CHAPTER

7 Geometry Review

This chapter reviews key skills and concepts of geometry that you need to know for the SAT. Throughout the chapter are sample questions in the style of SAT questions. Each sample SAT question is followed by an explanation of the correct answer.

Vocabulary

It is essential in geometry to recognize and understand the terminology used. Before you take the SAT, be sure you know and understand each geometry term in the following list.

acute angle	an angle that measures less than 90°
acute triangle	a triangle with every angle that measures less than 90°
adjacent angles	two angles that have the same vertex, share one side, and do not overlap
angle	two rays connected by a vertex
arc	a curved section of a circle
area	the number of square units inside a shape
bisect	divide into two equal parts
central angle	an angle formed by an arc in a circle

chord	a line segment that goes through a circle, with its endpoints on the circle
circumference	the distance around a circle
complementary angles	two angles whose sum is 90°
congruent	identical in shape and size; the geometric symbol for *congruent to* is ≅
coordinate plane	a grid divided into four quadrants by both a horizontal *x*-axis and a vertical *y*-axis
coordinate points	points located on a coordinate plane
diagonal	a line segment between two non-adjacent vertices of a polygon
diameter	a chord that passes through the center of a circle—the longest line you can draw in a circle. The term is used not only for this line segment, but also for its length.
equiangular polygon	a polygon with all angles of equal measure
equidistant	the same distance
equilateral triangle	a triangle with three equal sides and three equal angles
exterior angle	an angle on the outer sides of two lines cut by a transversal; or, an angle outside a triangle
hypotenuse	the longest leg of a right triangle. The hypotenuse is always opposite the right angle in a right triangle.
interior angle	an angle on the inner sides of two lines cut by a transversal
isosceles triangle	a triangle with two equal sides
line	a straight path that continues infinitely in two directions. The geometric notation for a line through points *A* and *B* is $\overline{AB}$.
line segment	the part of a line between (and including) two points. The geometric notation for the line segment joining points *A* and *B* is $\overline{AB}$. The notation $\overline{AB}$ is used both to refer to the segment itself and to its length.
major arc	an arc greater than or equal to 180°
midpoint	the point at the exact middle of a line segment
minor arc	an arc less than or equal to 180°
obtuse angle	an angle that measures greater than 90°
obtuse triangle	a triangle with an angle that measures greater than 90°
ordered pair	a location of a point on the coordinate plane in the form of (x,y). The *x* represents the location of the point on the horizontal *x*-axis, and the *y* represents the location of the point on the vertical *y*-axis.

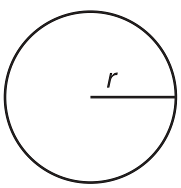
origin	coordinate point (0,0): the point on a coordinate plane at which the x-axis and y-axis intersect
parallel lines	two lines in a plane that do not intersect. Parallel lines are marked by a symbol $\parallel$.
parallelogram	a quadrilateral with two pairs of parallel sides
perimeter	the distance around a figure
perpendicular lines	lines that intersect to form right angles
polygon	a closed figure with three or more sides
Pythagorean theorem	the formula $a^2 + b^2 = c^2$, where a and b represent the lengths of the *legs* and c represents the length of the *hypotenuse* of a right triangle
Pythagorean triple	a set of three whole numbers that satisfies the Pythagorean theorem, $a^2 + b^2 = c^2$, such as 3:4:5 and 5:12:13
quadrilateral	a four-sided polygon
radius	a line segment inside a circle with one point on the radius and the other point at the center on the circle. The radius is half the diameter. This term can also be used to refer to the length of such a line segment. The plural of *radius* is *radii*.
ray	half of a line. A ray has one endpoint and continues infinitely in one direction. The geometric notation for a ray is with endpoint A and passing through point B is $\overrightarrow{AB}$.
rectangle	a parallelogram with four right angles
regular polygon	a polygon with all equal sides
rhombus	a parallelogram with four equal sides
right angle	an angle that measures exactly 90°
right triangle	a triangle with an angle that measures exactly 90°
scalene triangle	a triangle with no equal sides
sector	a slice of a circle formed by two radii and an arc
similar polygons	two or more polygons with equal corresponding angles and corresponding sides in proportion
slope	the steepness of a line, as determined by $\frac{\text{vertical change}}{\text{horizontal change}}$, or $\frac{y_2 - y_1}{x_2 - x_1}$, on a coordinate plane where (x_1, y_1) and (x_2, y_2) are two points on that line
solid	a three-dimensional figure
square	a parallelogram with four equal sides and four right angles
supplementary angles	two angles whose sum is 180°

surface area	the sum of the areas of the faces of a solid
tangent	a line that touches a curve (such as a circle) at a single point without cutting across the curve. A tangent line that touches a circle at point *P* is perpendicular to the circle's radius drawn to point *P*
transversal	a line that intersects two or more lines
vertex	a point at which two lines, rays, or line segments connect
vertical angles	two opposite congruent angles formed by intersecting lines
volume	the number of cubic units inside a three-dimensional figure

▶ Formulas

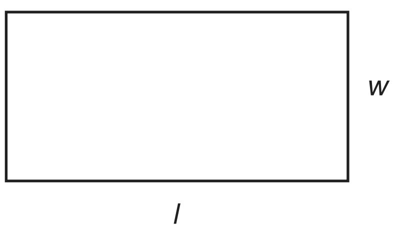
The formulas below for area and volume will be provided to you on the SAT. You do not need to memorize them (although it wouldn't hurt!). Regardless, be sure you understand them thoroughly.

Circle



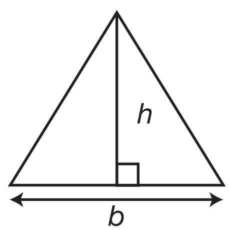
$C = 2\pi r$
$A = \pi r^2$

Rectangle



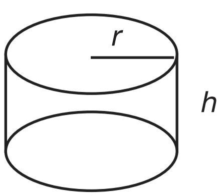
$A = lw$

Triangle



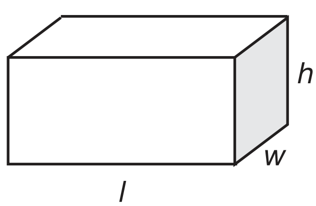
$A = \frac{1}{2}bh$

Cylinder



$V = \pi r^2 h$

Rectangle Solid

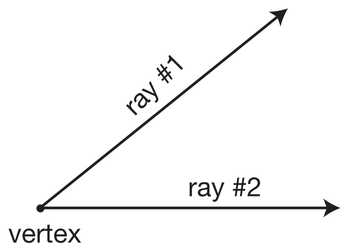


$V = lwh$

C =	Circumference	w =	Width
A =	Area	h =	Height
r =	Radius	V =	Volume
l =	Length	b =	Base

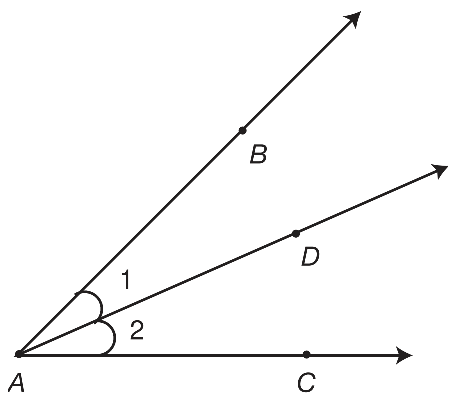
▶ Angles

An **angle** is formed by two rays and an endpoint or line segments that meet at a point, called the **vertex**.



Naming Angles

There are three ways to name an angle.



1. An angle can be named by the vertex when no other angles share the same vertex: $\angle A$.
2. An angle can be represented by a number or variable written across from the vertex: $\angle 1$ and $\angle 2$.
3. When more than one angle has the same vertex, three letters are used, with the vertex always being the middle letter: $\angle 1$ can be written as $\angle BAD$ or $\angle DAB$, and $\angle 2$ can be written as $\angle DAC$ or $\angle CAD$.

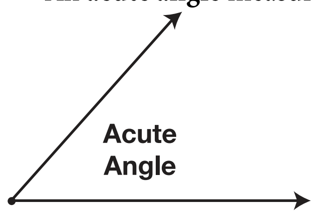
The Measure of an Angle

The notation $m\angle A$ is used when referring to the measure of an angle (in this case, angle A). For example, if $\angle D$ measures 100°, then $m\angle D = 100°$.

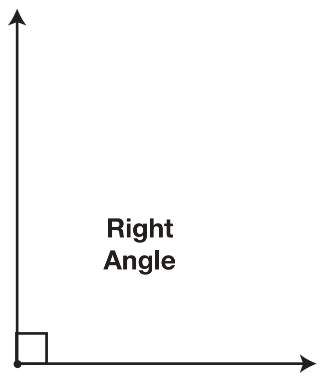
Classifying Angles

Angles are classified into four categories: acute, right, obtuse, and straight.

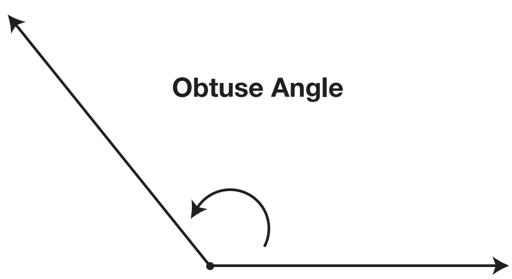
- An **acute angle** measures less than 90°.



- A **right angle** measures exactly 90°. A right angle is symbolized by a square at the vertex.

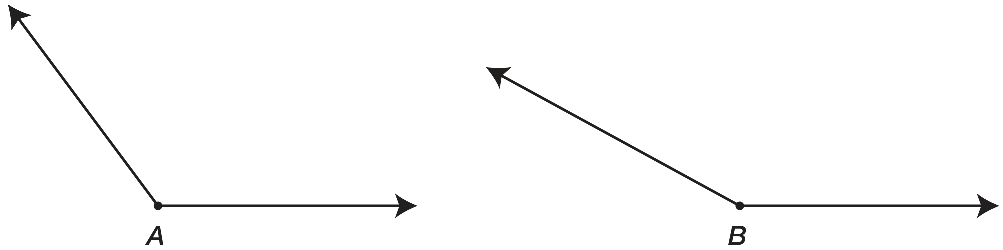


- An **obtuse angle** measures more than 90° but less then 180°.



- A **straight angle** measures exactly 180°. A straight angle forms a line.

Practice Question



Which of the following must be true about the sum of m∠*A* and m∠*B*?

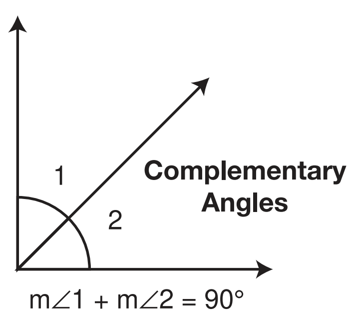
a. It is equal to 180°.
b. It is less than 180°.
c. It is greater than 180°.
d. It is equal to 360°.
e. It is greater than 360°.

Answer

c. Both ∠*A* and ∠*B* are obtuse, so they are both greater than 90°. Therefore, if 90° + 90° = 180°, then the sum of m∠*A* and m∠*B* must be greater than 180°.

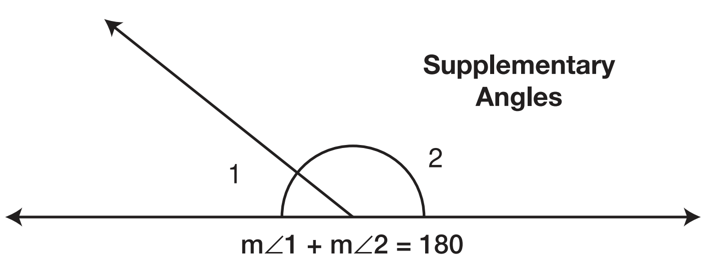
Complementary Angles

Two angles are **complementary** if the sum of their measures is 90°.

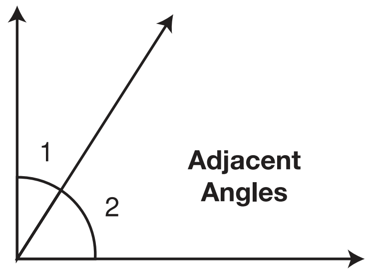


Supplementary Angles

Two angles are **supplementary** if the sum of their measures is 180°.

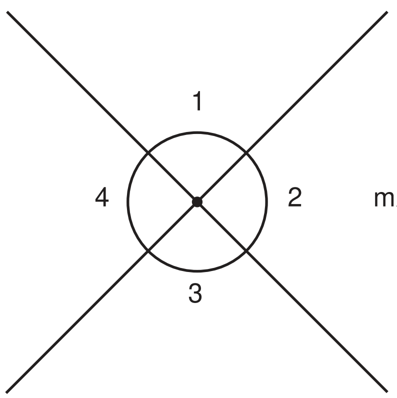


Adjacent angles have the same vertex, share one side, and do not overlap.



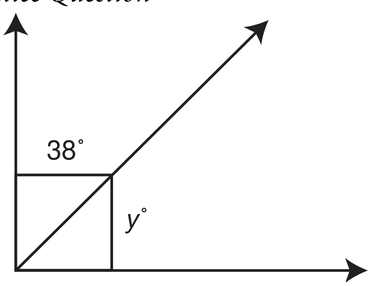
∠1 and ∠2 are adjacent

The sum of all adjacent angles around the same vertex is equal to 360°.



$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$

Practice Question



Which of the following must be the value of *y*?

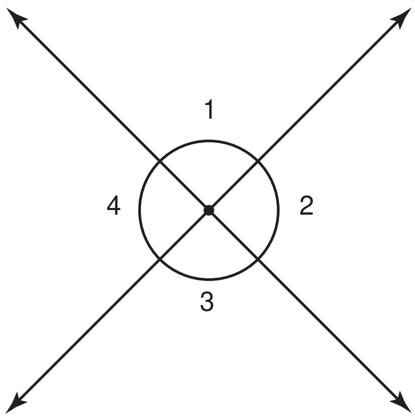
a. 38
b. 52
c. 90
d. 142
e. 180

Answer

b. The figure shows two complementary angles, which means the sum of the angles equals 90°. If one of the angles is 38°, then the other angle is (90° − 38°). Therefore, $y° = 90° - 38° = 52°$, so $y = 52$.

Angles of Intersecting Lines

When two lines intersect, **vertical angles** are formed. In the figure below, ∠1 and ∠3 are vertical angles and ∠2 and ∠4 are vertical angles.



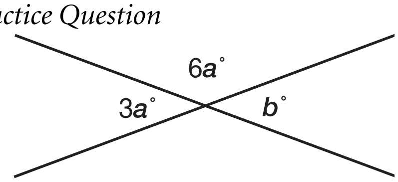
Vertical angles have equal measures:

- m∠1 = m∠3
- m∠2 = m∠4

Vertical angles are supplementary to adjacent angles. The sum of a vertical angle and its adjacent angle is 180°:

- m∠1 + m∠2 = 180°
- m∠2 + m∠3 = 180°
- m∠3 + m∠4 = 180°
- m∠1 + m∠4 = 180°

Practice Question



What is the value of b in the figure above?

a. 20
b. 30
c. 45
d. 60
e. 120

Answer

d. The drawing shows angles formed by intersecting lines. The laws of intersecting lines tell us that $3a° = b°$ because they are the measures of opposite angles. We also know that $3a° + 6a° = 180°$ because $3a°$ and $6a°$ are measures of supplementary angles. Therefore, we can solve for a:

$3a + 6a = 180$

$9a = 180$

$a = 20$

Because $3a° = b°$, we can solve for b by substituting 20 for a:

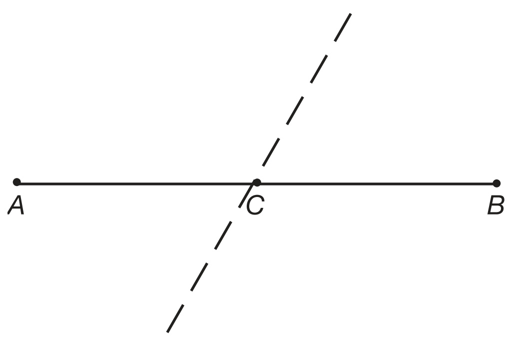
$3a = b$

$3(20) = b$

$60 = b$

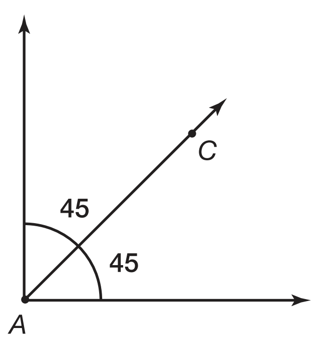
Bisecting Angles and Line Segments

A line or segment **bisects** a line segment when it divides the second segment into two equal parts.



The dotted line **bisects** segment $\overline{AB}$ at point C, so $\overline{AC} = \overline{CB}$.

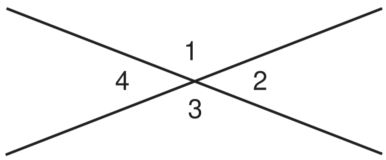
A line **bisects** an angle when it divides the angle into two equal smaller angles.



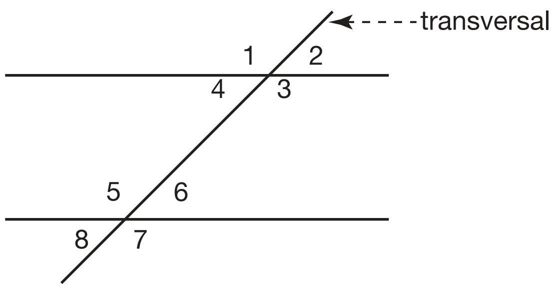
According to the figure, ray $\overrightarrow{AC}$ bisects $\angle A$ because it divides the right angle into two 45° angles.

Angles Formed with Parallel Lines

Vertical angles are the opposite angles formed by the intersection of any two lines. In the figure below, ∠1 and ∠3 are vertical angles because they are opposite each other. ∠2 and ∠4 are also vertical angles.



A special case of vertical angles occurs when a transversal line intersects two parallel lines.



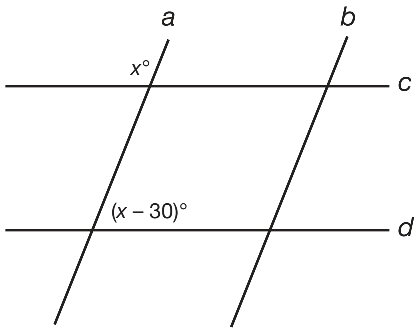
The following rules are true when a transversal line intersects two parallel lines.

- There are four sets of vertical angles:
  ∠1 and ∠3
  ∠2 and ∠4
  ∠5 and ∠7
  ∠6 and ∠8
- Four of these vertical angles are obtuse:
  ∠1, ∠3, ∠5, and ∠7
- Four of these vertical angles are acute:
  ∠2, ∠4, ∠6, and ∠8
- The obtuse angles are equal:
  $\angle 1 = \angle 3 = \angle 5 = \angle 7$
- The acute angles are equal:
  $\angle 2 = \angle 4 = \angle 6 = \angle 8$
- In this situation, any acute angle added to any obtuse angle is supplementary.
  $m\angle 1 + m\angle 2 = 180°$
  $m\angle 2 + m\angle 3 = 180°$
  $m\angle 3 + m\angle 4 = 180°$
  $m\angle 1 + m\angle 4 = 180°$
  $m\angle 5 + m\angle 6 = 180°$
  $m\angle 6 + m\angle 7 = 180°$
  $m\angle 7 + m\angle 8 = 180°$
  $m\angle 5 + m\angle 8 = 180°$

You can use these rules of vertical angles to solve problems.

Example

In the figure below, if $c \parallel d$, what is the value of x?



Because $c \parallel d$, you know that the sum of an acute angle and an obtuse angle formed by an intersecting line (line a) is equal to 180°. $\angle x$ is obtuse and $\angle(x - 30)$ is acute, so you can set up the equation $x + (x - 30) = 180$.

Now solve for x:

$x + (x - 30) = 180$

$2x - 30 = 180$

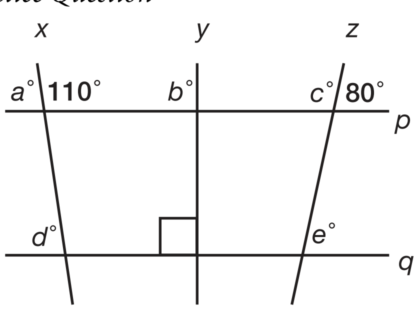
$2x - 30 + 30 = 180 + 30$

$2x = 210$

$x = 105$

Therefore, $m\angle x = 105°$. The acute angle is equal to $180 - 105 = 75°$.

Practice Question



If $p \parallel q$, which the following is equal to 80?

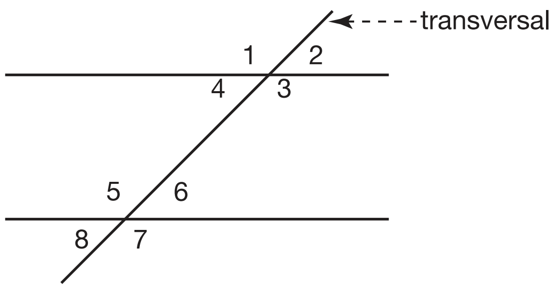
a. *a*
b. *b*
c. *c*
d. *d*
e. *e*

Answer

e. Because $p \parallel q$, the angle with measure 80° and the angle with measure $e°$ are corresponding angles, so they are equivalent. Therefore $e° = 80°$, and $e = 80$.

Interior and Exterior Angles

Exterior angles are the angles on the outer sides of two lines intersected by a transversal. **Interior angles** are the angles on the inner sides of two lines intersected by a transversal.



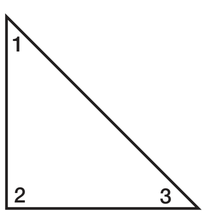
In the figure above:

∠1, ∠2, ∠7, and ∠8 are exterior angles.
∠3, ∠4, ∠5, and ∠6 are interior angles.

▶ Triangles

Angles of a Triangle

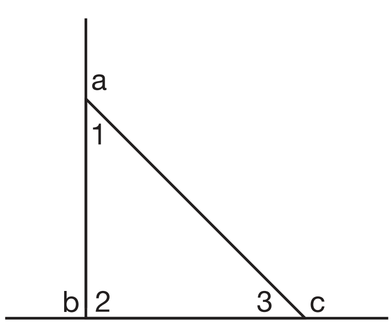
The measures of the three angles in a triangle always add up to 180°.



$m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Exterior Angles of a Triangle

Triangles have three exterior angles. $\angle a$, $\angle b$, and $\angle c$ are the exterior angles of the triangle below.



- An exterior angle and interior angle that share the same vertex are supplementary:

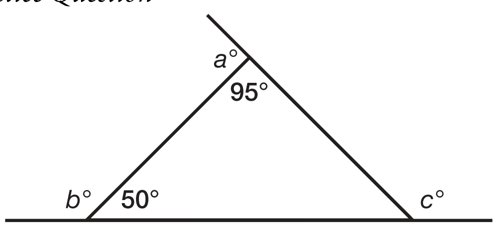
$m\angle 1 + m\angle a = 180°$
$m\angle 2 + m\angle b = 180°$
$m\angle 3 + m\angle c = 180°$

- An exterior angle is equal to the sum of the non-adjacent interior angles:

$m\angle a = m\angle 2 + m\angle 3$
$m\angle b = m\angle 1 + m\angle 3$
$m\angle c = m\angle 1 + m\angle 2$

The sum of the exterior angles of any triangle is 360°.

Practice Question



Based on the figure, which of the following must be true?

I. $a < b$
II. $c = 135°$
III. $b < c$

a. I only
b. III only
c. I and III only
d. II and III only
e. I, II, and III

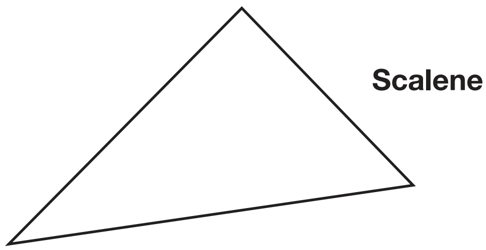
Answer

c. To solve, you must determine the value of the third angle of the triangle and the values of *a*, *b*, and *c*. The third angle of the triangle $= 180° - 95° - 50° = 35°$ (because the sum of the measures of the angles of a triangle are 180°).
$a = 180 - 95 = 85$ (because $\angle a$ and the angle that measures 95° are supplementary).
$b = 180 - 50 = 130$ (because $\angle b$ and the angle that measures 50° are supplementary).
$c = 180 - 35 = 145$ (because $\angle c$ and the angle that measures 35° are supplementary).
Now we can evaluate the three statements:
I: $a < b$ is TRUE because $a = 85$ and $b = 130$.
II: $c = 135°$ is FALSE because $c = 145°$.
III: $b < c$ is TRUE because $b = 130$ and $c = 145$.
Therefore, only I and III are true.

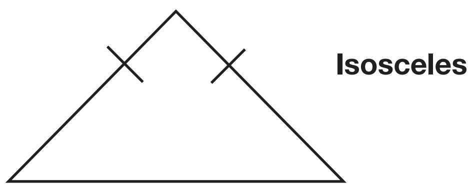
Types of Triangles

You can classify triangles into three categories based on the number of equal sides.

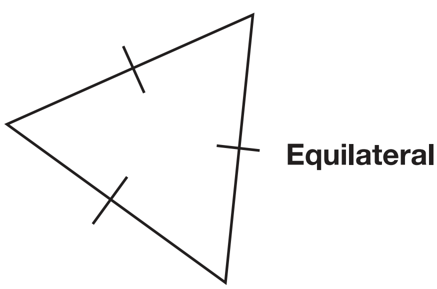
- **Scalene Triangle:** no equal sides



- **Isosceles Triangle:** two equal sides

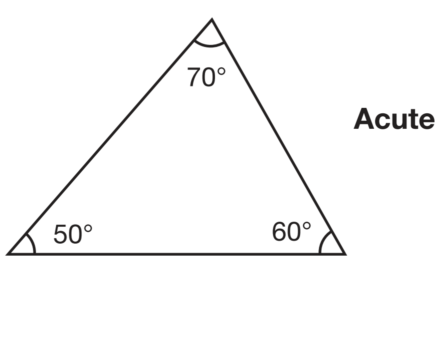


- **Equilateral Triangle:** all equal sides

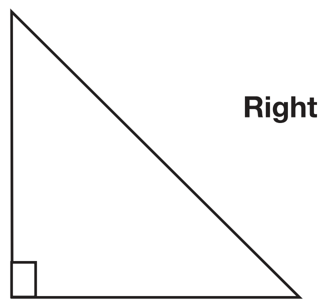


You also can classify triangles into three categories based on the measure of the greatest angle:

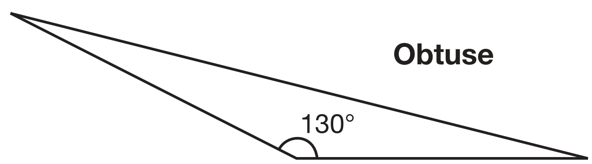
- **Acute Triangle:** greatest angle is acute



- **Right Triangle:** greatest angle is 90°



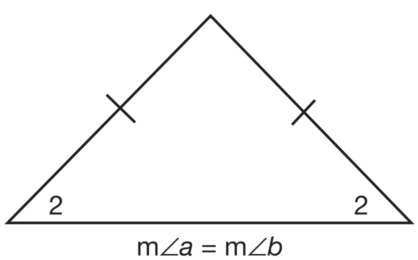
- **Obtuse Triangle:** greatest angle is obtuse



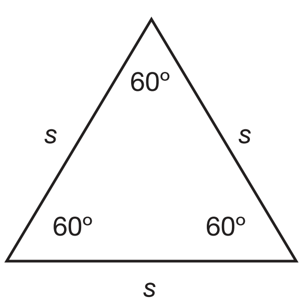
Angle-Side Relationships

Understanding the angle-side relationships in isosceles, equilateral, and right triangles is essential in solving questions on the SAT.

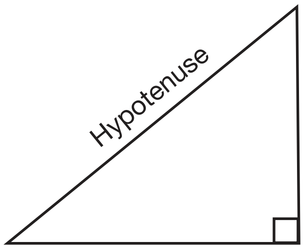
- In **isosceles triangles**, equal angles are opposite equal sides.



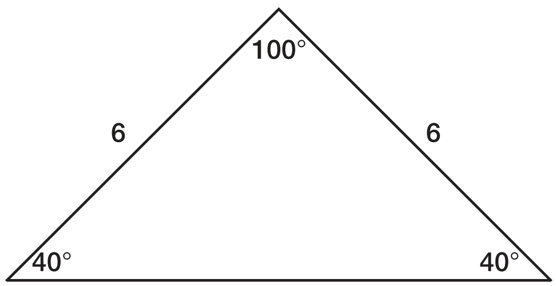
- In **equilateral triangles**, all sides are equal and all angles are 60°.



- In **right triangles**, the side opposite the right angle is called the hypotenuse.



Practice Question



Which of the following best describes the triangle above?

a. scalene and obtuse
b. scalene and acute
c. isosceles and right
d. isosceles and obtuse
e. isosceles and acute

Answer

d. The triangle has an angle greater than 90°, which makes it *obtuse*. Also, the triangle has two equal sides, which makes it *isosceles*.

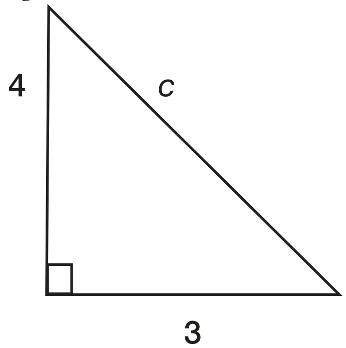
▶ Pythagorean Theorem

The **Pythagorean theorem** is an important tool for working with right triangles. It states:

$a^2 + b^2 = c^2$, where a and b represent the lengths of the *legs* and c represents the length of the *hypotenuse* of a right triangle.

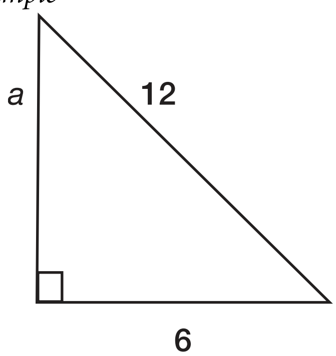
Therefore, if you know the lengths of two sides of a right triangle, you can use the Pythagorean theorem to determine the length of the third side.

Example



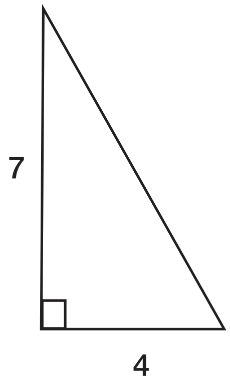
$a^2 + b^2 = c^2$
$3^2 + 4^2 = c^2$
$9 + 16 = c^2$
$25 = c^2$
$\sqrt{25} = \sqrt{c^2}$
$5 = c$

Example



$a^2 + b^2 = c^2$
$a^2 + 6^2 = 12^2$
$a^2 + 36 = 144$
$a^2 + 36 - 36 = 144 - 36$
$a^2 = 108$
$\sqrt{a^2} = \sqrt{108}$
$a = \sqrt{108}$

Practice Question



What is the length of the hypotenuse in the triangle above?

a. $\sqrt{11}$
b. 8
c. $\sqrt{65}$
d. 11
e. 65

Answer

c. Use the Pythagorean theorem: $a^2 + b^2 = c^2$, where $a = 7$ and $b = 4$.
$a^2 + b^2 = c^2$
$7^2 + 4^2 = c^2$
$49 + 16 = c^2$
$65 = c^2$
$\sqrt{65} = \sqrt{c^2}$
$\sqrt{65} = c$

Pythagorean Triples

A **Pythagorean triple** is a set of three positive integers that satisfies the Pythagorean theorem, $a^2 + b^2 = c^2$.

Example

The set 3:4:5 is a Pythagorean triple because:
$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
$25 = 25$

Multiples of Pythagorean triples are also Pythagorean triples.

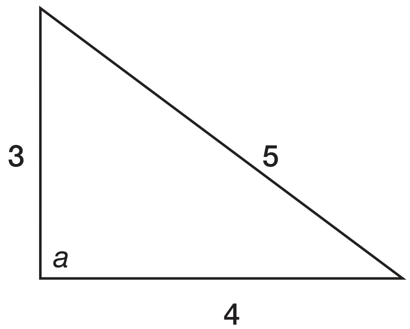
Example

Because set 3:4:5 is a Pythagorean triple, 6:8:10 is also a Pythagorean triple:
$6^2 + 8^2 = 10^2$
$36 + 64 = 100$
$100 = 100$

Pythagorean triples are important because they help you identify right triangles and identify the lengths of the sides of right triangles.

Example

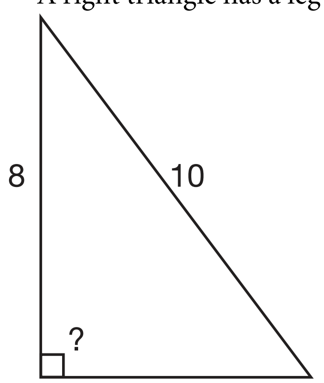
What is the measure of $\angle a$ in the triangle below?



Because this triangle shows a Pythagorean triple (3:4:5), you know it is a right triangle. Therefore, $\angle a$ must measure 90°.

Example

A right triangle has a leg of 8 and a hypotenuse of 10. What is the length of the other leg?

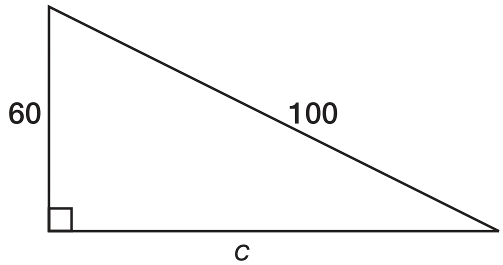


Because this triangle is a right triangle, you know its measurements obey the Pythagorean theorem. You could plug 8 and 10 into the formula and solve for the missing leg, but you don't have to. The triangle shows two parts of a Pythagorean triple (?:8:10), so you know that the missing leg must complete the triple. Therefore, the second leg has a length of 6.

It is useful to memorize a few of the smallest Pythagorean triples:

3:4:5	$3^2 + 4^2 = 5^2$
6:8:10	$6^2 + 8^2 = 10^2$
5:12:13	$5^2 + 12^2 = 13^2$
7:24:25	$7^2 + 24^2 = 25^2$
8:15:17	$8^2 + 15^2 = 17^2$

Practice Question



What is the length of *c* in the triangle above?

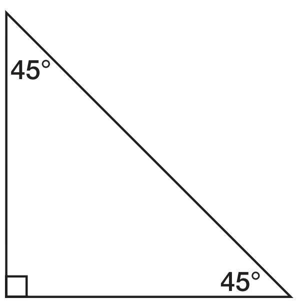
a. 30
b. 40
c. 60
d. 80
e. 100

Answer

d. You could use the Pythagorean theorem to solve this question, but if you notice that the triangle shows two parts of a Pythagorean triple, you don't have to. 60:*c*:100 is a multiple of 6:8:10 (which is a multiple of 3:4:5). Therefore, *c* must equal 80 because 60:80:100 is the same ratio as 6:8:10.

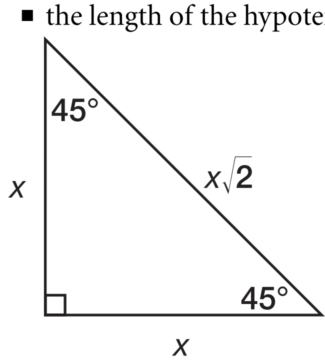
45-45-90 Right Triangles

An **isosceles right triangle** is a right triangle with two angles each measuring 45°.

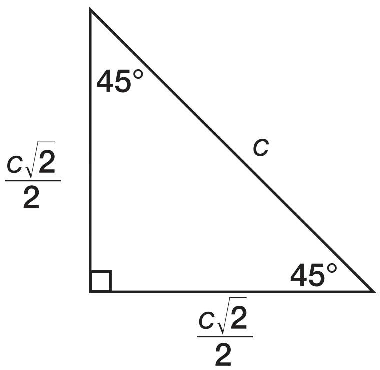


Special rules apply to isosceles right triangles:

- the length of the hypotenuse $= \sqrt{2} \times$ the length of a leg of the triangle



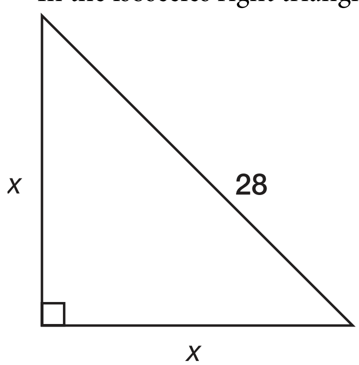
- the length of each leg is $\frac{\sqrt{2}}{2}$ × the length of the hypotenuse



You can use these special rules to solve problems involving isosceles right triangles.

Example

In the isosceles right triangle below, what is the length of a leg, *x*?



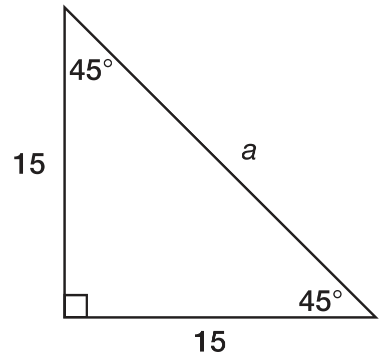
$x = \frac{\sqrt{2}}{2} \times$ the length of the hypotenuse

$x = \frac{\sqrt{2}}{2} \times 28$

$x = \frac{28\sqrt{2}}{2}$

$x = 14\sqrt{2}$

Practice Question



What is the length of *a* in the triangle above?

a. $\frac{15\sqrt{2}}{4}$

b. $\frac{15\sqrt{2}}{2}$

c. $15\sqrt{2}$

d. 30

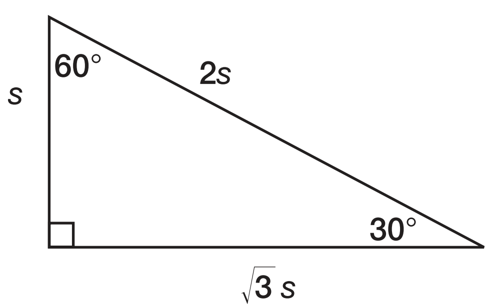
e. $30\sqrt{2}$

Answer

c. In an isosceles right triangle, the length of the hypotenuse = $\sqrt{2}$ × the length of a leg of the triangle. According to the figure, one leg = 15. Therefore, the hypotenuse is $15\sqrt{2}$.

30-60-90 Triangles

Special rules apply to right triangles with one angle measuring 30° and another angle measuring 60°.

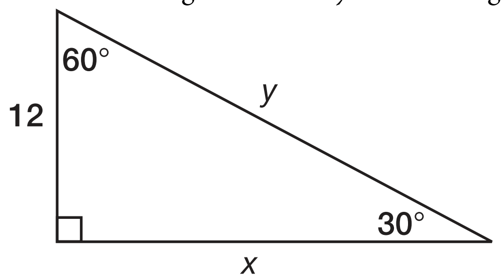


- the hypotenuse = 2 × the length of the leg opposite the 30° angle
- the leg opposite the 30° angle = $\frac{1}{2}$ × the length of the hypotenuse
- the leg opposite the 60° angle = $\sqrt{3}$ × the length of the other leg

You can use these rules to solve problems involving 30-60-90 triangles.

Example

What are the lengths of x and y in the triangle below?



The hypotenuse = 2 × the length of the leg opposite the 30° angle. Therefore, you can write an equation:

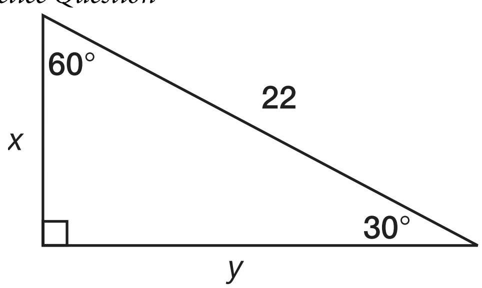
$y = 2 \times 12$

$y = 24$

The leg opposite the 60° angle = $\sqrt{3}$ × the length of the other leg. Therefore, you can write an equation:

$x = 12\sqrt{3}$

Practice Question



What is the length of y in the triangle above?

a. 11
b. $11\sqrt{2}$
c. $11\sqrt{3}$
d. $22\sqrt{2}$
e. $22\sqrt{3}$

Answer

c. In a 30-60-90 triangle, the leg opposite the 30° angle = half the length of the hypotenuse. The hypotenuse is 22, so the leg opposite the 30° angle = 11. The leg opposite the 60° angle = $\sqrt{3}$ × the length of the other leg. The other leg = 11, so the leg opposite the 60° angle = $11\sqrt{3}$.

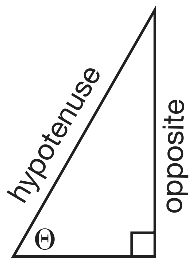
Triangle Trigonometry

There are special ratios we can use when working with right triangles. They are based on the trigonometric functions called **sine**, **cosine**, and **tangent**.

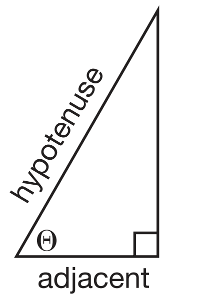
For an angle, Θ, within a right triangle, we can use these formulas:

$\sin \Theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos \Theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan \Theta = \frac{\text{opposite}}{\text{adjacent}}$

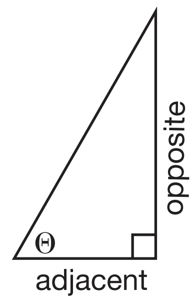
To find sin Θ...



To find cos Θ...



To find tan Θ...



The popular mnemonic to use to remember these formulas is **SOH CAH TOA**.

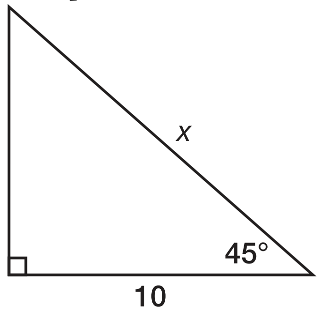
SOH stands for **S**in: **O**pposite/**H**ypotenuse
CAH stands for **C**os: **A**djacent/**H**ypotenuse
TOA stands for **T**an: **O**pposite/**A**djacent

TRIG VALUES OF SOME COMMON ANGLES

	SIN	COS	TAN
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Although trigonometry is tested on the SAT, all SAT trigonometry questions can also be solved using geometry (such as rules of 45-45-90 and 30-60-90 triangles), so knowledge of trigonometry is not essential. But if you don't bother learning trigonometry, be sure you understand triangle geometry completely.

Example



First, let's solve using trigonometry:
We know that $\cos 45° = \frac{\sqrt{2}}{2}$, so we can write an equation:

$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{2}}{2}$

$\frac{10}{x} = \frac{\sqrt{2}}{2}$ Find cross products.

$2 \times 10 = x\sqrt{2}$ Simplify.

$20 = x\sqrt{2}$

$\frac{20}{\sqrt{2}} = x$

Now, multiply $\frac{20}{\sqrt{2}}$ by $\frac{\sqrt{2}}{\sqrt{2}}$ (which equals 1), to remove the $\sqrt{2}$ from the denominator.

$\frac{\sqrt{2}}{\sqrt{2}} \times \frac{20}{\sqrt{2}} = x$

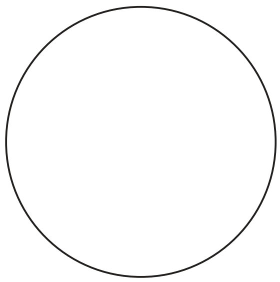
$\frac{20\sqrt{2}}{2} = x$

$10\sqrt{2} = x$

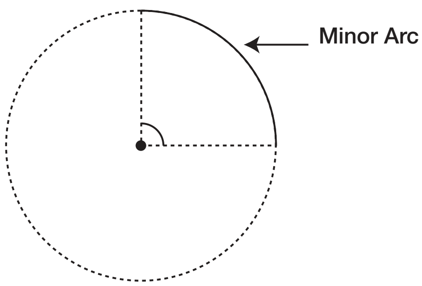
Now let's solve using rules of 45-45-90 triangles, which is a lot simpler:
The length of the hypotenuse $= \sqrt{2} \times$ the length of a leg of the triangle. Therefore, because the leg is 10, the hypotenuse is $\sqrt{2} \times 10 = 10\sqrt{2}$.

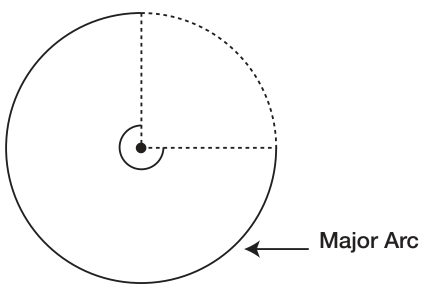
▶ Circles

A **circle** is a closed figure in which each point of the circle is the same distance from the center of the circle.

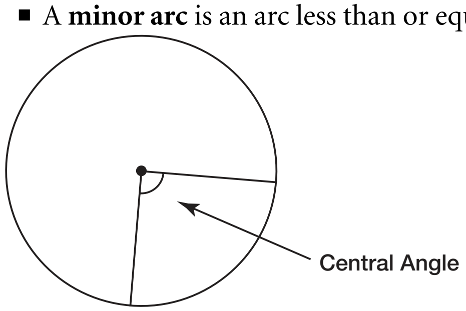


Angles and Arcs of a Circle





- An **arc** is a curved section of a circle.
- A **minor arc** is an arc less than or equal to 180°. A **major arc** is an arc greater than or equal to 180°.



- A **central angle** of a circle is an angle with its vertex at the center and sides that are radii. Arcs have the same degree measure as the central angle whose sides meet the circle at the two ends of the arc.

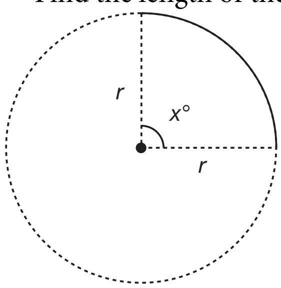
Length of an Arc

To find the length of an arc, multiply the circumference of the circle, $2\pi r$, where r = the radius of the circle, by the fraction $\frac{x}{360}$, with x being the degree measure of the central angle:

$$2\pi r \times \frac{x}{360} = \frac{2\pi rx}{360} = \frac{\pi rx}{180}$$

Example

Find the length of the arc if $x = 90$ and $r = 56$.



$L = \frac{\pi rx}{180}$

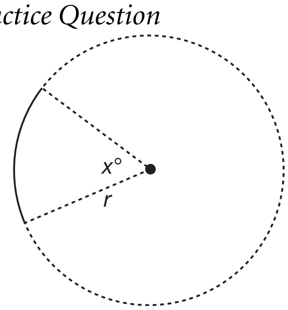
$L = \frac{\pi(56)(90)}{180}$

$L = \frac{\pi(56)}{2}$

$L = 28\pi$

The length of the arc is 28π.

Practice Question



If $x = 32$ and $r = 18$, what is the length of the arc shown in the figure above?

a. $\frac{16\pi}{5}$

b. $\frac{32\pi}{5}$

c. 36π

d. $\frac{288\pi}{5}$

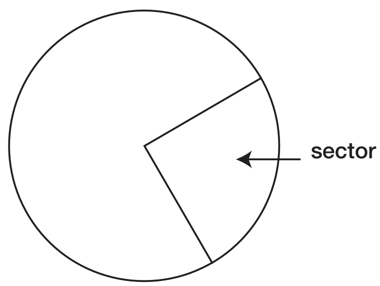
e. 576π

Answer

a. To find the length of an arc, use the formula $\frac{\pi rx}{180}$, where r = the radius of the circle and x = the measure of the central angle of the arc. In this case, $r = 18$ and $x = 32$.
$\frac{\pi rx}{180} = \frac{\pi(18)(32)}{180} = \frac{\pi(32)}{10} = \frac{\pi(16)}{5} = \frac{16\pi}{5}$

Area of a Sector

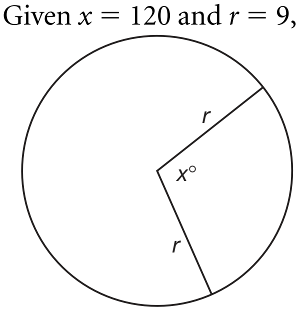
A **sector** of a circle is a slice of a circle formed by two radii and an arc.



To find the area of a sector, multiply the area of a circle, πr^2, by the fraction $\frac{x}{360}$, with x being the degree measure of the central angle: $\frac{\pi r^2 x}{360}$.

Example

Given $x = 120$ and $r = 9$, find the area of the sector:



$A = \frac{\pi r^2 x}{360}$

$A = \frac{\pi(9^2)(120)}{360}$

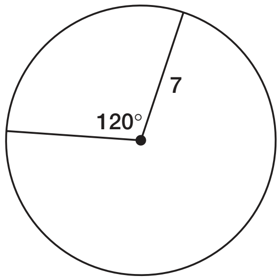
$A = \frac{\pi(9^2)}{3}$

$A = \frac{81\pi}{3}$

$A = 27\pi$

The area of the sector is 27π.

Practice Question



What is the area of the sector shown above?

a. $\frac{49\pi}{360}$

b. $\frac{7\pi}{3}$

c. $\frac{49\pi}{3}$

d. 280π

e. $5{,}880\pi$

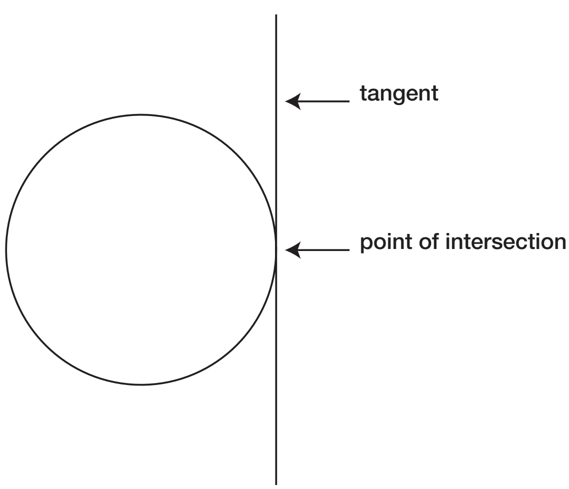
Answer

c. To find the area of a sector, use the formula $\frac{\pi r^2 x}{360}$, where r = the radius of the circle and x = the measure of the central angle of the arc. In this case, $r = 7$ and $x = 120$.

$\frac{\pi r^2 x}{360} = \frac{\pi(7^2)(120)}{360} = \frac{\pi(49)(120)}{360} = \frac{\pi(49)}{3} = \frac{49\pi}{3}$

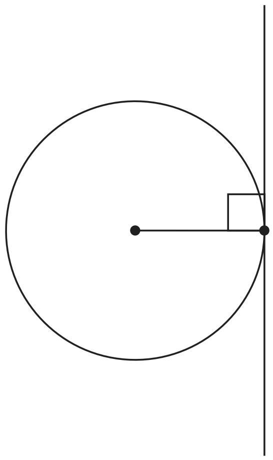
Tangents

A **tangent** is a line that intersects a circle at one point only.

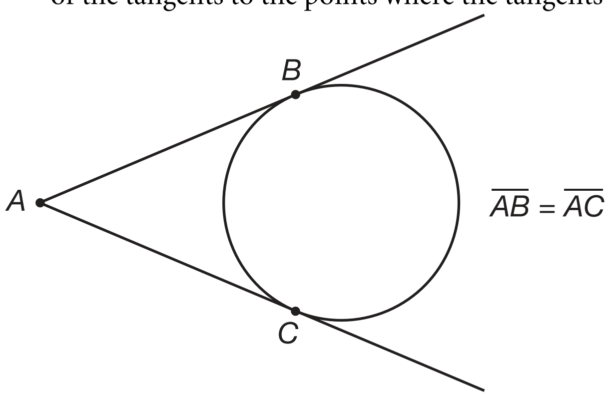


There are two rules related to tangents:

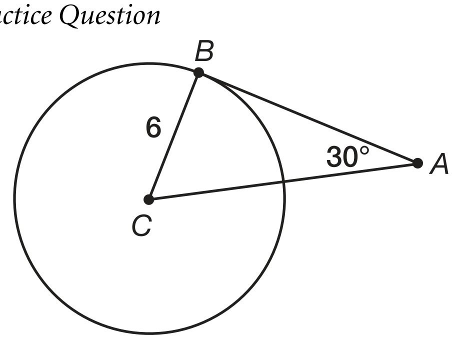
1. A radius whose endpoint is on the tangent is always perpendicular to the tangent line.



2. Any point outside a circle can extend exactly two tangent lines to the circle. The distances from the origin of the tangents to the points where the tangents intersect with the circle are equal.



Practice Question



What is the length of $\overline{AB}$ in the figure above if $\overline{BC}$ is the radius of the circle and $\overline{AB}$ is tangent to the circle?

a. 3
b. $3\sqrt{2}$
c. $6\sqrt{2}$
d. $6\sqrt{3}$
e. 12

Answer

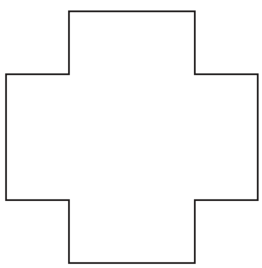
d. This problem requires knowledge of several rules of geometry. A tangent intersects with the radius of a circle at 90°. Therefore, ΔABC is a right triangle. Because one angle is 90° and another angle is 30°, then the third angle must be 60°. The triangle is therefore a 30-60-90 triangle.

In a 30-60-90 triangle, the leg opposite the 60° angle is $\sqrt{3} \times$ the leg opposite the 30° angle. In this figure, the leg opposite the 30° angle is 6, so $\overline{AB}$, which is the leg opposite the 60° angle, must be $6\sqrt{3}$.

▶ Polygons

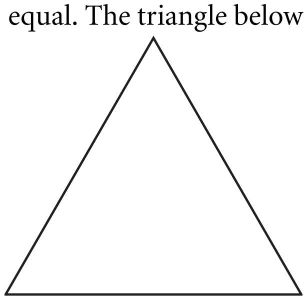
A **polygon** is a closed figure with three or more sides.

Example

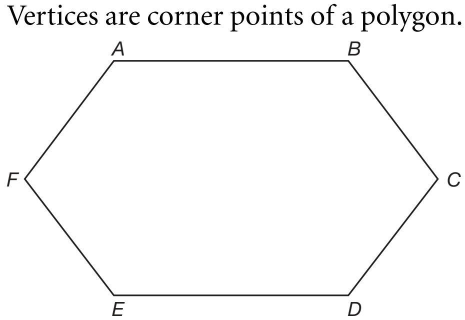


Terms Related to Polygons

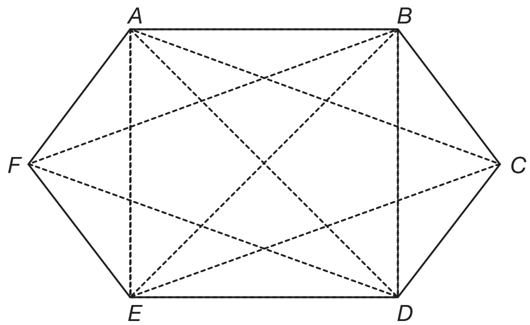
- A **regular** (or equilateral) polygon has sides that are all equal; an **equiangular** polygon has angles that are all equal. The triangle below is a regular and equiangular polygon:



- Vertices are corner points of a polygon. The vertices in the six-sided polygon below are: *A*, *B*, *C*, *D*, *E*, and *F*.

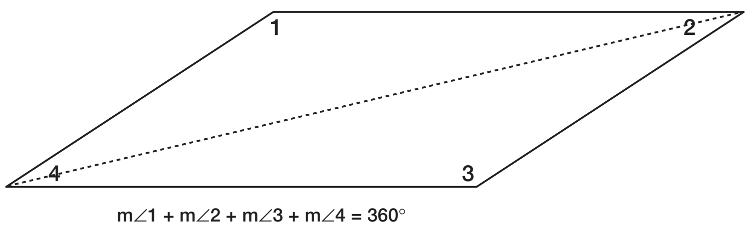


- A **diagonal** of a polygon is a line segment between two non-adjacent vertices. The diagonals in the polygon below are line segments $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{BD}$, $\overline{BE}$, $\overline{BF}$, $\overline{CE}$, $\overline{CF}$, and $\overline{DF}$.



Quadrilaterals

A **quadrilateral** is a four-sided polygon. Any quadrilateral can be divided by a diagonal into two triangles, which means the sum of a quadrilateral's angles is $180° + 180° = 360°$.



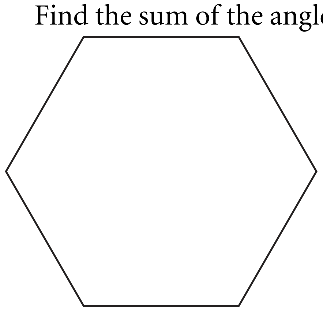
Sums of Interior and Exterior Angles

To find the sum of the **interior angles** of any polygon, use the following formula:

$S = 180(x - 2)$, with x being the number of sides in the polygon.

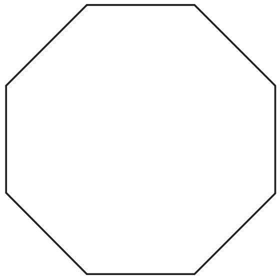
Example

Find the sum of the angles in the six-sided polygon below:



$S = 180(x - 2)$
$S = 180(6 - 2)$
$S = 180(4)$
$S = 720$
The sum of the angles in the polygon is 720°.

Practice Question



What is the sum of the interior angles in the figure above?

a. 360°
b. 540°
c. 900°
d. 1,080°
e. 1,260°

Answer

d. To find the sum of the interior angles of a polygon, use the formula $S = 180(x - 2)$, with x being the number of sides in the polygon. The polygon above has eight sides, therefore $x = 8$.
$S = 180(x - 2) = 180(8 - 2) = 180(6) = 1{,}080°$

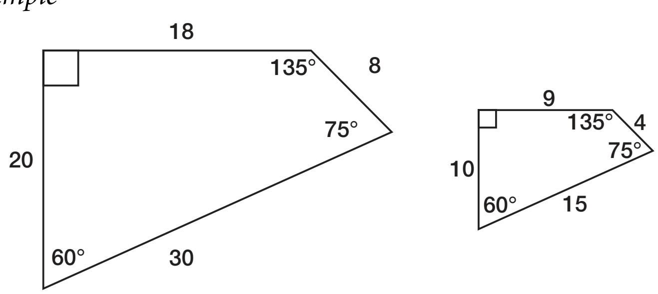
Exterior Angles

The sum of the exterior angles of *any* polygon (triangles, quadrilaterals, pentagons, hexagons, etc.) is 360°.

Similar Polygons

If two polygons are similar, their corresponding angles are equal, and the ratio of the corresponding sides is in proportion.

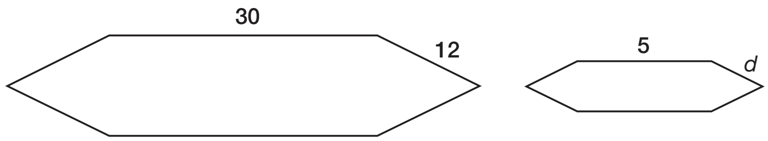
Example



These two polygons are similar because their angles are equal and the ratio of the corresponding sides is in proportion:

$\frac{20}{10} = \frac{2}{1}$ $\frac{18}{9} = \frac{2}{1}$ $\frac{8}{4} = \frac{2}{1}$ $\frac{30}{15} = \frac{2}{1}$

Practice Question



If the two polygons above are similar, what is the value of d?

a. 2
b. 5
c. 7
d. 12
e. 23

Answer

a. The two polygons are similar, which means the ratio of the corresponding sides are in proportion. Therefore, if the ratio of one side is 30:5, then the ration of the other side, 12:d, must be the same. Solve for d using proportions:

$\frac{30}{5} = \frac{12}{d}$ Find cross products.

$30d = (5)(12)$

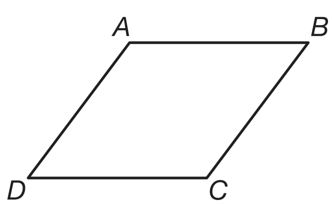
$30d = 60$

$d = \frac{60}{30}$

$d = 2$

Parallelograms

A **parallelogram** is a quadrilateral with two pairs of parallel sides.



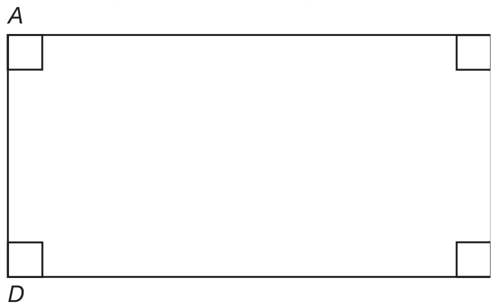
In the figure above, $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$.

Parallelograms have the following attributes:

- opposite sides that are equal
  $\overline{AD} = \overline{BC}$ $\overline{AB} = \overline{DC}$
- opposite angles that are equal
  $m\angle A = m\angle C$ $m\angle B = m\angle D$
- consecutive angles that are supplementary
  $m\angle A + m\angle B = 180°$ $m\angle B + m\angle C = 180°$
  $m\angle C + m\angle D = 180°$ $m\angle D + m\angle A = 180°$

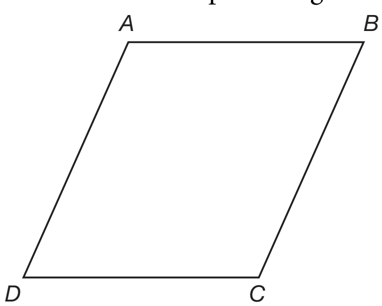
Special Types of Parallelograms

- A **rectangle** is a parallelogram with four right angles.



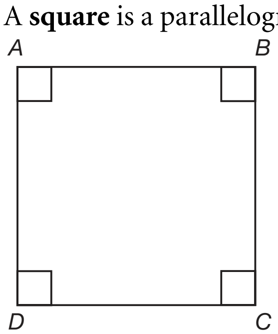
$\overline{AD} = \overline{BC}$ $\overline{AB} = \overline{DC}$

- A **rhombus** is a parallelogram with four equal sides.



$\overline{AB} = \overline{BC} = \overline{DC} = \overline{AD}$

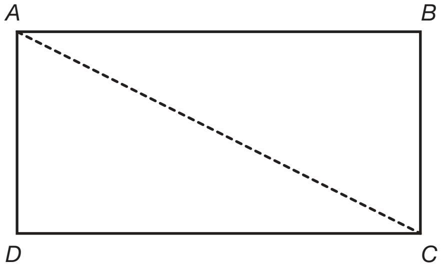
- A **square** is a parallelogram with four equal sides and four right angles.



$\overline{AB} = \overline{BC} = \overline{DC} = \overline{AD}$
$m\angle A = m\angle B = m\angle C = m\angle D = 90$

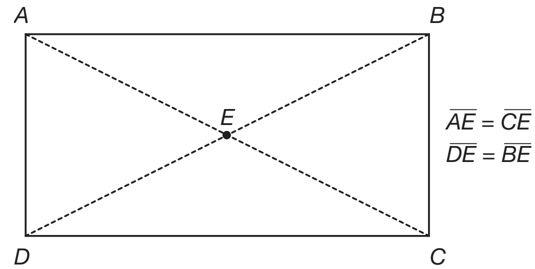
Diagonals

- A diagonal cuts a parallelogram into two equal halves.

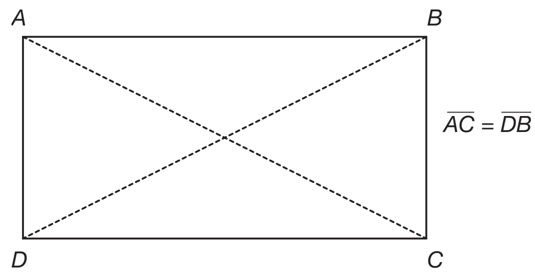


$\triangle ABC = \triangle ADC$

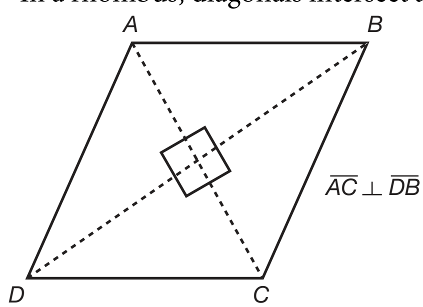
- In all parallelograms, diagonals cut each other into two equal halves.



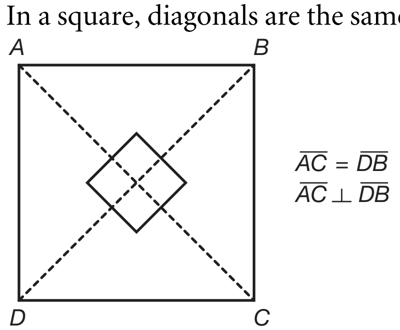
- In a rectangle, diagonals are the same length.



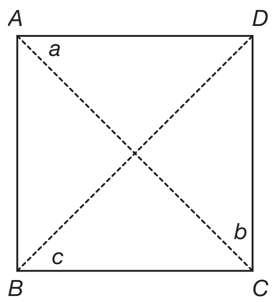
- In a rhombus, diagonals intersect at right angles.



- In a square, diagonals are the same length and intersect at right angles.



Practice Question



Which of the following must be true about the square above?

I. $a = b$

II. $\overline{AC} = \overline{BD}$

III. $b = c$

a. I only
b. II only
c. I and II only
d. II and III only
e. I, II, and III

Answer

e. $\overline{AC}$ and $\overline{BD}$ are diagonals. Diagonals cut parallelograms into two equal halves. Therefore, the diagonals divide the square into two 45-45-90 right triangles. Therefore, a, b, and c each equal 45°.

Now we can evaluate the three statements:

I: $a = b$ is TRUE because $a = 45$ and $b = 45$.

II: $\overline{AC} = \overline{BD}$ is TRUE because diagonals are equal in a square.

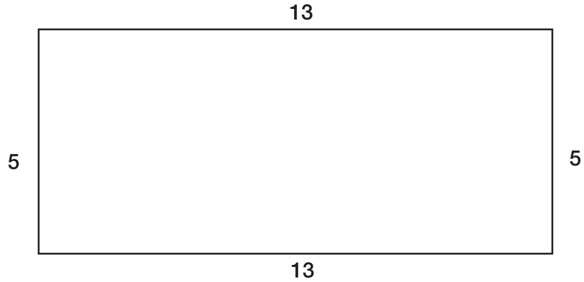
III: $b = c$ is TRUE because $b = 45$ and $c = 45$.

Therefore I, II, and III are ALL TRUE.

▶ Solid Figures, Perimeter, and Area

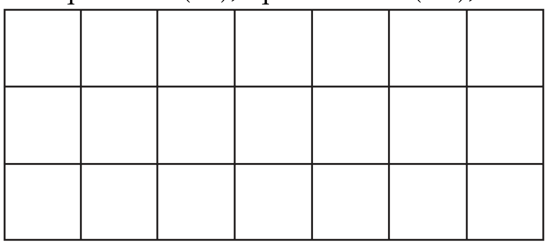
There are five kinds of measurement that you must understand for the SAT:

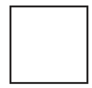
1. The **perimeter** of an object is the sum of all of its sides.



Perimeter = $5 + 13 + 5 + 13 = 36$

2. **Area** is the number of square units that can fit inside a shape. Square units can be square inches (in^2), square feet (ft^2), square meters (m^2), etc.

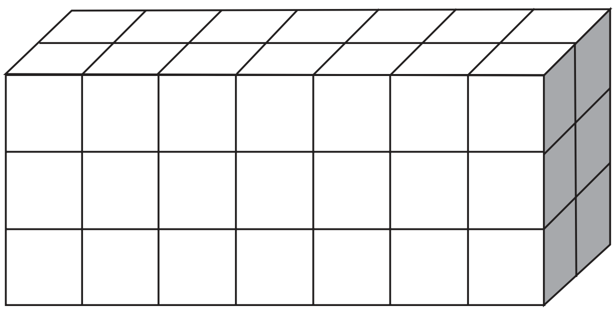


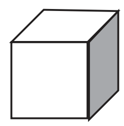


= 1 square unit

The area of the rectangle above is 21 square units. 21 square units fit inside the rectangle.

3. **Volume** is the number of cubic units that fit inside solid. Cubic units can be cubic inches (in^3), cubic feet (ft^2), cubic meters (m^3), etc.



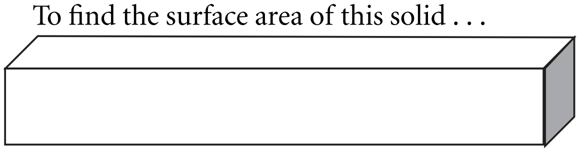


= 1 cubic unit

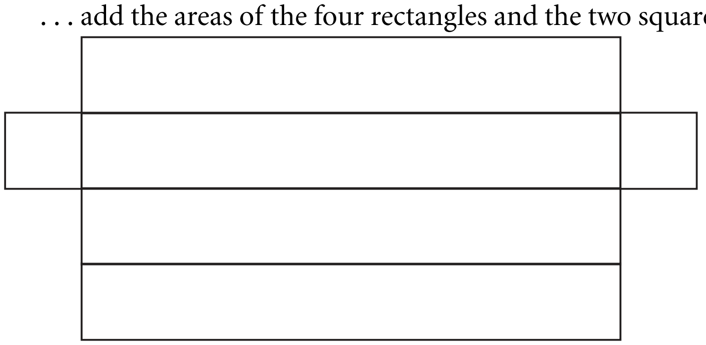
The volume of the solid above is 36 cubic units. 36 cubic units fit inside the solid.

4. The **surface area** of a solid is the sum of the areas of all its faces.

To find the surface area of this solid . . .

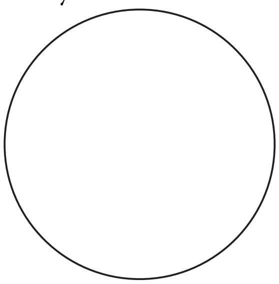


. . . add the areas of the four rectangles and the two squares that make up the surfaces of the solid.



5. Circumference is the distance around a circle.

If you uncurled this circle . . .



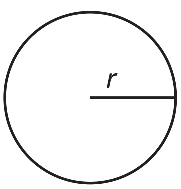
. . . you would have this line segment:

The circumference of the circle is the length of this line segment.

Formulas

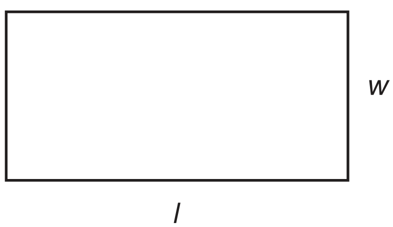
The following formulas are provided on the SAT. You therefore do not need to memorize these formulas, but you do need to understand when and how to use them.

Circle



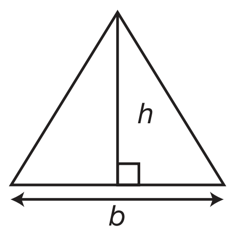
$C = 2\pi r$
$A = \pi r^2$

Rectangle



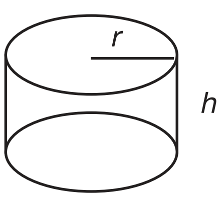
$A = lw$

Triangle



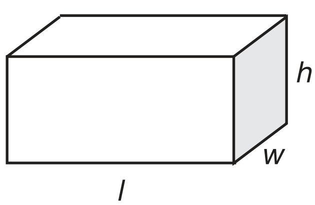
$A = \frac{1}{2}bh$

Cylinder



$V = \pi r^2 h$

Rectangle Solid



$V = lwh$

C	=	Circumference	w	=	Width
A	=	Area	h	=	Height
r	=	Radius	V	=	Volume
l	=	Length	b	=	Base

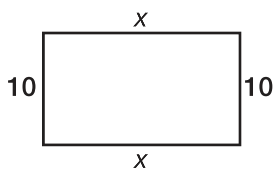
Practice Question

A rectangle has a perimeter of 42 and two sides of length 10. What is the length of the other two sides?

a. 10
b. 11
c. 22
d. 32
e. 52

Answer

b. You know that the rectangle has two sides of length 10. You also know that the other two sides of the rectangle are equal because rectangles have two sets of equal sides. Draw a picture to help you better understand:



Based on the figure, you know that the perimeter is $10 + 10 + x + x$. So set up an equation and solve for x:

$10 + 10 + x + x = 42$
$20 + 2x = 42$
$20 + 2x - 20 = 42 - 20$
$2x = 22$
$\frac{2x}{2} = \frac{22}{2}$
$x = 11$

Therefore, we know that the length of the other two sides of the rectangle is 11.

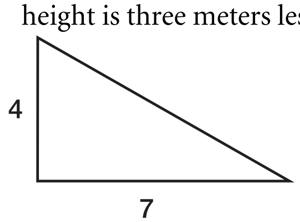
Practice Question

The height of a triangular fence is 3 meters less than its base. The base of the fence is 7 meters. What is the area of the fence in square meters?

a. 4
b. 10
c. 14
d. 21
e. 28

Answer

c. Draw a picture to help you better understand the problem. The triangle has a base of 7 meters. The height is three meters less than the base ($7 - 3 = 4$), so the height is 4 meters:



The formula for the area of a triangle is $\frac{1}{2}$(base)(height):

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(7)(4)$

$A = \frac{1}{2}(28)$

$A = 14$

The area of the triangular wall is 14 square meters.

Practice Question

A circular cylinder has a radius of 3 and a height of 5. Ms. Stewart wants to build a rectangular solid with a volume as close as possible to the cylinder. Which of the following rectangular solids has dimension closest to that of the circular cylinder?

a. $3 \times 3 \times 5$
b. $3 \times 5 \times 5$
c. $2 \times 5 \times 9$
d. $3 \times 5 \times 9$
e. $5 \times 5 \times 9$

Answer

d. First determine the approximate volume of the cylinder. The formula for the volume of a cylinder is $V = \pi r^2 h$. (Because the question requires only an approximation, use $\pi \approx 3$ to simplify your calculation.)

$V = \pi r^2 h$

$V \approx (3)(3^2)(5)$

$V \approx (3)(9)(5)$

$V \approx (27)(5)$

$V \approx 135$

Now determine the answer choice with dimensions that produce a volume closest to 135:

Answer choice **a:** $3 \times 3 \times 5 = 9 \times 5 = 45$

Answer choice **b:** $3 \times 5 \times 5 = 15 \times 5 = 75$

Answer choice **c:** $2 \times 5 \times 9 = 10 \times 9 = 90$

Answer choice **d:** $3 \times 5 \times 9 = 15 \times 9 = 135$

Answer choice **e:** $5 \times 5 \times 9 = 25 \times 9 = 225$

Answer choice **d** equals 135, which is the same as the approximate volume of the cylinder.

Practice Question

Mr. Suarez painted a circle with a radius of 6. Ms. Stone painted a circle with a radius of 12. How much greater is the circumference of Ms. Stone's circle than Mr. Suarez's circle?

a. 3π
b. 6π
c. 12π
d. 108π
e. 216π

Answer

c. You must determine the circumferences of the two circles and then subtract. The formula for the circumference of a circle is $C = 2\pi r$.

Mr. Suarez's circle has a radius of 6:
$C = 2\pi r$
$C = 2\pi(6)$
$C = 12\pi$

Ms. Stone's circle has a radius of 12:
$C = 2\pi r$
$C = 2\pi(12)$
$C = 24\pi$

Now subtract:
$24\pi - 12\pi = 12\pi$

The circumference of Ms. Stone's circle is 12π greater than Mr. Suarez's circle.

▶ Coordinate Geometry

A **coordinate plane** is a grid divided into four quadrants by both a horizontal x-axis and a vertical y-axis. **Coordinate points** can be located on the grid using **ordered pairs**. Ordered pairs are given in the form of (x,y). The x represents the location of the point on the horizontal x-axis, and the y represents the location of the point on the vertical y-axis. The x-axis and y-axis intersect at the **origin**, which is coordinate point (0,0).

Graphing Ordered Pairs

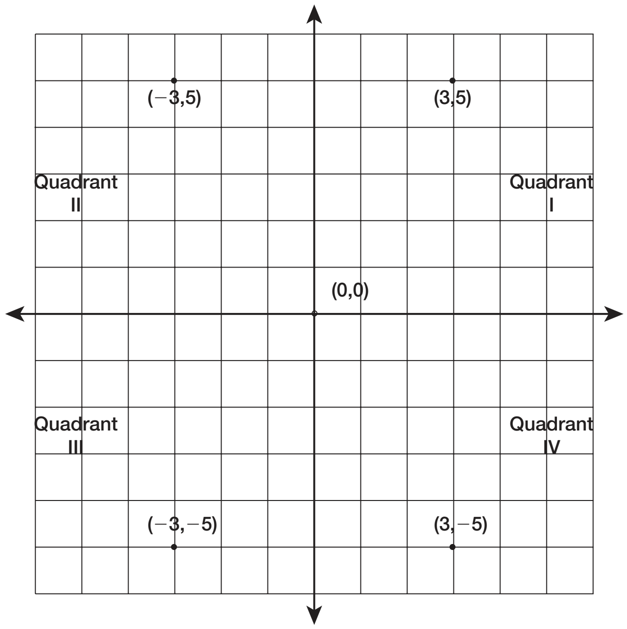
The ***x*-coordinate** is listed first in the ordered pair, and it tells you how many units to move to either the left or the right. If the x-coordinate is positive, move from the origin to the right. If the x-coordinate is negative, move from the origin to the left.

The ***y*-coordinate** is listed second and tells you how many units to move up or down. If the *y*-coordinate is positive, move up from the origin. If the *y*-coordinate is negative, move down from the origin.

Example

Graph the following points:

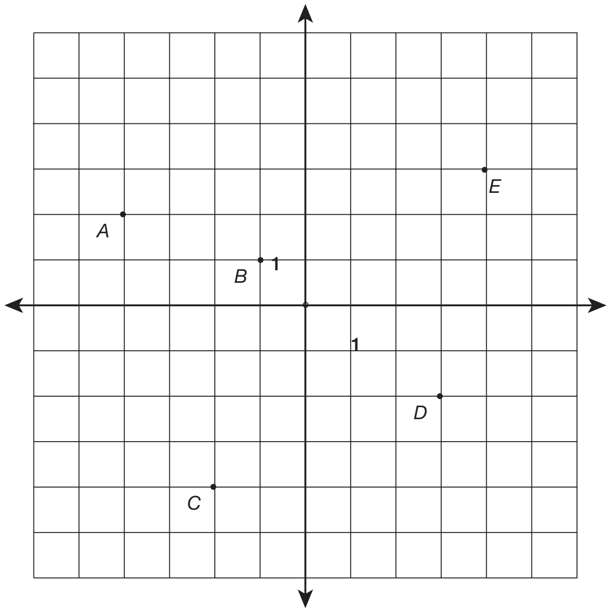
(0,0) (3,5) (3,−5) (−3,5) (−3,−5)



Notice that the graph is broken up into four quadrants with one point plotted in each one. The chart below indicates which quadrants contain which ordered pairs based on their signs:

POINT	SIGNS OF COORDINATES	QUADRANT
(3,5)	(+,+)	I
(–3,5)	(–,+)	II
(–3,–5)	(–,–)	III
(3,–5)	(+,–)	IV

Practice Question



Which of the five points on the graph above has coordinates (x,y) such that $x + y = 1$?

a. *A*
b. *B*
c. *C*
d. *D*
e. *E*

Answer

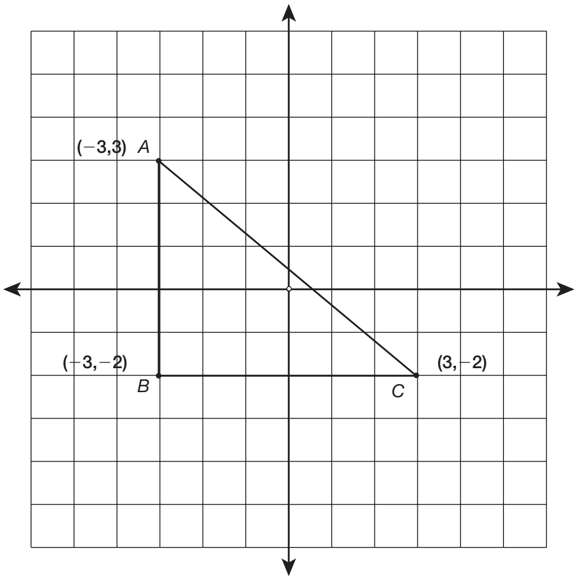
d. You must determine the coordinates of each point and then add them:
A $(2,-4)$: $2 + (-4) = -2$
B $(-1,1)$: $-1 + 1 = 0$
C $(-2,-4)$: $-2 + (-4) = -6$
D $(3,-2)$: $3 + (-2) = 1$
E $(4,3)$: $4 + 3 = 7$
Point *D* is the point with coordinates (x,y) such that $x + y = 1$.

Lengths of Horizontal and Vertical Segments

The length of a horizontal or a vertical segment on the coordinate plane can be found by taking the absolute value of the difference between the two coordinates, which are different for the two points.

Example

Find the length of $\overline{AB}$ and $\overline{BC}$.



$\overline{AB}$ is parallel to the y-axis, so subtract the absolute value of the y-coordinates of its endpoints to find its length:

$\overline{AB} = |3 - (-2)|$

$\overline{AB} = |3 + 2|$

$\overline{AB} = |5|$

$\overline{AB} = 5$

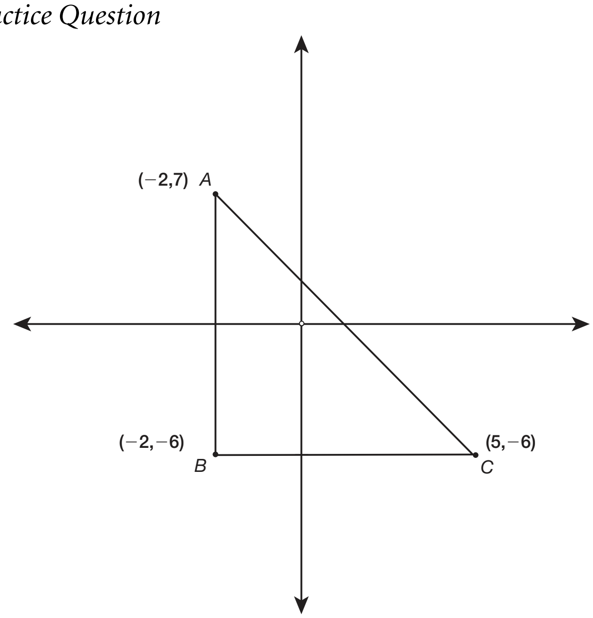
$\overline{BC}$ is parallel to the x-axis, so subtract the absolute value of the x-coordinates of its endpoints to find its length:

$\overline{BC} = |-3 - 3|$

$\overline{BC} = |-6|$

$\overline{BC} = 6$

Practice Question



What is the sum of the length of $\overline{AB}$ and the length of $\overline{BC}$?

a. 6
b. 7
c. 13
d. 16
e. 20

Answer

e. $\overline{AB}$ is parallel to the y-axis, so subtract the absolute value of the y-coordinates of its endpoints to find its length:
$\overline{AB} = |7 - (-6)|$
$\overline{AB} = |7 + 6|$
$\overline{AB} = |13|$
$\overline{AB} = 13$
$\overline{BC}$ is parallel to the x-axis, so subtract the absolute value of the x-coordinates of its endpoints to find its length:
$\overline{BC} = |5 - (-2)|$
$\overline{BC} = |5 + 2|$
$\overline{BC} = |7|$
$\overline{BC} = 7$
Now add the two lengths: $7 + 13 = 20$.

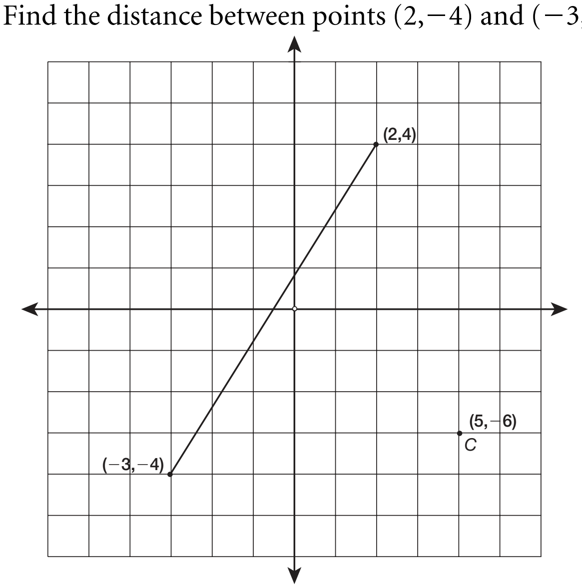
Distance between Coordinate Points

To find the distance between two points, use this variation of the Pythagorean theorem:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example

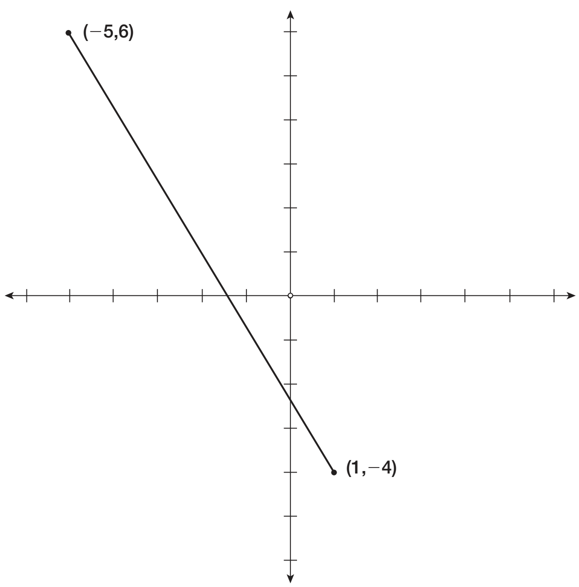
Find the distance between points $(2,-4)$ and $(-3,-4)$.



The two points in this problem are (2,−4) and (−3,−4).
$x_1 = 2$
$x_2 = -3$
$y_1 = -4$
$y_2 = -4$

Plug in the points into the formula:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(-3 - 2)^2 + (-4 - (-4))^2}$
$d = \sqrt{(-3 - 2)^2 + (-4 + 4)^2}$
$d = \sqrt{(-5)^2 + (0)^2}$
$d = \sqrt{25}$
$d = 5$
The distance is 5.

Practice Question



What is the distance between the two points shown in the figure above?
a. $\sqrt{20}$
b. 6
c. 10
d. $2\sqrt{34}$
e. $4\sqrt{34}$

Answer

d. To find the distance between two points, use the following formula:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The two points in this problem are $(-5,6)$ and $(1,-4)$.

$x_1 = -5$

$x_2 = 1$

$y_1 = 6$

$y_2 = -4$

Plug the points into the formula:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(1 - (-5))^2 + (-4 - 6)^2}$

$d = \sqrt{(1 + 5)^2 + (-10)^2}$

$d = \sqrt{(6)^2 + (-10)^2}$

$d = \sqrt{36 + 100}$

$d = \sqrt{136}$

$d = \sqrt{4 \times 34}$

$d = \sqrt{34}$

The distance is $2\sqrt{34}$.

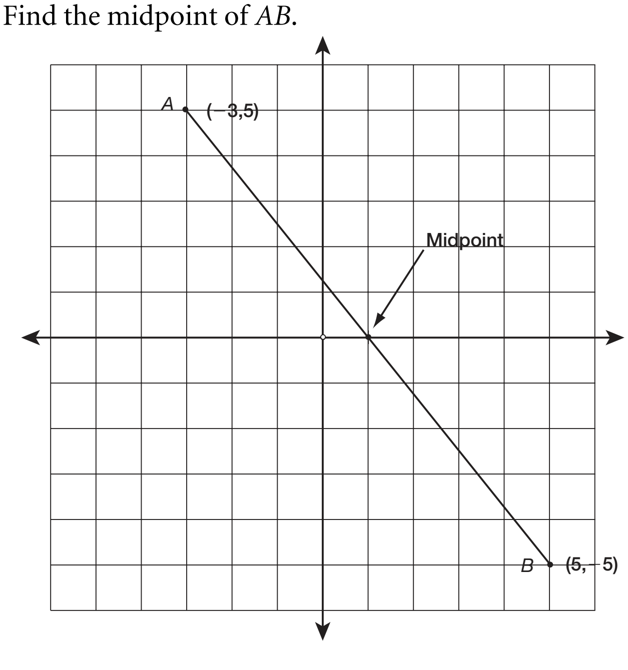
Midpoint

A **midpoint** is the point at the exact middle of a line segment. To find the midpoint of a segment on the coordinate plane, use the following formulas:

Midpoint $x = \frac{x_1 + x_2}{2}$ Midpoint $y = \frac{y_1 + y_2}{2}$

Example

Find the midpoint of $\overline{AB}$.



Midpoint $x = \frac{x_1 + x_2}{2} = \frac{-3 + 5}{2} = \frac{2}{2} = 1$
Midpoint $y = \frac{y_1 + y_2}{2} = \frac{5 + (-5)}{2} = \frac{0}{2} = 0$

Therefore, the midpoint of $\overline{AB}$ is (1,0).

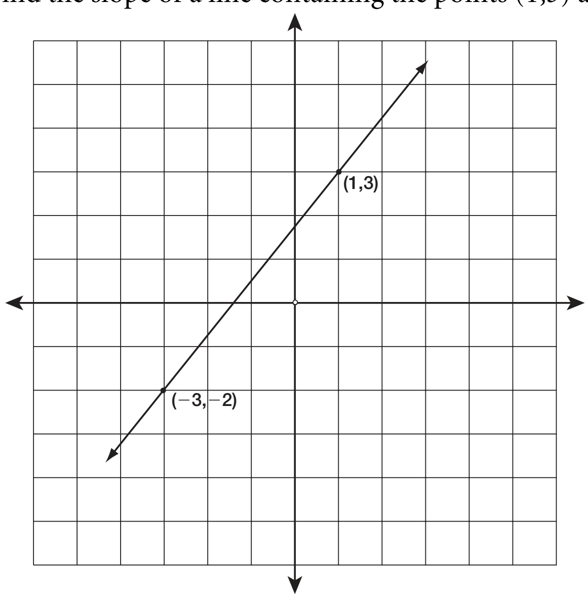
Slope

The **slope** of a line measures its steepness. Slope is found by calculating the ratio of the change in *y*-coordinates of any two points on the line, over the change of the corresponding *x*-coordinates:

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$$

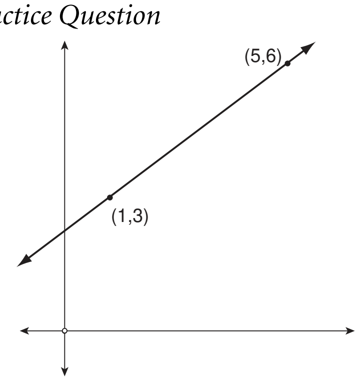
Example

Find the slope of a line containing the points (1,3) and (−3,−2).



Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-2)}{1 - (-3)} = \frac{3 + 2}{1 + 3} = \frac{5}{4}$
Therefore, the slope of the line is $\frac{5}{4}$.

Practice Question



What is the slope of the line shown in the figure on the previous page?

a. $\frac{1}{2}$

b. $\frac{3}{4}$

c. $\frac{4}{3}$

d. 2

e. 3

Answer

b. To find the slope of a line, use the following formula:
slope $= \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$
The two points shown on the line are (1,3) and (5,6).
$x_1 = 1$
$x_2 = 5$
$y_1 = 3$
$y_2 = 6$
Plug in the points into the formula:
slope $= \frac{6 - 3}{5 - 2}$
slope $= \frac{3}{4}$

Using Slope

If you know the slope of a line and one point on the line, you can determine other coordinate points on the line. Because slope tells you the ratio of $\frac{\text{vertical change}}{\text{horizontal change}}$, you can simply move from the coordinate point you know the required number of units determined by the slope.

Example

A line has a slope of $\frac{6}{5}$ and passes through point (3,4). What is another point the line passes through?
The slope is $\frac{6}{5}$, so you know there is a vertical change of 6 and a horizontal change of 5. So, starting at point (3,4), add 6 to the y-coordinate and add 5 to the x-coordinate:
y: $4 + 6 = 10$
x: $3 + 5 = 8$
Therefore, another coordinate point is (8,10).
If you know the slope of a line and one point on the line, you can also determine a point at a certain coordinate, such as the y-intercept $(x,0)$ or the x-intercept $(0,y)$.

Example

A line has a slope of $\frac{2}{3}$ and passes through point (1,4). What is the y-intercept of the line?
Slope $= \frac{y_2 - y_1}{x_2 - x_1}$, so you can plug in the coordinates of the known point (1,4) and the unknown point, the y-intercept $(x,0)$, and set up a ratio with the known slope, $\frac{2}{3}$, and solve for x:
$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{3}$
$\frac{0 - 4}{x - 1} = \frac{2}{3}$

$\frac{0-4}{x-1} = \frac{2}{3}$ Find cross products.

$(-4)(3) = 2(x - 1)$

$-12 = 2x - 2$

$-12 + 2 = 2x - 2 + 2$

$-\frac{10}{2} = \frac{2x}{2}$

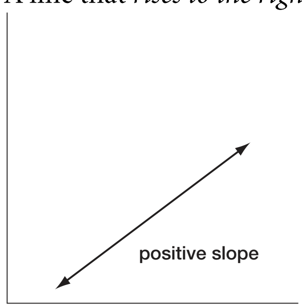
$-\frac{10}{2} = x$

$-5 = x$

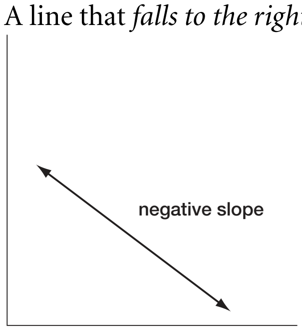
Therefore, the x-coordinate of the y-intercept is -5, so the y-intercept is $(-5,0)$.

Facts about Slope

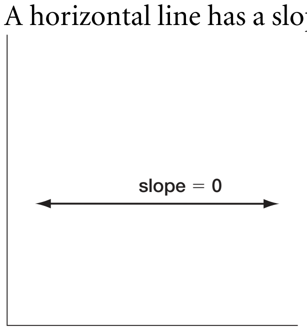
- A line that *rises to the right* has a positive slope.



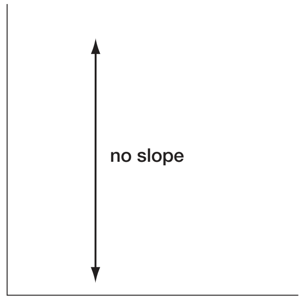
- A line that *falls to the right* has a negative slope.



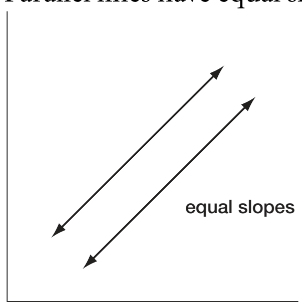
- A horizontal line has a slope of 0.



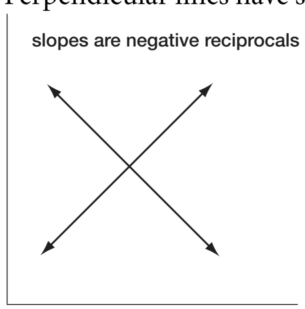
- A vertical line does not have a slope at all—it is undefined.



- Parallel lines have equal slopes.



- Perpendicular lines have slopes that are negative reciprocals of each other (e.g., 2 and $-\frac{1}{2}$).



Practice Question

A line has a slope of -3 and passes through point (6,3). What is the y-intercept of the line?

a. (7,0)
b. (0,7)
c. (7,7)
d. (2,0)
e. (15,0)

Answer

a. Slope $= \frac{y_2 - y_1}{x_2 - x_1}$, so you can plug in the coordinates of the known point (6,3) and the unknown point, the y-intercept (x,0), and set up a ratio with the known slope, −3, and solve for x:

$\frac{y_2 - y_1}{x_2 - x_1} = -3$

$\frac{0 - 3}{x - 6} = -3$

$\frac{-3}{x - 6} = -3$ Simplify.

$(x - 6)\frac{-3}{x - 6} = -3(x - 6)$

$-3 = -3x + 18$

$-3 - 18 = -3x + 18 - 18$

$-21 = -3x$

$\frac{-21}{-3} = \frac{-3x}{-3}$

$\frac{-21}{-3} = x$

$7 = x$

Therefore, the x-coordinate of the y-intercept is 7, so the y-intercept is (7,0).