CHAPTER

6 Algebra Review

This chapter reviews key skills and concepts of algebra that you need to know for the SAT. Throughout the chapter are sample questions in the style of SAT questions. Each sample SAT question is followed by an explanation of the correct answer.

Equations

To solve an algebraic **equation** with one variable, find the value of the unknown variable.

Rules for Working with Equations

1. The equal sign separates an equation into two sides.
2. Whenever an operation is performed on one side, the same operation must be performed on the other side.
3. To solve an equation, first move all of the variables to one side and all of the numbers to the other. Then simplify until only one variable (with a coefficient of 1) remains on one side and one number remains on the other side.

Example

$7x - 11 = 29 - 3x$	Move the variables to one side.
$7x - 11 + 3x = 29 - 3x + 3x$	Perform the same operation on both sides.
$10x - 11 = 29$	Now move the numbers to the other side.
$10x - 11 + 11 = 29 + 11$	Perform the same operation on both sides.
$10x = 40$	Divide both sides by the coefficient.
$\frac{10x}{10} = \frac{40}{10}$	Simplify.
$x = 4$	

Practice Question

If $13x - 28 = 22 - 12x$, what is the value of x?

a. -6
b. $-\frac{6}{25}$
c. 2
d. 6
e. 50

Answer

c. To solve for x:

$13x - 28 = 22 - 12x$
$13x - 28 + 12x = 22 - 12x + 12x$
$25x - 28 = 22$
$25x - 28 + 28 = 22 + 28$
$25x = 50$
$x = 2$

Cross Products

You can solve an equation that sets one fraction equal to another by finding **cross products** of the fractions. Finding cross products allows you to remove the denominators from each side of the equation by multiplying each side by a fraction equal to 1 that has the denominator from the opposite side.

Example

$\frac{a}{b} = \frac{c}{d}$	First multiply one side by $\frac{d}{d}$ and the other by $\frac{b}{b}$. The fractions $\frac{d}{d}$ and $\frac{b}{b}$ both equal 1, so they don't change the equation.
$\frac{a}{b} \times \frac{d}{d} = \frac{c}{d} \times \frac{b}{b}$	
$\frac{ad}{bd} = \frac{bc}{bd}$	The denominators are now the same. Now multiply both sides by the denominator and simplify.
$bd \times \frac{ad}{bd} = bd \times \frac{bc}{bd}$	
$ad = bc$	The example above demonstrates how finding cross products works. In the future, you can skip all the middle steps and just assume that $\frac{a}{b} = \frac{c}{d}$ is the same as $ad = bc$.

Example

$\frac{x}{6} = \frac{12}{36}$ Find cross products.

$36x = 6 \times 12$

$36x = 72$

$x = 2$

Example

$\frac{x}{4} = \frac{x + 12}{16}$ Find cross products.

$16x = 4(x + 12)$

$16x = 4x + 48$

$12x = 48$

$x = 4$

Practice Question

If $\frac{y}{9} = \frac{y - 7}{12}$, what is the value of y?

a. -28
b. -21
c. $-\frac{63}{11}$
d. $-\frac{7}{3}$
e. 28

Answer

b. To solve for y:

$\frac{y}{9} = \frac{y - 7}{12}$ Find cross products.

$12y = 9(y - 7)$

$12y = 9y - 63$

$12y - 9y = 9y - 63 - 9y$

$3y = -63$

$y = -21$

Checking Equations

After you solve an equation, you can check your answer by substituting your value for the variable into the original equation.

Example

We found that the solution for $7x - 11 = 29 - 3x$ is $x = 4$. To check that the solution is correct, substitute 4 for x in the equation:

$7x - 11 = 29 - 3x$

$7(4) - 11 = 29 - 3(4)$

$28 - 11 = 29 - 12$

$17 = 17$

This equation checks, so $x = 4$ is the correct solution!

Special Tips for Checking Equations on the SAT

1. If time permits, check all equations.
2. For questions that ask you to find the solution to an equation, you can simply substitute each answer choice into the equation and determine which value makes the equation correct. Begin with choice **c**. If choice **c** is not correct, pick an answer choice that is either larger or smaller.
3. Be careful to answer the question that is being asked. Sometimes, questions require that you solve for a variable and then perform an operation. For example, a question may ask the value of $x - 2$. You might find that $x = 2$ and look for an answer choice of 2. But the question asks for the value of $x - 2$ and the answer is not 2, but $2 - 2$. Thus, the answer is 0.

Equations with More Than One Variable

Some equations have more than one variable. To find the solution of these equations, solve for one variable in terms of the other(s). Follow the same method as when solving single-variable equations, but isolate only one variable.

Example

$3x + 6y = 24$	To isolate the x variable, move $6y$ to the other side.
$3x + 6y - 6y = 24 - 6y$	
$3x = 24 - 6y$	
$\frac{3x}{3} = \frac{24 - 6y}{3}$	Then divide both sides by 3, the coefficient of x.
$x = 8 - 2y$	Then simplify. The solution is for x in terms of y.

Practice Question

If $8a + 16b = 32$, what does a equal in terms of b?

a. $4 - 2b$
b. $2 - \frac{1}{2}b$
c. $32 - 16b$
d. $4 - 16b$
e. $24 - 16b$

Answer

a. To solve for a in terms of b:

$8a + 16b = 32$

$8a + 16b - 16b = 32 - 16b$

$8a = 32 - 16b$

$\frac{8a}{8} = \frac{32 - 16b}{8}$

$a = 4 - 2b$

▶ Monomials

A **monomial** is an expression that is a number, a variable, or a product of a number and one or more variables.

6 y $-5xy^2$ $19a^6b^4$

▶ Polynomials

A **polynomial** is a monomial or the sum or difference of two or more monomials.

$7y^5$ $-6ab^4$ $8x + y^3$ $8x + 9y - z$

Operations with Polynomials

To add polynomials, simply combine like terms.

Example

$(5y^3 - 2y + 1) + (y^3 + 7y - 4)$
First remove the parentheses:
$5y^3 - 2y + 1 + y^3 + 7y - 4$
Then arrange the terms so that like terms are grouped together:
$5y^3 + y^3 - 2y + 7y + 1 - 4$
Now combine like terms:
Answer: $6y^3 + 5y - 3$

Example

$(2x - 5y + 8z) - (16x + 4y - 10z)$
First remove the parentheses. Be sure to distribute the subtraction correctly to all terms in the second set of parentheses:
$2x - 5y + 8z - 16x - 4y + 10z$
Then arrange the terms so that like terms are grouped together:
$2x - 16x - 5y - 4y + 8z + 10z$

Three Kinds of Polynomials

- A **monomial** is a polynomial with one term, such as $5b^6$.
- A **binomial** is a polynomial with two unlike terms, such as $2x + 4y$.
- A **trinomial** is a polynomial with three unlike terms, such as $y^3 + 8z - 2$.

Now combine like terms:
$-14x - 9y + 18z$

To multiply monomials, multiply their coefficients and multiply like variables by adding their exponents.

Example
$(-4a^3b)(6a^2b^3) = (-4)(6)(a^3)(a^2)(b)(b^3) = -24a^5b^4$

To divide monomials, divide their coefficients and divide like variables by subtracting their exponents.

Example
$\frac{10x^5y^7}{15x^4y^2} = (\frac{10}{15})(\frac{x^5}{x^4})(\frac{y^7}{y^2}) = \frac{2xy^5}{3}$

To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial and add the products.

Example

$8x(12x - 3y + 9)$	Distribute.
$(8x)(12x) - (8x)\,(3y) + (8x)(9)$	Simplify.
$96x^2 - 24xy + 72x$	

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial and add the quotients.

Example
$\frac{6x - 18y + 42}{6} = \frac{6x}{6} - \frac{18y}{6} + \frac{42}{6} = x - 3y + 7$

Practice Question
Which of the following is the solution to $\frac{18x^8y^5}{24x^3y^4}$?

a. $\frac{3}{4x^5y}$
b. $\frac{18x^{11}y^9}{24}$
c. $42x^{11}y^9$
d. $\frac{3x^5y}{4}$
e. $\frac{x^5y}{6}$

Answer

d. To find the quotient:

$\frac{18x^8y^5}{24x^3y^4}$	Divide the coefficients and subtract the exponents.
$\frac{3x^{8-3}y^{5-4}}{4}$	
$\frac{3x^5y^1}{4}$	
$= \frac{3x^5y}{4}$	

FOIL

The FOIL method is used when multiplying binomials. FOIL represents the order used to multiply the terms: **F**irst, **O**uter, **I**nner, and **L**ast. To multiply binomials, you multiply according to the FOIL order and then add the products.

Example

$(4x + 2)(9x + 8)$

F: $4x$ and $9x$ are the **first** pair of terms.

O: $4x$ and 8 are the **outer** pair of terms.

I: 2 and $9x$ are the **inner** pair of terms.

L: 2 and 8 are the **last** pair of terms.

Multiply according to FOIL:

$(4x)(9x) + (4x)(8) + (2)(9x) + (2)(8) = 36x^2 + 32x + 18x + 16$

Now combine like terms:

$36x^2 + 50x + 16$

Practice Question

Which of the following is the product of $7x + 3$ and $5x - 2$?

a. $12x^2 - 6x + 1$
b. $35x^2 + 29x - 6$
c. $35x^2 + x - 6$
d. $35x^2 + x + 6$
e. $35x^2 + 11x - 6$

Answer

c. To find the product, follow the FOIL method:

$(7x + 3)(5x - 2)$

F: $7x$ and $5x$ are the **first** pair of terms.

O: $7x$ and -2 are the **outer** pair of terms.

I: 3 and $5x$ are the **inner** pair of terms.

L: 3 and -2 are the **last** pair of terms.

Now multiply according to FOIL:

$(7x)(5x) + (7x)(-2) + (3)(5x) + (3)(-2) = 35x^2 - 14x + 15x - 6$

Now combine like terms:

$35x^2 + x - 6$

Factoring

Factoring is the reverse of multiplication. When multiplying, you find the product of factors. When factoring, you find the factors of a product.

Multiplication: $3(x + y) = 3x + 3y$
Factoring: $3x + 3y = 3(x + y)$

Three Basic Types of Factoring

- Factoring out a common monomial:

$18x^2 - 9x = 9x(2x - 1)$ $\qquad$ $ab - cb = b(a - c)$

- Factoring a quadratic trinomial using FOIL in reverse:

$x^2 - x - 20 = (x - 4)\,(x + 4)$ $\qquad$ $x^2 - 6x + 9 = (x - 3)(x - 3) = (x - 3)^2$

- Factoring the difference between two perfect squares using the rule $a^2 - b^2 = (a + b)(a - b)$:

$x^2 - 81 = (x + 9)(x - 9)$ $\qquad$ $x^2 - 49 = (x + 7)(x - 7)$

Practice Question

Which of the following expressions can be factored using the rule $a^2 - b^2 = (a + b)(a - b)$ where b is an integer?

a. $x^2 - 27$
b. $x^2 - 40$
c. $x^2 - 48$
d. $x^2 - 64$
e. $x^2 - 72$

Answer

d. The rule $a^2 - b^2 = (a + b)(a - b)$ applies to only the difference between perfect squares. 27, 40, 48, and 72 are not perfect squares. 64 is a perfect square, so $x^2 - 64$ can be factored as $(x + 8)(x - 8)$.

Using Common Factors

With some polynomials, you can determine a **common factor** for each term. For example, $4x$ is a common factor of all three terms in the polynomial $16x^4 + 8x^2 + 24x$ because it can divide evenly into each of them. To factor a polynomial with terms that have common factors, you can divide the polynomial by the known factor to determine the second factor.

Example

In the binomial $64x^3 + 24x$, $8x$ is the greatest common factor of both terms.

Therefore, you can divide $64x^3 + 24x$ by $8x$ to find the other factor.

$\frac{64x^3 + 24x}{8x} = \frac{64x^3}{8x} + \frac{24x}{8x} = 8x^2 + 3$

Thus, factoring $64x^3 + 24x$ results in $8x(8x^2 + 3)$.

Practice Question

Which of the following are the factors of $56a^5 + 21a$?

a. $7a(8a^4 + 3a)$
b. $7a(8a^4 + 3)$
c. $3a(18a^4 + 7)$
d. $21a(56a^4 + 1)$
e. $7a(8a^5 + 3a)$

Answer

b. To find the factors, determine a common factor for each term of $56a^5 + 21a$. Both coefficients (56 and 21) can be divided by 7 and both variables can be divided by a. Therefore, a common factor is $7a$. Now, to find the second factor, divide the polynomial by the first factor:

$\frac{56a^5 + 21a}{7a}$

$\frac{8a^5 + 3a}{a^1}$ Subtract exponents when dividing.

$8a^{5-1} + 3a^{1-1}$

$8a^4 + 3a^0$ A base with an exponent of $0 = 1$.

$8a^4 + 3(1)$

$8a^4 + 3$

Therefore, the factors of $56a^5 + 21a$ are $7a(8a^4 + 3)$.

Isolating Variables Using Fractions

It may be necessary to use factoring in order to isolate a variable in an equation.

Example

If $ax - c = bx + d$, what is x in terms of a, b, c, and d?

First isolate the x terms on the same side of the equation:

$ax - bx = c + d$

Now factor out the common x term:

$x(a - b) = c + d$

Then divide both sides by $a - b$ to isolate the variable x:

$\frac{x(a-b)}{a-b} = \frac{c+d}{a-b}$

Simplify:

$x = \frac{c+d}{a-b}$

Practice Question

If $bx + 3c = 6a - dx$, what does x equal in terms of a, b, c, and d?

a. $b - d$
b. $6a - 5c - b - d$
c. $(6a - 5c)(b + d)$
d. $\frac{6a - d - 5c}{b}$
e. $\frac{6a - 5c}{b + d}$

Answer

e. Use factoring to isolate x:

$bx + 5c = 6a - dx$	First isolate the x terms on the same side.
$bx + 5c + dx = 6a - dx + dx$	
$bx + 5c + dx = 6a$	
$bx + 5c + dx - 5c = 6a - 5c$	Finish isolating the x terms on the same side.
$bx + dx = 6a - 5c$	Now factor out the common x term.
$x(b + d) = 6a - 5c$	Now divide to isolate x.
$\frac{x(b + d)}{b + d} = \frac{6a - 5c}{b + d}$	
$x = \frac{6a - 5c}{b + d}$	

▶ Quadratic Trinomials

A **quadratic trinomial** contains an x^2 term as well as an x term. For example, $x^2 - 6x + 8$ is a quadratic trinomial. You can factor quadratic trinomials by using the FOIL method in reverse.

Example

Let's factor $x^2 - 6x + 8$.

Start by looking at the last term in the trinomial: 8. Ask yourself, "What two integers, when multiplied together, have a product of positive 8?" Make a mental list of these integers:

1×8 $\quad -1 \times -8$ $\quad 2 \times 4$ $\quad -2 \times -4$

Next look at the middle term of the trinomial: $-6x$. Choose the two factors from the above list that also add up to the coefficient -6:

-2 and -4

Now write the factors using -2 and -4:

$(x - 2)(x - 4)$

Use the FOIL method to double-check your answer:

$(x - 2)(x - 4) = x^2 - 6x + 8$

The answer is correct.

Practice Question

Which of the following are the factors of $z^2 - 6z + 9$?

a. $(z + 3)(z + 3)$
b. $(z + 1)(z + 9)$
c. $(z - 1)(z - 9)$
d. $(z - 3)(z - 3)$
e. $(z + 6)(z + 3)$

Answer

d. To find the factors, follow the FOIL method in reverse:

$z^2 - 6z + 9$

The product of the **last** pair of terms equals $+9$. There are a few possibilities for these terms: 3 and 3 (because $3 \times 3 = +9$), -3 and -3 (because $-3 \times -3 = +9$), 9 and 1 (because $9 \times 1 = +9$), -9 and -1 (because $-9 \times -1 = +9$).

The sum of the product of the **outer** pair of terms and the **inner** pair of terms equals $-6z$. So we must choose the two last terms from the list of possibilities that would add up to -6. The only possibility is -3 and -3. Therefore, we know the last terms are -3 and -3.

The product of the **first** pair of terms equals z^2. The most likely two terms for the first pair is z and z because $z \times z = z^2$.

Therefore, the factors are $(z - 3)(z - 3)$.

Fractions with Variables

You can work with fractions with variables the same as you would work with fractions without variables.

Example

Write $\frac{x}{6} - \frac{x}{12}$ as a single fraction.

First determine the LCD of 6 and 12: The LCD is 12. Then convert each fraction into an equivalent fraction with 12 as the denominator:

$\frac{x}{6} - \frac{x}{12} = \frac{x \times 2}{6 \times 2} - \frac{x}{12} = \frac{2x}{12} - \frac{x}{12}$

Then simplify:

$\frac{2x}{12} - \frac{x}{12} = \frac{x}{12}$

Practice Question

Which of the following best simplifies $\frac{5x}{8} - \frac{2x}{5}$?

a. $\frac{9}{40}$
b. $\frac{9x}{40}$
c. $\frac{x}{5}$
d. $\frac{3x}{40}$
e. x

Answer

b. To simplify the expression, first determine the LCD of 8 and 5: The LCD is 40. Then convert each fraction into an equivalent fraction with 40 as the denominator:

$\frac{5x}{8} - \frac{2x}{5} = (5x \times \frac{5}{8} \times 5) - \frac{(2x \times 8)}{(5 \times 8)} = \frac{25x}{40} - \frac{16x}{40}$

Then simplify:

$\frac{25x}{40} - \frac{16x}{40} = \frac{9x}{40}$

Reciprocal Rules

There are special rules for the sum and difference of reciprocals. The following formulas can be memorized for the SAT to save time when answering questions about reciprocals:

- If x and y are not 0, then $\frac{1}{x} + y = \frac{x+y}{xy}$
- If x and y are not 0, then $\frac{1}{x} - \frac{1}{y} = \frac{y-x}{xy}$

Note: These rules are easy to figure out using the techniques of the last section, if you are comfortable with them and don't like having too many formulas to memorize.

Quadratic Equations

A **quadratic equation** is an equation in the form $ax^2 + bx + c = 0$, where a, b, and c are numbers and $a \neq 0$. For example, $x^2 + 6x + 10 = 0$ and $6x^2 + 8x - 22 = 0$ are quadratic equations.

Zero-Product Rule

Because quadratic equations can be written as an expression equal to zero, the zero-product rule is useful when solving these equations.

The **zero-product rule** states that if the product of two or more numbers is 0, then at least one of the numbers is 0. In other words, if $ab = 0$, then you know that either a or b equals zero (or they both might be zero). This idea also applies when a and b are factors of an equation. When an equation equals 0, you know that one of the factors of the equation must equal zero, so you can determine the two possible values of x that make the factors equal to zero.

Example

Find the two possible values of x that make this equation true: $(x + 4)(x - 2) = 0$

Using the zero-product rule, you know that either $x + 4 = 0$ or that $x - 2 = 0$.

So solve both of these possible equations:

$x + 4 = 0$	$x - 2 = 0$
$x + 4 - 4 = 0 - 4$	$x - 2 + 2 = 0 + 2$
$x = -4$	$x = 2$

Thus, you know that both $x = -4$ and $x = 2$ will make $(x + 4)(x - 2) = 0$.

The zero product rule is useful when solving quadratic equations because you can rewrite a quadratic equation as equal to zero and take advantage of the fact that one of the factors of the quadratic equation is thus equal to 0.

Practice Question

If $(x - 8)(x + 5) = 0$, what are the two possible values of x?

a. $x = 8$ and $x = -5$
b. $x = -8$ and $x = 5$
c. $x = 8$ and $x = 0$
d. $x = 0$ and $x = -5$
e. $x = 13$ and $x = -13$

Answer

a. If $(x - 8)(x + 5) = 0$, then one (or both) of the factors must equal 0.
$x - 8 = 0$ if $x = 8$ because $8 - 8 = 0$.
$x + 5 = 0$ if $x = -5$ because $-5 + 5 = 0$.

Therefore, the two values of x that make $(x - 8)(x + 5) = 0$ are $x = 8$ and $x = -5$.

Solving Quadratic Equations by Factoring

If a quadratic equation is not equal to zero, rewrite it so that you can solve it using the zero-product rule.

Example

If you need to solve $x^2 - 11x = 12$, subtract 12 from both sides:
$x^2 - 11x - 12 = 12 - 12$
$x^2 - 11x - 12 = 0$
Now this quadratic equation can be solved using the zero-product rule:
$x^2 - 11x - 12 = 0$
$(x - 12)(x + 1) = 0$
Therefore:

$x - 12 = 0$	or	$x + 1 = 0$
$x - 12 + 12 = 0 + 12$		$x + 1 - 1 = 0 - 1$
$x = 12$		$x = -1$

Thus, you know that both $x = 12$ and $x = -1$ will make $x^2 - 11x - 12 = 0$.

A quadratic equation must be factored before using the zero-product rule to solve it.

Example

To solve $x^2 + 9x = 0$, first factor it:
$x(x + 9) = 0$.
Now you can solve it.
Either $x = 0$ or $x + 9 = 0$.
Therefore, possible solutions are $x = 0$ and $x = -9$.

Practice Question

If $x^2 - 8x = 20$, which of the following could be a value of $x^2 + 8x$?

a. -20
b. 20
c. 28
d. 108
e. 180

Answer

e. This question requires several steps to answer. First, you must determine the possible values of x considering that $x^2 - 8x = 20$. To find the possible x values, rewrite $x^2 - 8x = 20$ as $x^2 - 8x - 20 = 0$, factor, and then use the zero-product rule.

$x^2 - 8x - 20 = 0$ is factored as $(x - 10)(x + 2)$.

Thus, possible values of x are $x = 10$ and $x = -2$ because $10 - 10 = 0$ and $-2 + 2 = 0$.

Now, to find possible values of $x^2 + 8x$, plug in the x values:

If $x = -2$, then $x^2 + 8x = (-2)^2 + (8)(-2) = 4 + (-16) = -12$. None of the answer choices is -12, so try $x = 10$.

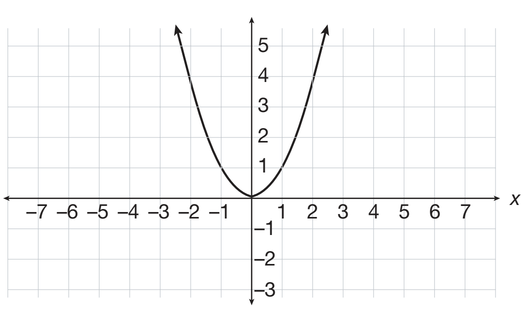
If $x = 10$, then $x^2 + 8x = 10^2 + (8)(10) = 100 + 80 = 180$.

Therefore, answer choice **e** is correct.

► Graphs of Quadratic Equations

The (x,y) solutions to quadratic equations can be plotted on a graph. It is important to be able to look at an equation and understand what its graph will look like. You must be able to determine what calculation to perform on each x value to produce its corresponding y value.

For example, below is the graph of $y = x^2$.



The equation $y = x^2$ tells you that for every x value, you must square the x value to find its corresponding y value. Let's explore the graph with a few x-coordinates:

An x value of 1 produces what y value? Plug $x = 1$ into $y = x^2$.

When $x = 1$, $y = 1^2$, so $y = 1$.
Therefore, you know a coordinate in the graph of $y = x^2$ is (1,1).

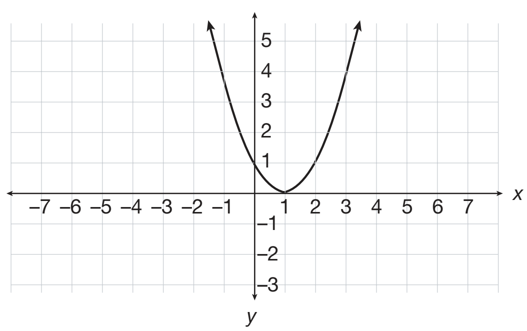
An x value of 2 produces what y value? Plug $x = 2$ into $y = x^2$.
When $x = 2$, $y = 2^2$, so $y = 4$.
Therefore, you know a coordinate in the graph of $y = x^2$ is (2,4).

An x value of 3 produces what y value? Plug $x = 3$ into $y = x^2$.
When $x = 3$, $y = 3^2$, so $y = 9$.
Therefore, you know a coordinate in the graph of $y = x^2$ is (3,9).

The SAT may ask you, for example, to compare the graph of $y = x^2$ with the graph of $y = (x - 1)^2$. Let's compare what happens when you plug numbers (x values) into $y = (x - 1)^2$ with what happens when you plug numbers (x values) into $y = x^2$:

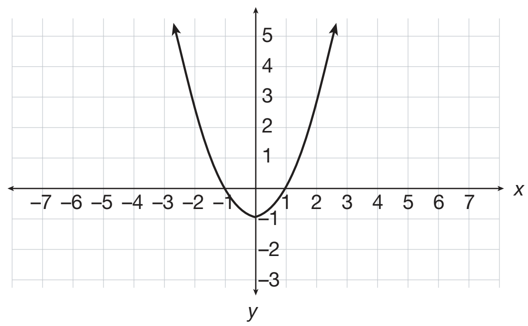
$y = x^2$	$y = (x - 1)^2$
If $x = 1$, $y = 1$.	If $x = 1$, $y = 0$.
If $x = 2$, $y = 4$.	If $x = 2$, $y = 1$.
If $x = 3$, $y = 9$.	If $x = 3$, $y = 4$.
If $x = 4$, $y = 16$.	If $x = 4$, $y = 9$.

The two equations have the same y values, but they match up with different x values because $y = (x - 1)^2$ subtracts 1 before squaring the x value. As a result, the graph of $y = (x - 1)^2$ looks identical to the graph of $y = x^2$ except that the base is shifted to the right (on the x-axis) by 1:

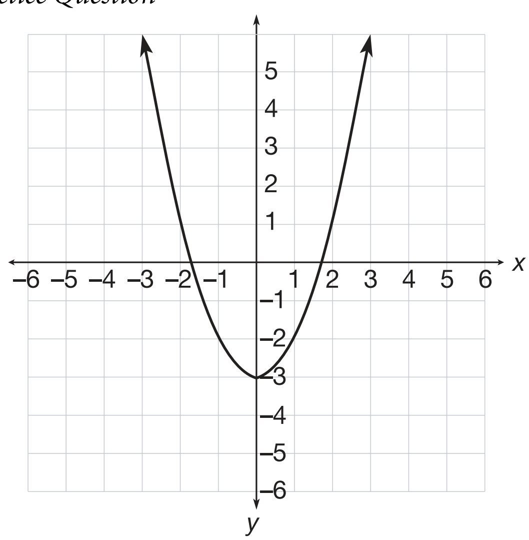


How would the graph of $y = x^2$ compare with the graph of $y = x^2 - 1$?

In order to find a y value with $y = x^2$, you square the x value. In order to find a y value with $y = x^2 - 1$, you square the x value and then subtract 1. This means the graph of $y = x^2 - 1$ looks identical to the graph of $y = x^2$ except that the base is shifted down (on the y-axis) by 1:



Practice Question



What is the equation represented in the graph above?

a. $y = x^2 + 3$
b. $y = x^2 - 3$
c. $y = (x + 3)^2$
d. $y = (x - 3)^2$
e. $y = (x - 1)^3$

Answer

b. This graph is identical to a graph of $y = x^2$ except it is moved down 3 so that the parabola intersects the y-axis at -3 instead of 0. Each y value is 3 less than the corresponding y value in $y = x^2$, so its equation is therefore $y = x^2 - 3$.

▶ Rational Equations and Inequalities

Rational numbers are numbers that can be written as fractions (and decimals and repeating decimals). Similarly, **rational equations** are equations in fraction form. **Rational inequalities** are also in fraction form and use the symbols $<$, $>$, $\leq$, and $\geq$ instead of $=$.

Example

Given $\frac{(x+5)(x^2+5x-14)}{(x^2+3x-10)} = 30$, find the value of x.

Factor the top and bottom:

$\frac{(x+5)(x+7)(x-2)}{(x+5)(x-2)} = 30$

You can cancel out the $(x + 5)$ and the $(x - 2)$ terms from the top and bottom to yield:

$x + 7 = 30$

Now solve for x:

$x + 7 = 30$

$x + 7 - 7 = 30 - 7$

$x = 23$

Practice Question

If $\frac{(x+8)(x^2+11x-26)}{(x^2+6x-16)} = 17$, what is the value of x?

a. -16
b. -13
c. -8
d. 2
e. 4

Answer

e. To solve for x, first factor the top and bottom of the fractions:

$\frac{(x+8)(x^2+11x-26)}{(x^2+6x-16)} = 17$

$\frac{(x+8)(x+13)(x-2)}{(x+8)(x-2)} = 17$

You can cancel out the $(x + 8)$ and the $(x - 2)$ terms from the top and bottom:

$x + 13 = 17$

Solve for x:

$x + 13 - 13 = 17 - 13$

$x = 4$

► Radical Equations

Some algebraic equations on the SAT include the square root of the unknown. To solve these equations, first isolate the radical. Then square both sides of the equation to remove the radical sign.

Example

$5\sqrt{c} + 15 = 35$

To isolate the variable, subtract 15 from both sides:

$5\sqrt{c} + 15 - 15 = 35 - 15$

$5\sqrt{c} = 20$

Next, divide both sides by 5:

$\frac{5\sqrt{c}}{5} = \frac{20}{5}$

$\sqrt{c} = 4$

Last, square both sides:

$(\sqrt{c})^2 = 4^2$

$c = 16$

Practice Question

If $6\sqrt{d} - 10 = 32$, what is the value of d?

a. 7
b. 14
c. 36
d. 49
e. 64

Answer

d. To solve for d, isolate the variable:

$6\sqrt{d} - 10 = 32$

$6\sqrt{d} - 10 + 10 = 32 + 10$

$6\sqrt{d} = 42$

$\frac{6\sqrt{d}}{6} = \frac{42}{6}$

$\sqrt{d} = 7$

$(\sqrt{d})^2 = 7^2$

$d = 49$

► Sequences Involving Exponential Growth

When analyzing a sequence, try to find the mathematical operation that you can perform to get the next number in the sequence. Let's try an example. Look carefully at the following sequence:

2, 4, 8, 16, 32, . . .

Notice that each successive term is found by multiplying the prior term by 2. ($2 \times 2 = 4$, $4 \times 2 = 8$, and so on.) Since each term is multiplied by a constant number (2), there is a constant ratio between the terms. Sequences that have a constant ratio between terms are called **geometric sequences**.

On the SAT, you may be asked to determine a specific term in a sequence. For example, you may be asked to find the thirtieth term of a geometric sequence like the previous one. You could answer such a question by writing out 30 terms of a sequence, but this is an inefficient method. It takes too much time. Instead, there is a formula to use. Let's determine the formula:

First, let's evaluate the terms.
2, 4, 8, 16, 32, . . .
Term 1 = 2
Term 2 = 4, which is 2×2
Term 3 = 8, which is $2 \times 2 \times 2$
Term 4 = 16, which is $2 \times 2 \times 2 \times 2$

You can also write out each term using exponents:

Term 1 = 2
Term 2 = 2×2^1
Term 3 = 2×2^2
Term 4 = 2×2^3

We can now write a formula:

Term $n = 2 \times 2^{n-1}$

So, if the SAT asks you for the thirtieth term, you know that:

Term $30 = 2 \times 2^{30-1} = 2 \times 2^{29}$

The generic formula for a geometric sequence is Term $n = a_1 \times r^{n-1}$, where n is the term you are looking for, a_1 is the first term in the series, and r is the ratio that the sequence increases by. In the above example, $n = 30$ (the thirtieth term), $a_1 = 2$ (because 2 is the first term in the sequence), and $r = 2$ (because the sequence increases by a ratio of 2; each term is two times the previous term).

You can use the formula Term $n = a_1 \times r^{n-1}$ when determining a term in any geometric sequence.

Practice Question

1, 3, 9, 27, 81, . . .

What is the thirty-eighth term of the sequence above?

a. 3^{38}
b. 3×1^{37}
c. 3×1^{38}
d. 1×3^{37}
e. 1×3^{38}

Answer

d. 1, 3, 9, 27, 81, . . . is a geometric sequence. There is a constant ratio between terms. Each term is three times the previous term. You can use the formula Term $n = a_1 \times r^{n-1}$ to determine the nth term of this geometric sequence.

First determine the values of n, a_1, and r:

$n = 38$ (because you are looking for the thirty-eighth term)

$a_1 = 1$ (because the first number in the sequence is 1)

$r = 3$ (because the sequence increases by a ratio of 3; each term is three times the previous term.)

Now solve:

Term $n = a_1 \times r^{n-1}$

Term $38 = 1 \times 3^{38-1}$

Term $38 = 1 \times 3^{37}$

▶ Systems of Equations

A system of equations is a set of two or more equations with the same solution. If $2c + d = 11$ and $c + 2d = 13$ are presented as a system of equations, we know that we are looking for values of c and d, which will be the same in both equations and will make both equations true.

Two methods for solving a system of equations are **substitution** and **linear combination**.

Substitution

Substitution involves solving for one variable in terms of another and then substituting that expression into the second equation.

Example

Here are the two equations with the same solution mentioned above:

$2c + d = 11$ and $c + 2d = 13$

To solve, first choose one of the equations and rewrite it, isolating one variable in terms of the other. It does not matter which variable you choose.

$2c + d = 11$ becomes $d = 11 - 2c$

Next substitute $11 - 2c$ for d in the other equation and solve:

$c + 2d = 13$
$c + 2(11 - 2c) = 13$
$c + 22 - 4c = 13$
$22 - 3c = 13$
$22 = 13 + 3c$
$9 = 3c$
$c = 3$
Now substitute this answer into either original equation for c to find d.
$2c + d = 11$
$2(3) + d = 11$
$6 + d = 11$
$d = 5$
Thus, $c = 3$ and $d = 5$.

Linear Combination

Linear combination involves writing one equation over another and then adding or subtracting the like terms so that one letter is eliminated.

Example

$x - 7 = 3y$ and $x + 5 = 6y$
First rewrite each equation in the same form.
$x - 7 = 3y$ becomes $x - 3y = 7$
$x + 5 = 6y$ becomes $x - 6y = -5$.
Now subtract the two equations so that the x terms are eliminated, leaving only one variable:

$$\begin{aligned} x - 3y &= 7 \\ \underline{-(x - 6y} &\underline{= -5)} \\ (x - x) + (-3y + 6y) &= 7 - (-5) \\ 3y &= 12 \\ y &= 4 \text{ is the answer.} \end{aligned}$$

Now substitute 4 for y in one of the original equations and solve for x.
$x - 7 = 3y$
$x - 7 = 3(4)$
$x - 7 = 12$
$x - 7 + 7 = 12 + 7$
$x = 19$
Therefore, the solution to the system of equations is $y = 4$ and $x = 19$.

Systems of Equations with No Solution

It is possible for a system of equations to have no solution if there are no values for the variables that would make all the equations true. For example, the following system of equations has no solution because there are no values of x and y that would make both equations true:

$3x + 6y = 14$
$3x + 6y = 9$
In other words, one expression cannot equal both 14 and 9.

Practice Question

$5x + 3y = 4$
$15x + dy = 21$
What value of d would give the system of equations NO solution?

a. -9
b. -3
c. 1
d. 3
e. 9

Answer

e. The first step in evaluating a system of equations is to write the equations so that the coefficients of one of the variables are the same. If we multiply $5x + 3y = 4$ by 3, we get $15x + 9y = 12$. Now we can compare the two equations because the coefficients of the x variables are the same:
$15x + 9y = 12$
$15x + dy = 21$
The only reason there would be no solution to this system of equations is if the system contains the same expressions equaling different numbers. Therefore, we must choose the value of d that would make $15x + dy$ identical to $15x + 9y$. If $d = 9$, then:
$15x + 9y = 12$
$15x + 9y = 21$
Thus, if $d = 9$, there is no solution. Answer choice **e** is correct.

▶ Functions, Domain, and Range

A **function** is a relationship in which one value depends upon another value. Functions are written in the form beginning with the following symbols:

$f(x) =$

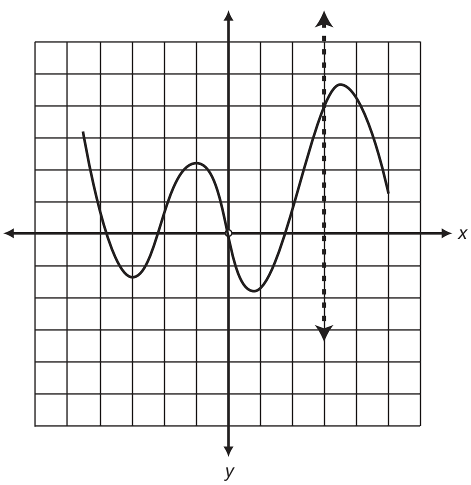
For example, consider the function $f(x) = 8x - 2$. If you are asked to find $f(3)$, you simply substitute the 3 into the given function equation.

$f(x) = 8x - 2$
becomes
$f(3) = 8(3) - 2f(3) = 24 - 2 = 22$

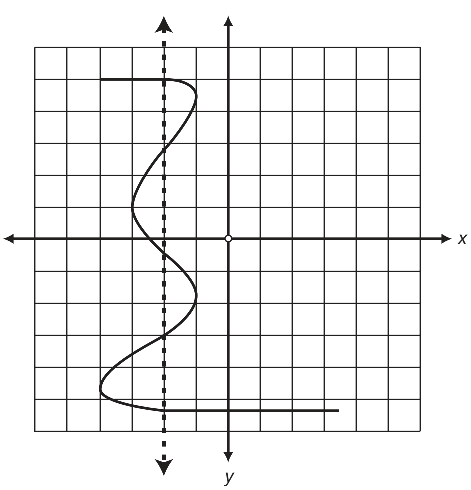
So, when $x = 3$, the value of the function is 22.

Potential functions must pass the **vertical line test** in order to be considered a function. The vertical line test is the following: Does any vertical line drawn through a graph of the potential function pass through only one point of the graph? If YES, then any vertical line drawn passes through only one point, and the potential function is a function. If NO, then a vertical line can be drawn that passes through more than one point, and the potential function is *not* a function.

The graph below shows a function because any vertical line drawn on the graph (such as the dotted vertical line shown) passes through the graph of the function only once:



The graph below does NOT show a function because the dotted vertical line passes five times through the graph:



All of the x values of a function, collectively, are called its **domain**. Sometimes there are x values that are outside of the domain, but these are the x values for which the function is not defined.

All of the values taken on by $f(x)$ are collectively called the **range**. Any values that $f(x)$ cannot be equal to are said to be outside of the range.

The x values are known as the **independent variables**. The y values *depend* on the x values, so the y values are called the **dependent variables**.

Practice Question

If the function f is defined by $f(x) = 9x + 3$, which of the following is equal to $f(4b)$?

a. $36b + 12b$

b. $36b + 12$

c. $36b + 3$

d. $\frac{9}{4b + 3}$

e. $\frac{4b}{9 + 3}$

Answer

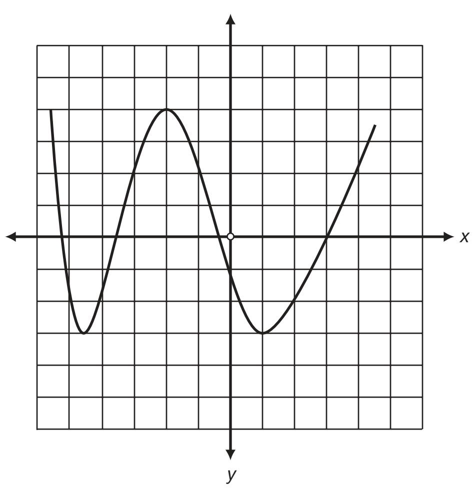
c. If $f(x) = 9x + 3$, then, for $f(4b)$, $4b$ simply replaces x in $9x + 3$. Therefore, $f(4b) = 9(4b) + 3 = 36b + 3$.

Qualitative Behavior of Graphs and Functions

For the SAT, you should be able to analyze the graph of a function and interpret, qualitatively, something about the function itself.

Example

Consider the portion of the graph shown below. Let's determine how many values there are for $f(x) = 2$.



When $f(x) = 2$, the y value equals 2. So let's draw a horizontal line through $y = 2$ to see how many times the line intersects with the function. These points of intersection tell us the x values for $f(x) = 2$. As shown below, there are 4 such points, so we know there are four values for $f(x) = 2$.