the decimal point two units to the right and add a percentage symbol.

$$0.65 = 65\% \quad 0.04 = 4\% \quad 0.3 = 30\%$$

- One method of converting a fraction to a percentage is to first change the fraction to a decimal (by dividing the numerator by the denominator) and to then change the decimal to a percentage.

$$\frac{3}{5} = 0.60 = 60\% \quad \frac{1}{5} = 0.2 = 20\% \quad \frac{3}{8} = 0.375 = 37.5\%$$

- Another method of converting a fraction to a percentage is to, if possible, convert the fraction so that it has a denominator of 100. The percentage is the new numerator followed by a percentage symbol.

$$\frac{3}{5} = \frac{60}{100} = 60\% \quad \frac{6}{25} = \frac{24}{100} = 24\%$$

- To change a percentage to a decimal, move the decimal point two places to the left and eliminate the percentage symbol.

$$64\% = 0.64 \quad 87\% = 0.87 \quad 7\% = 0.07$$

- To change a percentage to a fraction, divide by 100 and reduce.

$$44\% = \frac{44}{100} = \frac{11}{25} \quad 70\% = \frac{70}{100} = \frac{7}{10} \quad 52\% = \frac{52}{100} = \frac{26}{50}$$

- Keep in mind that any percentage that is 100 or greater converts to a number greater than 1, such as a whole number or a mixed number.

$$500\% = 5 \quad 275\% = 2.75 \text{ or } 2\frac{3}{4}$$

Here are some conversions you should be familiar with:

FRACTION	DECIMAL	PERCENTAGE
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{1}{3}$	0.333 ...	33. $\bar{3}$ %
$\frac{2}{3}$	0.666 ...	66. $\bar{6}$ %
$\frac{1}{10}$	0.1	10%
$\frac{1}{8}$	0.125	12.5%
$\frac{1}{6}$	0.1666 ...	16. $\bar{6}$ %
$\frac{1}{5}$	0.2	20%

Practice Question

If $\frac{7}{25} < x < 0.38$, which of the following could be a value of x ?

- a. 20%
- b. 26%
- c. 34%
- d. 39%
- e. 41%

Answer

c. $\frac{7}{25} = \frac{28}{100} = 28\%$
 $0.38 = 38\%$

Therefore, $28\% < x < 38\%$.

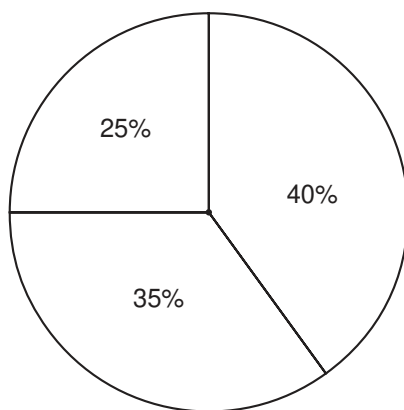
Only answer choice c, 34%, is greater than 28% and less than 38%.

► Graphs and Tables

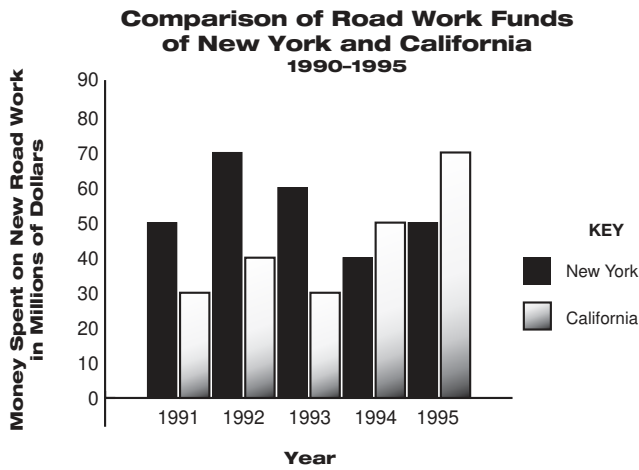
The SAT includes questions that test your ability to analyze graphs and tables. Always read graphs and tables carefully before moving on to read the questions. Understanding the graph will help you process the information that is presented in the question. Pay special attention to headings and units of measure in graphs and tables.

Circle Graphs or Pie Charts

This type of graph is representative of a whole and is usually divided into percentages. Each section of the chart represents a portion of the whole. All the sections added together equal 100% of the whole.

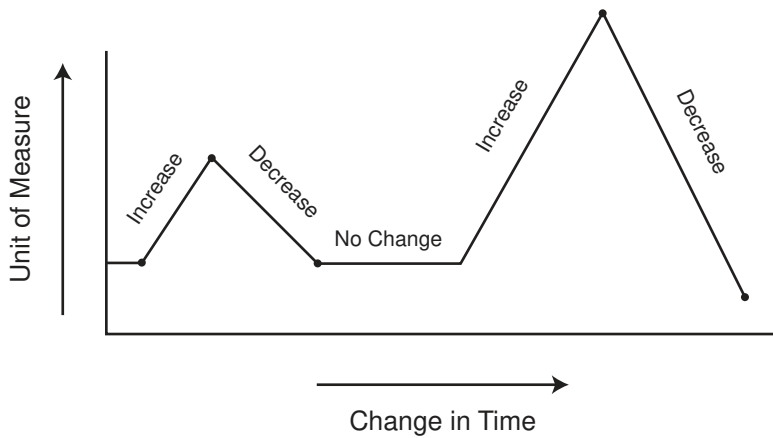
**Bar Graphs**

Bar graphs compare similar things with different length bars representing different values. On the SAT, these graphs frequently contain differently shaded bars used to represent different elements. Therefore, it is important to pay attention to both the size and shading of the bars.

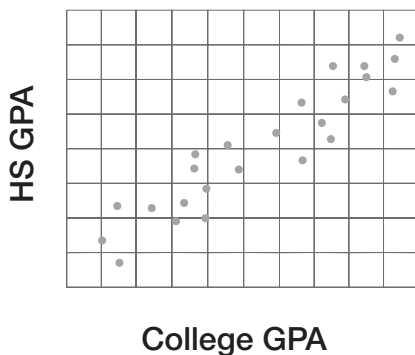


Broken-Line Graphs

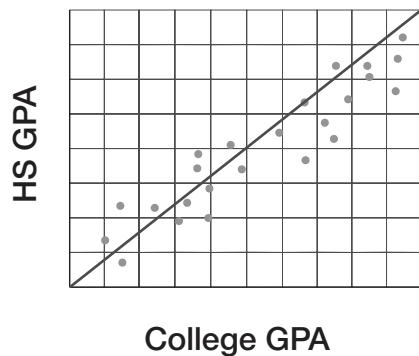
Broken-line graphs illustrate a measurable change over time. If a line is slanted up, it represents an increase whereas a line sloping down represents a decrease. A flat line indicates no change as time elapses.



Scatterplots illustrate the relationship between two quantitative variables. Typically, the values of the independent variables are the x -coordinates, and the values of the dependent variables are the y -coordinates. When presented with a scatterplot, look for a trend. Is there a line that the points seem to cluster around? For example:

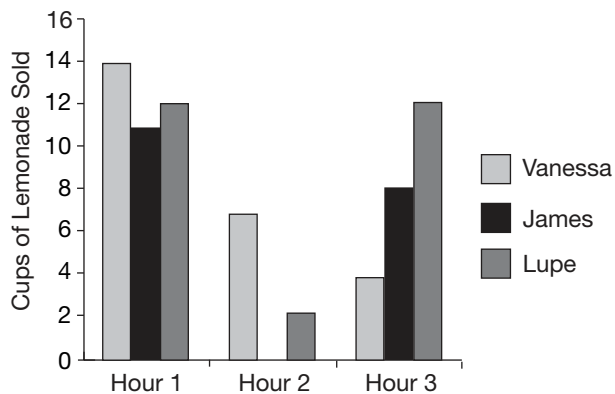


In the previous scatterplot, notice that a “line of best fit” can be created:



Practice Question

Lemonade Sold



Based on the graph above, which of the following statements are true?

- I. In the first hour, Vanessa sold the most lemonade.
 - II. In the second hour, Lupe didn't sell any lemonade.
 - III. In the third hour, James sold twice as much lemonade as Vanessa.
- a. I only
 - b. II only
 - c. I and II
 - d. I and III
 - e. I, II, and III

Answer

- d. Let's evaluate the three statements:

Statement I: *In the first hour, Vanessa sold the most lemonade.*

In the graph, Vanessa's bar for the first hour is highest, which means she sold the most lemonade in the first hour. Therefore, statement I is TRUE.

Statement II: *In the second hour, Lupe didn't sell any lemonade.*

In the second hour, there is no bar for James, which means he sold no lemonade. However, the bar for Lupe is at 2, so Lupe sold 2 cups of lemonade. Therefore, statement II is FALSE.

Statement III: *In the third hour, James sold twice as much lemonade as Vanessa.*

In the third hour, James's bar is at 8 and Vanessa's bar is at 4, which means James sold twice as much lemonade as Vanessa. Therefore, statement III is TRUE.

Answer: Only statements I and III are true.

► Matrices

Matrices are rectangular arrays of numbers. Below is an example of a 2 by 2 matrix:

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$$

Review the following basic rules for performing operations on 2 by 2 matrices.

Addition

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 & a_2 + b_2 \\ a_3 + b_3 & a_4 + b_4 \end{bmatrix}$$

Subtraction

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} - \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1 - b_1 & a_2 - b_2 \\ a_3 - b_3 & a_4 - b_4 \end{bmatrix}$$

Multiplication

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \times \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1b_1 + a_2b_3 & a_1b_2 + a_2b_4 \\ a_3b_1 + a_4b_3 & a_3b_2 + a_4b_4 \end{bmatrix}$$

Scalar Multiplication

$$k \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} = \begin{bmatrix} ka_1 & ka_2 \\ ka_3 & ka_4 \end{bmatrix}$$

Practice Question

$$\begin{bmatrix} 4 & 3 \\ 7 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ 5 & 2 \end{bmatrix} =$$

Which of the following shows the correct solution to the problem above?

a. $\begin{bmatrix} 7 & 8 \\ 8 & 7 \end{bmatrix}$

b. $\begin{bmatrix} 11 & 11 \\ 4 & 4 \end{bmatrix}$

c. $\begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}$

d. $\begin{bmatrix} 24 & 6 \\ 35 & 2 \end{bmatrix}$

e. $\begin{bmatrix} 10 & 5 \\ 12 & 3 \end{bmatrix}$

Answer

e. $\begin{bmatrix} 4 & 3 \\ 7 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 4+6 & 3+2 \\ 7+5 & 1+2 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 12 & 3 \end{bmatrix}$