Answers and Solutions Setion 1:

1.	**D**	**6.**	**B**	**11.**	**C**	**16.**	**C**
2.	**B**	**7.**	**A**	**12.**	**D**	**17.**	**C**
3.	**A**	**8.**	**B**	**13.**	**E**	**18.**	**D**
4.	**A**	**9.**	**D**	**14.**	**B**	**19.**	**B**
5.	**B**	**10.**	**D**	**15.**	**A**	**20.**	**B**

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1. We will use the *Substitution Method* to solve this problem. Substitution is a very useful technique for solving SAT math problems. It often reduces hard problems to routine ones. In the substitution method, we choose numbers that have the properties given in the problem and plug them into the answer-choices.

Now, we are told that n is an odd integer. So choose an odd integer for n, say, 1 and substitute it into each answer-choice.

Now, n^3 becomes $1^3 = 1$, which is not an even integer. So eliminate (A).

Next, $n/4 = 1/4$ is not an even integer—eliminate (B).

Next, $2n + 3 = 2 \cdot 1 + 3 = 5$ is not an even integer—eliminate (C).

Next, $n(n + 3) = 1(1 + 3) = 4$ is even and hence the answer is possibly (D).

Finally, $\sqrt{n} = \sqrt{1} = 1$, which is not even—eliminate (E). The answer is (D).

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2. We call this type of problem a *Defined Function*. Defined functions are very common on the SAT, and at first most students struggle with them. Yet once you get used to them, defined functions can be some of the easiest problems on the test. In this type of problem, you will be given a symbol (in this case, ∇) and a property that defines the symbol.

From the given definition, we know that $x\nabla y = xy - y$. So, all we have to do is replace x with 2 and y with 3 in the definition:

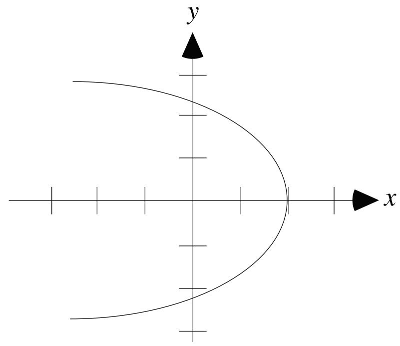
$$2\nabla 3 = 2 \cdot 3 - 3 = 3$$

Hence, the answer is (B).

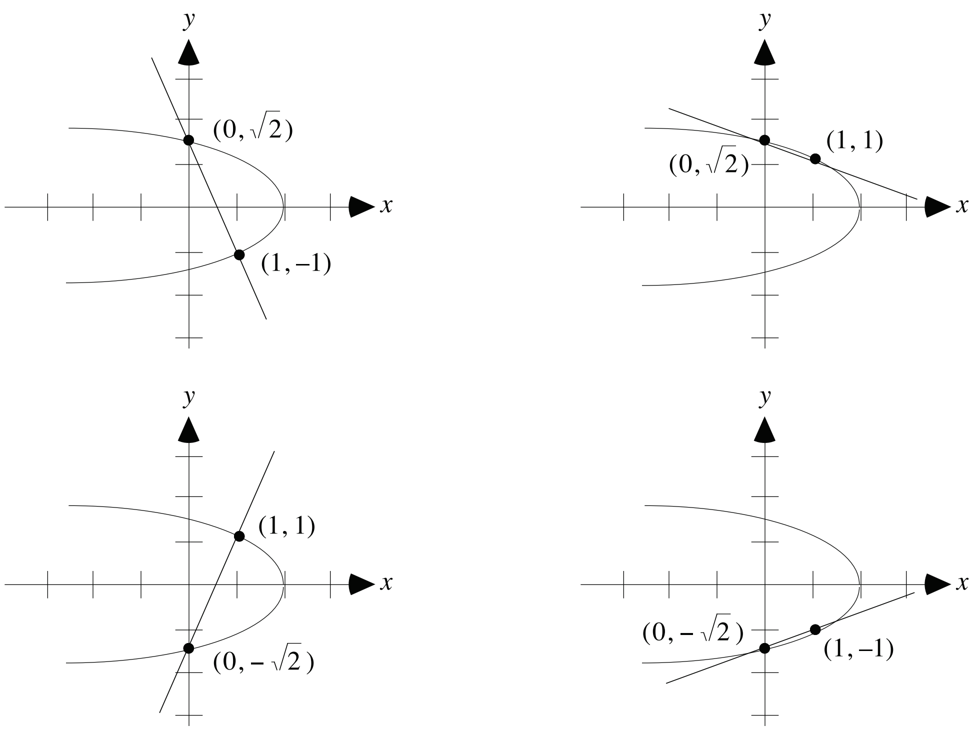
3. Let's make a rough sketch of the graphs. Expressing $x = -y^2 + 2$ in standard form yields

$$x = -1y^2 + 0 \cdot y + 2$$

Since $a = -1$, $b = 0$, and $c = 2$, the graph opens to the left and its vertex is at $(2, 0)$.



Since p and q can be positive or negative, there are four possible positions for line k (the y-coordinates in the graphs below can be calculated by plugging $x = 0$ and $x = 1$ into the function $x = -y^2 + 2$):



Since the line in the first graph has the steepest negative slope, it is the smallest possible slope. Calculating the slope yields

$$m = \frac{\sqrt{2} - (-1)}{0 - 1} = \frac{\sqrt{2} + 1}{-1} = -\left(\sqrt{2} + 1\right) = -\sqrt{2} - 1$$

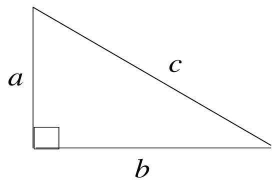
The answer is (A).

4. Since the triangle is a right triangle, the Pythagorean Theorem applies: $h^2 + 3^2 = 5^2$, where h is the height of the triangle (see summary of the Pythagoran Theorem below). Solving for h yields $h = 4$. Hence, the area of the triangle is

$$\frac{1}{2}(base)(height) = \frac{1}{2}(3)(4) = 6$$

The answer is (A).

- **Pythagorean Theorem (For right triangles only):**



$$c^2 = a^2 + b^2$$

5. Before we begin solving this problem, let's review the definition of division:

- ***"The remainder is r when p is divided by k"* means $p = kq + r$; the integer q is called the quotient. For instance, *"The remainder is 1 when 7 is divided by 3"* means $7 = 3 \cdot 2 + 1$.**

Solution: Translating *"When the integer n is divided by 2, the quotient is u and the remainder is 1"* into an equation gives

$$n = 2u + 1$$

Translating *"When the integer n is divided by 5, the quotient is v and the remainder is 3"* into an equation gives

$$n = 5v + 3$$

Since both expressions equal n, we can set them equal to each other:

$$2u + 1 = 5v + 3$$

Rearranging and then combining like terms yields

$$2u - 5v = 2$$

The answer is (B).

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6. Since a number raised to an even exponent is greater than or equal to zero, we know that y^2 is positive (it cannot be zero because the product xy^2z would then be zero). Hence, we can divide both sides of the inequality $xy^2z < 0$ by y^2:

$$\frac{xy^2z}{y^2} < \frac{0}{y^2}$$

Simplifying yields

$$xz < 0$$

Therefore, I is true, which eliminates (A), (C), and (E). Now, the following illustrates that $z < 0$ is not necessarily true:

$$-1 \cdot 2^2 \cdot 3 = -12 < 0$$

This eliminates (D). Hence, the answer is (B).

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7. To solve this problem, note the following strategy:

- **To compare two fractions, cross-multiply. The larger number will be on the same side as the larger fraction.**

Solution: Cross-multiplying the fractions 9/10 and 10/11 gives $9 \cdot 11$ versus $10 \cdot 10$, which reduces to 99 versus 100. Now, 100 is greater than 99. Hence, 10/11 is greater than 9/10. Continuing in this manner shows that 10/11 is the largest fraction in the group. Hence, the answer is (A).

8.

- **In Algebra, you solve an equation for, say, *y* by isolating *y* on one side of the equality symbol. On the SAT, however, you are often asked to solve for an entire term, say, 3 – *y* by isolating it on one side.**

Solution: Translating the sentence into an equation gives

$$a + 3a = b + 3b - 4$$

Combining like terms gives

$$4a = 4b - 4$$

Subtracting $4b$ from both sides gives

$$4a - 4b = -4$$

Finally, dividing by 4 gives

$$a - b = -1$$

Hence, the answer is (B).

9. First, let's review the definition of an average:

- **The *average* of *N* numbers is their sum divided by *N*, that is,** $average = \frac{sum}{N}$.

Solution: By the definition of an average, we get

$$\frac{x+2x+6}{3} = \frac{3x+6}{3} = \frac{3(x+2)}{3} = x+2$$

Hence, the answer is (D).

10. Before presenting the solution, let's review the concept of a ratio.

A ratio is simply a fraction. The following notations all express the ratio of x to y:

$$x : y,\ x \div y, \text{ or } x/y$$

Writing two numbers as a ratio provides a convenient way to compare their sizes. For example, since $3/\pi < 1$, we know that 3 is less than π. A ratio compares two numbers. Just as you cannot compare apples and oranges, so to must the numbers you are comparing have the same units. For example, you cannot form the ratio of 2 feet to 4 yards because the two numbers are expressed in different units—feet vs. yards. It is quite common for the SAT to ask for the ratio of two numbers with different units. Before you form any ratio, make sure the two numbers are expressed in the same units.

Solution: The ratio in cannot be formed until the numbers are expressed in the same units. Let's turn the yards into feet. Since there are 3 feet in a yard, 4 yards = 4 × 3 feet = 12 feet. Forming the ratio yields

$$\frac{2\ feet}{12\ feet} = \frac{1}{6}\ or\ 1:6$$

The answer is (D).

Note, taking the reciprocal of a fraction usually changes its size. For example, $\frac{3}{4} \neq \frac{4}{3}$. So order is important in a ratio: $3 : 4 \neq 4 : 3$.

11. First, apply the rule $\left(x^a\right)^b = x^{ab}$ to the expression $\frac{x\left(x^5\right)^2}{x^4}$:

$$\frac{x \cdot x^{5 \cdot 2}}{x^4} = \frac{x \cdot x^{10}}{x^4}$$

Next, apply the rule $x^a \cdot x^b = x^{a+b}$:

$$\frac{x \cdot x^{10}}{x^4} = \frac{x^{11}}{x^4}$$

Finally, apply the rule $\frac{x^a}{x^b} = x^{a-b}$:

$$\frac{x^{11}}{x^4} = x^{11-4} = x^7$$

The answer is (C).

Let's review the rules of exponents.

- **Exponents afford a convenient way of expressing long products of the same number. The expression b^n is called a power and it stands for $b \times b \times b \times \cdots \times b$, where there are n factors of b. b is called the base, and n is called the exponent. By definition, $b^0 = 1$.**

There are six rules that govern the behavior of exponents:

Rule 1: $x^a \cdot x^b = x^{a+b}$ Example, $2^3 \cdot 2^2 = 2^{3+2} = 2^5 = 32$. Caution, $x^a + x^b \neq x^{a+b}$

Rule 2: $\left(x^a\right)^b = x^{ab}$ Example, $\left(2^3\right)^2 = 2^{3 \cdot 2} = 2^6 = 64$

Rule 3: $(xy)^a = x^a \cdot y^a$ Example, $(2y)^3 = 2^3 \cdot y^3 = 8y^3$

Rule 4: $\left(\dfrac{x}{y}\right)^a = \dfrac{x^a}{y^a}$ Example, $\left(\dfrac{x}{3}\right)^2 = \dfrac{x^2}{3^2} = \dfrac{x^2}{9}$

Rule 5: $\dfrac{x^a}{x^b} = x^{a-b}$, if $a > b$. Example, $\dfrac{2^6}{2^3} = 2^{6-3} = 2^3 = 8$

$\dfrac{x^a}{x^b} = \dfrac{1}{x^{b-a}}$, if $b > a$. Example, $\dfrac{2^3}{2^6} = \dfrac{1}{2^{6-3}} = \dfrac{1}{2^3} = \dfrac{1}{8}$

Rule 6: $x^{-a} = \dfrac{1}{x^a}$ Example, $z^{-3} = \dfrac{1}{z^3}$ Caution, a negative exponent does not make the number negative; it merely indicates that the base should be reciprocated. For example, $3^{-2} \neq -\dfrac{1}{3^2}$ *or* $-\dfrac{1}{9}$.

Problems involving these six rules are common on the test, and they are often listed as hard problems. However, the process of solving these problems is quite mechanical: simply apply the six rules until they can no longer be applied.

Note: Typically, there are many ways of solving these types of problems. For this problem, we could have begun with Rule 5, $\dfrac{x^a}{x^b} = \dfrac{1}{x^{b-a}}$:

$$\frac{x\left(x^5\right)^2}{x^4} = \frac{\left(x^5\right)^2}{x^{4-1}} = \frac{\left(x^5\right)^2}{x^3}$$

Then apply Rule 2, $\left(x^a\right)^b = x^{ab}$:

$$\frac{\left(x^5\right)^2}{x^3} = \frac{x^{10}}{x^3}$$

Finally, apply the other version of Rule 5, $\dfrac{x^a}{x^b} = x^{a-b}$:

$$\frac{x^{10}}{x^3} = x^7$$

12.

$$\left(x-\frac{y}{3}\right)-\left(y-\frac{x}{3}\right)=$$

$x-\frac{y}{3}-y+\frac{x}{3}=$ by distributing the negative sign

$\frac{4}{3}x-\frac{4}{3}y=$ by combining the fractions

$\frac{4}{3}(x-y)=$ by factoring out the common factor 4/3

$\frac{4}{3}(9)=$ since $x - y = 9$

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The answer is (D).

The Distributive Rule was used twice in this solution (the first and third steps). Let's review this important factoring rule.

DISTRIBUTIVE RULE

The most basic type of factoring involves the distributive rule:

$$ax + ay = a(x + y)$$

When this rule is applied from left to right, it is called factoring. When the rule is applied from right to left, it is called distributing.

For example, $3h + 3k = 3(h + k)$, and $5xy + 45x = 5xy + 9 \cdot 5x = 5x(y + 9)$. The distributive rule can be generalized to any number of terms. For three terms, it looks like

$$ax + ay + az = a(x + y + z)$$

For example, $2x + 4y + 8 = 2x + 2 \cdot 2y + 2 \cdot 4 = 2(x + 2y + 4)$.

For another example, $x^2y^2 + xy^3 + y^5 = y^2\left(x^2 + xy + y^3\right)$.

13. $2-\left(5-3^3[4\div 2+1]\right)=$

$2-\left(5-3^3[2+1]\right)=$ By performing the division within the innermost parentheses

$2-\left(5-3^3[3]\right)=$ By performing the addition within the innermost parentheses

$2-(5-27[3])=$ By performing the exponentiation

$2-(5-81)=$ By performing the multiplication within the parentheses

$2-(-76)=$ By performing the subtraction within the parentheses

$2+76=$ By multiplying the two negatives

78

The answer is (E).

Let's review the concepts of Algebraic Expressions, Use of Parentheses, Order of Operations, and FOIL Multiplication.

A mathematical expression that contains a variable is called an algebraic expression. Some examples of algebraic expressions are x^2, $3x - 2y$, $2z\left(y^3 - \frac{1}{z^2}\right)$. Two algebraic expressions are called like terms if both the variable parts and the exponents are identical. That is, the only parts of the expressions that can differ are the coefficients. For example, $5y^3$ and $\frac{3}{2}y^3$ are like terms, as are $x + y^2$ and $-7(x + y^2)$. However, x^3 and y^3 are not like terms, nor are $x - y$ and $2 - y$.

ADDING & SUBTRACTING ALGEBRAIC EXPRESSIONS

Only like terms may be added or subtracted. To add or subtract like terms, merely add or subtract their coefficients:

$$x^2 + 3x^2 = (1+3)x^2 = 4x^2$$

$$2\sqrt{x} - 5\sqrt{x} = (2-5)\sqrt{x} = -3\sqrt{x}$$

$$.5\left(x + \frac{1}{y}\right)^2 + .2\left(x + \frac{1}{y}\right)^2 = (.5 + .2)\left(x + \frac{1}{y}\right)^2 = .7\left(x + \frac{1}{y}\right)^2$$

$$\left(3x^3 + 7x^2 + 2x + 4\right) + \left(2x^2 - 2x - 6\right) = 3x^3 + (7+2)x^2 + (2-2)x + (4-6) = 3x^3 + 9x^2 - 2$$

You may add or multiply algebraic expressions in any order. This is called the commutative property:

$$\boxed{x + y = y + x}$$

$$\boxed{xy = yx}$$

For example, $-2x + 5x = 5x + (-2x) = (5 - 2)x = 3x$ and $(x - y)(-3) = (-3)(x - y) = (-3)x - (-3)y = -3x + 3y$.

Caution: the commutative property does not apply to division or subtraction: $2 = 6 \div 3 \neq 3 \div 6 = 1/2$ and $-1 = 2 - 3 \neq 3 - 2 = 1$.

When adding or multiplying algebraic expressions, you may regroup the terms. This is called the associative property:

$$\boxed{x + (y + z) = (x + y) + z}$$

$$\boxed{x(yz) = (xy)z}$$

Notice in these formulas that the variables have not been moved, only the way they are grouped has changed: on the left side of the formulas the last two variables are grouped together, and on the right side of the formulas the first two variables are grouped together.

For example,

$$(x - 2x) + 5x = (x + [-2x]) + 5x = x + (-2x + 5x) = x + 3x = 4x$$

and

$$2(12x) = (2 \cdot 12)x = 24x$$

The associative property doesn't apply to division or subtraction:

$$4 = 8 \div 2 = 8 \div (4 \div 2) \neq (8 \div 4) \div 2 = 2 \div 2 = 1$$

and

$$-6 = -3 - 3 = (-1 - 2) - 3 \neq -1 - (2 - 3) = -1 - (-1) = -1 + 1 = 0$$

Notice in the first example that we changed the subtraction into negative addition: $(x - 2x) = (x + [-2x])$. This allowed us to apply the associative property over addition.

PARENTHESES

When simplifying expressions with nested parentheses, work from the inner most parentheses out:

$$5x + (y - (2x - 3x)) = 5x + (y - (-x)) = 5x + (y + x) = 6x + y$$

Sometimes when an expression involves several pairs of parentheses, one or more pairs are written as brackets. This makes the expression easier to read:

$$2x(x - [y + 2(x - y)]) =$$
$$2x(x - [y + 2x - 2y]) =$$
$$2x(x - [2x - y]) =$$
$$2x(x - 2x + y) =$$
$$2x(-x + y) =$$
$$-2x^2 + 2xy$$

ORDER OF OPERATIONS: (PEMDAS)

When simplifying algebraic expressions, perform operations within parentheses first and then exponents and then multiplication and then division and then addition and lastly subtraction. This can be remembered by the mnemonic:

PEMDAS
Please **E**xcuse **M**y **D**ear **A**unt **S**ally

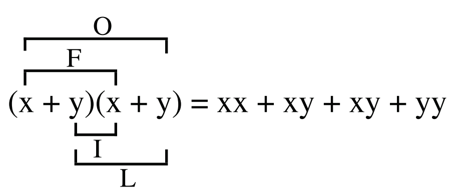
This mnemonic isn't quite precise enough. Multiplication and division are actually tied in order of operation, as is the pair addition and subtraction. When multiplication and division, or addition and subtraction, appear at the same level in an expression, perform the operations from left to right. For example, $6 \div 2 \times 4 = (6 \div 2) \times 4 = 3 \times 4 = 12$. To emphasize this left-to-right order, we can use parentheses in the mnemonic:

PE(MD)(AS)

FOIL MULTIPLICATION

You may recall from algebra that when multiplying two expressions you use the FOIL method:

First, **O**uter, **I**nner, **L**ast:



Simplifying the right side yields

$$(x + y)(x + y) = x^2 + 2xy + y^2$$

For the product $(x-y)(x-y)$, we get

$$(x-y)(x-y) = x^2 - 2xy + y^2$$

These types of products occur often, so it is worthwhile to memorize the formulas (They are called *Perfect Square Trinomials*). Nevertheless, you should still learn the FOIL method of multiplying because the formulas do not apply in all cases.

Examples (FOIL):

$$(2-y)(x-y^2) = 2x - 2y^2 - xy + yy^2 = 2x - 2y^2 - xy + y^3$$

$$\left(\frac{1}{x} - y\right)\left(x - \frac{1}{y}\right) = \frac{1}{x}x - \frac{1}{x}\frac{1}{y} - xy + y\frac{1}{y} = 1 - \frac{1}{xy} - xy + 1 = 2 - \frac{1}{xy} - xy$$

$$\left(\frac{1}{2} - y\right)^2 = \left(\frac{1}{2} - y\right)\left(\frac{1}{2} - y\right) = \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)y + y^2 = \frac{1}{4} - y + y^2$$

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14. Translate the sentence into a mathematical equation as follows:

What	percent	of	25	is	5
↓	↓	↓	↓	↓	↓
x	$\frac{1}{100}$	$\cdot$	25	=	5

$$\frac{25}{100}x = 5$$

$$\frac{1}{4}x = 5$$

$$x = 20$$

The answer is (B).

Let's review the concept of a percent since problems involving percent are common on the SAT. The word *percent* means "divided by one hundred." When you see the word "percent," or the symbol %, remember it means 1/100. For example,

25 percent
↓ ↓
$$25 \times \frac{1}{100} = \frac{1}{4}$$

To convert a decimal into a percent, move the decimal point two places to the right. For example,

0.25 = 25%
0.023 = 2.3%
1.3 = 130%

Conversely, to convert a percent into a decimal, move the decimal point two places to the left. For example,

47% = .47
3.4% = .034
175% = 1.75

To convert a fraction into a percent, first change it into a decimal (by dividing the denominator [bottom] into the numerator [top]) and then move the decimal point two places to the right. For example,

$$\frac{7}{8} = 0.875 = 87.5\%$$

Conversely, to convert a percent into a fraction, first change it into a decimal and then change the decimal into a fraction. For example,

$$80\% = .80 = \frac{80}{100} = \frac{4}{5}$$

Following are the most common fractional equivalents of percents:

$$33\frac{1}{3}\% = \frac{1}{3} \qquad 20\% = \frac{1}{5}$$

$$66\frac{2}{3}\% = \frac{2}{3} \qquad 40\% = \frac{2}{5}$$

$$25\% = \frac{1}{4} \qquad 60\% = \frac{3}{5}$$

$$50\% = \frac{1}{2} \qquad 80\% = \frac{4}{5}$$

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15. Questions involving graphs are more common on the new SAT. Rarely do these questions involve any significant calculating. Usually, the solution is merely a matter of interpreting the graph. Reading from the graph, we see that in 1985 the company's earnings were \$8 million and its sales were \$80 million. This gives

$$\frac{8}{10} = \frac{1}{10} = \frac{10}{100} = 10\%$$

The answer is (A).

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16. The graph yields the following information:

Year	**Earnings**
1986	\$5 million
1987	\$10 million
1988	\$12 million

Forming the average yields

$$\frac{5 + 10 + 12}{3} = \frac{27}{3} = 9$$

The answer is (C).

17. To find the percentage increase (or decrease), divide the numerical change by the original amount. This yields

Year	Percentage increase
86	$\frac{70-80}{80}=\frac{-10}{80}=\frac{-1}{8}=-12.5\%$
87	$\frac{50-70}{70}=\frac{-20}{70}=\frac{-2}{7}\approx -29\%$
88	$\frac{80-50}{50}=\frac{30}{50}=\frac{3}{5}=60\%$
89	$\frac{90-80}{80}=\frac{10}{80}=\frac{1}{8}=12.5\%$
90	$\frac{100-90}{90}=\frac{10}{90}=\frac{1}{9}\approx 11\%$

The largest number in the right-hand column, 60%, corresponds to the year 1988. The answer is (C).

18. Calculating 10 percent of the sales for each year yields

Year	10% of Sales (millions)	Earnings (millions)
85	.10 × 80 = 8	8
86	.10 × 70 = 7	5
87	.10 × 50 = 5	10
88	.10 × 80 = 8	12
89	.10 × 90 = 9	11
90	.10 × 100 = 10	8

Comparing the right columns shows that earnings were 10 percent or less of sales in 1985, 1986, and 1990. The answer is (D).

19. Following Guideline 1 (see list below), we let r = *Scott's rate*. Then $2r - 1$ = *Garrett's rate*. Turning to Guideline 2, we look for two quantities that are equal to each other. When Garrett overtakes Scott, they will have traveled the same distance. Now, from the formula $D = R \times T$, Scott's distance is

$$D = r \times 2\frac{1}{2}$$

and Garrett's distance is

$$D = (2r - 1)2 = 4r - 2$$

Setting these expressions equal to each other gives

$$4r - 2 = r \times 2\frac{1}{2}$$

Solving this equation for r gives

$$r = 4/3$$

Hence, Garrett will have traveled $D = 4r - 2 = 4\left(\frac{4}{3}\right) - 2 = 3\frac{1}{3}$ miles. The answer is (B).

Let's develop some guidelines for solving word problems.

First, we need to be very comfortable with translating words into mathematical symbols. Following is a partial list of words and their mathematical equivalents.

Concept	Symbol	Words	Example	Translation
equality	=	is	2 plus 2 is 4	$2 + 2 = 4$
		equals	x minus 5 equals 2	$x - 5 = 2$
		is the same as	multiplying x by 2 is the same as dividing x by 7	$2x = x/7$
addition	+	sum	the sum of y and π is 20	$y + \pi = 20$
		plus	x plus y equals 5	$x + y = 5$
		add	how many marbles must John add to collection P so that he has 13 marbles	$x + P = 13$
		increase	a number is increased by 10%	$x + 10\%x$
		more	the perimeter of the square is 3 more than the area	$P = 3 + A$
subtraction	–	minus	x minus y	$x - y$
		difference	the difference of x and y is 8	$\|x - y\| = 8$
		subtracted	x subtracted from y	$y - x$ *
		less than	the circumference is 5 less than the area	$C = A - 5$
multiplication	× or •	times	the acceleration is 5 times the velocity	$a = 5v$
		product	the product of two consecutive integers	$x(x + 1)$
		of	x is 125% of y	$x = 125\%y$
division	÷	quotient	the quotient of x and y is 9	$x \div y = 9$
		divided	if x is divided by y, the result is 4	$x \div y = 4$

Although exact steps for solving word problems cannot be given, the following guidelines will help:

(1) First, choose a variable to stand for the least unknown quantity, and then try to write the other unknown quantities in terms of that variable.

For example, suppose we are given that Sue's age is 5 years less than twice Jane's and the sum of their ages is 16. Then Jane's age would be the least unknown, and we let $x = \textit{Jane's age}$. Expressing Sue's age in terms of x gives $\textit{Sue's age} = 2x - 5$.

(2) Second, write an equation that involves the expressions in Step 1. Most (though not all) word problems pivot on the fact that two quantities in the problem are equal. Deciding which two quantities should be set equal is usually the hardest part in solving a word problem since it can require considerable ingenuity to discover which expressions are equal.

For the example above, we would get $(2x - 5) + x = 16$.

(3) Third, solve the equation in Step 2 and interpret the result.

For the example above, we would get by adding the x's: $3x - 5 = 16$

Then adding 5 to both sides gives $3x = 21$

Finally, dividing by 3 gives $x = 7$

Hence, Jane is 7 years old and Sue is $2x - 5 = 2 \cdot 7 - 5 = 9$ years old.

MOTION PROBLEMS

Virtually, all motion problems involve the formula $\textit{Distance} = \textit{Rate} \times \textit{Time}$, or

$$D = R \times T$$

* Notice that with "minus" and "difference" the terms are subtracted in the same order as they are written, from left to right (x minus y —> $x - y$). However, with "subtracted" and "less than," the order of subtraction is reversed (x subtracted from y —> $y - x$). Many students translate "subtracted from" in the wrong order.

20. We know *"the 3rd term of S is 4,"* and that *"the 3rd term is four times the 2nd."* This is equivalent to saying the 2nd term is 1/4 the 3rd term: $\frac{1}{4} \cdot 4 = 1$. Further, we know *"the 2nd term is three times the 1st."* This is equivalent to saying the 1st term is 1/3 the 2nd term: $\frac{1}{3} \cdot 1 = \frac{1}{3}$. Hence, the first term of the sequence is fully determined:

1/3, 1, 4

The answer is (B).

Let's briefly discuss the concept of a sequence.

SEQUENCES

A sequence is an ordered list of numbers. The following is a sequence of odd numbers:

1, 3, 5, 7, . . .

A term of a sequence is identified by its position in the sequence. In the above sequence, 1 is the first term, 3 is the second term, etc. The ellipsis symbol (. . .) indicates that the sequence continues forever.

Answers and Solutions Section 2:

1. D	**6. A**	**11. 23**	**16. 2**
2. C	**7. C**	**12. 11/12**	**17. 86**
3. E	**8. E**	**13. 1**	**18. 60**
4. D	**9. 53**	**14. 30**	
5. B	**10. 200**	**15. 40**	

---

1. Choose n to be 1. Then $2n + 2 = 2(1) + 2 = 4$, which is even. So eliminate (A). Next, $n - 5 = 1 - 5 = -4$. Eliminate (B). Next, $2n = 2(1) = 2$. Eliminate (C). Next, $2n + 3 = 2(1) + 3 = 5$ is not even—it *may* be our answer. However, $5n + 2 = 5(1) + 2 = 7$ is not even as well. So we choose another number, say, 2. Then $5n + 2 = 5(2) + 2 = 12$ is even, which eliminates (E). Thus, choice (D), $2n + 3$, is the answer.

- **When using the substitution method to solve a problem, be sure to check every answer-choice because the number you choose may work for more than one answer-choice. If this does occur, then choose another number and plug it in, and so on, until you have eliminated all but the answer. This may sound like a lot of computing, but the calculations can usually be done in a few seconds.**

---

2. First, we must determine whether $2k - 1$ is odd or even. (It cannot be both—why?) To this end, let $k = 1$. Then $2k - 1 = 2 \cdot 1 - 1 = 1$, which is an odd number. Therefore, we use the bottom-half of the definition given above. That is, $(2k - 1)^* = 4(2k - 1) = 8k - 4$. The answer is (C).

You may be wondering how defined functions differ from the functions, $f(x)$, you studied in Intermediate Algebra and more advanced math courses. They *don't* differ. They are the same old concept you dealt with in your math classes. The function in this problem could just as easily be written $f(x) = \sqrt{x}$ and $f(x) = 4x$. The purpose of defined functions is to see how well you can adapt to unusual structures. Once you realize that defined functions are evaluated and manipulated just as regular functions, they become much less daunting.

---

3. For a fractional expression to be an integer, the denominator must divide evenly into the numerator. Now, Statement I cannot be an integer. Since q is odd and p is even, $p + q$ is odd. Further, since $p + q$ is odd, it cannot be divided evenly by the even number p. Hence, $(p + q)/p$ cannot be an integer. Next, Statement II can be an integer. For example, if $p = 2$ and $q = 3$, then $\frac{pq}{3} = \frac{2 \cdot 3}{3} = 2$. Finally, Statement III cannot be an integer. $p^2 = p \cdot p$ is even since it is the product of two even numbers. Further, since q is odd, it cannot be divided evenly by the even integer p^2. The answer is (E).

Let's discuss some of the concepts from Number Theory.

- **A number n is even if the remainder is zero when n is divided by 2: $n = 2z + 0$, or $n = 2z$.**

- **A number n is odd if the remainder is one when n is divided by 2: $n = 2z + 1$.**

- **The following properties for odd and even numbers are very useful—you should memorize them:**

$$even \times even = even$$
$$odd \times odd = odd$$
$$even \times odd = even$$

$$even + even = even$$
$$odd + odd = even$$
$$even + odd = odd$$

4. Statement I could be true because $-|0| = -(+0) = -(0) = 0$. Statement II could be true because the right side of the equation is always negative:

$$-|x| = -(\text{a positive number}) = \text{a negative number}$$

Now, if one side of an equation is always negative, then the other side must always be negative, otherwise the opposite sides of the equation would not be equal. Since Statement III is the opposite of Statement II, it must be false. But let's show this explicitly: Suppose x were positive. Then $|x| = x$, and the equation $|x| = -x$ becomes $x = -x$. Dividing both sides of this equation by x yields $1 = -1$. This is a contradiction. Hence, x cannot be positive. The answer is (D).

Let's discuss some of the properties of inequalities.

Inequalities are manipulated algebraically the same way as equations with one exception:

- **Multiplying or dividing both sides of an inequality by a negative number reverses the inequality. That is, if $x > y$ and $c < 0$, then $cx < cy$.**

Example: For which values of x is $4x + 3 > 6x - 8$?

As with equations, our goal is to isolate x on one side:

Subtracting $6x$ from both sides yields

$$-2x + 3 > -8$$

Subtracting 3 from both sides yields

$$-2x > -11$$

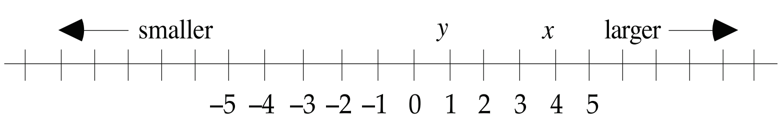
Dividing both sides by –2 and reversing the inequality yields

$$x < 11/2$$

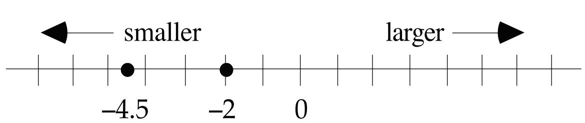
Positive & Negative Numbers

A number greater than 0 is positive. On the number line, positive numbers are to the right of 0. A number less than 0 is negative. On the number line, negative numbers are to the left of 0. Zero is the only number that is neither positive nor negative; it divides the two sets of numbers. On the number line, numbers increase to the right and decrease to the left.

The expression $x > y$ means that x is greater than y. In other words, x is to the right of y on the number line:



We usually have no trouble determining which of two numbers is larger when both are positive or one is positive and the other negative (e.g., $5 > 2$ and $3.1 > -2$). However, we sometimes hesitate when both numbers are negative (e.g., $-2 > -4.5$). When in doubt, think of the number line: if one number is to the right of the number, then it is larger. As the number line below illustrates, –2 is to the right of –4.5. Hence, –2 is larger than –4.5.

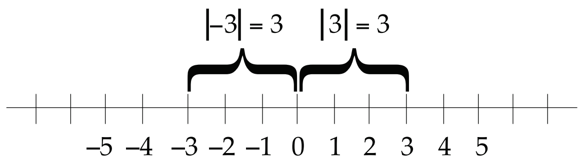


Miscellaneous Properties of Positive and Negative Numbers

1. The product (quotient) of positive numbers is positive.
2. The product (quotient) of a positive number and a negative number is negative.
3. The product (quotient) of an even number of negative numbers is positive.
4. The product (quotient) of an odd number of negative numbers is negative.
5. The sum of negative numbers is negative.
6. A number raised to an even exponent is greater than or equal to zero.

Absolute Value

The absolute value of a number is its distance on the number line from 0. Since distance is a positive number, absolute value of a number is positive. Two vertical bars denote the absolute value of a number: $|x|$. For example, $|3| = 3$ and $|-3| = 3$. This can be illustrated on the number line:



Students rarely struggle with the absolute value of numbers: if the number is negative, simply make it positive; and if it is already positive, leave it as is. For example, since –2.4 is negative, $|-2.4| = 2.4$ and since 5.01 is positive $|5.01| = 5.01$.

Further, students rarely struggle with the absolute value of positive variables: if the variable is positive, simply drop the absolute value symbol. For example, if $x > 0$, then $|x| = x$.

However, negative variables can cause students much consternation. If x is negative, then $|x| = -x$. This often confuses students because the absolute value is positive but the $-x$ appears to be negative. It is actually positive—it is the negative of a negative number, which is positive. To see this more clearly let $x = -k$, where k is a positive number. Then x is a negative number. So $|x| = -x = -(-k) = k$. Since k is positive so is $-x$. Another way to view this is

$$|x| = -x = (-1) \cdot x = (-1)(\text{a negative number}) = \text{a positive number}$$

5.

- **Sometimes on the SAT, a system of 3 equations will be written as one long "triple" equation. For example, the three equations $x = y$, $y = z$, $x = z$, can be written more compactly as $x = y = z$.**

The equation $w = 2x = \sqrt{2}y$ stands for three equations: $w = 2x$, $2x = \sqrt{2}y$, and $w = \sqrt{2}y$. From the last equation, we get $w = \sqrt{2}y$; and from the second equation, we get $x = \frac{\sqrt{2}}{2}y$. Hence,

$$w - x = \sqrt{2}y - \frac{\sqrt{2}}{2}y = \frac{2}{2}\sqrt{2}y - \frac{\sqrt{2}}{2}y = \frac{2\sqrt{2}y - \sqrt{2}y}{2} = \frac{\sqrt{2}y}{2}$$

Hence, the answer is (B).

Let's discuss some of the properties of equations.

When simplifying algebraic expressions, we perform operations within parentheses first and then exponents and then multiplication and then division and then addition and lastly subtraction. This can be remembered by the mnemonic:

PEMDAS
Please **E**xcuse **M**y **D**ear **A**unt **S**ally

When solving equations, however, we apply the mnemonic in reverse order: **SADMEP**. This is often expressed as follows: inverse operations in inverse order. The goal in solving an equation is to isolate the variable on one side of the equal sign (usually the left side). This is done by identifying the main operation—addition, multiplication, etc.—and then performing the opposite operation.

Example: Solve the following equation for x: $2x + y = 5$

Solution: The main operation is addition (remember addition now comes before multiplication, SADMEP), so subtracting y from both sides yields

$$2x + y - y = 5 - y$$

Simplifying yields

$$2x = 5 - y$$

The only operation remaining on the left side is multiplication. Undoing the multiplication by dividing both sides by 2 yields

$$\frac{2x}{2} = \frac{5-y}{2}$$

Canceling the 2 on the left side yields

$$x = \frac{5-y}{2}$$

Example: Solve the following equation for x: $3x - 4 = 2(x - 5)$

Solution: Here x appears on both sides of the equal sign, so let's move the x on the right side to the left side. But the x is trapped inside the parentheses. To release it, distribute the 2:

$$3x - 4 = 2x - 10$$

Now, subtracting $2x$ from both sides yields*

$$x - 4 = -10$$

Finally, adding 4 to both sides yields

$$x = -6$$

* Note, students often mistakenly add $2x$ to both sides of this equation because of the minus symbol between $2x$ and 10. But $2x$ is positive, so we subtract it. This can be seen more clearly by rewriting the right side of the equation as $-10 + 2x$.

We often manipulate equations without thinking about what the equations actually say. The SAT likes to test this oversight. Equations are packed with information. Take for example the simple equation $3x + 2 = 5$. Since 5 is positive, the expression $3x + 2$ must be positive as well. An equation means that the terms on either side of the equal sign are equal in every way. Hence, any property one side of an equation has the other side will have as well. Following are some immediate deductions that can be made from simple equations.

Equation	**Deduction**
$y - x = 1$	$y > x$
$y^2 = x^2$	$y = \pm x$, or $\|y\| = \|x\|$. That is, x and y can differ only in sign.
$y^3 = x^3$	$y = x$
$y = x^2$	$y \ge 0$
$\frac{y}{x^2} = 1$	$y > 0$
$\frac{y}{x^3} = 2$	Both x and y are positive or both x and y are negative.
$x^2 + y^2 = 0$	$y = x = 0$
$3y = 4x$ and $x > 0$	$y > x$ and y is positive.
$3y = 4x$ and $x < 0$	$y < x$ and y is negative.
$y = \sqrt{x+2}$	$y \ge 0$ and $x \ge -2$
$y = 2x$	y is even
$y = 2x + 1$	y is odd
$yx = 0$	$y = 0$ or $x = 0$, or both

---

6. Canceling the common factor 3 yields $\frac{1\cdot1\cdot1\cdot1}{3\cdot3\cdot3\cdot3}$, or $\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}$. Now, by the definition of a power,

$$\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} = \left(\frac{1}{3}\right)^4$$

Hence, the answer is (A).

7.

$$\frac{2^{20} - 2^{19}}{2^{11}} = \frac{2^{19+1} - 2^{19}}{2^{11}} =$$

$$\frac{2^{19}\cdot 2^1 - 2^{19}}{2^{11}} = \qquad \text{by the rule } x^a \cdot x^b = x^{a+b}$$

$$\frac{2^{19}(2-1)}{2^{11}} = \qquad \text{by the distributive property } ax + ay = a(x+y)$$

$$\frac{2^{19}}{2^{11}} =$$

$$2^8 \qquad \text{by the rule } \frac{x^a}{x^b} = x^{a-b}$$

The answer is (C).

---

8. Since *"every number in the sequence* –1, 3, –3, . . . *is the product of the two immediately preceding numbers,"* the forth term of the sequence is –9 = 3(–3). The first 6 terms of this sequence are

–1, 3, –3, –9, 27, –243, . . .

At least six numbers in this sequence are odd: –1, 3, –3, –9, 27, –243. The answer is (E).

9.

- **The number of integers between two integers <u>inclusive</u> is one more than their difference.**

Solution: By the principle stated above, the number of integers between 49 and 101 inclusive is (101 – 49) + 1 = 53. Grid-in 53. To see this more clearly, choose smaller numbers, say, 9 and 11. The difference between 9 and 11 is 2. But there are three numbers between them inclusive—9, 10, and 11—one more than their difference.

---

10. Let n be the number of marbles in the bowl, and let c be the capacity of the bowl. Then translating *"if 1/4 of the marbles were removed, the bowl would be filled to 1/2 of its capacity"* into an equation yields

$$n - \frac{1}{4}n = \frac{1}{2}c \text{, or } \frac{3}{2}n = c \text{ .}$$

Next, translating *"if 100 marbles were added, the bowl would be full"* into an equation yields

$$100 + n = c$$

Hence, we have the system:

$$\frac{3}{2}n = c$$

$$100 + n = c$$

Combining the two above equations yields

$$\frac{3}{2}n = 100 + n$$

$$3n = 200 + 2n$$

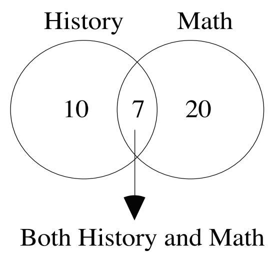
$$n = 200$$

Grid-in 200.

11. Counting may have been one of humankind's first thought processes; nevertheless, counting can be deceptively hard. In part, because we often forget some of the principles of counting, but also because counting can be inherently difficult.

- **When counting elements that are in overlapping sets, the total number will equal the number in one group plus the number in the other group minus the number common to both groups. Venn diagrams are very helpful with these problems.**

Solution:



By the principle stated above, we add 10 and 20 and then subtract 7 from the result. Thus, there are (10 + 20) – 7 = 23 students. Grid-in 23.

12. There are 11 (= 5 + 6) people who selected a number 2 or number 3 marble, and there are 15 total people. Hence, the probability of selecting a number 2 or number 3 marble is 11/15. Grid-in 11/15.

Let's discuss some of the concepts from Statistics.

STATISTICS

Statistics is the study of the patterns and relationships of numbers and data. There are four main concepts that may appear on the test:

Median

When a set of numbers is arranged in order of size, the *median* is the middle number. For example, the median of the set {8, 9, 10, 11, 12} is 10 because it is the middle number. In this case, the median is also the mean (average). But this is usually not the case. For example, the median of the set {8, 9, 10, 11, 17} is 10 because it is the middle number, but the mean is $11 = \frac{8+9+10+11+17}{5}$. If a set contains an even number of elements, then the median is the average of the two middle elements. For example, the median of the set {1, 5, 8, 20} is $6.5\left(=\frac{5+8}{2}\right)$.

Example: What is the median of $0, -2, 256, 18, \sqrt{2}$?

Arranging the numbers from smallest to largest (we could also arrange the numbers from the largest to smallest; the answer would be the same), we get $-2, 0, \sqrt{2}, 18, 256$. The median is the middle number, $\sqrt{2}$.

Mode

The *mode* is the number or numbers that appear most frequently in a set. Note that this definition allows a set of numbers to have more than one mode.

Example: What is the mode of $3, -4, 3, 7, 9, 7.5$?

The number 3 is the mode because it is the only number that is listed more than once.

Example: What is the mode of 2, π, 2, –9, π, 5 ?

Both 2 and π are modes because each occurs twice, which is the greatest number of occurrences for any number in the list.

Range

The *range* is the distance between the smallest and largest numbers in a set. To calculate the range, merely subtract the smallest number from the largest number.

Example: What is the range of 2, 8, 1 , –6, π, 1/2 ?

The largest number in this set is 8, and the smallest number is –6. Hence, the range is 8 – (–6) = 8 + 6 = 14.

Standard Deviation

On the test, you are not expected to know the definition of standard deviation. However, you may be presented with the definition of standard deviation and then be asked a question based on the definition. To make sure we cover all possible bases, we'll briefly discuss this concept.

Standard deviation measures how far the numbers in a set vary from the set's mean. If the numbers are scattered far from the set's mean, then the standard deviation is large. If the numbers are bunched up near the set's mean, then the standard deviation is small.

Example: Which of the following sets has the larger standard deviation?

A = {1, 2, 3, 4, 5}
B = {1, 4, 15, 21, 34}

All the numbers in Set A are within 2 units of the mean, 3. All the numbers in Set B are greater than 5 units from the mean, 15 (except, or course, the mean itself). Hence, the standard deviation of Set B is greater.

---

13. Since we are being asked to evaluate $f(v)$ and we are told that $v = f(-1)$, we are just being asked to compose f(x) with itself. That is, we need to calculate $f(f(-1))$. From the graph, $f(-1) = 3$. So $f(f(-1)) = f(3)$. Again, from the graph, $f(3) = 1$. So $f(f(-1)) = f(3) = 1$. Grid-in 1.

Let's discuss some of the concepts of a function.

DEFINITION OF A FUNCTION

A function is a special relationship (correspondence) between two sets such that for each element x in its domain there is assigned one and only one element y in its range.

Notice that the correspondence has two parts:

1) For each x there is assigned *one* y. (This is the ordinary part of the definition.)
2) For each x there is assigned *only one* y. (This is the special part of the definition.)

The second part of the definition of a function creates the uniqueness of the assignment: There cannot be assigned two values of y to one x. In mathematics, uniqueness is very important. We know that 2 + 2 = 4, but it would be confusing if 2 + 2 could also equal something else, say 5. In this case, we could never be sure that the answer to a question was the *right* answer.

The correspondence between x and y is usually expressed with the function notation: $y = f(x)$, where y is called the dependent variable and x is called the independent variable. In other words, the value of y

depends on the value of x plugged into the function. For example, the square root function can be written as $y = f(x) = \sqrt{x}$. To calculate the correspondence for $x = 4$, we get $y = f(4) = \sqrt{4} = 2$. That is, the square root function assigns the unique y value of 2 to the x value of 4. Most expressions can be turned into functions. For example, the expression $2^x - \frac{1}{x}$ becomes the function

$$f(x) = 2^x - \frac{1}{x}$$

DOMAIN AND RANGE

We usually identify a function with its correspondence, as in the example above. However, a function consists of three parts: a domain, a range, and correspondence between them.

- **The *domain* of a function is the set of *x* values for which the function is defined.**

For example, the function $f(x) = \frac{1}{x-1}$ is defined for all values of $x \neq 1$, which causes division by zero. There is an infinite variety of functions with restricted domains, but only two types of restricted domains appear on the SAT: division by zero and even roots of negative numbers. For example, the function $f(x) = \sqrt{x-2}$ is defined only if $x - 2 \geq 0$, or $x \geq 2$. The two types of restrictions can be combined. For example, $f(x) = \frac{1}{\sqrt{x-2}}$. Here, $x - 2 \geq 0$ since it's under the square root symbol. Further $x - 2 \neq 0$, or $x \neq 2$, because that would cause division by zero. Hence, the domain is all $x > 2$.

- **The *range* of a function is the set of *y* values that are assigned to the *x* values in the domain.**

For example, the range of the function $y = f(x) = x^2$ is $y \geq 0$ since a square is never negative. The range of the function $y = f(x) = x^2 + 1$ is $y \geq 1$ since $x^2 + 1 \geq 1$. You can always calculate the range of a function algebraically, but it is usually better to graph the function and read off its range from the y values of the graph.

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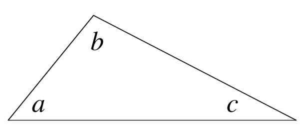
14. Since a triangle has 180°, we get $100 + 50 + c = 180$. Solving for c yields $c = 30$. Grid-in 30.

- **The angle sum of a triangle is 180°:**



$$a + b + c = 180^\circ$$

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15. Since a and b form a straight angle, $a + b = 180$. Now, translating "the quotient of a and b is 7/2" into an equation gives $a/b = 7/2$. Solving for a yields $a = 7b/2$. Plugging this into the equation $a + b = 180$ yields

$$7b/2 + b = 180$$
$$7b + 2b = 360$$
$$9b = 360$$
$$b = 40$$

Grid-in 40.

Let's discuss some of the properties of Geometry.

About one-fourth of the math problems on the SAT involve geometry. (There are no proofs.) Fortunately, the figures on the SAT are usually drawn to scale. Hence, you can check your work and in some cases even solve a problem by "eyeballing" the drawing.

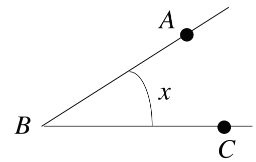
Following is a discussion of the basic properties of geometry. You probably know many of these properties. Memorize any that you do not know.

Lines & Angles

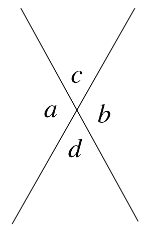
When two straight lines meet at a point, they form an angle. The point is called the vertex of the angle, and the lines are called the sides of the angle.

The angle shown can be identified in three ways:

1. $\angle x$
2. $\angle B$
3. $\angle ABC$ or $\angle CBA$

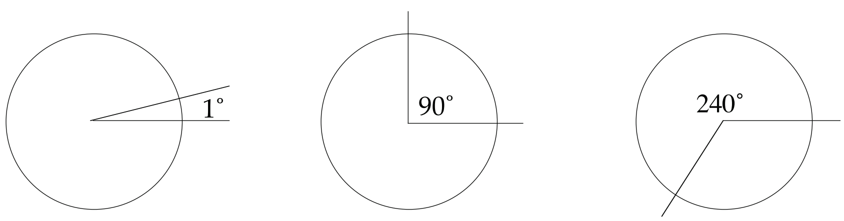


When two straight lines meet at a point, they form four angles. The angles opposite each other are called vertical angles, and they are congruent (equal). In the figure, $a = b$, and $c = d$.



$a = b$ and $c = d$

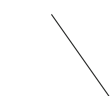
Angles are measured in degrees, ˚. By definition, a circle has 360˚. So an angle can be measured by its fractional part of a circle. For example, an angle that is 1/360 of the arc of a circle is 1˚. And an angle that is 1/4 of the arc of a circle is $\frac{1}{4} \times 360 = 90^\circ$.



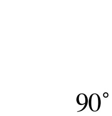
1/360 of an arc of a circle | 1/4 of an arc of a circle | 2/3 of an arc of a circle

There are four major types of angle measures:

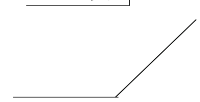
An **acute angle** has measure less than 90˚:



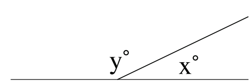
A **right angle** has measure 90˚:



An **obtuse angle** has measure greater than 90˚:



A **straight angle** has measure 180˚:



$x + y = 180^\circ$

16. Factor out the 2 in the expression:

$$\frac{2\left(x^2+2x+1\right)}{(x+1)^2}$$

Factor the quadratic expressions:

$$\frac{2(x+1)(x+1)}{(x+1)(x+1)}$$

Finally, canceling the (x + 1)'s gives 2. Grid-in 2.

Let's discuss some of the properties of fractions.

Fractions

A fraction consists of two parts: a numerator and a denominator.

$$\frac{\textit{numerator}}{\textit{denominator}}$$

If the numerator is smaller than the denominator, the fraction is called *proper* and is less than one. For example: 1/2, 4/5, and 3/π are all proper fractions and therefore less than 1.

If the numerator is larger than the denominator, the fraction is called *improper* and is greater than 1. For example: 3/2, 5/4, and π/3 are all improper fractions and therefore greater than 1.

An improper fraction can be converted into a *mixed fraction* by dividing its denominator into its numerator. For example, since 2 divides into 7 three times with a remainder of 1, we get

$$\frac{7}{2}=3\frac{1}{2}$$

To convert a mixed fraction into an improper fraction, multiply the denominator and the integer and then add the numerator. Then, write the result over the denominator. For example, $5\frac{2}{3}=\frac{3\cdot 5+2}{3}=\frac{17}{3}$.

In a negative fraction, the negative symbol can be written on the top, in the middle, or on the bottom; however, when a negative symbol appears on the bottom, it is usually moved to the top or the middle: $\frac{5}{-3}=\frac{-5}{3}=-\frac{5}{3}$. If both terms in the denominator of a fraction are negative, the negative symbol is often factored out and moved to the top or middle of the fraction:

$$\frac{1}{-x-2}=\frac{1}{-(x+2)}=-\frac{1}{x+2} \text{ or } \frac{-1}{x+2}$$

17.

- ***Weighted Average:*** **The average between two sets of numbers is closer to the set with more numbers.**

Solution: If on a test three people answered 90% of the questions correctly and two people answered 80% correctly, then the average for the group is not 85% but rather

$$\frac{3\cdot 90+2\cdot 80}{5}=\frac{430}{5}=86$$

Here, 90 has a weight of 3—it occurs 3 times. Whereas 80 has a weight of 2—it occurs 2 times. So the average is closer to 90 than to 80 as we have just calculated. Grid-in 86.

---

18. Translating *"the ratio of y to x is equal to 3"* into an equation yields

$$\frac{y}{x}=3$$

Translating *"the sum of y and x is 80"* into an equation yields

$$y+x=80$$

Solving the first equation for y gives $y = 3x$. Substituting this into the second equation yields

$$3x+x=80$$
$$4x=80$$
$$x=20$$

Hence, $y = 3x = 3(20) = 60$. Grid-in 60.

Let's discuss some of the properties of proportions.

PROPORTION

A proportion is simply an equality between two ratios (fractions). For example, the ratio of x to y is equal to the ratio of 3 to 2 is translated as

$$\frac{x}{y}=\frac{3}{2}$$

or in ratio notation,

$$x:y::3:2$$

Two variables are *directly proportional* if one is a constant multiple of the other:

$$y=kx$$

where k is a constant.

The above equation shows that as x increases (or decreases) so does y. This simple concept has numerous applications in mathematics. For example, in constant velocity problems, distance is directly proportional to time: $d = vt$, where v is a constant. Note, sometimes the word *directly* is suppressed.

Answers and Solutions Section 3:

1.	**D**	**5.**	**B**	**9.**	**A**	**13.**	**A**
2.	**C**	**6.**	**C**	**10.**	**C**	**14.**	**D**
3.	**A**	**7.**	**D**	**11.**	**D**	**15.**	**D**
4.	**B**	**8.**	**A**	**12.**	**D**	**16.**	**D**

---

1. Let's solve this problem by substitution. We must choose x and y so that $x/y > 1$. So choose $x = 3$ and $y = 2$. Now, $\frac{3y}{x} = \frac{3 \cdot 2}{3} = 2$ is greater than 1, so eliminate (A).

Next, $\frac{x}{3y} = \frac{3}{3 \cdot 2} = \frac{1}{2}$, which is less than 1—it may be our answer. Next, $\sqrt{\frac{x}{y}} = \sqrt{\frac{3}{2}} > 1$; eliminate (C).

Now, $\frac{y}{x} = \frac{2}{3} < 1$. So it too may be our answer. Next, $y = 2 > 1$; eliminate (E).

Hence, we must decide between answer-choices (B) and (D).

Let $x = 6$ and $y = 2$. Then $\frac{x}{3y} = \frac{6}{3 \cdot 2} = 1$, which eliminates (B).

Therefore, the answer is (D).

---

2. Working from the inner parentheses out, we get

$$((-\pi)^*)^* = (\pi - (-\pi))^* = (\pi + \pi)^* = (2\pi)^* = \pi - 2\pi = -\pi.$$

Hence, the answer is (C).

Method II: We can rewrite this problem using ordinary function notation. Replacing the odd symbol x^* with $f(x)$ gives $f(x) = \pi - x$. Now, the expression $((-\pi)^*)^*$ becomes the ordinary composite function

$$f(f(-\pi)) = f(\pi - (-\pi)) = f(\pi + \pi) = f(2\pi) = \pi - 2\pi = -\pi$$

---

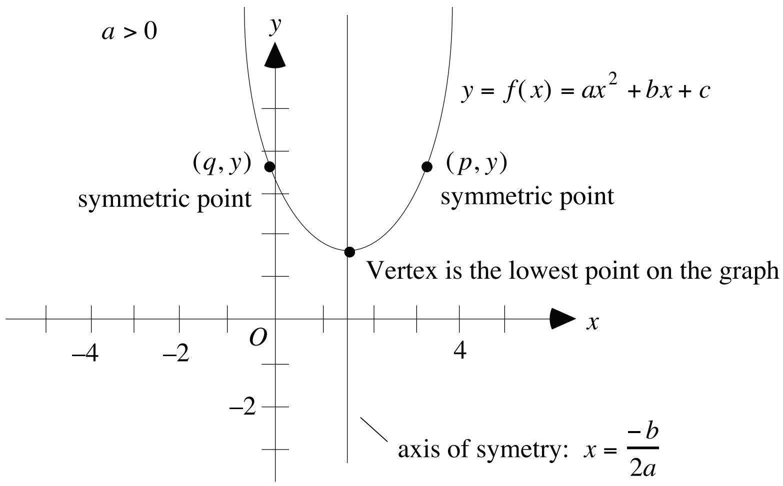
3. Since the graph is symmetric about the x-axis, its base graph is $x = y^2$. Since the graph opens to the left, we know that the exterior of the base function is multiplied by negative one: $-y^2$. Since the graph is shifted one unit to the left, we know that one is subtracted from the exterior of the function: $x = -y^2 - 1$. The answer is (A).

Let's discuss some of the properties of quadratic functions.

QUADRATIC FUNCTIONS

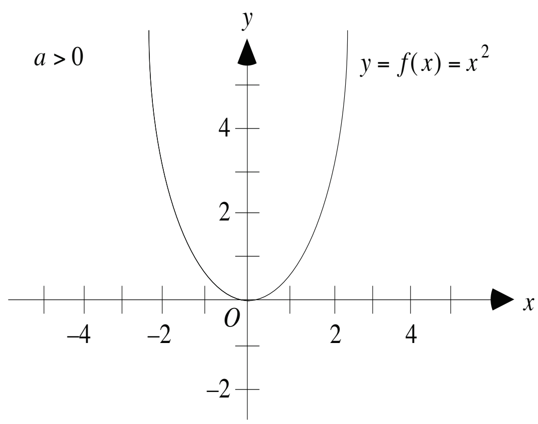
Quadratic functions (parabolas) have the following form:

$$y = f(x) = ax^2 + bx + c$$

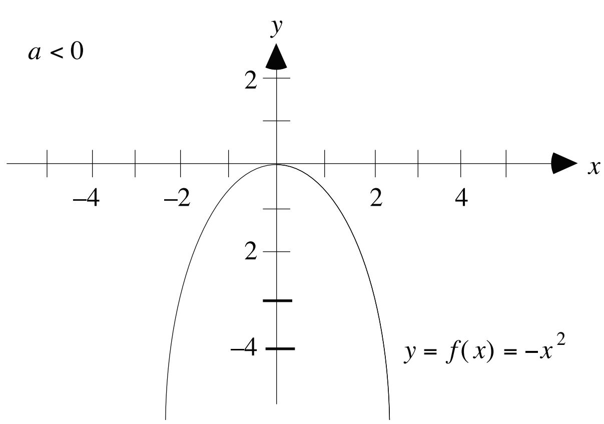


The lowest or highest point on a quadratic graph is called the vertex. The x–coordinate of the vertex occurs at $x = -b/2a$. This vertical line also forms the axis of symmetry of the graph, which means that if the graph were folded along its axis, the left and right sides of the graph would coincide.

In graphs of the form $y = f(x) = ax^2 + bx + c$ if $a > 0$, then the graph opens up.



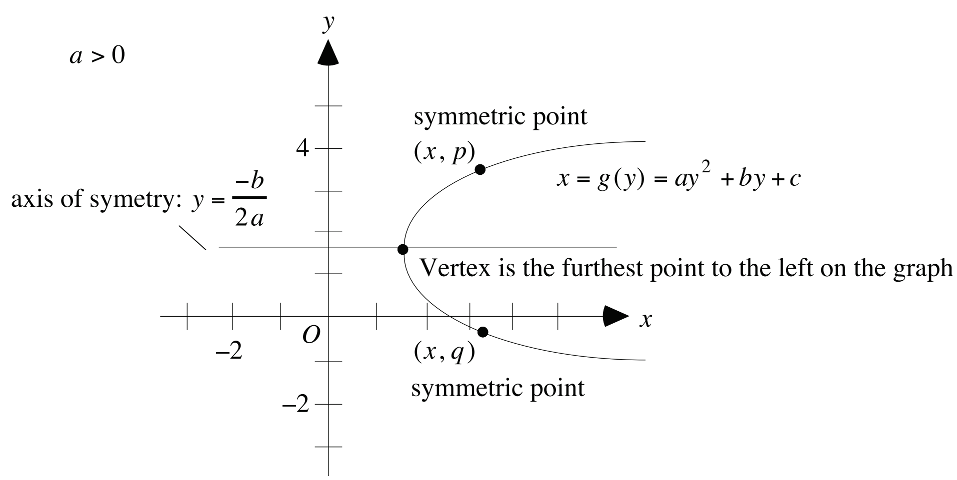
If $a < 0$, then the graph opens down.



By completing the square, the form $y = ax^2 + bx + c$ can be written as $y = a(x-h)^2 + k$. You are not expected to know this form on the test. But it is a convenient form since the vertex occurs at the point (h, k) and the axis of symmetry is the line $x = h$.

We have been analyzing quadratic functions that are vertically symmetric. Though not as common, quadratic functions can also be horizontally symmetric. They have the following form:

$$x = g(y) = ay^2 + by + c$$



The furthest point to the left on this graph is called the vertex. The y-coordinate of the vertex occurs at $y = -b/2a$. This horizontal line also forms the axis of symmetry of the graph, which means that if the graph were folded along its axis, the top and bottom parts of the graph would coincide.

In graphs of the form $x = ay^2 + by + c$ if $a > 0$, then the graph opens to the right and if $a < 0$ then the graph opens to the left.

---

4. Let x, $x + 1$, $x + 2$ stand for the consecutive integers a, b, and c, in that order. Plugging this into Statement I yields

$$b - c = (x + 1) - (x + 2) = -1$$

Hence, Statement I is false.

As to Statement II, since a, b, and c are three consecutive integers, one of them must be divisible by 3. Hence, $abc/3$ is an integer, and Statement II is true.

As to Statement III, suppose a is even, b is odd, and c is even. Then $a + b$ is odd since

$$even + odd = odd$$

Hence,

$$a + b + c = (a + \mathrm{b}) + c = (odd) + even = odd$$

Thus, Statement III is not necessarily true. The answer is (B).

Let's discuss some of the concepts from number theory.

- **Consecutive integers are written as $x, x + 1, x + 2, \ldots$**
- **Consecutive even or odd integers are written as $x, x + 2, x + 4, \ldots$**
- **The integer zero is neither positive nor negative, but it is even: $0 = 2 \times 0$.**
- **A *prime number* is an integer that is divisible only by itself and 1.**

  The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, . . .

- **A number is divisible by 3 if the sum of its digits is divisible by 3.**

  For example, 135 is divisible by 3 because the sum of its digits (1 + 3 + 5 = 9) is divisible by 3.

- **A *common multiple* is a multiple of two or more integers.**

  For example, some common multiples of 2 and 5 are 0, 10, 20, 40, and 50.

- **The *least common multiple* (LCM) of two integers is the smallest positive integer that is a multiple of both.**

  For example, the LCM of 4 and 10 is 20. The standard method of calculating the LCM is to prime factor the numbers and then form a product by selecting each factor the greatest number of times it occurs. For 4 and 10, we get

  $4 = 2^2$
  $10 = 2 \bullet 5$

  In this case, select 2^2 instead of 2 because it has the greater number of factors of 2, and select 5 by default since there are no other factors of 5. Hence, the LCM is $2^2 \bullet 5 = 4 \bullet 5 = 20$.

  For another example, let's find the LCM of 8, 36, and 54. Prime factoring yields

  $8 = 2^3$
  $36 = 2^2 \bullet 3^2$
  $54 = 2 \bullet 3^3$

  In this case, select 2^3 because it has more factors of 2 than 2^2 or 2 itself, and select 3^3 because is has more factors of 3 than 3^2 does. Hence, the LCM is $2^3 \bullet 3^3 = 8 \bullet 27 = 216$.

  A shortcut for finding the LCM is to just keep adding the largest number to itself until the other numbers divide into it evenly. For 4 and 10, we would add 10 to itself: 10 + 10 = 20. Since 4 divides evenly in 20, the LCM is 20. For 8, 36, and 54, we would add 54 to itself: 54 + 54 + 54 + 54 = 216. Since both 8 and 36 divide evenly into 216, the LCM is 216.

- **The absolute value of a number, $| \ |$, is always positive. In other words, the absolute value symbol eliminates negative signs.**

  For example, $|-7| = 7$ and $|-\pi| = \pi$. Caution, the absolute value symbol acts only on what is inside the symbol, $| \ |$. For example, $-|-(7 - \pi)| = -(7 - \pi)$. Here, only the negative sign inside the absolute value symbol but outside the parentheses is eliminated.

- **The product (quotient) of positive numbers is positive.**
- **The product (quotient) of a positive number and a negative number is negative.**

  For example, $-5(3) = -15$ and $\frac{6}{-3} = -2$.

- **The product (quotient) of an even number of negative numbers is positive.**

For example, $(-5)(-3)(-2)(-1) = 30$ is positive because there is an even number, 4, of positives.
$\dfrac{-9}{-2} = \dfrac{9}{2}$ is positive because there is an even number, 2, of positives.

- **The product (quotient) of an odd number of negative numbers is negative.**

  For example, $(-2)(-\pi)(-\sqrt{3}) = -2\pi\sqrt{3}$ is negative because there is an odd number, 3, of negatives.
  $\dfrac{(-2)(-9)(-6)}{(-12)\left(-18/2\right)} = -1$ is negative because there is an odd number, 5, of negatives.

- **The sum of negative numbers is negative.**

  For example, $-3 - 5 = -8$. Some students have trouble recognizing this structure as a sum because there is no plus symbol, +. But recall that subtraction is defined as negative addition. So $-3 - 5 = -3 + (-5)$.

- **A number raised to an even exponent is greater than or equal to zero.**

  For example, $(-\pi)^4 = \pi^4 \geq 0$, and $x^2 \geq 0$, and $0^2 = 0 \cdot 0 = 0 \geq 0$.

---

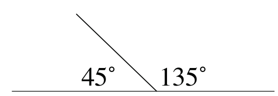
5. Since $4x$ and $2y - 40$ represent vertical angles, $4x = 2y - 40$. Since $3x$ and y form a straight angle, $3x + y = 180$. This yields the following system:

$$4x = 2y - 40$$
$$3x + y = 180$$

Solving this system for y yields $y = 84$. Hence, the answer is (B).

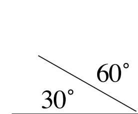
Here are some more geometric properties that you should know.

Two angles are supplementary if their angle sum is 180°:



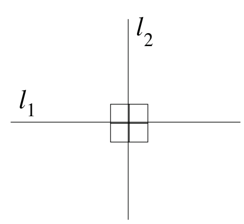
45 + 135 = 180

Two angles are complementary if their angle sum is 90°:



30 + 60 = 90

Perpendicular lines meet at right angles:

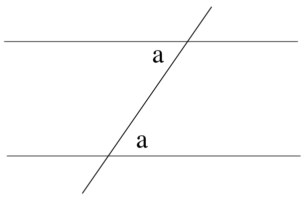


$l_1 \perp l_2$

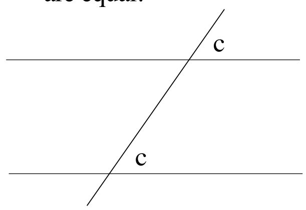
Two lines in the same plane are parallel if they never intersect. Parallel lines have the same slope.

When parallel lines are cut by a transversal, three important angle relationships exist:

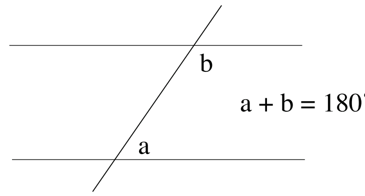
Alternate interior angles are equal.



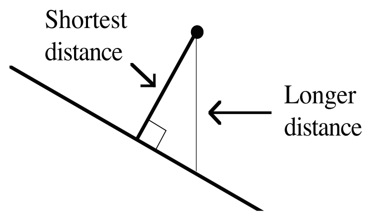
Corresponding angles are equal.



Interior angles on the same side of the transversal are supplementary.



The shortest distance from a point to a line is along a new line that passes through the point and is perpendicular to the original line.



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6. We'll use the transitive property to solve this problem:

Transitive Property

If $x < y$ and $y < z$, then $x < z$

Since $1/Q > 1$ and $1 > 0$, we know from the transitive property that $1/Q$ is positive. Hence, Q is positive. Therefore, we can multiply both sides of $1/Q > 1$ by Q without reversing the inequality:

$$Q \cdot \frac{1}{Q} > 1 \cdot Q$$

Reducing yields

$$1 > Q$$

Multiplying both sides again by Q yields

$$Q > Q^2$$

Using the transitive property to combine the last two inequalities yields

$$1 > Q^2$$

The answer is (C).

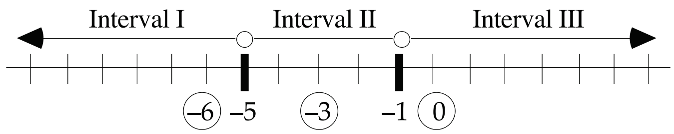
Higher Order Inequalities

These inequalities have variables whose exponents are greater than 1. For example, $x^2 + 4 < 2$ and $x^3 - 9 > 0$. The number line is often helpful in solving these types of inequalities.

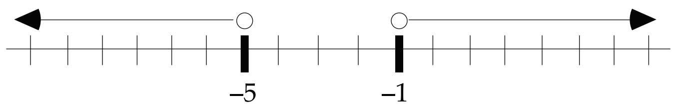
Example: For which values of x is $x^2 > -6x - 5$?

First, replace the inequality symbol with an equal symbol:	$x^2 = -6x - 5$
Adding $6x$ and 5 to both sides yields	$x^2 + 6x + 5 = 0$
Factoring yields (see General Trinomials in the chapter Factoring)	$(x + 5)(x + 1) = 0$
Setting each factor to 0 yields	$x + 5 = 0$ and $x + 1 = 0$
Or	$x = -5$ and $x = -1$

Now, the only numbers at which the expression can change sign are –5 and –1. So –5 and –1 divide the number line into three intervals. Let's set up a number line and choose test points in each interval:



When $x = -6$, $x^2 > -6x - 5$ becomes $36 > 31$. This is true. Hence, all numbers in Interval I satisfy the inequality. That is, $x < -5$. When $x = -3$, $x^2 > -6x - 5$ becomes $9 > 13$. This is false. Hence, no numbers in Interval II satisfy the inequality. When $x = 0$, $x^2 > -6x - 5$ becomes $0 > -5$. This is true. Hence, all numbers in Interval III satisfy the inequality. That is, $x > -1$. The graph of the solution follows:



Note, if the original inequality had included the greater-than-or-equal symbol, $\geq$, the solution set would have included both –5 and –1. On the graph, this would have been indicated by filling in the circles above –5 and –1. The open circles indicate that –5 and –1 are not part of the solution.

Summary of steps for solving higher order inequalities:

1. Replace the inequality symbol with an equal symbol.
2. Move all terms to one side of the equation (usually the left side).
3. Factor the equation.
4. Set the factors equal to 0 to find zeros.
5. Choose test points on either side of the zeros.
6. If a test point satisfies the original inequality, then all numbers in that interval satisfy the inequality. Similarly, if a test point does not satisfy the inequality, then no numbers in that interval satisfy the inequality.

7.

- **To solve a fractional equation, multiply both sides by the LCD (lowest common denominator) to clear fractions.**

First, multiply both sides of the equation by $x - 3$: $\quad (x-3)\frac{x+3}{x-3} = (x-3)y$

Cancel the $(x - 3)$'s on the left side of the equation: $\quad x + 3 = (x - 3)y$

Distribute the y: $\quad x + 3 = xy - 3y$

Subtract xy and 3 from both sides: $\quad x - xy = -3y - 3$

Factor out the x on the left side of the equation: $\quad x(1 - y) = -3y - 3$

Finally, divide both sides of the equation by $1 - y$: $\quad x = \frac{-3y-3}{1-y}$

Hence, the answer is (D).

8.

- **Often on the SAT, you can solve a system of two equations in two unknowns by merely adding or subtracting the equations—instead of solving for one of the variables and then substituting it into the other equation.**

Solution: Subtract the second equation from the first:

$$\begin{array}{rl} & p^2 + q^2 = 16 \\ (-) & \underline{p^2 - q^2 = 8} \\ & 2q^2 = 8 \end{array}$$

Dividing both sides of the equation by 2 gives $q^2 = 4$

Finally, taking the square root of both sides gives $q = \pm 2$

Hence, the answer is (A).

<u>METHOD OF SUBSTITUTION</u> (Four-Step Method)

Although on the SAT you can usually solve a system of two equations in two unknowns by merely adding or subtracting the equations, you still need to know a standard method for solving these types of systems.

The four-step method will be illustrated with the following system:

$$2x + y = 10$$
$$5x - 2y = 7$$

1) *Solve one of the equations for one of the variables*:

   Solving the top equation for y yields $y = 10 - 2x$.

2) *Substitute the result from Step 1 into the other equation*:

   Substituting $y = 10 - 2x$ into the bottom equation yields $5x - 2(10 - 2x) = 7$.

3) *Solve the resulting equation*:

$$5x - 2(10 - 2x) = 7$$
$$5x - 20 + 4x = 7$$
$$9x - 20 = 7$$
$$9x = 27$$
$$x = 3$$

4) *Substitute the result from Step 3 into the equation derived in Step 1*:

   Substituting $x = 3$ into $y = 10 - 2x$ yields $y = 10 - 2(3) = 10 - 6 = 4$.

Hence, the solution of the system of equations is the ordered pair (3, 4).

9. Let the five numbers be a, b, c, d, e. Then their average is $\frac{a+b+c+d+e}{5} = -10$. Now three of the numbers have a sum of 16, say, $a + b + c = 16$. So substitute 16 for $a + b + c$ in the average above: $\frac{16+d+e}{5} = -10$. Solving this equation for $d + e$ gives $d + e = -66$. Finally, dividing by 2 (to form the average) gives $\frac{d+e}{2} = -33$. Hence, the answer is (A).

10. In many word problems, as one quantity increases (decreases), another quantity also increases (decreases). This type of problem can be solved by setting up a *direct* proportion.

Solution: As time increases so does the number of shaped surfboards. Hence, we set up a direct proportion. First, convert 5 hours into minutes: 5 *hours* = 5 × 60 *minutes* = 300 *minutes*. Next, let x be the number of surfboards shaped in 5 hours. Finally, forming the proportion yields

$$\frac{3}{50} = \frac{x}{300}$$

$$\frac{3 \cdot 300}{50} = x$$

$$18 = x$$

The answer is (C).

11. First, factor the top of the fraction:

$$\frac{(2 \cdot 3)^4}{3^2}$$

Next, apply the rule $(xy)^a = x^a \cdot y^a$:

$$\frac{2^4 \cdot 3^4}{3^2}$$

Finally, apply the rule $\frac{x^a}{x^b} = x^{a-b}$:

$$2^4 \cdot 3^2$$

Hence, the answer is (D).

12. $y^3 = -8$ yields one cube root, $y = -2$. However, $x^2 = 4$ yields two square roots, $x = \pm 2$. Now, if $x = 2$, then $x > y$; but if $x = -2$, then $x = y$. Hence, choice (D) is not necessarily true. The answer is (D).

Let's discuss some of the properties of roots.

ROOTS

The symbol $\sqrt[n]{b}$ is read the nth root of b, where n is called the index, b is called the base, and $\sqrt{\ }$ is called the radical. $\sqrt[n]{b}$ denotes that number which raised to the nth power yields b. In other words, a is the nth root of b if $a^n = b$. For example, $\sqrt{9} = 3$* because $3^2 = 9$, and $\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$. Even roots occur in pairs: both a positive root and a negative root. For example, $\sqrt[4]{16} = 2$ since $2^4 = 16$, and $\sqrt[4]{16} = -2$ since $(-2)^4 = 16$. Odd roots occur alone and have the same sign as the base: $\sqrt[3]{-27} = -3$ since $(-3)^3 = -27$. If given an even root, you are to assume it is the positive root. However, if you introduce even roots by solving an equation, then you <u>must</u> consider both the positive and negative roots:

$$x^2 = 9$$

$$\sqrt{x^2} = \pm\sqrt{9}$$

$$x = \pm 3$$

* With square roots, the index is not written, $\sqrt[2]{9} = \sqrt{9}$.

Square roots and cube roots can be simplified by removing perfect squares and perfect cubes, respectively. For example,

$$\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$$
$$\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27}\sqrt[3]{2} = 3\sqrt[3]{2}$$

Radicals are often written with fractional exponents. The expression $\sqrt[n]{b}$ can be written as $b^{1/n}$. This can be generalized as follows:

$$b^{m/n} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$$

Usually, the form $\left(\sqrt[n]{b}\right)^m$ is better when calculating because the part under the radical is smaller in this case. For example, $27^{2/3} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$. Using the form $\sqrt[n]{b^m}$ would be much harder in this case: $27^{2/3} = \sqrt[3]{27^2} = \sqrt[3]{729} = 9$. Most students know the value of $\sqrt[3]{27}$, but few know the value of $\sqrt[3]{729}$.

If n is even, then

$$\sqrt[n]{x^n} = |x|$$

For example, $\sqrt[4]{(-2)^4} = |-2| = 2$. With odd roots, the absolute value symbol is not needed. For example, $\sqrt[3]{(-2)^3} = \sqrt[3]{-8} = -2$.

To solve radical equations, just apply the rules of exponents to undo the radicals. For example, to solve the radical equation $x^{2/3} = 4$, we cube both sides to eliminate the cube root:

$$\left(x^{2/3}\right)^3 = 4^3$$
$$x^2 = 64$$
$$\sqrt{x^2} = \sqrt{64}$$
$$|x| = 8$$
$$x = \pm 8$$

Even roots of negative numbers do not appear on the SAT. For example, you will not see expressions of the form $\sqrt{-4}$; expressions of this type are called complex numbers.

The following rules are useful for manipulating roots:

$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$ For example, $\sqrt{3x} = \sqrt{3}\sqrt{x}$.

$\sqrt[n]{\dfrac{x}{y}} = \dfrac{\sqrt[n]{x}}{\sqrt[n]{y}}$ For example, $\sqrt[3]{\dfrac{x}{8}} = \dfrac{\sqrt[3]{x}}{\sqrt[3]{8}} = \dfrac{\sqrt[3]{x}}{2}$.

Caution: $\sqrt[n]{x+y} \ne \sqrt[n]{x} + \sqrt[n]{y}$. For example, $\sqrt{x+5} \ne \sqrt{x} + \sqrt{5}$. Also, $\sqrt{x^2+y^2} \ne x+y$. This common mistake occurs because it is similar to the following valid property: $\sqrt{(x+y)^2} = x+y$ (If $x + y$ can be negative, then it must be written with the absolute value symbol: $|x + y|$). Note, in the valid formula, it's the whole term, $x + y$, that is squared, not the individual x and y.

To add two roots, both the index and the base must be the same. For example, $\sqrt[3]{2}+\sqrt[4]{2}$ cannot be added because the indices are different, nor can $\sqrt{2}+\sqrt{3}$ be added because the bases are different. However, $\sqrt[3]{2}+\sqrt[3]{2}=2\sqrt[3]{2}$. In this case, the roots can be added because both the indices and bases are the same. Sometimes radicals with different bases can actually be added once they have been simplified to look alike. For example, $\sqrt{28}+\sqrt{7}=\sqrt{4\cdot 7}+\sqrt{7}=\sqrt{4}\sqrt{7}+\sqrt{7}=2\sqrt{7}+\sqrt{7}=3\sqrt{7}$.

You need to know the approximations of the following roots:

$$\sqrt{2}\approx 1.4 \qquad \sqrt{3}\approx 1.7 \qquad \sqrt{5}\approx 2.2$$

13.

DIFFERENCE OF SQUARES

One of the most important formulas on the SAT is the difference of squares:

$$\boxed{x^2-y^2=(x+y)(x-y)}$$

Caution: a sum of squares, x^2+y^2, does not factor.

Solution: In most algebraic expressions involving multiplication or division, you won't actually multiply or divide, rather you will factor and cancel, as in this problem.

$$\frac{8x^2-32}{4x+8}=$$

$$\frac{8(x^2-4)}{4(x+2)}=$$ by the distributive property $ax+ay=a(x+y)$

$$\frac{8(x+2)(x-2)}{4(x+2)}=$$ by the difference of squares $x^2-y^2=(x+y)(x-y)$

$$2(x-2)$$ by canceling common factors

The answer is (A).

---

14. Translate the sentence into a mathematical equation as follows:

What	percent	of	a	is	$3a$
↓	↓	↓	↓	↓	↓
x	$\frac{1}{100}$	$\cdot$	a	$=$	$3a$

$$\frac{x}{100}\cdot a=3a$$

$$\frac{x}{100}=3 \quad \text{(by canceling the } a\text{'s)}$$

$$x=300$$

The answer is (D).

15. Solving the formula $D = R \times T$ for T yields $T = \frac{D}{R}$. For the first half of the trip, this yields $T = 20/15 = 4/3$ hours. Since the entire trip takes 2 hours, the return trip takes 2 – 4/3 hours, or 2/3 hours. Now, the return trip is also 20 miles, so solving the formula $D = R \times T$ for R yields

$$R = \frac{D}{T} = \frac{20}{2/3} = 20 \cdot \frac{3}{2} = 30$$

The answer is (D).

16.

Arithmetic Progressions

An arithmetic progression is a sequence in which the difference between any two consecutive terms is the same. This is the same as saying: each term exceeds the previous term by a fixed amount. For example, 0, 6, 12, 18, . . . is an arithmetic progression in which the common difference is 6. The sequence 8, 4, 0, –4, . . . is arithmetic with a common difference of –4.

Solution: Since each number *"in the sequence is 4 less than the number immediately preceding it,"* the sixth term is 31 + 4 = 35; the fifth number in the sequence is 35 + 4 = 39; and the fourth number in the sequence is 39 + 4 = 43. The answer is (D). Following is the sequence written out:

55, 51, 47, 43, 39, 35, 31, 27, 23, 19, 15, 11, . . .

<u>Advanced concepts</u>: (Sequence Formulas)

Students with strong backgrounds in mathematics may prefer to solve sequence problems by using formulas. Note, none of the formulas in this section are necessary to answer questions about sequences on the SAT.

Since each term of an arithmetic progression *"exceeds the previous term by a fixed amount,"* we get the following:

first term	$a + 0d$	where a is the first term and d is the common difference
second term	$a + 1d$	
third term	$a + 2d$	
fourth term	$a + 3d$	
	. . .	
nth term	$a + (n - 1)d$	This formula generates the nth term

The sum of the first n terms of an arithmetic sequence is

$$\frac{n}{2}\left[2a + (n-1)d\right]$$

Answers and Solutions Section 1:

1.	**C**	**6.**	**A**	**11.**	**D**	**16.**	**C**
2.	**A**	**7.**	**A**	**12.**	**B**	**17.**	**B**
3.	**E**	**8.**	**D**	**13.**	**E**	**18.**	**C**
4.	**A**	**9.**	**E**	**14.**	**D**	**19.**	**C**
5.	**E**	**10.**	**E**	**15.**	**C**	**20.**	**A**

---

1. Let's solve this problem by substitution. Choose $n = 1$. Then $n/2 = 1/2$, which is not even—eliminate (A). Next, $4n + 3 = 4 \cdot 1 + 3 = 7$, which is not even—eliminate (B). Next, $2n = 2 \cdot 1 = 2$, which is even and may therefore be the answer. Next, both (D) and (E) equal 1, which is not even. Hence, the answer is (C).

---

2. Since $3u$ is odd, u is odd. (Proof: Suppose u were even, then $3u$ would be even as well. But we are given that $3u$ is odd. Hence, u must be odd.) Since $7 - v$ is odd, v must be even. (Proof: Suppose v were odd, then $7 - v$ would be even [the difference of two odd numbers is an even number]. But we are given that $7 - v$ is odd. Hence, v must be even.)

Since u is odd, the top part of the definition gives /u\ = 5. Since v is even, the bottom part of the definition gives /v\ = 10. Hence, /u\ – /v\ = 5 – 10 = –5. The answer is (A).

---

3. Evaluating the function $h(x) = \sqrt{x} + 2$ at v yields $h(v) = \sqrt{v} + 2$. Plugging this into the equation $3h(v) = 18$ yields

$$3\left(\sqrt{v} + 2\right) = 18$$

$$\sqrt{v} + 2 = 6 \qquad \text{by dividing both sides by 3}$$

$$\sqrt{v} = 4 \qquad \text{by subtracting 2 from both sides}$$

$$\left(\sqrt{v}\right)^2 = 4^2 \qquad \text{by squaring both sides}$$

$$v = 16 \qquad \text{since } \left(\sqrt{v}\right)^2 = v$$

Plugging $v = 16$ into $h\left(\frac{v}{4}\right)$ yields

$$h\left(\frac{v}{4}\right) = h\left(\frac{16}{4}\right) = h(4) = \sqrt{4} + 2 = 2 + 2 = 4$$

The answer is (E).

EVALUATION AND COMPOSITION OF FUNCTIONS

We have been using the function notation $f(x)$ intuitively; we also need to study what it actually means. You can think of the letter f in the function notation $f(x)$ as the name of the function. Instead of using the equation $y = x^3 - 1$ to describe the function, we can write $f(x) = x^3 - 1$. Here, f is the name of the function and $f(x)$ is the value of the function at x. So $f(2) = 2^3 - 1 = 8 - 1 = 7$ is the value of the function at 2. As you can see, this notation affords a convenient way of prompting the evaluation of a function for a particular value of x.

Any letter can be used as the independent variable in a function. So the previous function could be written $f(p) = p^3 - 1$. This indicates that the independent variable in a function is just a "placeholder." The function could be written without a variable as follows:

$$f(\ ) = (\ )^3 - 1$$

In this form, the function can be viewed as an input/output operation. If 2 is put into the function $f(2)$, then $2^3 - 1$ is returned.

In addition to plugging numbers into functions, we can plug expressions into functions. Plugging $y + 1$ into the function $f(x) = x^2 - x$ yields

$$f(y+1) = (y+1)^2 - (y+1)$$

You can also plug other expressions in terms of x into a function. Plugging $2x$ into the function $f(x) = x^2 - x$ yields

$$f(2x) = (2x)^2 - 2x$$

This evaluation can be troubling to students because the variable x in the function is being replaced by the same variable. But the x in function is just a placeholder. If the placeholder were removed from the function, the substitution would appear more natural. In $f(\ ) = (\ )^2 - (\ )$, we plug $2x$ into the left side $f(2x)$ and it returns the right side $(2x)^2 - 2x$.

COMPOSITION

We have plugged numbers into functions and expressions into functions; now let's plug in other functions. Since a function is identified with its expression, we have actually already done this. In the example above with $f(x) = x^2 - x$ and $2x$, let's call $2x$ by the name $g(x)$. In other words, $g(x) = 2x$. Then the composition of f with g (that is plugging g into f) is

$$f(g(x)) = f(2x) = (2x)^2 - 2x$$

You probably won't see the notation $f(g(x))$ on the test. But you probably will see one or more problems that ask you perform the substitution. For another example, let $f(x) = \frac{1}{x+1}$ and let $g(x) = x^2$. Then $f(g(x)) = \frac{1}{x^2+1}$ and $g(f(x)) = \left(\frac{1}{x+1}\right)^2$. Once you see that the composition of functions merely substitutes one function into another, these problems can become routine. Notice that the composition operation $f(g(x))$ is performed from the inner parentheses out, not from left to right. In the operation $f(g(2))$, the number 2 is first plugged into the function g and then that result is plugged in the function f.

A function can also be composed with itself. That is, substituted into itself. Let $f(x) = \sqrt{x} - 2$. Then $f(f(x)) = \sqrt{\sqrt{x} - 2} - 2$.

4.

- **Taking the square root of a fraction between 0 and 1 makes it larger.**

  Example: $\sqrt{\frac{1}{4}} = \frac{1}{2}$ and 1/2 is greater than 1/4.

  Caution: This is not true for fractions greater than 1. For example, $\sqrt{\frac{9}{4}} = \frac{3}{2}$. But $\frac{3}{2} < \frac{9}{4}$.

- **Squaring a fraction between 0 and 1 makes it smaller.**

  Example: $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ and 1/4 is less than 1/2.

Solution: Using this information, we know that squaring a fraction between 0 and 1 makes it smaller, and taking the square root of it makes it larger. Hence, Choice (A) is smaller than Choice (B). Choices (C), (D), (E) are all greater than one since $8/7 > 1$. The answer is (A).

5. Both 3 and 2 are prime, and $3 - 2 = 1$. This eliminates (A). Next, both 5 and 2 are prime, and $5 - 2 = 3$. This eliminates (B). Next, both 11 and 2 are prime, and $11 - 2 = 9$. This eliminates (C). Next, both 17 and 2 are prime, and $17 - 2 = 15$. This eliminates (D). Hence, by process of elimination, the answer is (E).

6.

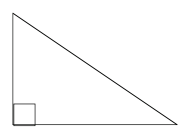
$x + 150 = 180$	since x and 150 form a straight angle
$x = 30$	solving for x
$z + x + 90 = 180$	since the angle sum of a triangle is 180°
$z + 30 + 90 = 180$	replacing x with 30
$z = 60$	solving for z
$z = y = 60$	since y and z are vertical angles
$w + y + 90 = 180$	since the angle sum of a triangle is 180°
$w + 60 + 90 = 180$	replacing y with 60
$w = 30$	solving for w

The answer is (A).

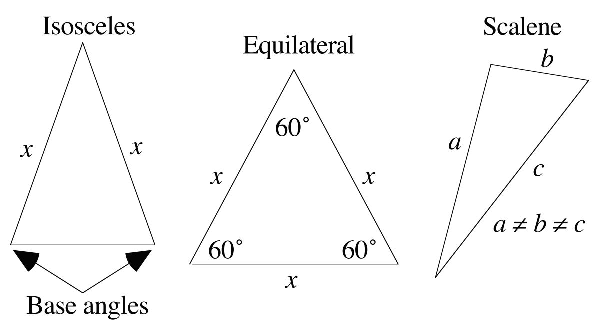
Here are some properties of triangles.

Triangles

A triangle containing a right angle is called a *right triangle*. The right angle is denoted by a small square:

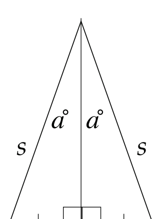


A triangle with two equal sides is called *isosceles*. The angles opposite the equal sides are called the base angles, and they are congruent (equal). A triangle with all three sides equal is called *equilateral*, and each angle is 60°. A triangle with no equal sides (and therefore no equal angles) is called *scalene*:

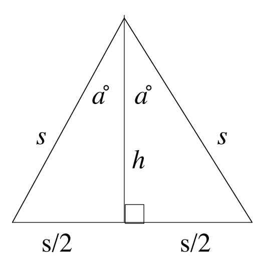


The altitude to the base of an isosceles or equilateral triangle bisects the base and bisects the vertex angle:

Isosceles:

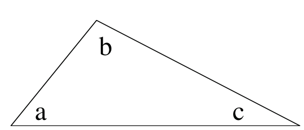


Equilateral:



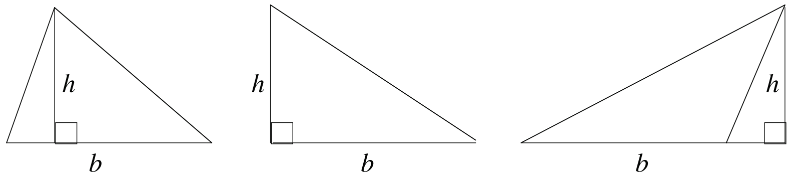
$$h = \frac{s\sqrt{3}}{2}$$

The angle sum of a triangle is 180°:



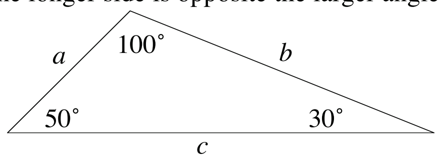
$$a + b + c = 180°$$

The area of a triangle is $\frac{1}{2}bh$, where b is the base and h is the height. Sometimes the base must be extended in order to draw the altitude, as in the third drawing directly below:



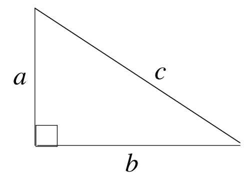
$$A = \frac{1}{2}bh$$

In a triangle, the longer side is opposite the larger angle, and vice versa:



50˚ is larger than 30˚, so side b is longer than side a.

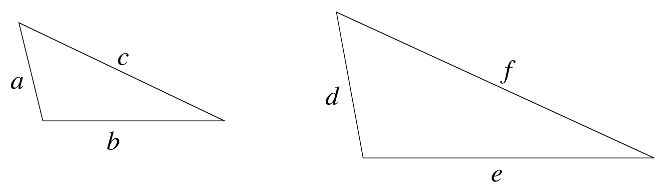
Pythagorean Theorem (right triangles only): The square of the hypotenuse is equal to the sum of the squares of the legs.



$$c^2 = a^2 + b^2$$

Pythagorean triples: The numbers 3, 4, and 5 can always represent the sides of a right triangle and they appear very often: $5^2 = 3^2 + 4^2$. Another, but less common, Pythagorean Triple is 5, 12, 13: $13^2 = 5^2 + 12^2$.

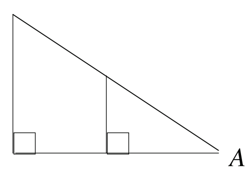
Two triangles are similar (same shape and usually different sizes) if their corresponding angles are equal. If two triangles are similar, their corresponding sides are proportional:



$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

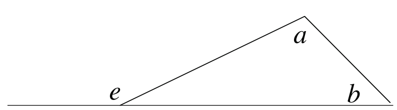
If two angles of a triangle are congruent to two angles of another triangle, the triangles are similar.

In the figure, the large and small triangles are similar because both contain a right angle and they share $\angle A$..



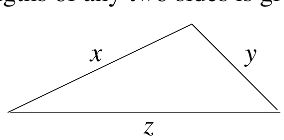
Two triangles are congruent (identical) if they have the same size and shape.

In a triangle, an exterior angle is equal to the sum of its remote interior angles and is therefore greater than either of them:



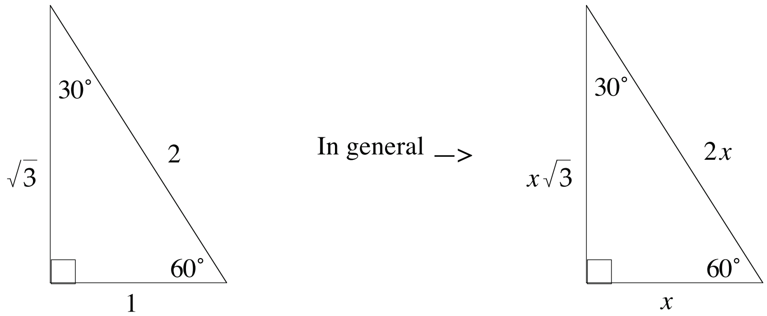
$e = a + b$ and $e > a$ and $e > b$

In a triangle, the sum of the lengths of any two sides is greater than the length of the remaining side:

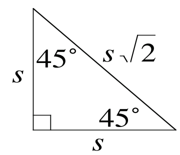


$$x + y > z$$
$$y + z > x$$
$$x + z > y$$

In a 30°–60°–90° triangle, the sides have the following relationships:



In a 45°–45°–90° triangle, the sides have the following relationships:



7.

Like Inequalities Can Be Added

If $x < y$ and $w < z$, then $x + w < y + z$

Solution: Multiplying both sides of $3 < y < 5$ by -1 yields $-3 > -y > -5$. Now, we usually write the smaller number on the left side of the inequality. So $-3 > -y > -5$ becomes $-5 < -y < -3$. Add this inequality to the like inequality $2 < x < 5$:

$$\begin{array}{rr} & 2 < x < 5 \\ (+) & -5 < -y < -3 \\ \hline & -3 < x - y < 2 \end{array}$$

The answer is (A).

8.

- **Complex Fractions: When dividing a fraction by a whole number (or vice versa), you must keep track of the main division bar:**

$$\frac{a}{b/c} = a \cdot \frac{c}{b} = \frac{ac}{b}.\ \text{But}\ \frac{a/b}{c} = \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{bc}.$$

Solution: $\dfrac{1-\frac{1}{2}}{3} = \dfrac{\frac{2}{2}-\frac{1}{2}}{3} = \dfrac{\frac{2-1}{2}}{3} = \dfrac{\frac{1}{2}}{3} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$. The answer is (D).

---

9. Dividing both sides of the equation $6a = 5b$ by 6 gives $a = \frac{5}{6}b$. Thus, a is a fraction of b. But b is greater than zero and therefore b is greater than a. (Note, had we been given that a was less than zero, then a would have been greater than b.) The answer is (E).

10. Here is the definition of *Average Speed*:

$$\textbf{\textit{Average Speed}} = \frac{\textbf{\textit{Total Distance}}}{\textbf{\textit{Total Time}}}$$

Although the definition for average speed is simple, few people solve these problems correctly because most fail to find both the total distance and the total time.

Solution: The total distance is $1 \cdot 50 + 3 \cdot 60 = 230$. And the total time is 4 hours. Hence,

$$Average\ Speed = \frac{Total\ Distance}{Total\ Time} = \frac{230}{4} = 57\,{}^1\!/_2$$

The answer is (E). Note, the answer is not the mere average of 50 and 60. Rather the average is closer to 60 because he traveled longer at 60 mph (3 hrs) than at 50 mph (1 hr).

11. As the distance on the map increases so does the actual distance. Hence, we set up a direct proportion. Let x be the actual distance between the cities. Forming the proportion yields

$$\frac{1\,in}{150\,mi} = \frac{3\,{}^1\!/_2\,in}{x\,mi}$$

$$x = 3\,{}^1\!/_2 \times 150$$

$$x = 525$$

The answer is (D).

Note, you need not worry about how you form the direct proportion so long as the order is the same on both sides of the equal sign. This proportion could have been written as

$$\frac{1\,in}{3\,{}^1\!/_2\,in} = \frac{150\,mi}{x\,mi}$$

In this case, the order is inches to inches and miles to miles. However, the following is not a direct proportion because the order is not the same on both sides of the equal sign:

$$\frac{1\,in}{150\,mi} = \frac{x\,mi}{3\,{}^1\!/_2\,in}$$

In this case, the order is inches to miles on the left side of the equal sign but miles to inches on the right side.

12. Translating the expression *"y is 5 more than the square of x"* into an equation yields:

$$y = x^2 + 5$$
$$y - 5 = x^2$$
$$\pm\sqrt{y-5} = x$$

Since we are given that $x < 0$, we take the negative root, $-\sqrt{y-5} = x$. The answer is (B).

RATIONALIZING

A fraction is not considered simplified until all the radicals have been removed from the denominator. If a denominator contains a single term with a square root, it can be rationalized by multiplying both the numerator and denominator by that square root. If the denominator contains square roots separated by a plus or minus sign, then multiply both the numerator and denominator by the conjugate, which is formed by merely changing the sign between the roots.

Example: Rationalize the fraction $\frac{2}{3\sqrt{5}}$.

Multiply top and bottom of the fraction by $\sqrt{5}$: $\frac{2}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{3 \cdot \sqrt{25}} = \frac{2\sqrt{5}}{3 \cdot 5} = \frac{2\sqrt{5}}{15}$

Example: Rationalize the fraction $\frac{2}{3-\sqrt{5}}$.

Multiply top and bottom of the fraction by the conjugate $3+\sqrt{5}$:

$$\frac{2}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2\left(3+\sqrt{5}\right)}{3^2 + 3\sqrt{5} - 3\sqrt{5} - \left(\sqrt{5}\right)^2} = \frac{2\left(3+\sqrt{5}\right)}{9-5} = \frac{2\left(3+\sqrt{5}\right)}{4} = \frac{3+\sqrt{5}}{2}$$

13.

PERFECT SQUARE TRINOMIALS

Like the difference of squares formula, perfect square trinomial formulas are very common on the SAT.

$$x^2 + 2xy + y^2 = (x+y)^2$$
$$x^2 - 2xy + y^2 = (x-y)^2$$

For example, $x^2 + 6x + 9 = x^2 + 2(3x) + 3^2 = (x+3)^2$. Note, in a perfect square trinomial, the middle term is twice the product of the square roots of the outer terms.

Solution:

$$r^2 - 2rs + s^2 = 4$$

$(r-s)^2 = 4$ by the formula $x^2 - 2xy + y^2 = (x-y)^2$

$\left[(r-s)^2\right]^3 = 4^3$ by cubing both sides of the equation

$(r-s)^6 = 64$ by the rule $\left(x^a\right)^b = x^{ab}$

The answer is (E).

14.

$\frac{x^2 - 2x + 1}{x - 1} = \frac{(x-1)(x-1)}{x-1} = x - 1$. The answer is (D).

DIVISION OF ALGEBRAIC EXPRESSIONS

When dividing algebraic expressions, the following formula is useful:

$$\frac{x+y}{z} = \frac{x}{z} + \frac{y}{z}$$

This formula generalizes to any number of terms.

Examples:

$$\frac{x^2 + y}{x} = \frac{x^2}{x} + \frac{y}{x} = x^{2-1} + \frac{y}{x} = x + \frac{y}{x}$$

$$\begin{aligned}\frac{x^2 + 2y - x^3}{x^2} &= \frac{x^2}{x^2} + \frac{2y}{x^2} - \frac{x^3}{x^2} \\ &= x^{2-2} + \frac{2y}{x^2} - x^{3-2} \\ &= x^0 + \frac{2y}{x^2} - x \\ &= 1 + \frac{2y}{x^2} - x\end{aligned}$$

When there is more than a single variable in the denomination, we usually factor the expression and then cancel, instead of using the above formula.

15. Translate the sentence into a mathematical equation as follows:

2	is	10	%	of	what number
↓	↓	↓	↓	↓	↓
2	=	10	$\frac{1}{100}$	·	x

$$2 = \frac{10}{100}x$$

$$2 = \frac{1}{10}x$$

$$20 = x$$

The answer is (C).

16. Remember, rarely does a graph question involve significant computation. For this question, we need merely to read the bar graph. The Total Profit graph shows that in 1993 approximately 680 thousand was earned, and in 1990 approximately 560 thousand was earned. Subtracting these numbers yields 680 – 560 = 120. The answer is (C).

17. The Total Revenue graph indicates that in 1990 the revenue from copying was about $2,600,000. The Total Profit graph shows the profit from copying in that same year was about $270,000. The profit margin is

$$\frac{\text{Profit}}{\text{Revenue}} = \frac{270{,}000}{2{,}600{,}000} \approx 10\%$$

The answer is (B).

18. From the chart, the profit in 1992 was approximately $700,000 of which 35% x $700,000 = $245,000 was from corporate customers and 20% x $700,000 = $140,000 was from government customers. Subtracting these amounts yields

$$\$245{,}000 - \$140{,}000 = \$105{,}000$$

The answer is (C).

19. The Total Profit graph shows that 1992 and 1993 are clearly the two years in which total profit was most nearly equal. Turning to the Total Revenue graph, we see that in 1992 the revenue from printing sales was approximately 2.5 million, and that in 1993 the revenue from printing sales was approximately 2 million. This gives a total of 4.5 million in total printing sales revenue for the period. The answer is (C).

20. The Total Profit graph shows that Zippy Printing earned about $340,000 from copying in 1992. The Pie Chart indicates that 20% of this was earned from government sales. Multiplying these numbers gives

$$\$340{,}000 \times 20\% \approx \$70{,}000$$

The answer is (A).

Answers and Solutions Section 2:

1. C	**6. C**	**11. 20**	**16. 285**
2. D	**7. B**	**12. –8**	**17. 42**
3. C	**8. C**	**13. –1/4**	**18. 1/4**
4. D	**9. 100/3**	**14. 6**	
5. B	**10. 6**	**15. 50**	

1.
Geometric Progressions

A geometric progression is a sequence in which the ratio of any two consecutive terms is the same. Thus, each term is generated by multiplying the preceding term by a fixed number. For example, –3, 6, –12, 24, . . . is a geometric progression in which the common ratio is –2. The sequence 32, 16, 8, 4, . . . is geometric with common ratio 1/2.

Solution: Since the common ratio between any two consecutive terms in this problem is –1/3, the fifth term is

$$\frac{10}{9} = \left(-\frac{1}{3}\right)\cdot\left(-\frac{10}{3}\right)$$

Hence, the sixth number in the sequence is $-\frac{10}{27} = \left(-\frac{1}{3}\right)\cdot\left(\frac{10}{9}\right)$. The answer is (C).

Advanced concepts: (Sequence Formulas)

Note, none of the formulas in this section are necessary to answer questions about sequences on the SAT.

Since each term of a geometric progression *"is generated by multiplying the preceding term by a fixed number,"* we get the following:

first term	a	
second term	ar^1	where r is the common ratio
third term	ar^2	
fourth term	ar^3	
	. . .	
nth term	$a_n = ar^{n-1}$	This formula generates the nth term

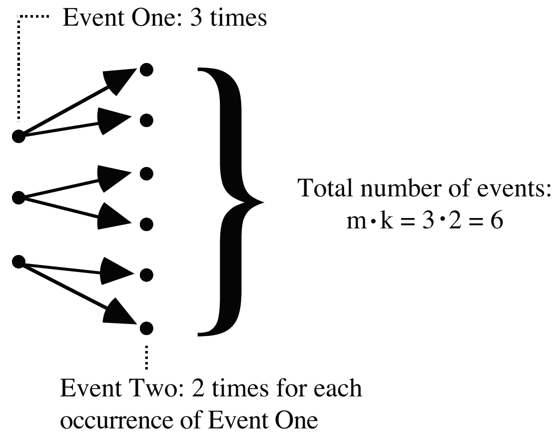
The sum of the first n terms of an geometric sequence is

$$\frac{a\left(1-r^n\right)}{1-r}$$

2.

- ***Fundamental Principle of Counting*: If an event occurs *m* times, and each of the *m* events is followed by a second event which occurs *k* times, then the second event follows the first event $m \cdot k$ times.**

The following diagram illustrates the fundamental principle of counting for an event that occurs 3 times with each occurrence being followed by a second event that occurs 2 times for a total of $3 \cdot 2 = 6$ events:



Solution: There is at most 5 jars each of which contains at most 40 marbles; so by the fundamental counting principle, there is at most $5 \cdot 40 = 200$ marbles in the drum. Since 10 percent of the marbles are flawed, there is at most $20 = 10\% \cdot 200$ flawed marbles. The answer is (D).

---

3. Randomly guessing either of the last two digits does not affect the choice of the other, which means that these events are independent and we are dealing with consecutive probabilities. Since each of the last two digits is greater than 5, Sarah has four digits to choose from: 6, 7, 8, 9. Her chance of guessing correctly on the first choice is 1/4, and on the second choice also 1/4. Her chance of guessing correctly on both choices is

$$\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

Since she gets three tries, the total probability is

$$\frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$$

The answer is (C).

---

4. Let's solve this problem by substitution. Choose $x = 4$ and $y = 9$. Then $x^2 = 4^2 = 16$, which is a perfect square. (Note, we cannot eliminate x^2 because it may not be a perfect square for another choice of x.) Next, $xy = 4 \cdot 9 = 36$ which is a perfect square. Next, $4x = 4 \cdot 4 = 16$, which is a perfect square. Next, $x + y = 4 + 9 = 13$, which is <u>not</u> a perfect square. Hence, the answer is (D).

5.

Substitution (Plugging In):
Sometimes instead of making up numbers to substitute into the problem, we can use the actual answer-choices. This is called "Plugging In." It is a very effective technique, but not as common as Substitution.

Solution: First, check to see which of the answer-choices has a sum of digits equal to 18.

For choice (A), $2 + 4 + 6 \neq 18$. Eliminate.

For choice (B), $3 + 6 + 9 = 18$. This may be the answer.

For choice (C), $5 + 3 + 1 \neq 18$. Eliminate.

For choice (D), $8 + 5 + 5 = 18$. This too may be the answer.

For choice (E), $8 + 9 + 3 \neq 18$. Eliminate.

Now, in choice (D), the ten's digit is <u>not</u> twice the hundred's digit, $5 \neq 2 \cdot 8$. Eliminate. Hence, by process of elimination, the answer is (B). Note that we did not need the fact that the hundred's digit is 1/3 the unit's digit.

6. Let's change the fractional notation to radical notation: $g(x) = \sqrt[4]{2x-3} + 1$. Since we have an even root, the expression under the radical must be greater than or equal to zero. Hence, $2x - 3 \geq 0$. Adding 3 to both sides of this inequality yields $2x \geq 3$. Dividing both sides by 2 yields $x \geq 3/2$. The answer is (C).

7.

- $\boldsymbol{ax^2 \neq (ax)^2}$. **In fact,** $\boldsymbol{a^2x^2 = (ax)^2}$.
  Example: $3 \cdot 2^2 = 3 \cdot 4 = 12$. But $(3 \cdot 2)^2 = 6^2 = 36$. This mistake is often seen in the following form: $-x^2 = (-x)^2$. To see more clearly why this is wrong, write $-x^2 = (-1)x^2$, which is negative. But $(-x)^2 = (-x)(-x) = x^2$, which is positive.

Solution: From the formula $a^2x^2 = (ax)^2$, we see that

$$(2x)^2 = 2^2 \cdot x^2 = 4x^2$$

Now, since $x \neq 0$, $4x^2$ is clearly larger than $2x^2$. Hence, the answer is (B).

8. Working from the innermost parentheses out, we get

$$-x = -|-(-2 + 5)|$$
$$-x = -|-(+3)|$$
$$-x = -|-3|$$
$$-x = -(+3)$$
$$-x = -3$$
$$x = 3$$

The answer is (C).

9.

- **Often you will need to find the percent of increase (or decrease). To find it, calculate the increase (or decrease) and divide it by the original amount:**

$$\textbf{Percent of change: } \frac{Amount\,of\ change}{Original\ amount} \times 100\%$$

Solution: The population increased from 12,000 to 16,000. Hence, the change in population was 4,000. Now, translate the main part of the sentence into a mathematical equation:

Percent of change:

$$\frac{Amount\,of\ change}{Original\ amount} \times 100\% =$$

$$\frac{4000}{12000} \times 100\% =$$

$$\frac{1}{3} \times 100\% = \qquad \text{(by canceling 4000)}$$

$$33\frac{1}{3}\%$$

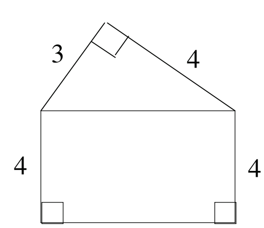
After converting this mixed fraction to an improper fraction, grid-in 100/3.

10. Let r be the rate of the slower jogger. Then the rate of the faster jogger is $r + 2$. Since they are jogging for 3 hours, the distance traveled by the slower jogger is $D = rt = 3r$, and the distance traveled by the faster jogger is $3(r + 2)$. Since they are 30 miles apart, adding the distances traveled gives

$$3r + 3(r + 2) = 30$$
$$3r + 3r + 6 = 30$$
$$6r + 6 = 30$$
$$6r = 24$$
$$r = 4$$

Hence, the rate of the faster jogger is $r + 2 = 4 + 2 = 6$. Grid-in 6.

11.
Add the following line to the figure:



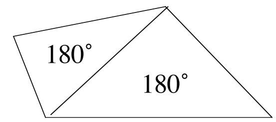
Since the legs of the right triangle formed are of lengths 3 and 4, the triangle must be a 3-4-5 right triangle. Hence, the added line has length 5. Since the bottom figure is a rectangle, the length of the base of the figure is also 5. Hence, the perimeter of the pentagon is 3 + 4 + 4 + 5 + 4 = 20. Grid-in 20.

Let's discuss some of the properties of quadrilaterals.

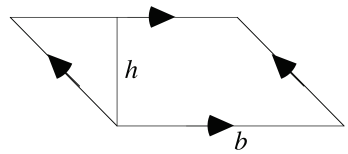
Quadrilaterals

A *quadrilateral* is a four-sided closed figure, where each side is a straight line.

The angle sum of a quadrilateral is 360°. You can view a quadrilateral as being composed of two 180-degree triangles:

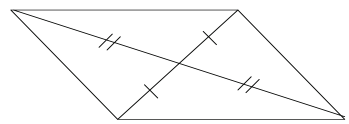


A *parallelogram* is a quadrilateral in which the opposite sides are both parallel and congruent. Its area is *base* × *height*:

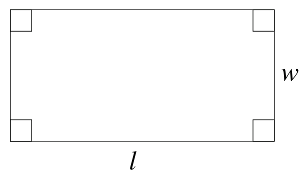


$A = bh$

The diagonals of a parallelogram bisect each other:



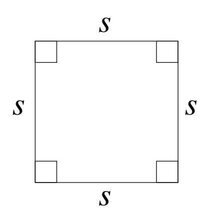
A parallelogram with four right angles is a *rectangle*. If w is the width and l is the length of a rectangle, then its area is $A = l \cdot w$ and its perimeter is $P = 2w + 2l$.



$A = l \cdot w$

$P = 2w + 2l$

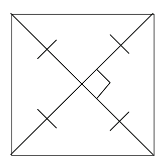
If the opposite sides of a rectangle are equal, it is a square and its area is $A = s^2$ and its perimeter is $P = 4s$, where s is the length of a side:



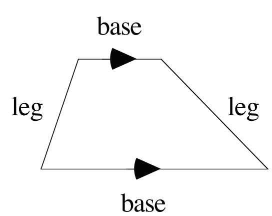
$A = s^2$

$P = 4s$

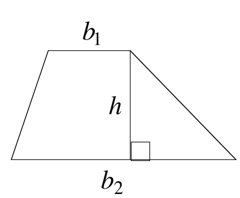
The diagonals of a square bisect each other and are perpendicular to each other:



A quadrilateral with only one pair of parallel sides is a *trapezoid*. The parallel sides are called *bases*, and the non-parallel sides are called *legs*:



The area of a trapezoid is the average of the two bases times the height:



$$A = \left(\frac{b_1 + b_2}{2}\right)h$$

12. Since y is less than 9 and $y = -2x - 8$, we get

$$-2x - 8 < 9$$

Adding 8 to both sides of this inequality yields

$$-2x < 17$$

Dividing by –2 and reversing the inequality yields

$$x > -17/2 = -8.5$$

Since x is an integer and is to be as small as possible,

$$x = -8$$

Grid in –8.

13.

- **Two fractions can be added quickly by cross-multiplying:** $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$

Solution: Cross multiplying the expression $\frac{1}{2} - \frac{3}{4}$ yields $\frac{1 \cdot 4 - 2 \cdot 3}{2 \cdot 4} = \frac{4-6}{8} = \frac{-2}{8} = -\frac{1}{4}$. Grid in –1/4.

14. Adding the two equations $\begin{matrix} p - q + r = 4 \\ p + q + r = 8 \end{matrix}$ gives

$$2p + 2r = 12$$

Then dividing by 2 gives

$$p + r = 6$$

Hence, grid in 6.

15.

MIXTURE PROBLEMS

The key to these problems is that the combined total of the concentrations in the two parts must be the same as the whole mixture.

Solution: Let x be the ounces of the 30 percent solution. Then $30\%x$ is the amount of salt in that solution. The final solution will be $50 + x$ ounces, and its concentration of salt will be $20\%(50 + x)$. The original amount of salt in the solution is $10\% \cdot 50$. Now, the concentration of salt in the original solution plus the concentration of salt in the added solution must equal the concentration of salt in the resulting solution:

$$10\% \cdot 50 + 30\%x = 20\%(50 + x)$$

Multiply this equation by 100 to clear the percent symbol and then solving for x yields $x = 50$. Hence, grid in 50.

16.

SERIES

A series is simply the sum of the terms of a sequence. The following is a series of even numbers formed from the sequence 2, 4, 6, 8, . . . :

$$2 + 4 + 6 + 8 + \cdots$$

A term of a series is identified by its position in the series. In the above series, 2 is the first term, 4 is the second term, etc. The ellipsis symbol (. . .) indicates that the series continues forever.

Solution: We are given a formula for the sum of the squares of the first n positive integers. Plugging $n = 9$ into this formula yields

$$\frac{n(n+1)(2n+1)}{6} = \frac{9(9+1)(2\cdot 9+1)}{6} = \frac{9(10)(19)}{6} = 285$$

Hence, grid in 285.

---

17. Let D be the number of Democrats and let R be the number of Republicans. "One fifth of the legislators are neither Republican nor Democrat," so there are 200/5 = 40 legislators who are neither Republican nor Democrat. Hence, there are 200 – 40 = 160 Democrats and Republicans, or $D + R = 160$. Translating the clause "the number of Democrats is 50 less than 4 times the number of Republicans" into an equation yields $D = 4R - 50$. Plugging this into the equation $D + R = 160$ yields

$$4R - 50 + R = 160$$
$$5R - 50 = 160$$
$$5R = 210$$
$$R = 42$$

Hence, grid in 42

18.

Geometric Probability

In this type of problem, you will be given two figures, with one inside the other. You'll then be asked what is the probability that a randomly selected point will be in the smaller figure. These problems are solved with the following principle: $Probability = \frac{desired\ outcome}{possible\ outcomes}$.

Solution: Applying the probability principle, we get

$$Probability = \frac{\text{area of the small square}}{\text{area of the large square}} = \frac{2^2}{4^2} = \frac{4}{16} = \frac{1}{4}$$

Hence, grid in 1/4.

Let's discuss some of the concepts in probability theory.

PROBABILITY

We know what probability means, but what is its formal definition? Let's use our intuition to define it. If there is no chance that an event will occur, then its probability of occurring should be 0. On the other extreme, if an event is certain to occur, then its probability of occurring should be 100%, or 1. Hence, our *probability* should be a number between 0 and 1, inclusive. But, what kind of number? Suppose your favorite actor has a 1 in 3 chance of winning the Oscar for best actor. This can be measured by forming the fraction 1/3. Hence, a *probability* is a fraction where the top is the number of ways an event can occur and the bottom is the total number of possible events:

$$P = \frac{\text{Number of ways an event can occur}}{\text{Number of total possible events}}$$

Example: ***Flipping a coin***

What's the probability of getting heads when flipping a coin?

There is only one way to get heads in a coin toss. Hence, the top of the probability fraction is 1. There are two possible results: heads or tails. Forming the probability fraction gives 1/2.

Example: ***Tossing a die***

What's the probability of getting a 3 when tossing a die?

A die (a cube) has six faces, numbered 1 through 6. There is only one way to get a 3. Hence, the top of the fraction is 1. There are 6 possible results: 1, 2, 3, 4, 5, and 6. Forming the probability fraction gives 1/6.

Example: ***Drawing a card from a deck***

What's the probability of getting a king when drawing a card from a deck of cards?

A deck of cards has four kings, so there are 4 ways to get a king. Hence, the top of the fraction is 4. There are 52 total cards in a deck. Forming the probability fraction gives 4/52, which reduces to 1/13. Hence, there is 1 chance in 13 of getting a king.

Example: ***Drawing marbles from a bowl***

What's the probability of drawing a blue marble from a bowl containing 4 red marbles, 5 blue marbles, and 5 green marbles?

There are five ways of drawing a blue marble. Hence, the top of the fraction is 5. There are 14 (= 4 + 5 + 5) possible results. Forming the probability fraction gives 5/14.

Example: ***Drawing marbles from a bowl (second drawing)***

What's the probability of drawing a red marble from the same bowl, given that the first marble drawn was blue and was not placed back in the bowl?

There are four ways of drawing a red marble. Hence, the top of the fraction is 4. Since the blue marble from the first drawing was not replaced, there are only 4 blue marbles remaining. Hence, there are 13 (= 4 + 4 + 5) possible results. Forming the probability fraction gives 4/13.

<u>Consecutive Probabilities</u>

What's the probability of getting heads twice in a row when flipping a coin twice? Previously we calculated the probability for the first flip to be 1/2. Since the second flip is not affected by the first (these are called *independent* events), its probability is also 1/2. Forming the product yields the probability of two heads in a row:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

What's the probability of drawing a blue marble and then a red marble from a bowl containing 4 red marbles, 5 blue marbles, and 5 green marbles? (Assume that the marbles are not replaced after being selected.) As calculated before, there is a 5/14 likelihood of selecting a blue marble first and a 4/13 likelihood of selecting a red marble second. Forming the product yields the probability of a red marble immediately followed by a blue marble:

$$\frac{5}{14} \times \frac{4}{13} = \frac{20}{182} = \frac{10}{91}$$

These two examples can be generalized into the following rule for calculating consecutive probabilities:

To calculate consecutive probabilities, multiply the individual probabilities.

This rule applies to two, three, or any number of consecutive probabilities.

Either-Or **Probabilities**

What's the probability of getting either heads or tails when flipping a coin once? Since the only possible outcomes are heads or tails, we expect the probability to be 100%, or 1: $\frac{1}{2}+\frac{1}{2}=1$. Note that the events heads and tails are independent. That is, if heads occurs, then tails cannot (and vice versa).

What's the probability of drawing a red marble or a green marble from a bowl containing 4 red marbles, 5 blue marbles, and 5 green marbles? There are 4 red marbles out of 14 total marbles. So the probability of selecting a red marble is 4/14 = 2/7. Similarly, the probability of selecting a green marble is 5/14. So the probability of selecting a red or green marble is $\frac{2}{7}+\frac{5}{14}=\frac{9}{14}$. Note again that the events are independent. For instance, if a red marble is selected, then neither a blue marble nor a green marble is selected.

These two examples can be generalized into the following rule for calculating *either-or* probabilities:

To calculate *either-or* probabilities, add the individual probabilities (only if the events are independent).

The probabilities in the two immediately preceding examples can be calculated more naturally by adding up the events that occur and then dividing by the total number of possible events. For the coin example, we get 2 events (heads or tails) divided by the total number of possible events, 2 (heads and tails): 2/2 = 1. For the marble example, we get 9 (= 4 + 5) ways the event can occur divided by 14 (= 4 + 5 + 5) possible events: 9/14.

If it's more natural to calculate the *either-or* probabilities above by adding up the events that occur and then dividing by the total number of possible events, why did we introduce a second way of calculating the probabilities? Because in some cases, you may have to add the individual probabilities. For example, you may be given the individual probabilities of two independent events and be asked for the probability that either could occur. You now know to merely add their individual probabilities.

Answers and Solutions Section 3:

1.	**A**	**5.**	**A**	**9.**	**C**	**13.**	**B**
2.	**B**	**6.**	**B**	**10.**	**C**	**14.**	**E**
3.	**D**	**7.**	**B**	**11.**	**A**	**15.**	**E**
4.	**C**	**8.**	**E**	**12.**	**E**	**16.**	**C**

---

1. Let's solve this problem by substitution. Choose $x = 1$ and $y = 2$. Then $3x + y/2 = 3 \cdot 1 + 2/2 = 4$, which is even. The answer is (A). Note: We don't need to check the other answer-choices because the problem asked for the expression that *could be* even. Thus, the first answer-choice that turns out even is the answer.

---

2. Statement I is false. For instance, $1 \Diamond 2 = 1^2 = 1$, but $2 \Diamond 1 = 2^1 = 2$. This eliminates (A) and (D).

Statement II is true: $(-a)\Diamond(-a) = (-a)^{-a} = (-1 \cdot a)^{-a} = (-1)^{-a}(a)^{-a} = \dfrac{(-1)^{-a}}{a^a}$. This eliminates (C).

Unfortunately, we have to check Statement III. It is false: $(2\Diamond 2)\Diamond 3 = 2^2 \Diamond 3 = 4 \Diamond 3 = 4^3 = 64$ and $2\Diamond(2\Diamond 3) = 2 \Diamond 2^3 = 2 \Diamond 8 = 2^8 = 256$. This eliminates (E), and the answer is (B).

Note: The expression $a \cdot b \neq 0$ insures that neither a nor b equals 0: if $a \cdot b = 0$, then either $a = 0$ or $b = 0$, or both. This prevents division by zero from occurring in the problem, otherwise if $a = 0$ and $b = -1$, then $0 \Diamond (-1) = 0^{-1} = \dfrac{1}{0}$.

---

3. The figure shows that the graphs intersect at three points. At each of these points, both graphs have a height, or y-coordinate, of 1. The points are approximately (–.8, 1), (1.2, 1), and (4, 1). Hence, $f(x) = 1$ for three x values. The answer is (D).

---

4. The statement *"the remainder is 1 when m is divided by 2"* translates into

$$m = 2u + 1$$

The statement *"the remainder is 3 when n is divided by 4"* translates into

$$n = 4v + 3$$

Forming the sum of m and n gives

$$m + n = 2u + 1 + 4v + 3 = 2u + 4v + 4 = 2(u + 2v + 2)$$

Since we have written $m + n$ as a multiple of 2, it is even. The answer is (C).

Method II (Substitution)
Let $m = 3$ and $n = 7$. Then

$$3 = 2 \cdot 1 + 1$$

and

$$7 = 4 \cdot 1 + 3$$

Now, both 3 and 7 are odd, which eliminates (A) and (B). Further, $3 \cdot 7 = 21$ is odd, which eliminates (D). Finally, 3/7 is not an integer, which eliminates (E). Hence, by process of elimination, the answer is (C).

5. Let e be the length of an edge of the cube. Recall that the volume of a cube is e^3 and its surface area is $6e^2$. Since we are given that both the volume and the surface area are x, these expressions are equal:

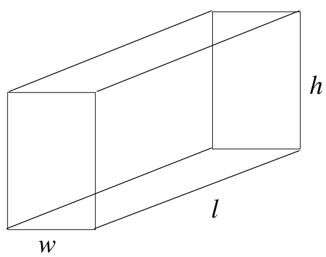
$$e^3 = 6e^2$$
$$e^3 - 6e^2 = 0$$
$$e^2(e - 6) = 0$$
$$e^2 = 0 \text{ or } e - 6 = 0$$
$$e = 0 \text{ or } e = 6$$

We reject $e = 0$ since in that case no cube would exist. Hence, $e = 6$ and the answer is (A).

Let's discuss some of the formulas for common geometric objects.

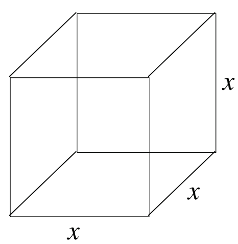
Volume

The volume of a rectangular solid (a box) is the product of the length, width, and height. The surface area is the sum of the area of the six faces:



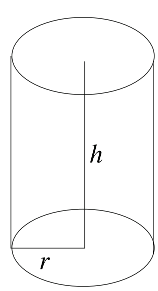
$$V = l \cdot w \cdot h$$
$$S = 2wl + 2hl + 2wh$$

If the length, width, and height of a rectangular solid (a box) are the same, it is a cube. Its volume is the cube of one of its sides, and its surface area is the sum of the areas of the six faces:



$$V = x^3$$
$$S = 6x^2$$

The volume of a cylinder is $V = \pi r^2 h$, and the lateral surface (excluding the top and bottom) is $S = 2\pi rh$, where r is the radius and h is the height:



$$V = \pi r^2 h$$
$$S = 2\pi rh + 2\pi r^2$$

6. From $1 < x < y$, we know that both x and y are positive. So dividing both sides of $x < y$ by x yields $1 < y/x$; and dividing both sides of $x < y$ by y yields $x/y < 1$. Hence, $\frac{x}{y} < 1 < \frac{y}{x}$. By the transitive property of inequalities, $\frac{x}{y} < \frac{y}{x}$. The answer is (B).

7. The average of x and $1/x$ is $\dfrac{x + \frac{1}{x}}{2} = \dfrac{\frac{x^2 + 1}{x}}{2} = \dfrac{x^2 + 1}{x} \cdot \dfrac{1}{2} = \dfrac{x^2 + 1}{2x}$. Thus, the answer is (B).

- **To add three or more fractions with different denominators, you need to form a common denominator of all the fractions.**

For example, to add the fractions in the expression $\frac{1}{3} + \frac{1}{4} + \frac{1}{18}$, we have to change the denominator of each fraction into the common denominator 36 (note, 36 is a common denominator because 3, 4, and 18 all divide into it evenly). This is done by multiply the top and bottom of each fraction by an appropriate number (this does not change the value of the expression because any number divided by itself equals 1):

$$\frac{1}{3}\left(\frac{12}{12}\right) + \frac{1}{4}\left(\frac{9}{9}\right) + \frac{1}{18}\left(\frac{2}{2}\right) = \frac{12}{36} + \frac{9}{36} + \frac{2}{36} = \frac{12 + 9 + 2}{36} = \frac{23}{36}$$

You may remember from algebra that to find a common denominator of a set of fractions, you prime factor the denominators and then select each factor the greatest number of times it occurs in any of the factorizations. That is too cumbersome, however. A better way is to simply add the largest denominator to itself until all the other denominators divide into it evenly. In the above example, we just add 18 to itself to get the common denominator 36.

- **To find a common denominator of a set of fractions, simply add the largest denominator to itself until all the other denominators divide into it evenly.**

- **Fractions often behave in unusual ways: Squaring a fraction makes it smaller, and taking the square root of a fraction makes it larger.** (**Caution:** This is true only for proper fractions, that is, fractions between 0 and 1.)

Example: $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ and 1/9 is less than 1/3. Also $\sqrt{\frac{1}{4}} = \frac{1}{2}$ and 1/2 is greater than 1/4.

- **You can cancel only over multiplication, not over addition or subtraction.**

For example, the c's in the expression $\frac{c + x}{c}$ cannot be canceled. However, the c's in the expression $\frac{cx + c}{c}$ can be canceled as follows: $\frac{cx + c}{c} = \frac{\not{c}(x + 1)}{\not{c}} = x + 1$.

8. Clearing fractions in the equation $y - 2 = \frac{y + 5}{2}$ gives $2(y - 2) = y + 5$

Distributing the 2 gives	$2y - 4 = y + 5$
Subtracting y and adding 4 to both sides gives	$y = 9$
Now, replacing y with 9 in the equation $x = y - 2$ gives	$x = y - 2 = 9 - 2 = 7$

Hence, the answer is (E).

9. Since the average of p and $4p$ is 10, we get

$$\frac{p+4p}{2}=10$$

Combining the p's gives

$$\frac{5p}{2}=10$$

Multiplying by 2 yields

$$5p = 20$$

Finally, dividing by 5 gives

$$p=4$$

The answer is (C).

---

10. If one quantity increases (or decreases) while another quantity decreases (or increases), the quantities are said to be *inversely* proportional. The statement "y is inversely proportional to x" is written as

$$y=\frac{k}{x}$$

where k is a constant.

Multiplying both sides of $y=\frac{k}{x}$ by x yields

$$yx=k$$

Hence, in an inverse proportion, the product of the two quantities is constant. Therefore, instead of setting ratios equal, we set products equal.

In many word problems, as one quantity increases (decreases), another quantity decreases (increases). This type of problem can be solved by setting up a product of terms.

Solution: As the number of workers increases, the amount of time required to assemble the car decreases. Hence, we set the products of the terms equal. Let x be the time it takes the 12 workers to assemble the car. Forming the equation yields

$$7\cdot 8=12\cdot x$$
$$\frac{56}{12}=x$$
$$4\tfrac{2}{3}=x$$

The answer is (C).

To summarize: if one quantity increases (decreases) as another quantity also increases (decreases), set ratios equal. If one quantity increases (decreases) as another quantity decreases (increases), set products equal.

11.

$$\left(\frac{2y^3}{x^2}\right)^4 \cdot x^{10} = \frac{\left(2y^3\right)^4}{\left(x^2\right)^4} \cdot x^{10} = \qquad \text{by the rule } \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

$$\frac{2^4 \cdot \left(y^3\right)^4}{\left(x^2\right)^4} \cdot x^{10} = \qquad \text{by the rule } (xy)^a = x^a \cdot y^a$$

$$\frac{2^4 \cdot y^{12}}{x^8} \cdot x^{10} = \qquad \text{by the rule } \left(x^a\right)^b = x^{ab}$$

$$2^4 \cdot y^{12} \cdot x^2 = \qquad \text{by the rule } \frac{x^a}{x^b} = x^{a-b}$$

$$16 \cdot y^{12} \cdot x^2$$

The answer is (A).

RATIONALIZING

A fraction is not considered simplified until all the radicals have been removed from the denominator. If a denominator contains a single term with a square root, it can be rationalized by multiplying both the numerator and denominator by that square root. If the denominator contains square roots separated by a plus or minus sign, then multiply both the numerator and denominator by the conjugate, which is formed by merely changing the sign between the roots.

Example: Rationalize the fraction $\frac{2}{3\sqrt{5}}$.

Multiply top and bottom of the fraction by $\sqrt{5}$:

$$\frac{2}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{3 \cdot \sqrt{25}} = \frac{2\sqrt{5}}{3 \cdot 5} = \frac{2\sqrt{5}}{15}$$

Example: Rationalize the fraction $\frac{2}{3-\sqrt{5}}$.

Multiply top and bottom of the fraction by the conjugate $3+\sqrt{5}$:

$$\frac{2}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2\left(3+\sqrt{5}\right)}{3^2 + 3\sqrt{5} - 3\sqrt{5} - \left(\sqrt{5}\right)^2} = \frac{2\left(3+\sqrt{5}\right)}{9-5} = \frac{2\left(3+\sqrt{5}\right)}{4} = \frac{3+\sqrt{5}}{2}$$

12.

GENERAL TRINOMIALS

$$\boldsymbol{x^2 + (a+b)x + ab = (x+a)(x+b)}$$

The expression $x^2 + (a + b)x + ab$ tells us that we need two numbers whose product is the last term and whose sum is the coefficient of the middle term. Consider the trinomial $x^2 + 5x + 6$. Now, two factors of 6 are 1 and 6, but $1 + 6 \neq 5$. However, 2 and 3 are also factors of 6, and $2 + 3 = 5$. Hence, $x^2 + 5x + 6 = (x + 2)(x + 3)$.

Solution: Now, both 2 and –9 are factors of 18, and $2 + (-9) = -7$. Hence, $x^2 - 7x - 18 = (x + 2)(x - 9) = 0$. Setting each factor equal to zero yields $x + 2 = 0$ and $x - 9 = 0$. Solving these equations yields $x = -2$ and 9. The answer is (E).

13. Solution:

$$\left(x^2+2\right)\left(x-x^3\right) = x^2x - x^2x^3 + 2x - 2x^3$$
$$= x^3 - x^5 + 2x - 2x^3$$
$$= -x^5 - x^3 + 2x$$

Thus, the answer is (B).

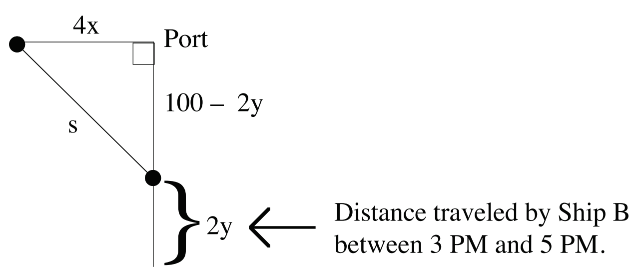
14. The total number of students in the class is 15 + 25 = 40. Now, translate the main part of the sentence into a mathematical equation:

what	percent	of	<u>the class</u>	is	<u>boys</u>
↓	↓	↓	↓	↓	↓
x	$\frac{1}{100}$	$\cdot$	40	=	15

$$\frac{40}{100}x = 15$$
$$\frac{2}{5}x = 15$$
$$2x = 75$$
$$x = 37.5$$

The answer is (E).

15. Since Ship A is traveling at x miles per hour, its distance traveled at 5 PM is $D = rt = 4x$. The distance traveled by Ship B is $D = rt = 2y$. This can be represented by the following diagram:



Applying the Pythagorean Theorem yields

$$s^2 = (4x)^2 + (100 - 2y)^2$$

Taking the square root of this equation gives

$$s = \sqrt{(4x)^2 + (100-2y)^2}$$

The answer is (E).

16. $<3> = 2(3) + (2 \cdot 3 - 1) + (2 \cdot 3 - 2) + (2 \cdot 3 - 3) + (2 \cdot 3 - 4) + (2 \cdot 3 - 5) = 6 + 5 + 4 + 3 + 2 + 1 = 21$

$<2> = 2(2) + (2 \cdot 2 - 1) + (2 \cdot 2 - 2) + (2 \cdot 2 - 3) = 4 + 3 + 2 + 1 = 10$

Hence, $<3> \cdot <2> = 21 \cdot 10 = 210$, and the answer is (C).

Answers and Solutions Section 1:

1.	**E**	**6.**	**B**	**11.**	**C**	**16.**	**C**
2.	**E**	**7.**	**A**	**12.**	**C**	**17.**	**A**
3.	**C**	**8.**	**E**	**13.**	**B**	**18.**	**A**
4.	**A**	**9.**	**A**	**14.**	**E**	**19.**	**D**
5.	**A**	**10.**	**D**	**15.**	**B**	**20.**	**A**

---

1. Let's use substitution to solve this problem. Choose $k = 1/4$. Then $\frac{3}{2}k = \frac{3}{2} \cdot \frac{1}{4} = \frac{3}{8} > \frac{1}{4}$; eliminate (A).

Next, $\frac{1}{k} = \frac{1}{1/4} = 4 > \frac{1}{4}$; eliminate (B).

Next, $|k| = \left|\frac{1}{4}\right| = \frac{1}{4}$; eliminate (C).

Next, $\sqrt{k} = \sqrt{\frac{1}{4}} = \frac{1}{2} > \frac{1}{4}$; eliminate (D).

Thus, by process of elimination, the answer is (E).

---

2. $(x @ y) @ z = \left(x^y\right) @ z = \left(x^y\right)^z$. Hence, the answer is (E). Note, though it might appear that choices (A) and (E) are equivalent, they are not. $\left(x^y\right)^z = x^{yz}$, which is not equal to x^{y^z}.

---

3. As you read the graph from left to right, it shows that sales initially increase rapidly and then slow to a maximum of about 300,000. From there, sales drop precipitously and then slowly approach zero as the price continues to increase. From the graph, sales of 300,000 units on the *y*-axis correspond to a price of about $10 on the *x*-axis. The answer is (C).

---

4. Begin by comparing 15/16 to each of the other answer-choices. Cross-multiplying 15/16 and 7/9 gives 135 vs. 112. Now, 135 is greater than 112, so 15/16 is greater than 7/9. Using this procedure to compare 15/16 to each of the remaining answer-choices shows that 15/16 is the greatest fraction listed. The answer is (A).

---

5. Since *x* and *y* are prime and greater than 2, *xy* is the product of two odd numbers and is therefore odd. Hence, 2 cannot be a divisor of *xy*. The answer is (A).

6. The circumference of the circle is $2\pi r = 2\pi(2) = 4\pi$. A central angle has by definition the same degree measure as its intercepted arc. Hence, arc *ACB* is also 60°. Now, the circumference of the circle has 360°. So arc *ACB* is $\frac{1}{6}$ (= 60/360) of the circle's circumference. Hence, arc $ACB = \frac{1}{6}(4\pi) = \frac{2}{3}\pi$. The answer is (B).

Let's discuss some of the properties of circles.

Circles

A circle is a set of points in a plane equidistant from a fixed point (the center of the circle). The perimeter of a circle is called the *circumference*.

A line segment from a circle to its center is a *radius*.

A line segment with both end points on a circle is a *chord*.

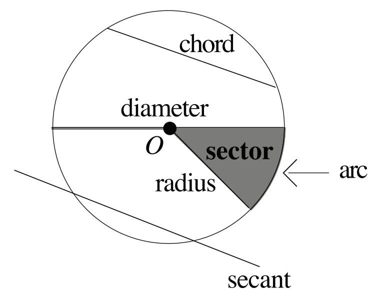
A chord passing though the center of a circle is a *diameter*.

A diameter can be viewed as two radii, and hence a diameter's length is twice that of a radius.

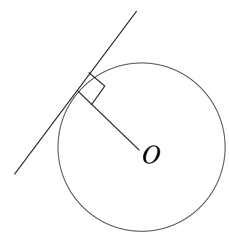
A line passing through two points on a circle is a *secant*.

A piece of the circumference is an *arc*.

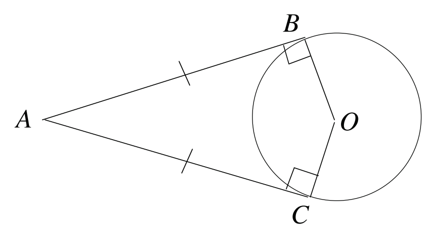
The area bounded by the circumference and an angle with vertex at the center of the circle is a *sector*.



A tangent line to a circle intersects the circle at only one point. The radius of the circle is perpendicular to the tangent line at the point of tangency:

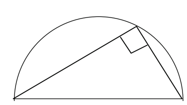


Two tangents to a circle from a common exterior point of the circle are congruent:

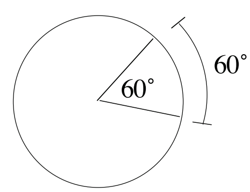


$AB \cong AC$

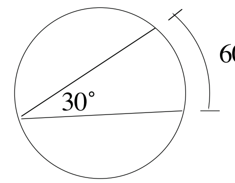
An angle inscribed in a semicircle is a right angle:



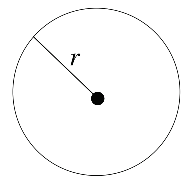
A central angle has by definition the same measure as its intercepted arc:



An inscribed angle has one-half the measure of its intercepted arc:



The area of a circle is πr^2, and its circumference (perimeter) is $2\pi r$, where r is the radius:



$$A = \pi r^2$$
$$C = 2\pi r$$

On the test, $\pi \approx 3$ is a sufficient approximation for π. You don't need $\pi \approx 3.14$.

---

7. Multiplying both sides of $3 < y < 7$ by -1 yields $-3 > -y > -7$. Now, we usually write the smaller number on the left side of an inequality. So $-3 > -y > -7$ becomes $-7 < -y < -3$. Add this inequality to the like inequality $-3 < x < -1$:

$$\begin{array}{rr} & -3 < x < -1 \\ (+) & -7 < -y < -3 \\ \hline & -10 < x - y < -4 \end{array}$$

Dividing $-10 < x - y < -4$ by 2 yields $\frac{-10}{2} < \frac{x-y}{2} < \frac{-4}{2}$, or $-5 < \frac{x-y}{2} < -2$. The answer is (A).

---

8. Recall that percent means to divide by 100. So .1 percent equals $\frac{.1}{100} = .001$. To convert 1/5 to a decimal, divide 5 into 1:

$$\begin{array}{r} .2 \\ 5\overline{)1.0} \\ \underline{10} \\ 0 \end{array}$$

In percent problems, "of" means multiplication. So multiplying .2 and .001 yields

$$\begin{array}{r} .001 \\ \times \quad .2 \\ \hline .0002 \end{array}$$

Hence, the answer is (E). Note, you may be surprised to learn that the SAT would consider this to be a hard problem.

Let's discuss some of the properties of decimals.

Decimals

If a fraction's denominator is a power of 10, it can be written in a special form called a *decimal fraction*. Some common decimals are $\frac{1}{10} = .1$, $\frac{2}{100} = .02$, $\frac{3}{1000} = .003$. Notice that the number of decimal places corresponds to the number of zeros in the denominator of the fraction. Also note that the value of the decimal place decreases to the right of the decimal point:

.1 — tenths
2 — hundredths
3 — thousandths
4 — ten-thousandths

This decimal can be written in expanded form as follows:

$$.1234 = \frac{1}{10} + \frac{2}{100} + \frac{3}{1000} + \frac{4}{10000}$$

Sometimes a zero is placed before the decimal point to prevent misreading the decimal as a whole number. The zero has no affect on the value of the decimal. For example, .2 = 0.2.

Fractions can be converted to decimals by dividing the denominator into the numerator. For example, to convert 5/8 to a decimal, divide 8 into 5 (note, a decimal point and as many zeros as necessary are added after the 5):

```
     .625
  8)5.000
    48
     20
     16
      40
      40
       0
```

The procedures for adding, subtracting, multiplying, and dividing decimals are the same as for whole numbers, except for a few small adjustments.

- **Adding and Subtracting Decimals:** To add or subtract decimals, merely align the decimal points and then add or subtract as you would with whole numbers.

```
   1.369       12.45
+  9.7       −  6.367
-------      --------
  11.069        6.083
```

- **Multiplying Decimals:** Multiply decimals as you would with whole numbers. The answer will have as many decimal places as the sum of the number of decimal places in the numbers being multiplied.

```
   1.23     2 decimal places
×  2.4      1 decimal place
------
   492
  246
------
  2.952     3 decimal places
```

- **Dividing Decimals:** Before dividing decimals, move the decimal point of the divisor all the way to the right and move the decimal point of the dividend the same number of spaces to the right (adding zeros if necessary). Then divide as you would with whole numbers.

```
                 2.5
.24).6  =  24)60.0
              48
              120
              120
                0
```

9. Replacing p with 3^{q+1} in the expression $\frac{p}{3^2}$ gives

$$\frac{p}{3^2} = \frac{3^{q+1}}{3^2} = 3^{q+1-2} = 3^{q-1}$$

Now, replacing q with $2r$ in the expression 3^{q-1} gives

$$3^{q-1} = 3^{2r-1}$$

Hence, the answer is (A).

---

10. We have six consecutive integers whose average is 9½, so we have the first three integers less than 9½ and the first three integers greater than 9½. That is, we are dealing with the numbers 7, 8, 9, 10, 11, 12. Clearly, the average of the last three numbers in this list is 11. Hence, the answer is (D).

---

11. The concept of proportion can be generalized to three or more ratios. A, B, and C are in the ratio 3:4:5 means $\frac{A}{B} = \frac{3}{4}$, $\frac{A}{C} = \frac{3}{5}$, and $\frac{B}{C} = \frac{4}{5}$.

Solution: Since the angle sum of a triangle is 180°, $A + B + C = 180$. Forming two of the ratios yields

$$\frac{A}{B} = \frac{5}{12} \qquad \frac{A}{C} = \frac{5}{13}$$

Solving the first equation for B yields

$$B = \frac{12}{5}A$$

Solving the second equation for C yields

$$C = \frac{13}{5}A$$

Hence, $180 = A + B + C = A + \frac{12}{5}A + \frac{13}{5}A = 6A$. Therefore, $180 = 6A$, or $A = 30$. The answer is choice (C).

12.

$$\begin{aligned}&\sqrt{(31-6)(16+9)} = \\ &\sqrt{25 \cdot 25} = \\ &\sqrt{25}\sqrt{25} = \\ &5 \cdot 5 = \\ &25\end{aligned}$$

The answer is (C).

---

13. First, interchanging 5 and $7x$ in the expression $3y + 5 = 7x$ yields $3y - 7x = -5$. Next, factoring $21y - 49x$ yields

$$\begin{aligned} 21y - 49x &= \\ 7 \cdot 3y - 7 \cdot 7x &= \\ 7(3y - 7x) &= \\ 7(-5) &= \qquad \text{since } 3y - 7x = -5 \\ -35 & \end{aligned}$$

The answer is (B).

14.

$$\begin{aligned} -2\left(3 - x\left[\frac{5 + y - 2}{x}\right] - 7 + 2 \cdot 3^2\right) &= \\ -2\left(3 - x\left[\frac{3 + y}{x}\right] - 7 + 2 \cdot 3^2\right) &= \\ -2\left(3 - [3 + y] - 7 + 2 \cdot 3^2\right) &= \\ -2\left(3 - 3 - y - 7 + 2 \cdot 3^2\right) &= \\ -2\left(3 - 3 - y - 7 + 2 \cdot 9\right) &= \\ -2(3 - 3 - y - 7 + 18) &= \\ -2(-y + 11) &= \\ 2y - 22 & \end{aligned}$$

The answer is (E).

---

15. Consider the first sentence: John spent $25, which is 15 percent of his monthly wage. Now, translate the main part of the sentence into a mathematical equation as follows:

25	is	15	%	of	<u>his monthly wage</u>
↓	↓	↓	↓	↓	↓
25	=	15	$\frac{1}{100}$	$\cdot$	x

$$25 = \frac{15}{100}x$$

$$2500 = 15x$$

$$x = \frac{2500}{15} = \frac{500}{3} = 166\frac{2}{3}$$

The answer is (B).

---

16. From the projected-crime graph, we see that the criminal population will be 20 million and of these 30 percent are projected to be involved in white-collar crime. Hence, the number of white-collar criminals is

(30%)(20 million) = (.30)(20 million) = 6 million

The answer is (C).

17. In 2010, there were 10 million criminals and 20% were robbers. Thus, the number of robbers in 2010 was

$$(20\%)(10 \text{ million}) = (.20)(10 \text{ million}) = 2 \text{ million}$$

In 2020, there are projected to be 20 million criminals of which 25% are projected to be robbers. Thus, the number of robbers in 2020 is projected to be

$$(25\%)(20 \text{ million}) = (.25)(20 \text{ million}) = 5 \text{ million}$$

Forming the ratio of the above numbers yields

$$\frac{\textit{number of robbers in } 2010}{\textit{number of robbers in } 2020} = \frac{2}{5}$$

The answer is (A).

18. The following table lists the number of criminals by category for 2010 and 2020 and the projected increase or decrease:

Category	**Number in 2010 (millions)**	**Number in 2020 (millions)**	**Projected increase (millions)**	**Projected decrease (millions)**
Vice	1.7	3	1.3	None
Assault	2	4	2	None
White Collar	3.8	6	2.2	None

As the table displays, there is a projected increase (not decrease) in all three categories. Hence, the answer is (A).

19. Remember, to calculate the percentage increase, find the absolute increase and divide it by the original number. Now, in 2010, the number of criminals in vice was 1.7 million, and in 2020 it is projected to be 3 million. The absolute increase is thus:

$$3 - 1.7 = 1.3$$

Hence the projected percent increase in the number of criminals in vice is

$$\frac{\textit{absolute increase}}{\textit{original number}} = \frac{1.3}{1.7} \approx 75\%.$$

The answer is (D).

20. In 2010, the number of white-collar criminals was (38%)(10 million) = 3.8 million. From the projected-crime graph, we see that the criminal population in the year 2020 will be 20 million and of these (25%)(20 million) = 5 million will be robbers. Hence, the projected number of Robbers in 2020 will exceed the number of white-collar criminals in 2010 by 5 – 3.8 = 1.2 million. The answer is (A).

Answers and Solutions Section 2:

1. E	**6. B**	**11. 16.9**	**16. 60**
2. A	**7. D**	**12. 3/8**	**17. 0**
3. E	**8. D**	**13. 16**	**18. 8**
4. A	**9. 3/2**	**14. 7**	
5. E	**10. 20**	**15. 2420**	

---

1. Without substitution, this is a hard problem. With substitution, it's quite easy. Suppose you begin reading on page 1 and stop on page 2. Then you will have read 2 pages. Now, merely substitute $h = 1$ and $k = 2$ into the answer-choices to see which one(s) equal 2. Only $k - h + 1 = 2 - 1 + 1 = 2$ does. (Verify this.) The answer is (E).

---

2. Setting $x \# y$ equal to $-x$ yields

$$(xy)^2 - x + y^2 = -x$$

Canceling $-x$ from both sides of the equation yields

$$(xy)^2 + y^2 = 0$$

Expanding the first term yields

$$x^2y^2 + y^2 = 0$$

Factoring out y^2 yields

$$y^2(x^2 + 1) = 0$$

Setting each factor equal to zero yields

$$y^2 = 0 \text{ or } x^2 + 1 = 0$$

Now, $x^2 + 1$ is greater than or equal to 1 (why?). Hence,

$$y^2 = 0$$

Taking the square root of both sides of this equation yields

$$y = 0$$

Hence, the answer is (A).

---

3. We need to plug the x table values into each given function to find the one that returns the function values in the bottom row of the table. Let's start with $x = 0$ since zero is the easiest number to calculate with. According to the table $f(0) = 3$. This eliminates Choice (A) since $f(0) = -2(0)^2 = -2(0) = 0$; and it eliminates Choice (D) since $f(0) = -2(0)^2 - 3 = -2 \cdot 0 - 3 = 0 - 3 = -3$. Now, choose $x = 1$. The next easiest number to calculate with. According to the table $f(1) = 1$. This eliminates Choice (B) since $f(1) = 1^2 + 3 = 1 + 3 = 4$; and it eliminates Choice (C) since $f(1) = -(1)^2 + 3 = -1 + 3 = 2$. Hence, by process of elimination, the answer is (E).

---

4. $1 + \dfrac{1}{1 - \frac{1}{2}} = 1 + \dfrac{1}{1/2} = 1 + 2 = 3$. The answer is (A).

5. Since 2 divides evenly into x, we get $x = 2z$. Hence, $5x = 5(2z) = 10z$. In other words, $5x$ is divisible by 10. A similar analysis shows that $5y$ is also divisible by 10. Since 10 is the greatest number listed, the answer is (E).

6.

Shaded Regions

To find the area of the shaded region of a figure, subtract the area of the unshaded region from the area of the entire figure.

Solution: To find the area of the shaded region subtract the area of the circle from the area of the rectangle:

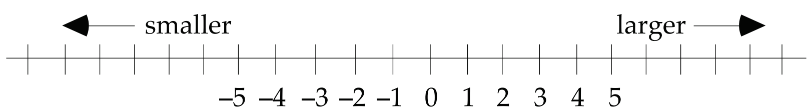
area of rectangle	–	area of circle
$3 \cdot 5$	–	$\pi \cdot 1^2$
15	–	$\boldsymbol{\pi}$

The answer is (B).

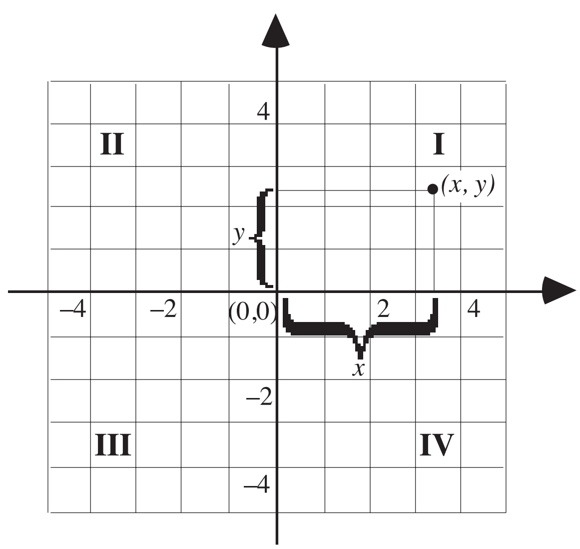
7. Since the y-coordinate of point B is 4, line segment CO has length 4. Since figure $ABCO$ is a square, line segment AO also has length 4. Since point B is in the second quadrant, the x-coordinate of B is –4. The answer is (D). Be careful not to choose 4. h is the x-coordinate of point B, not the length of the square's side.

Let's discuss some of the concepts from Coordinate Geometry.

On a number line, the numbers increase in size to the right and decrease to the left:



If we draw a line through the point 0 perpendicular to the number line, we will form a grid:

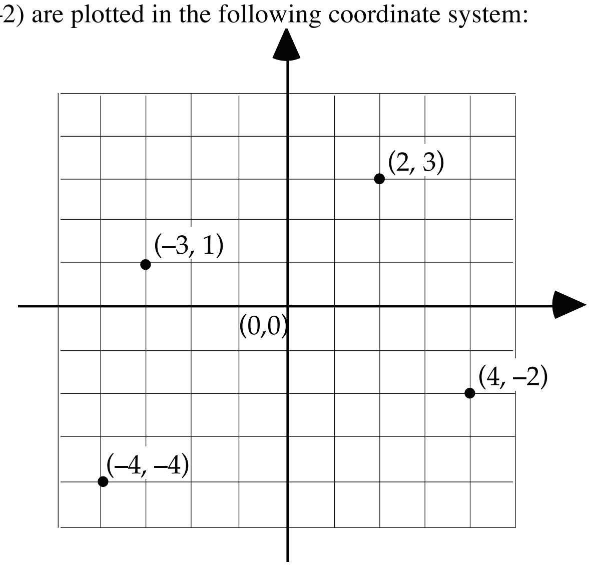


The thick horizontal line in the above diagram is called the x-axis, and the thick vertical line is called the y-axis. The point at which the axes meet, (0, 0), is called the origin. On the x-axis, positive numbers are to the right of the origin and increase in size to the right; further, negative numbers are to the left of the origin and decrease in size to the left. On the y-axis, positive numbers are above the origin and ascend in size; further, negative numbers are below the origin and descend in size.

As shown in the diagram, the point represented by the ordered pair (x, y) is reached by moving x units along the x-axis from the origin and then moving y units vertically. In the ordered pair (x, y), x is called the *abscissa* and y is called the *ordinate*; collectively they are called coordinates.

The x and y axes divide the plane into four quadrants, numbered I, II, III, and IV counterclockwise.

Note, if $x \neq y$, then (x, y) and (y, x) represent different points on the coordinate system. The points (2, 3), (–3, 1), (–4, –4), and (4, –2) are plotted in the following coordinate system:



8. Multiplying both sides of the inequality by –2 yields

$$-2(3 - 6x) \geq 4x + 2$$

Distributing the –2 yields

$$-6 + 12x \geq 4x + 2$$

Subtracting $4x$ and adding 6 to both sides yields

$$8x \geq 8$$

Dividing both sides of the inequality by 8 yields

$$x \geq 1$$

The answer is (D).

9. $\dfrac{2}{\frac{4}{3}} = 2 \cdot \dfrac{3}{4} = \dfrac{6}{4} = \dfrac{3}{2}$. Grid in 3/2.

10. Substituting $u = 18$ and $v = 2$ into the equation $\frac{u - v}{k} = 8$ gives $\frac{18 - 2}{k} = 8$

Subtracting gives $\frac{16}{k} = 8$

Multiplying both sides of this equation by k gives $16 = 8k$

Dividing by 8 gives $2 = k$

With this value for k, the original equation becomes $\frac{u - v}{2} = 8$

Now, we are asked to find u when $v = 4$.

Replacing v with 4 in the equation $\frac{u - v}{2} = 8$ gives $\frac{u - 4}{2} = 8$

Multiplying by 2 gives $u - 4 = 16$

Adding 4 gives $u = 20$

Grid in 20.

---

11. Forming the average of the five numbers gives

$$\frac{v + w + x + y + z}{5} = 6.9$$

Let the deleted number be z. Then forming the average of the remaining four numbers gives

$$\frac{v + w + x + y}{4} = 4.4$$

Multiplying both sides of this equation by 4 gives

$$v + w + x + y = 17.6$$

Plugging this value into the original average gives

$$\frac{17.6 + z}{5} = 6.9$$

Solving this equation for z gives

$$z = 16.9$$

Grid in 16.9.

---

12. First change all the units to inches: 2 ft. 3 in. = 27 in., and 2 yds. = 72 in. Forming the ratio yields

$$\frac{2\,ft.\,3\,in.}{2\,yds.} = \frac{27\,in.}{72\,in.} = \frac{3}{8}$$

Grid in 3/8.

13. Begin by completely factoring 20:

$$20^8 = (2 \cdot 2 \cdot 5)^8 =$$
$$2^8 \cdot 2^8 \cdot 5^8 = \quad \text{by the rule, } (xy)^a = x^a \cdot y^a\,*$$
$$2^{16} \cdot 5^8 \quad \text{by the rule, } x^a \cdot x^b = x^{a+b}$$

The expression 2^{16} represents all the factors of 20^8 of the form 2^n. Hence, 16 is the largest such number. Grid in 16.

14.

$$\frac{7x^2 + 28x + 28}{(x+2)^2} =$$
$$\frac{7\left(x^2 + 4x + 4\right)}{(x+2)^2} = \quad \text{by factoring out 7}$$
$$\frac{7(x+2)^2}{(x+2)^2} = \quad \text{by the formula } x^2 + 2xy + y^2 = (x+y)^2$$
$$7 \quad \text{by canceling the common factor } (x+2)^2$$

Grid in 7.

COMPLETE FACTORING

When factoring an expression, first check for a common factor, then check for a difference of squares, then for a perfect square trinomial, and then for a general trinomial.

Example: Factor the expression $2x^3 - 2x^2 - 12x$ completely.

Solution: First check for a common factor: $2x$ is common to each term. Factoring $2x$ out of each term yields $2x(x^2 - x - 6)$. Next, there is no difference of squares, and $x^2 - x - 6$ is not a perfect square trinomial since x does not equal twice the product of the square roots of x^2 and 6. Now, –3 and 2 are factors of –6 whose sum is –1. Hence, $2x(x^2 - x - 6)$ factors into $2x(x - 3)(x + 2)$.

15. Since the population increased at a rate of 10% per year, the population of any year is the population of the previous year + 10% of that same year. Hence, the population in 2011 is the population of 2010 + 10% of the population of 2010:

2000 + 10% of 2000 =

2000 + 200 =

2200

Similarly, the population in 2012 is the population of 2011 + 10% of the population of 2011:

2200 + 10% of 2200 =

2200 + 220 =

2420

Grid in 2420.

* Note, This rule can be extended to any number of terms by repeatedly applying the rule. For example, $(xyz)^a = ([xy]z)^a = [xy]^a \cdot z^a = x^a y^a z^a$.

16.

WORK PROBLEMS

The formula for work problems is *Work* = *Rate* × *Time*, or $W = R \times T$. The amount of work done is usually 1 unit. Hence, the formula becomes $1 = R \times T$. Solving this for R gives $R = \frac{1}{T}$.

Solution: Let $r = 1/t$ be Bobby's rate. Now, the rate at which they work together is merely the sum of their rates:

Total Rate = Johnny's Rate + Bobby's Rate

$$\frac{1}{20} = \frac{1}{30} + \frac{1}{t}$$
$$\frac{1}{20} - \frac{1}{30} = \frac{1}{t}$$
$$\frac{30 - 20}{30 \cdot 20} = \frac{1}{t}$$
$$\frac{1}{60} = \frac{1}{t}$$
$$t = 60$$

Hence, working alone, Bobby can do the job in 60 minutes. Grid in 60.

17. The sixth digit following the decimal point is the number zero: 0.47619**0**476190 . . . Since the digits repeat in blocks of six numbers, 0 will appear in the space for all multiples of six. Since 54 is a multiple of six, the 54th digit following the decimal point is 0. Grid in 0.

18. Since y is the middle number, it is the median. Forming the average of x, y, and z and setting it equal to 3 times the median yields

$$\frac{x + y + z}{3} = 3y$$

Replacing x with 0 and z with ky yields

$$\frac{0 + y + ky}{3} = 3y$$

Multiplying both sides of this equation by 3 yields

$$y + ky = 9y$$

Subtracting $9y$ from both sides yields

$$-8y + ky = 0$$

Factoring out y yields

$$y(-8 + k) = 0$$

Since $y \neq 0$ (why?), $-8 + k = 0$. Hence, $k = 8$. Grid in 8.

Answers and Solutions Section 3:

1. B	**5. B**	**9. A**	**13. A**
2. A	**6. B**	**10. C**	**14. C**
3. D	**7. D**	**11. E**	**15. A**
4. C	**8. D**	**12. E**	**16. E**

1. Begin with 0:

$$x^4 - 2x^2 = 0^4 - 2 \cdot 0^2 = 0 - 0 = 0$$

Hence, eliminate (A). Next, plug in 1:

$$x^4 - 2x^2 = 1^4 - 2 \cdot 1^2 = 1 - 2 = -1$$

Hence, the answer is (B).

2. "The ratio of 1/5 to 1/4 is equal to the ratio of 1/4 to x" means

$$\frac{1/5}{1/4} = \frac{1/4}{x}$$

or

$$\frac{1}{5} \cdot \frac{4}{1} = \frac{1}{4} \cdot \frac{1}{x}$$

This in turn reduces to

$$\frac{4}{5} = \frac{1}{4x}$$

Cross-multiplying yields

$$16x = 5$$

dividing by 16 gives

$$x = 5/16$$

The answer is (A).

3. Since the question asks for the *smallest* prime greater than 53, we start with the smallest answer-choice. 54 is not prime since 54 = 2(27). 55 is not prime since 55 = 5(11). 57 is not prime since 57 = 3(19). Now, 59 *is* prime. Hence, the answer is (D).

4. Since we are not given the radii of the circles, we can choose any two positive numbers such that one is three times the other. Let the outer radius be 3 and the inner radius be 1. Then the area of the outer circle is $\pi 3^2 = 9\pi$, and the area of the inner circle is $\pi 1^2 = \pi$. So the area of the shaded region is $9\pi - \pi = 8\pi$. Hence, the ratio of the area of the shaded region to the area of the smaller circle is $\frac{8\pi}{\pi} = \frac{8}{1}$. Therefore, the answer is (C).

5. Since the circle is centered at the origin and passes through the point (0, –3), the radius of the circle is 3. Now, if any other point is on the circle, the distance from that point to the center of the circle (the radius) must also be 3. Look at choice (B). Using the distance formula (see discussion below) to calculate the distance between $\left(-2\sqrt{2}, -1\right)$ and (0, 0) (the origin) yields

$$d = \sqrt{\left(-2\sqrt{2} - 0\right)^2 + \left(-1 - 0\right)^2} = \sqrt{\left(-2\sqrt{2}\right)^2 + \left(-1\right)^2} = \sqrt{8 + 1} = \sqrt{9} = 3$$

Hence, $\left(-2\sqrt{2}, -1\right)$ is on the circle, and the answer is (B).

Let's derive the distance formula.

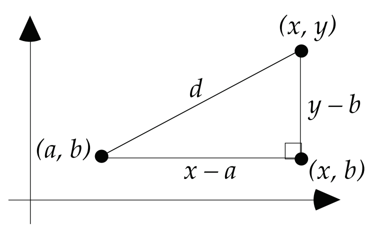
Distance Formula:

The distance formula is derived by using the Pythagorean theorem. Notice in the figure below that the distance between the points (x, y) and (a, b) is the hypotenuse of a right triangle. The difference $y - b$ is the measure of the height of the triangle, and the difference $x - a$ is the length of base of the triangle. Applying the Pythagorean theorem yields

$$d^2 = (x - a)^2 + (y - b)^2$$

Taking the square root of both sides this equation yields

$$\boldsymbol{d = \sqrt{(x - a)^2 + (y - b)^2}}$$



6. Since the sum of negative numbers is negative, $x + y$ is negative. Since the quotient of an even number of negative numbers is positive, y/x is positive. Hence, $\frac{y}{x} > x + y$. The answer is (B).

7. Converting .001 to a fraction gives $\frac{1}{1000}$. This fraction, in turn, can be written as $\frac{1}{10^3}$, or 10^{-3}. Cubing this expression yields $(.001)^3 = \left(10^{-3}\right)^3 = 10^{-9}$. Now, dividing the larger number, .1, by the smaller number, $(.001)^3$, yields

$$\frac{.1}{(.001)^3} = \frac{10^{-1}}{10^{-9}} = 10^{-1-(-9)} = 10^{-1+9} = 10^8$$

Hence, .1 is 10^8 times as large as $(.001)^3$. The answer is (D).

8. The equation $x = 3y = 4z$ contains three equations:

$$x = 3y$$
$$3y = 4z$$
$$x = 4z$$

Multiplying both sides of the equation $x = 3y$ by 6 gives $6x = 18y$. Hence, Statement I is true. This eliminates (B) and (C). Next, $3y + 20z = 3y + 5(4z)$. Substituting x for $3y$ and for $4z$ in this equation gives $3y + 20z = 3y + 5(4z) = x + 5x = 6x$. Hence, Statement II is true. This eliminates (A) and (E). Hence, by process of elimination, the answer is (D).

9. The average of the consecutive positive integers 1 through n is $A = \frac{1+2+\ldots+n}{n}$. Now, we are given that S denotes the sum of the consecutive positive integers 1 through n, that is, $S = 1 + 2 + \cdots + n$. Plugging this into the formula for the average gives $A = S/n$. Hence, Statement I is true, which eliminates (B) and (C). Next, solving the equation $A = S/n$ for S yields $S = A \cdot n$. Thus, Statement II is false, which eliminates (D) and (E). Therefore, the answer is (A).

10. Let x and y denote the numbers. Then $x/y = 10$ and $x - y = 18$. Solving the first equation for x and plugging it into the second equation yields

$$10y - y = 18$$

$$9y = 18$$

$$y = 2$$

Plugging this into the equation $x - y = 18$ yields $x = 20$. Hence, y is the smaller number. The answer is (A).

11. Begin by factoring 55 in the top of the fraction:

$$\frac{55^5}{5^{55}} = \frac{(5 \cdot 11)^5}{5^{55}} =$$

$$\frac{5^5 \cdot 11^5}{5^{55}} = \qquad \text{by Rule 3, } (xy)^a = x^a \cdot y^a$$

$$\frac{11^5}{5^{50}} \qquad \text{by Rule 5, } \frac{x^a}{x^b} = \frac{1}{x^{b-a}}$$

The answer is (C).

12.

$$2x^2 - 4xy + 2y^2 =$$
$$2(x^2 - 2xy + y^2) = \quad \text{by factoring out the common factor 2}$$
$$2(x - y)^2 = \quad \text{by the formula } x^2 - 2xy + y^2 = (x - y)^2$$
$$2p^2 \quad \text{since } x - y = p$$

The answer is (E).

13. $a◊b = ab - 1 = ba - 1 = b◊a$. Thus, I is true, which eliminates (B) and (C).

$\frac{a◊a}{a} = \frac{aa - 1}{a} \neq 1 \cdot 1 - 1 = 1 - 1 = 0 = 1◊1$. Thus, II is false, which eliminates (D).

$(a◊b)◊c = (ab - 1)◊c = (ab - 1)c - 1 = abc - c - 1 \neq a◊(bc - 1) = a(bc - 1) - 1 = abc - a - 1 = a◊(b◊c)$. Thus, III is false, which eliminates (E). Hence, the answer is (A).

14. Translate the main part of the sentence into a mathematical equation as follows:

What	percent	of	$2a$	is	$2b$
↓	↓	↓	↓	↓	↓
x	$\frac{1}{100}$	$\cdot$	$2a$	$=$	$2b$

$\frac{x}{100} \cdot 2a = 2b$

$\frac{x}{100} \cdot 2(4b) = 2b$ (substituting $a = 4b$)

$\frac{x}{100} \cdot 8 = 2$ (canceling b from both sides)

$\frac{8x}{100} = 2$

$8x = 200$

$x = 25$

The answer is (C).

15.

Circular Motion: In this type of problem, the key is often the arc length formula $S = R\theta$, where S is the arc length (or distance traveled), R is the radius of the circle, and θ is the angle.

When calculating distance, degree measure must be converted to radian measure. To convert degree measure to radian measure, multiply by the conversion factor $\pi/180$. Multiplying 60° by $\pi/180$ yields $60 \cdot \frac{\pi}{180} = \frac{\pi}{3}$. Now, the length of arc traveled by the car in moving from point A to point B is S. Plugging this information into the formula $S = R\theta$ yields $S = \frac{1}{2} \cdot \frac{\pi}{3} = \frac{\pi}{6}$. The answer is (A).

16. We know that T is 6; and therefore from the fact that *"each successive integer is one more than the preceding integer"* we see that S is 5. Continuing in this manner yields the following unique sequence:

P	Q	R	S	T
2	3	4	5	6

Hence, the value of R is 4. The answer is (E).

Answers and Solutions Section 1:

1. C	**6. A**	**11. A**	**16. E**
2. C	**7. D**	**12. C**	**17. A**
3. B	**8. B**	**13. E**	**18. B**
4. A	**9. C**	**14. C**	**19. D**
5. D	**10. C**	**15. A**	**20. A**

---

1. Forming a system from the two given equations yields

$$x + 3y = 5$$
$$3x + y = 7$$

Adding the two equations yields

$$4x + 4y = 12$$

$4(x + y) = 12$ by factoring out 4

$x + y = 12/4 = 3$ by dividing by 4

The answer is (C).

---

2. Since the number of integers between two integers inclusive is one more than their difference, we get $69 - 29 + 1 = 41$ integers. The answer is (C).

---

3. We could solve the equation, but it is much faster to just plug in the answer-choices. Begin with 1:

$$\frac{1^6 - 5(1)^3 - 16}{8} = \frac{1 - 5 - 16}{8} = \frac{-20}{8}$$

Hence, eliminate (A).

Next, plug in 2:

$$\frac{2^6 - 5(2)^3 - 16}{8} = \frac{64 - 5(8) - 16}{8} = \frac{64 - 40 - 16}{8} = \frac{8}{8} = 1$$

Hence, the answer is (B).

---

4. $\dfrac{1}{1-(.2)^2} = \dfrac{1}{1-.04} = \dfrac{1}{.96} = \dfrac{1}{96/100} = 1 \cdot \dfrac{100}{96} = \dfrac{100}{96} = \dfrac{25}{24}$. The answer is (A).

5. Working from the innermost parentheses out, we get

$$-x - 2 = -|-(6 - 2)|$$
$$-x - 2 = -|-4|$$
$$-x - 2 = -(+4)$$
$$-x - 2 = -4$$
$$-x = -2$$
$$x = 2$$

The answer is (D).

6. Since $2x + 60$ is an exterior angle, it is equal to the sum of the remote interior angles. That is, $2x + 60 = x + 90$. Solving for x gives $x = 30$. The answer is (A).

7.

"Birds-Eye" View

Most geometry problems on the SAT require straightforward calculations. However, some problems measure your insight into the basic rules of geometry (like this one). For this type of problem, you should step back and take a "birds-eye" view of the problem.

Solution: The diagonals of a square are equal. Hence, line segment *OR* (not shown) is equal to *SP*. Now, *OR* is a radius of the circle and therefore $OR = 2$. Hence, $SP = 2$ as well, and the answer is (D).

8. Forming an equation from $x^2 < 2x$ yields

$$x^2 = 2x$$

Subtracting $2x$ from both sides yields

$$x^2 - 2x = 0$$

Factoring yields

$$x(x - 2) = 0$$

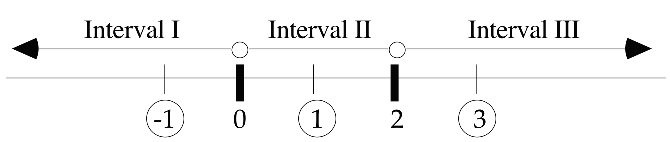
Setting each factor to zero yields

$$x = 0 \text{ and } x - 2 = 0$$

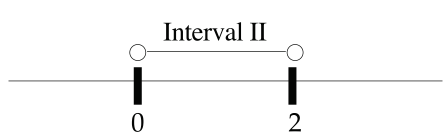
Solving yields

$$x = 0 \text{ and } x = 2$$

Setting up a number line and choosing test points (the circled numbers on the number line below) yields



Now, if $x = -1$, the inequality $x^2 < 2x$ becomes $(-1)^2 < 2(-1)$, or $1 < -2$. This is false. Hence, Interval I is not a solution. If $x = 1$, the inequality $x^2 < 2x$ becomes $1^2 < 2(1)$, or $1 < 2$. This is true. Hence, Interval II is a solution. If $x = 3$, the inequality $x^2 < 2x$ becomes $3^2 < 2(3)$, or $9 < 6$. This is false. Hence, Interval III is not a solution. Thus, only Interval II is a solution:



The answer is (B).

9. Converting .99 into a fraction gives 99/100. Since 99/100 is between 0 and 1, squaring it will make it smaller and taking its square root will make it larger. Hence, $(.99)^2 < .99 < \sqrt{.99}$. The answer is (C). Note, this property holds for all proper decimals (decimals between 0 and 1) just as it does for all proper fractions.

10. Each arc forms a quarter of a circle. Taken together the four arcs constitute one whole circle. From the drawing, we see that the radii of the arcs are each length 3, so the area of the four arcs together is $\pi(3)^2 = 9\pi$. Since the square has sides of length 6, its area is 36. Hence, the area of the shaded region is $36 - 9\pi$. The answer is (C).

11. Since x is raised to an even exponent, it is greater than or equal to zero. Further, since $x^4y \neq 0$, we know that neither x nor y is zero (otherwise $x^4y = 0$). Hence, we may divide $x^4y < 0$ by x^4 without reversing the inequality:

$$\frac{x^4 y}{x^4} < \frac{0}{x^4}$$

Simplifying yields

$$y < 0$$

A similar analysis of the inequality $xy^4 > 0$ shows that $x > 0$. Hence, $x > y$. The answer is (A).

12. First, factor the expression $\frac{x^2 + 6x + 9}{x + 3} \cdot \frac{x^2 - 9}{x - 3}$:

$$\frac{(x+3)(x+3)}{x+3} \cdot \frac{(x+3)(x-3)}{x-3}$$

Next, cancel the $x + 3$ and the $x - 3$:

$$(x + 3) \cdot (x - 3)$$

or

$$(x + 3)^2$$

The answer is (C).

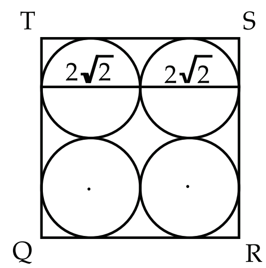
13. Setting the area of a circle equal to 2π gives $\pi r^2 = 2\pi$

Dividing both sides of this equation by π gives $r^2 = 2$

Taking the square root of both sides gives $r = \sqrt{2}$

Hence, the diameter of each circle is $d = 2r = 2\sqrt{2}$

Adding the diameters to the diagram gives



Clearly, in this diagram, the sides of the square are length $2\sqrt{2} + 2\sqrt{2} = 4\sqrt{2}$. Hence, the area of the square is $4\sqrt{2} \cdot 4\sqrt{2} = 16 \cdot 2 = 32$. The answer is (E).

14.

$$\left(x + \frac{1}{2}\right)^2 - (2x - 4)^2 =$$

$$x^2 + 2x\frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left[(2x)^2 - 2(2x)4 + 4^2\right] =$$

$$x^2 + x + \frac{1}{4} - 4x^2 + 16x - 16 =$$

$$-3x^2 + 17x - \frac{63}{4}$$

Hence, the answer is (C).

15. Since more than one letter is used in this question, we need to substitute one of the letters for the other to minimize the number of unknown quantities (letters).

40	percent	of	$3p$
↓	↓	↓	↓
40	$\frac{1}{100}$	×	$3p$

$= \frac{40}{100} \times 3p$

$= \frac{40}{100} \times 3(5q)$ (substituting $p = 5q$)

$= \frac{600q}{100}$

$= 6q$

The answer is (A).

16. The graph shows that nonfiction sales exceeded fiction sales in ‘81, ‘82, ‘83, ‘84, ‘85, and ‘87. The answer is (E).

17. The graph shows that the increase in sales of fiction titles from 1985 to 1986 was approximately 40 thousand and the increase in sales of fiction titles from 1983 to 1984 was approximately 10 thousand. Hence, the difference is 40 – 10 = 30. Choice (A) is the only answer-choice close to 30 thousand.

18. According to the chart, sales of fiction increased from 15,000 to 20,000 to 30,000 between 1982 and 1984. The answer is (B).

19. he following chart summarizes the sales for the years 1984 to 1988:

Year	Sales
1984	30 thousand
1985	11 thousand
1986	52 thousand
1987	52 thousand
1988	95 thousand

Forming the average yields:

$$\frac{30+11+52+52+95}{5}=48$$

The answer is (D).

Note, it is important to develop a feel for how the writers of the SAT approximate when calculating. We used 52 thousand to calculate the sales of fiction in 1986, which is the actual number. But from the chart, it is difficult to tell whether the actual number is 51, 52, or 53 thousand. However, using any of the these numbers, the average would still be nearer to 40 than to any other answer-choice.

20. Recall that the percentage increase (decrease) is formed by dividing the absolute increase (decrease) by the original amount:

$$\frac{57-40}{40}=.425$$

The answer is (A).

Answers and Solutions Section 2:

1. D	**6. B**	**11. 12**	**16. 1/2**
2. E	**7. C**	**12. 13**	**17. 5**
3. D	**8. C**	**13. 7/8**	**18. 25**
4. D	**9. 8**	**14. 400**	
5. C	**10. 75**	**15. 51**	

---

1. $\dfrac{1}{\frac{4}{3}-1} = \dfrac{1}{\frac{4}{3}-\frac{3}{3}} = \dfrac{1}{\frac{1}{3}} = 3$. The answer is (D).

---

2. Aligning the system of equations vertically yields

$$7x - y = 23$$
$$7y - x = 31$$

Adding the system of equations yields

$(7x - y) + (7y - x) = 23 + 31$	
$(7x - x) + (7y - y) = 54$	by collecting like terms
$6x + 6y = 54$	by adding like terms
$6(x + y) = 54$	by factoring out 6
$x + y = 9$	by dividing both sides by 6

The answer is (E).

---

3. OS and OT are equal since they are radii of the circle. Hence, ΔSOT is isosceles. Therefore, $S = T = 51^\circ$. Recalling that the angle sum of a triangle is 180°, we get $S + T + y = 51^\circ + 51^\circ + y = 180^\circ$. Solving for y gives $y = 78^\circ$. The answer is (D).

---

4. Dividing the equation $5x = 6y$ by $5y$ yields

$$\frac{x}{y} = \frac{6}{5} \quad \text{ratio of } x \text{ to } y$$

or in ratio notation

$$x : y = 6 : 5$$

The answer is (D).

---

5. $\sqrt{x} - x^2 = \sqrt{\frac{1}{9}} - \left(\frac{1}{9}\right)^2 = \frac{1}{3} - \frac{1}{81} = \frac{27}{27}\cdot\frac{1}{3} - \frac{1}{81} = \frac{27-1}{81} = \frac{26}{81}$. The answer is (C).

---

6. Since the two horizontal lines are parallel (Why?), angle a and the angle with measure 29 are alternate interior angles and therefore are equal. Further, from the drawing, angle b is 90°. Hence, $a + b = 29 + 90 = 119$. The answer is (B).

---

7. Since $l_1 \| l_2$, s and x are corresponding angles and therefore are congruent.

Now, about any point there are 360°. Hence,

$$5x + s = 360$$

Substituting x for s in this equation gives

$$5x + x = 360$$

Combining like terms gives

$$6x = 360$$

Dividing by 6 gives

$$x = 60$$

The answer is (C).

---

8. Suppose $m = 2$, an even integer. Then $4m + 1 = 9$, which is odd. Hence, the next even integer greater than 9 is 10. And the next even integer after 10 is 12. Now, 10 + 12 = 22. So look for an answer-choice which equals 22 when $m = 2$.

Begin with choice (A). Since $m = 2$, $8m + 2 = 18$—eliminate (A). Next, $8m + 4 = 20$—eliminate (B). Next, $8m + 6 = 22$. Hence, the answer is (C).

---

9. Substituting $p = 3$ into the equation $p^* = \frac{p+5}{p-2}$ gives $3^* = \frac{3+5}{3-2} = \frac{8}{1} = 8$. Grid in 8.

---

10. Since lines l and k are parallel, we know that the corresponding angles are equal. Hence, $y = 2y - 75$. Solving this equation for y gives $y = 75$. Grid in 75.

11.

50	%	of	<u>the blue balls</u>
↓	↓	↓	↓
50	$\frac{1}{100}$	×	24

$= \frac{50 \times 24}{100}$

$= \frac{1200}{100}$

$= 12$

Grid in 12.

12.

COIN PROBLEMS

The key to these problems is to keep the quantity of coins distinct from the value of the coins. An example will illustrate.

Solution: Let D stand for the number of dimes, and let Q stand for the number of quarters. Since the total number of coins in 20, we get $D + Q = 20$, or $Q = 20 - D$. Now, each dime is worth 10¢, so the value of the dimes is $10D$. Similarly, the value of the quarters is $25Q = 25(20 - D)$. Summarizing this information in a table yields

	Dimes	**Quarters**	**Total**
Number	D	$20 - D$	20
Value	$10D$	$25(20 - D)$	305

Notice that the total value entry in the table was converted from \$3.05 to 305¢. Adding up the value of the dimes and the quarters yields the following equation:

$$10D + 25(20 - D) = 305$$
$$10D + 500 - 25D = 305$$
$$-15D = -195$$
$$D = 13$$

Hence, there are 13 dimes. Grid in 13.

13. Since the height and base of the larger triangle are the same, the slope of the hypotenuse is 45°. Hence, the base of the smaller triangle is the same as its height, 3/2. Thus, the area of the shaded region equals

(area of the larger triangle) – (area of the smaller triangle) =

$$\left(\frac{1}{2}\cdot 2\cdot 2\right) - \left(\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{3}{2}\right) = 2 - \frac{9}{8} = \frac{7}{8}$$

Grid in 7/8.

14. Forming the series for u and v yields

$$u = 1 + 2 + \cdots + 19 + 20$$
$$v = 21 + 22 + \cdots + 39 + 40$$

Subtracting the series for u from the series for v yields

$$v - u = \underbrace{20 + 20 + \cdots + 20 + 20}_{\text{20 times}} = 20 \cdot 20 = 400$$

Grid in 400.

15. Since the road is 1015 feet long and the speed bumps are 20 feet apart, there are 1015/20 = 50.75, or 50 full sections in the road. If we ignore the first speed bump and associate the speed bump at the end of each section with that section, then there are 50 speed bumps (one for each of the fifty full sections). Counting the first speed bump gives a total of 51 speed bumps. Grid in 51.

16. Let x stand for the distances TP and TS. Applying the Pythagorean Theorem to the right triangle PST gives

$$TP^2 + TS^2 = PS^2$$

Substituting x for TP and TS and substituting 2 for PS gives

$$x^2 + x^2 = 2^2$$

Squaring and combining like terms gives

$$2x^2 = 4$$

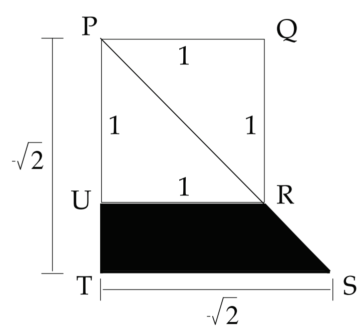
Dividing by 2 gives

$$x^2 = 2$$

Finally, taking the square root gives

$$x = \sqrt{2}$$

Adding this information to the diagram gives



Now, the area of the shaded region equals

(area of triangle PST) – (area of triangle PRU) =

$$\left(\frac{1}{2}\cdot\sqrt{2}\cdot\sqrt{2}\right)-\left(\frac{1}{2}\cdot1\cdot1\right)=\left(\frac{1}{2}\cdot2\right)-\left(\frac{1}{2}\right)=1-\frac{1}{2}=\frac{1}{2}$$

Grid in 1/2.

17. Plugging $y = x + 2$ into the equation $z = y + 2$ gives $z = (x + 2) + 2 = x + 4$. Hence, in terms of x, the three numbers x, y, and z are

$$x, x + 2, x + 4$$

Clearly, x is the smallest number. Further, since $x + 2$ is smaller than $x + 4$, $x + 2$ is the median. Subtracting the median from the product of the smallest number and the median and setting the result equal to 0 yields

$$x(x + 2) - (x + 2) = 0$$

Factoring out the common factor $x + 2$ yields

$$(x + 2)(x - 1) = 0$$

Setting each factor equal to 0 yields

$$x + 2 = 0 \text{ or } x - 1 = 0$$

Hence, $x = -2$ or $x = 1$. Since the three numbers are positive, x must be 1. Hence, the largest number is

$$x + 4 = 1 + 4 = 5$$

Grid in 5.

---

18. The area of triangle PQS is $\frac{1}{2} \cdot 5 \cdot 6 = 15$. Now, (the area of ΔQRS) equals

$$(\text{the area of } \Delta PQR) - (\text{the area of } \Delta PQS) = 40 - 15 = 25$$

Grid in 25.

Answers and Solutions Section 3:

1.	**C**	**5.**	**B**	**9.**	**D**	**13.**	**C**
2.	**C**	**6.**	**D**	**10.**	**E**	**14.**	**C**
3.	**A**	**7.**	**C**	**11.**	**D**	**15.**	**C**
4.	**B**	**8.**	**E**	**12.**	**D**	**16.**	**A**

---

1. Noticing that $x - y$ is a common factor, we factor it out:

$$x(x - y) - z(x - y) = (x - y)(x - z)$$

The answer is (C).

Method II
Sometimes a complicated expression can be simplified by making a substitution. In the expression $x(x - y) - z(x - y)$ replace $x - y$ with w:

$$xw - zw$$

Now, the structure appears much simpler. Factoring out the common factor w yields

$$w(x - z)$$

Finally, re-substitute $x - y$ for w:

$$(x - y)(x - z)$$

---

2. Suppose $x = 8$. Then x is divisible by 8 and is not divisible by 3. Now, $x/2 = 4$, $x/4 = 2$, $x/8 = 1$ and $x = 8$, which are all integers—eliminate (A), (B), (D), and (E). Hence, by process of elimination, the answer is (C).

---

3. Since M is the midpoint of side PQ, the length of MQ is 2. Hence, the area of triangle MQR is $\frac{1}{2} \cdot 2 \cdot 4 = 4$. A similar analysis shows that the area of triangle NSR is 4. Thus, the unshaded area of the figure is 4 + 4 = 8. Subtracting this from the area of the square gives 16 – 8 = 8. The answer is (A).

---

4. The area of a square with sides of length x is x^2. This formula yields

$$[9] \div [3] = 9^2 \div 3^2 = 81 \div 9 = 9$$

Now, $[3] = 3^2 = 9$. Hence, the answer is (B).

5. Since the area of the circle is 9π, we get

$$\pi r^2 = 9\pi$$
$$r^2 = 9$$
$$r = 3$$

Now, the circumference of the circle is

$$C = 2\pi r = 2\pi 3 = 6\pi$$

Since the central angle is 30°, the length of arc PRQ is

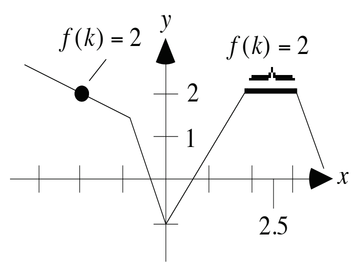
$$\frac{30}{360} C = \frac{1}{12} \cdot 6\pi = \frac{1}{2}\pi$$

Hence, the perimeter of the sector is

$$\frac{1}{2}\pi + 3 + 3 = \frac{1}{2}\pi + 6$$

The answer is (B).

6. The graph has a height of 2 for every value of x between 2 and 3; it also has a height of 2 at about $x = -2$. The only number offered in this interval is 2.5. This is illustrated by the dot and the thick line in the following graph:



The answer is (D).

7. $x \# (-y) = -(-y)^4 = -y^4$. Note: The exponent applies only to the negative inside the parentheses. The answer is (C).

8. Let a, $a + 2$, $a + 4$ stand for the consecutive even integers a, b, and c, in that order. Forming the average of a, b, and c yields

$$\frac{a+b+c}{3} = \frac{a+a+2+a+4}{3} = \frac{3a+6}{3} = a+2$$

Setting this less than $a/3$ gives

$$a + 2 < a/3$$

Multiplying by 3 yields

$$3a + 6 < a$$

Subtracting 6 and a from both sides yields

$$2a < -6$$

Dividing by 2 yields

$$a < -3$$

Hence, a is negative, and the best answer is (E).

9. Replacing 0.01 with its fractional equivalent, 1/100, yields

$$\frac{1}{3^n} < \frac{1}{100}$$

Multiplying both sides by 3^n and 100 and then simplifying yields

$$100 < 3^n$$

Beginning with $n = 2$, we plug in larger and larger values of n until we reach one that makes $100 < 3^n$ true. The table below summarizes the results:

n	**$100 < 3^n$**	
2	$100 < 3^2 = 9$	False
3	$100 < 3^3 = 27$	False
4	$100 < 3^4 = 81$	False
5	$100 < 3^5 = 243$	True

Since 5 is the first integer to work, the answer is (D).

10. Since A denotes the area of the circular region, we get

$$A = \pi r^2$$

$$\frac{A}{\pi} = r^2$$

$$\sqrt{\frac{A}{\pi}} = r$$

Hence, the circumference is $C = 2\pi r = 2\pi\sqrt{\frac{A}{\pi}}$

The answer is (E).

11. Since squaring a fraction between 0 and 1 makes it smaller, we know Statement I is true. This eliminates both (B) and (C).

Also, since taking the square root of a fraction between 0 and 1 makes it larger, we know Statement III is false. This eliminates (E).

To analyze Statement II, we'll use substitution. Since $0 < x < 1$, we need only check one fraction, say, $x = 1/2$. Then

$$\frac{1}{x^2} = \frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\left(\frac{1}{4}\right)} = 1 \cdot \frac{4}{1} = 4$$

Now, $1/2 < 4$. Hence, Statement II is true, and the answer is (D).

12.

Midpoint Formula:

The midpoint M between points (x, y) and (a, b) is given by

$$M = \left(\frac{x+a}{2}, \frac{y+b}{2}\right)$$

In other words, to find the midpoint, simply average the corresponding coordinates of the two points.

Solution: Since point R is on the x-axis, its y-coordinate is 0. Further, since $PQRO$ is a square and the x-coordinate of Q is 2, the x-coordinate of R is also 2. Since T is the midpoint of side QR, the midpoint formula yields

$$T = \left(\frac{2+2}{2}, \frac{2+0}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = (2,1)$$

The answer is (D).

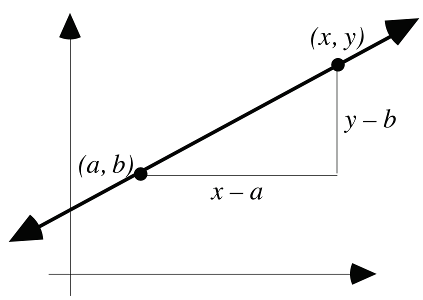
13. Plugging $P = 10$ and $k = 3$ into the equation $P = (x + y)k$ gives $10 = (x + y)3$. Dividing by 3 gives $x + y = 10/3$. Finally, to form the average, divide both sides of this equation by 2: $\frac{x+y}{2} = \frac{10}{6} = \frac{5}{3}$. Hence, the answer is (C).

14. The average of p, q, and r is $\frac{p+q+r}{3}$. Replacing $p + q$ with r gives $\frac{r+r}{3} = \frac{2r}{3}$. The answer is (C).

15.

Slope Formula:

The slope of a line measures the inclination of the line. By definition, it is the ratio of the vertical change to the horizontal change (see figure below). The vertical change is called the *rise*, and the horizontal change is called the *run*. Thus, the slope is the *rise over the run*.



Forming the *rise over the run* in the above figure yields

$$m = \frac{y-b}{x-a}$$

Solution: The slope formula yields $m = \frac{4-2}{5-1} = \frac{2}{4} = \frac{1}{2}$. The answer is (C).

16. From (1), ABC is a code word.

From (2), the C in the code word ABC can be moved to the front of the word: CAB.

Hence, CAB is a code word and the answer is (A).

Answers and Solutions Section 1:

1.	**B**	**6.**	**D**	**11.**	**D**	**16.**	**A**
2.	**A**	**7.**	**E**	**12.**	**B**	**17.**	**C**
3.	**D**	**8.**	**E**	**13.**	**C**	**18.**	**B**
4.	**A**	**9.**	**A**	**14.**	**E**	**19.**	**C**
5.	**B**	**10.**	**D**	**15.**	**E**	**20.**	**C**

1. Since the number of integers between two integers inclusive is one more than their difference, (91 – 49) + 1 = 43 callers won a prize. The answer is (B).

2.

$x^2 - y^2 =$

$(x + y)(x - y) =$ since $x^2 - y^2$ is a difference of squares

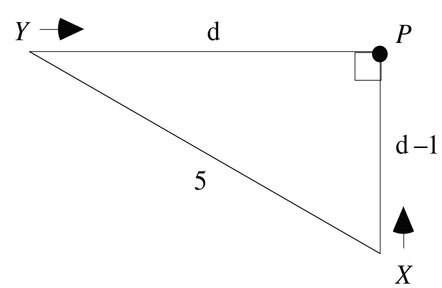
$(10)(5) =$ since $x + y = 10$ and $x - y = 5$

50

The answer is (A).

This problem can also be solved by adding the two equations. However, that approach will lead to long, messy fractions. Writers of the SAT put questions like this one on the SAT to see whether you will discover the shortcut. The premise being that those students who do not see the shortcut will take longer to solve the problem and therefore will have less time to finish the test.

3. Let d be the distance ship Y is from the point of collision. Then the distance ship X is from the point of collision is d – 1. The following diagram depicts the situation:



Applying the Pythagorean Theorem to the diagram yields

$$d^2 + (d-1)^2 = 5^2$$

$$d^2 + d^2 - 2d + 1 = 25$$

$$2d^2 - 2d - 24 = 0$$

$$d^2 - d - 12 = 0$$

$$(d-4)(d+3) = 0$$

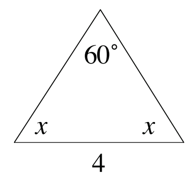
$$d = 4 \quad \text{or} \quad d = -3$$

Since d denotes distance, we reject $d = -3$. Hence, $d = 4$ and the answer is (D).

4. Subtracting $3x$ from both sides of $3x + y < 4$ yields $y < 4 - 3x$. Now, multiplying both sides of $x > 3$ by -3 yields $-3x < -9$. Adding 4 to both sides yields $4 - 3x < -5$. Now, using the transitive property to combine $y < 4 - 3x$ and $4 - 3x < -5$ yields $y < 4 - 3x < -5$. Hence, $y < -5$. The answer is (A).

5. $\frac{6^4 - 6^3}{5} = \frac{6^3(6-1)}{5} = \frac{6^3 \cdot 5}{5} = 6^3$. The answer is (B).

6. Since two sides of the triangle are radii of the circle, they are equal. Hence, the triangle is isosceles, and the base angles are equal:



Since the angle sum of a triangle is 180, we get

$$x + x + 60 = 180$$

$$2x = 120$$

$$x = 60$$

Hence, the triangle is equilateral. Therefore, the radius of the circle is 4, and the circumference is $C = 2\pi r = 2\pi 4 = 8\pi$. Now, the portion of the perimeter formed by the circle has length

$$\frac{360 - 60}{360} \cdot C = \frac{5}{6} \cdot 8\pi = \frac{20}{3}\pi$$

Adding the three sides of the square to this expression gives $\frac{20}{3}\pi + 12$. The answer is (D).

7.

$$4(xy)^3 + \left(x^3 - y^3\right)^2 =$$

$$4x^3y^3 + \left(x^3\right)^2 - 2x^3y^3 + \left(y^3\right)^2 =$$

$$\left(x^3\right)^2 + 2x^3y^3 + \left(y^3\right)^2 =$$

$$\left(x^3 + y^3\right)^2$$

The answer is (E).

---

8. Since $AB = AC$, ΔABC is isosceles. Hence, its base angles are equal: $y = z$. Since the angle sum of a triangle is 180°, we get $x + y + z = 180$. Replacing z with y and x with 30 in this equation and then simplifying yields

$$30 + y + y = 180$$
$$30 + 2y = 180$$
$$2y = 150$$
$$y = 75$$

The answer is (E).

---

9. If $x = 1$ and $y = 3$, then

$$y \neq 5x$$

and

$$\frac{x+5}{y} = \frac{1+5}{3} = \frac{6}{3} = 2$$

which is prime and not odd. Hence, Statements I and III are not necessarily true. Next, let $x = 3$ and $y = 4$. Then y is not prime and

$$\frac{x+5}{y} = \frac{3+5}{4} = \frac{8}{4} = 2$$

which is prime. Hence, Statement II is not necessarily true. The answer is (A).

---

10. Since female employees are 108 out of 180, there are 180 – 108 = 72 male employees. Now, translate the main part of the sentence into a mathematical equation as follows:

What	percent	of	<u>the employees</u>	are	male
↓	↓	↓	↓	↓	↓
x	$\frac{1}{100}$	$\cdot$	180	=	72

$$\frac{180}{100}x = 72$$

$$\frac{100}{180}\cdot\frac{180}{100}x = \frac{100}{180}\cdot 72$$

$$x = 40$$

The answer is (D).

---

11. Since the total surface area of the cube is 22 and each of the cube's six faces has the same area, the area of each face is 22/6, or 11/3. Now, each face of the cube is a square with area 11/3, so the length of a side of the cube is $\sqrt{\frac{11}{3}}$. Hence, the volume of the cube is

$$\sqrt{\frac{11}{3}}\cdot\sqrt{\frac{11}{3}}\cdot\sqrt{\frac{11}{3}} = \frac{11}{3}\cdot\sqrt{\frac{11}{3}}$$

The answer is (D).

12. Suppose $x^2 = 4$. Then $x = 2$ or $x = -2$. In either case, x is even. Hence, Statement I need not be true, which eliminates (A) and (D). Further, $x^3 = 8$ or $x^3 = -8$. In either case, x^3 is even. Hence, Statement III need not be true, which eliminates (C) and (E). Therefore, by process of elimination, the answer is (B).

13.

AGE PROBLEMS

Typically, in these problems, we start by letting x be a person's current age and then the person's age a years ago will be $x - a$ and the person's age a years in future will be $x + a$.

Solution: Steve's age is the most unknown quantity. So we let x = Steve's age and then $x + 20$ is John's age. Ten years from now, Steve and John's ages will be $x + 10$ and $x + 30$, respectively. Summarizing this information in a table yields

	Age now	**Age in 10 years**
Steve	x	$x + 10$
John	$x + 20$	$x + 30$

Since "in 10 years, Steve's age will be half that of John's," we get

$$\frac{1}{2}(x+30) = x + 10$$
$$x + 30 = 2(x + 10)$$
$$x + 30 = 2x + 20$$
$$x = 10$$

Hence, Steve is 10 years old, and the answer is (C).

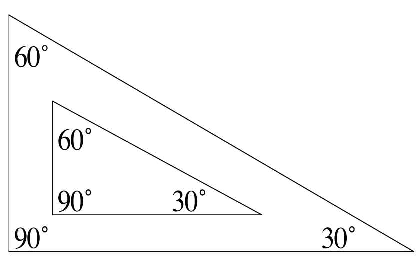
14. From the information given, we can determine the measures of the angles:

$$a + b + c = x + 2x + 3x = 6x = 180$$

Dividing the last equation by 6 gives

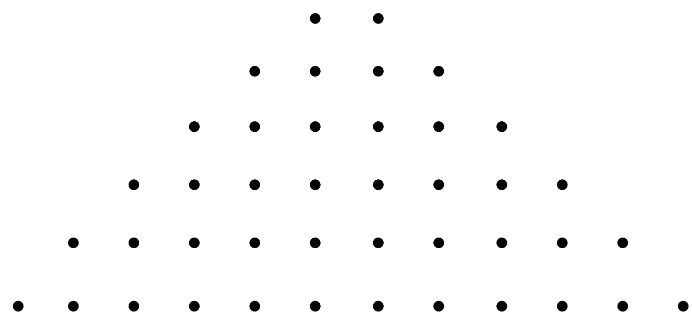
$$x = 30$$

Hence, $a = 30$, $b = 60$, and $c = 90$. However, different size triangles can have these angle measures, as the diagram below illustrates:



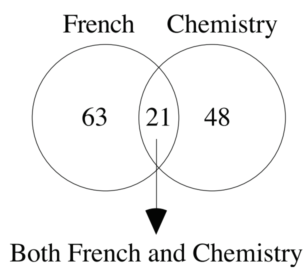
Hence, the information given is not sufficient to determine the area of the triangle. The answer is (E).

15. Extending the dots to six rows yields



Row 6 has twelve dots. Hence, the answer is (E).

16. First display the information in a Venn diagram:



Adding the number of students taking French and the number of students taking chemistry and then subtracting the number of students taking both yields (63 + 48) – 21 = 90. This is the number of students enrolled in *either* French or chemistry or both. Since the total school enrollment is 150, there are 150 – 90 = 60 students enrolled in *neither* French nor chemistry. The answer is (A).

17. Recall from geometry that a triangle inscribed in a semicircle is a right triangle. Hence, we can use the Pythagorean Theorem to calculate the length of *AB*:

$$AC^2 + BC^2 = AB^2$$

or

$$3^2 + 4^2 = AB^2$$

or

$$25 = AB^2$$

or

$$5 = AB$$

Hence, the radius of the circle is

$$\frac{diameter}{2} = \frac{5}{2}$$

The answer is (C).

18. Multiply each term of the inequality $-1 < x \le 2$ by -3 (this is done because the original expression involves $-3x$):

$$3 > -3x \ge -6$$

Add 2 to each term of this inequality (this is done because the original expression adds 2 and $-3x$):

$$5 > 2 - 3x \ge -4$$

Rewrite the inequality in standard form (with the smaller number on the left and the larger number on the right):

$$-4 \le 2 - 3x < 5$$

The answer is (B).

19. First, let's calculate the probability of selecting a red or a blue paper clip. This is an either-or probability and is therefore the sum of the individual probabilities:

$$1/4 + 1/6 = 5/12$$

Now, since there are only three types of objects, the sum of their probabilities must be 1 (Remember that the sum of the probabilities of all possible outcomes is always 1):

$$P(r) + P(b) + P(s) = 1,$$

where r stands for red, b stands for blue, and s stands for silver.

Replacing $P(r) + P(b)$ with 5/12 yields

$$5/12 + P(s) = 1$$

Subtracting 5/12 from both sides of this equation yields

$$P(s) = 1 - 5/12$$

Performing the subtraction yields

$$P(s) = 7/12$$

The answer is (C).

20. Since the area of the square is 16, the length of a side is

$$\sqrt{16} = 4$$

Since the circle is inscribed in the square, a diameter of the circle has the same length as a side of the square. Hence, the radius of the circle is

$$\frac{diameter}{2} = \frac{4}{2} = 2$$

Therefore, the area of the circle is

$$\pi \cdot 2^2 = 4\pi$$

and the area of the shaded region is

$$16 - 4\pi$$

The answer is (C).

Answers and Solutions Section 2:

1. B	**6. B**	**11. 3**	**16. 2**
2. A	**7. E**	**12. 1**	**17. 199**
3. A	**8. E**	**13. 8**	**18. 2**
4. C	**9. 64**	**14. 40**	
5. E	**10. 16**	**15. 475**	

1.

$$y^2 - \left(x - \left[y + \frac{1}{2}\right]\right) - 2 \cdot 3 =$$

$$(-3)^2 - \left(2 - \left[-3 + \frac{1}{2}\right]\right) - 2 \cdot 3 =$$

$$(-3)^2 - \left(2 - \left[-\frac{5}{2}\right]\right) - 2 \cdot 3 =$$

$$(-3)^2 - \left(2 + \frac{5}{2}\right) - 2 \cdot 3 =$$

$$(-3)^2 - \frac{9}{2} - 2 \cdot 3 =$$

$$9 - \frac{9}{2} - 2 \cdot 3 =$$

$$9 - \frac{9}{2} - 6 =$$

$$3 - \frac{9}{2} =$$

$$-\frac{3}{2}$$

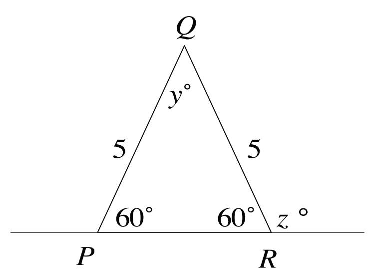
The answer is (B).

---

2. Since x is a fraction between 0 and 1, $\sqrt{x}$ is greater than either x^3 or x^4. It's also greater than $\frac{1}{\pi}x$ since $\frac{1}{\pi}x$ is less than x. To tell which is greater between $\sqrt{x}$ and $\frac{1}{\sqrt{x}}$, let $x = 1/4$ and plug it into each expression: $\sqrt{x} = \sqrt{\frac{1}{4}} = \frac{1}{2}$ and $\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{1/4}} = \frac{1}{1/2} = 2$. Hence, $\frac{1}{\sqrt{x}}$ is greater than $\sqrt{x}$. The answer is (A).

---

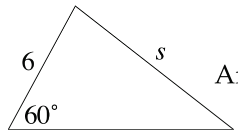
3. Since ΔPQR is isosceles, its base angles are equal:

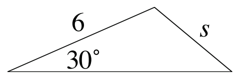


Remembering that the angle sum of a triangle is 180˚, we see y is also 60˚. The answer is (A).

4. Since $x > 0$, $|x| = x$. And the equation $|x| = \frac{1}{x}$ becomes $x = \frac{1}{x}$. Multiplying both sides of this equation by x yields $x^2 = 1$. Taking the square root of both sides gives $x = \pm 1$. Since we are given that $x > 0$, x must equal 1. The answer is (C).

5. Since we do not know the value of z, the triangle can vary in size. Each of the triangles illustrated below satisfies the given information, yet one has an area greater than the other:





The answer is (E).

6. Since the *"the first term in the sequence is* $\sqrt{2}$*"* and *"all odd numbered terms are equal,"* all odd numbered terms equal $\sqrt{2}$. Since the *"the second term is –2"* and *"all even numbered terms are equal,"* all even numbered terms equal –2. Hence, the sum of any two consecutive terms of the sequence is $\sqrt{2} + (-2) \approx -0.6$ (remember, $\sqrt{2} \approx 1.4$). The answer is (B).

7. Since there are 60 seconds in a minute and the press prints 5 pages every 2 seconds, the press prints $5 \cdot 30 = 150$ pages in one minute. Hence, in 7 minutes, the press will print $7 \cdot 150 = 1050$ pages. The answer is (E).

8. The following list shows all 12 ways of selecting the two marbles:

(0, 1)	(1, 0)	(2, 0)	**(3, 0)**
(0, 2)	**(1, 2)**	**(2, 1)**	(3, 1)
(0, 3)	(1, 3)	(2, 3)	(3, 2)

The four pairs in bold are the only ones whose sum is 3. Hence, the probability that two randomly drawn marbles will have a sum of 3 is

$$4/12 = 1/3$$

The answer is (E).

9. Since x is both a cube and between 2 and 200, we are looking at the integers:

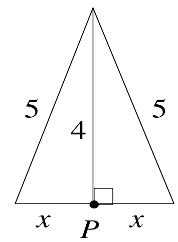
$$2^3, 3^3, 4^3, 5^3$$

which reduce to

$$8, 27, 64, 125$$

There is only one perfect square, $64 = 8^2$, in this set. Grid in 64.

10. Add the height to the diagram:



Applying the Pythagorean Theorem to either of the right triangles formed above yields

$$x^2 + 4^2 = 5^2$$

Solving for x yields

$$x = 3$$

Hence, the base of the triangle is $2x = 2(3) = 6$, and therefore the perimeter is $5 + 5 + 6 = 16$. Grid in 16.

11. Combining the inequalities $c > a$ and $a > d$ gives $c > a > d$. Since $b = 2$, a, c, and d must represent the remaining numbers 1, 3, and 4—not necessarily in that order. In order to satisfy the condition $c > a > d$, c must be 4, a must be 3, and d must be 1. Grid in 3.

12. $\dfrac{\dfrac{2x^2-2}{x-1}}{2(x+1)} = \dfrac{2x^2-2}{x-1} \cdot \dfrac{1}{2(x+1)} = \dfrac{2(x^2-1)}{x-1} \cdot \dfrac{1}{2(x+1)} = \dfrac{2(x+1)(x-1)}{x-1} \cdot \dfrac{1}{2(x+1)} = \dfrac{2}{2} \cdot \dfrac{x+1}{x+1} \cdot \dfrac{x-1}{x-1} = 1.$

Grid in 1.

13.

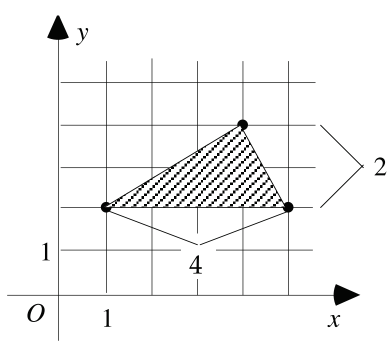
Areas and Perimeters:

Often, you will be given a geometric figure drawn on a coordinate system and will be asked to find its area or perimeter. In these problems, use the properties of the coordinate system to deduce the dimensions of the figure and then calculate the area or perimeter. For complicated figures, you may need to divide the figure into simpler forms, such as squares and triangles.

Solution: If the quadrilateral is divided horizontally through the line $y = 2$, two congruent triangles are formed. As the figure shows, the top triangle has height 2 and base 4. Hence, its area is

$$A = \frac{1}{2}bh = \frac{1}{2} \cdot 4 \cdot 2 = 4$$

The area of the bottom triangle is the same, so the area of the quadrilateral is $4 + 4 = 8$. Grid in 8.



14. Since the sum of the digits is 4, x must be 13, 22, 31, or 40. Further, since the difference of the digits is 4, x must be

$$40, 51, 15, 62, 26, 73, 37, 84, 48, 95, \text{ or } 59$$

We see that 40 and only 40 is common to the two sets of choices for x. Hence, x must be 40. Grid in 40.

15. Let C be the cost of the computer without the hard drive, and let H be the cost of the hard drive. Then translating *"The computer with the hard drive costs 2,900 dollars"* into an equation yields

$$C + H = 2{,}900$$

Next, translating *"The computer without the hard drive costs 1,950 dollars more than the hard drive alone"* into an equation yields

$$C = H + 1{,}950$$

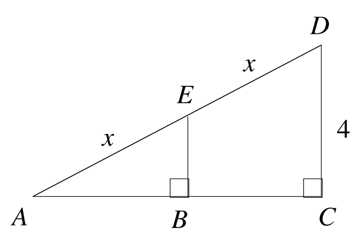
Combining these equations, we get the system:

$$C + H = 2{,}900$$
$$C = H + 1{,}950$$

Solving this system for H, yields $H = 475$. Grid in 475.

16. Recall from geometry that if two angles of one triangle are equal to two angles of another triangle then the triangles are similar. Hence, ΔACD is similar to ΔABE since they share angle A and both are right triangles.

Since E is the midpoint of AD, the diagram becomes



Since ΔABE and ΔACD are similar, their corresponding sides are proportional:

$$\frac{EB}{EA} = \frac{DC}{DA}$$

or

$$\frac{EB}{x} = \frac{4}{2x}$$

Solving for EB yields

$$EB = 2$$

Grid in 2.

17. Translating the clause “4 percent of $(p + q)$ is 8” into a mathematical expression yields

$$.04(p + q) = 8$$

Dividing both sides of this equation by .04 yields

$$p + q = 8/.04 = 200$$

Subtracting p from both sides yields

$$q = 200 - p$$

This expression will be greatest when p is as small as possible. This is when $p = 1$:

$$q = 200 - 1 = 199$$

Grid in 199.

18. Taking the square root of both sides of the equation $(bh)^2 = 16$ gives

$$bh = 4$$

Plugging this into the area formula gives

$$Area = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 4 = 2$$

Grid in 2.

Answers and Solutions Section 3:

1.	**E**	**5.**	**B**	**9.**	**D**	**13.**	**D**
2.	**C**	**6.**	**B**	**10.**	**D**	**14.**	**E**
3.	**A**	**7.**	**D**	**11.**	**C**	**15.**	**C**
4.	**B**	**8.**	**E**	**12.**	**D**	**16.**	**D**

---

1. Solution:

$(2 + x)(2 + y) - (2 + x) - (2 + y) =$
$4 + 2y + 2x + xy - 2 - x - 2 - y =$
$x + y + xy$

The answer is (E).

---

2. Writing the system of given equations vertically yields

$x + y = 4a/5$
$y + z = 7a/5$
$z + x = 9a/5$

Adding the three equations yields

$(x + y) + (y + z) + (z + x) = 4a/5 + 7a/5 + 9a/5$
$2x + 2y + 2z = 20a/5$ by adding like terms
$2(x + y + z) = 4a$
$x + y + z = 2a$ by dividing both sides by 2

The answer is (C).

3.

Slope-Intercept Form:

Multiplying both sides of the equation $m = \frac{y - b}{x - a}$ by $x - a$ yields

$$y - b = m(x - a)$$

Now, if the line passes through the y-axis at $(0, b)$, then the equation becomes

$$y - b = m(x - 0)$$

or

$$y - b = mx$$

or

$$y = mx + b$$

This is called the slope-intercept form of the equation of a line, where m is the slope and b is the y-intercept. This form is convenient because it displays the two most important bits of information about a line: its slope and its y-intercept.

Solution: Since $y = \frac{9}{10}x + k$ is in slope-intercept form, we know the slope of the line is 9/10. Now, the ratio of *BO* to *AO* is the slope of the line (rise over run). Hence,

$$\frac{BO}{AO} = \frac{9}{10}$$

Multiplying both sides of this equation by *AO* yields

$$BO = \frac{9}{10}AO$$

In other words, *BO* is 9/10 the length of *AO*. Hence, *AO* is longer. The answer is (A).

---

4. First, bring the negative symbol in the expression $\frac{x-y}{-z}$ to the top:

$$\frac{-(x-y)}{z}$$

Then distribute the negative symbol:

$$\frac{y-x}{z}$$

To make this expression as small as possible, we need to make both the $y - x$ and z as small as possible. To make $y - x$ as small as possible, let $y = 1$ and $x = 19$. Then $y - x = 1 - 19 = -18$. With these choices for y and x, the smallest remaining value for z is 2. This gives

$$\frac{y-x}{z} = \frac{1-19}{2} = \frac{-18}{2} = -9$$

In this case, we made the numerator as small as possible. Now, let's make the denominator as small as possible. To that end, chose $z = 1$ and $y = 2$ and $x = 19$. This gives

$$\frac{y-x}{z} = \frac{2-19}{1} = \frac{-17}{1} = -17$$

The answer is (B).

---

5. $\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{2}}} = \dfrac{1}{1-\dfrac{1}{\dfrac{2}{2}-\dfrac{1}{2}}} = \dfrac{1}{1-\dfrac{1}{\dfrac{1}{2}}} = \dfrac{1}{1-2} = \dfrac{1}{-1} = -1$. The answer is (B).

---

6. Since opposite angles of a parallelogram are equal, $\angle ABC = \angle ADC$. Further, since there are 360° in a parallelogram,

$$\angle ABC + \angle ADC + \angle BAD + \angle BCD = 360$$

$$\angle ABC + \angle ADC + 140 = 360$$

$$\angle ABC + \angle ABC = 220$$

$$2\angle ABC = 220$$

$$\angle ABC = 110$$

The answer is (B).

7. Let the four numbers be a, b, c, and d. Since their average is 20, we get

$$\frac{a+b+c+d}{4} = 20$$

Let d be the number that is removed. Since the average of the remaining numbers is 15, we get

$$\frac{a+b+c}{3} = 15$$

Solving for $a + b + c$ yields

$$a + b + c = 45$$

Substituting this into the first equation yields

$$\frac{45+d}{4} = 20$$

Multiplying both sides of this equation by 4 yields

$$45 + d = 80$$

Subtracting 45 from both sides of this equation yields

$$d = 35$$

The answer is (D).

8. This is a direct proportion: as the distance increases, the gallons of fuel consumed also increases. Setting ratios equal yields

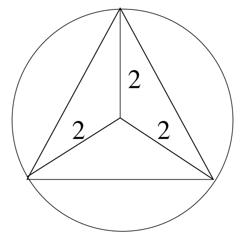
$$\frac{80\,gal.}{320\,mi.} = \frac{x\,gal.}{700\,mi.}$$

$$\frac{700 \cdot 80}{320} = x$$

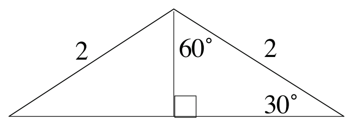
$$175 = x$$

The answer is (E).

9. Adding radii to the diagram yields



Now, viewing the bottom triangle in isolation yields



Recall, in a 30°–60°–90° triangle, the side opposite the 30° angle is 1/2 the length of the hypotenuse, and the side opposite the 60° angle is $\frac{\sqrt{3}}{2}$ times the length of the hypotenuse. Hence, the altitude of the above triangle is 1, and the base is $\sqrt{3}+\sqrt{3}=2\sqrt{3}$. Thus, the area of the triangle is $A=\frac{1}{2}\cdot 2\sqrt{3}\cdot 1=\sqrt{3}$. By symmetry, the area of the inscribed triangle is $3A=3\sqrt{3}$. The answer is (D).

10.

- **Strategy: On hard problems, if you are asked to find the least (or greatest) number, then eliminate the least (or greatest) answer-choice.**

This rule also applies to easy and medium problems. When people guess on these types of problems, they most often choose either the least or the greatest number. But if the least or the greatest number were the answer, most people would answer the problem correctly, and it therefore would not be a hard problem.

Note: 45% of the time the second smallest (or second largest) number is the answer. For easy and medium problems, this is true 40% of the time.

Solution: Clearly, there are more than four 3 × 3 squares in the checkerboard—eliminate (A). Next, eliminate (B) since it merely repeats a number from the problem. Further, eliminate (E) since it is the greatest. This leaves choices (C) and (D). If you count carefully, you will find sixteen 3 × 3 squares in the checkerboard. The answer is (D).

---

11. Translating the clause "the average of 10, 14, and n is greater than or equal to 8 and less than or equal to 12" into an inequality yields

$$8 \le \frac{10 + 14 + n}{3} \le 12$$

Adding 10 and 14 yields

$$8 \le \frac{24 + n}{3} \le 12$$

Multiplying each term by 3 yields

$$24 \le 24 + n \le 36$$

Subtracting 24 from each term yields

$$0 \le n \le 12$$

Hence, the least possible value of n is 0. The answer is (C).

---

12. From the equation $\left(x^2 - 4\right)\left(\frac{4}{x} - 5\right) = 0$, we get $x^2 - 4 = 0$ or $\frac{4}{x} - 5 = 0$. Consider the equation $x^2 - 4 = 0$ first. Factoring gives

$$(x + 2)(x - 2) = 0$$

Setting each factor to zero gives

$$x + 2 = 0 \quad \text{or} \quad x - 2 = 0$$

Hence, $x = 2$ or $x = -2$. But neither number is offered as an answer-choice. So we turn to the equation $\frac{4}{x} - 5 = 0$. Adding 5 to both sides yields

$$\frac{4}{x} = 5$$

Multiplying both sides by x gives

$$4 = 5x$$

Dividing both sides by 5 gives

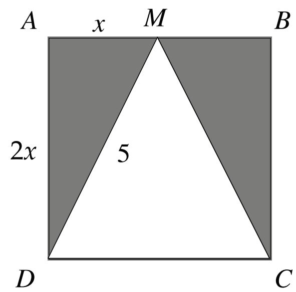
$$\frac{4}{5} = x$$

The answer is (D).

---

13. $27^5 = \left(3^3\right)^5 = 3^{15}$. Hence, $x = 15$ and the answer is (D).

14. Adding the given information to the diagram gives



Applying the Pythagorean Theorem yields

$$x^2 + (2x)^2 = 5^2$$
$$x^2 + 4x^2 = 5^2$$
$$5x^2 = 5^2$$
$$x^2 = 5$$
$$x = \sqrt{5}$$

Hence, the area of the square is $2x \cdot 2x = 2\sqrt{5} \cdot 2\sqrt{5} = 20$. Since the height of the unshaded triangle is the same as the length of a side of the square, the area of the triangle is

$$A = \frac{1}{2}\left(2\sqrt{5}\right)\left(2\sqrt{5}\right) = 10$$

Subtracting this from the area of the square gives

$$20 - 10 = 10$$

The answer is (E).

15.

- **Strategy: On hard problems, eliminate the answer-choice "not enough information."**

When people cannot solve a problem, they most often choose the answer-choice "not enough information." But if this were the answer, then it would not be a "hard" problem.

Solution: Since this is a hard problem, we eliminate (E), "not enough information."

Since 5 is prime, its only factors are 1 and 5. So the constant C in the expression $(x + 1)(x + C)$ must be 5:

$$(x + 1)(x + 5)$$

Multiplying out this expression yields

$$(x + 1)(x + 5) = x^2 + 5x + x + 5$$

Combining like terms yields

$$(x + 1)(x + 5) = x^2 + 6x + 5$$

Hence, $K = 6$, and the answer is (C).

16. Since the angle sum of a triangle is 180°, $x + y + z = 180$. Plugging this into the expression $\frac{x+y+z}{15}$ yields

$$\frac{x+y+z}{15} = \frac{180}{15} = 12$$

The answer is (D)

Answers and Solutions Section 1:

1.	**D**	**6.**	**E**	**11.**	**D**	**16.**	**E**
2.	**D**	**7.**	**B**	**12.**	**B**	**17.**	**D**
3.	**A**	**8.**	**A**	**13.**	**A**	**18.**	**C**
4.	**A**	**9.**	**C**	**14.**	**A**	**19.**	**E**
5.	**E**	**10.**	**A**	**15.**	**C**	**20.**	**E**

---

1. There are 60 minutes in an hour. Hence, there are $1\frac{1}{3}\cdot 60 = 80$ minutes in 1 1/3 hours. The answer is (D).

---

2. The ten's digit must be twice the unit's digit. This eliminates (A), (C), and (E). Now reversing the digits in choice (B) yields 12. But 21 – 12 ≠ 27. This eliminates (B). Hence, by process of elimination, the answer is (D). (63 – 36 = 27.)

---

3. Δ*OPQ* is isosceles. (Why?). Hence, $P = Q = 59^\circ$. Now, the angle sum of a triangle is 180. So

$$O + P + Q = 180.$$

Substituting $P = Q = 59^\circ$ into this equation gives

$$O + 59 + 59 = 180.$$

Solving for *O* gives

$$O = 62.$$

Now, since *O* is the largest angle in Δ*OPQ*, the side opposite it, *PQ*, is the longest side of the triangle. The answer is (A).

---

4. SAT answer-choices are usually listed in ascending order of size—occasionally they are listed in descending order. Hence, start with choice (C). If it is less than 2, then turn to choice (D). If it is greater than 2, then turn to choice (B).

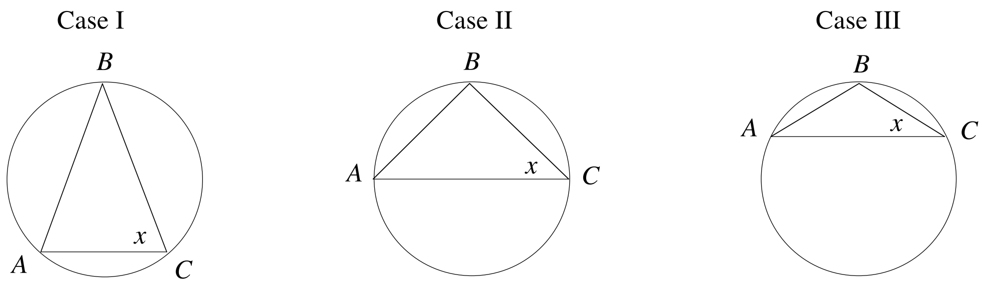
Now, $[6] = \frac{6^2}{2} = \frac{36}{2} = 18$, which is greater than 2. So we next check choice (B). Now, $[4] = \frac{4^2}{2} = \frac{16}{2} = 8$, which is greater than 2. Therefore, by process of elimination, the answer is (A). Let's verify this:

$$[2] = \frac{2^2}{2} = \frac{4}{2} = 2$$

5.

- **Strategy: When Drawing a Geometric Figure or Checking a Given One, Be Sure to Include Drawings of Extreme Cases As Well As Ordinary Ones.**

Solution: Although in the drawing *AC* looks to be a diameter, that cannot be assumed. All we know is that *AC* is a chord. Hence, numerous cases are possible, three of which are illustrated below:



In Case I, x is greater than 45˚; in Case II, x equals 45˚; in Case III, x is less than 45˚. Hence, the answer is (E).

6. Since there are twice as many townhomes as ranch-style homes, the probability of selecting a townhome is 2/3.* Now, "there are 3 times as many townhomes with pools than without pools." So the probability that a townhome will have a pool is 3/4. Hence, the probability of selecting a townhome with a pool is

$$\frac{2}{3}\cdot\frac{3}{4}=\frac{1}{2}$$

The answer is (E).

7. The expression $x^5 = 4$ can be rewritten as

$$x \cdot x^4 = 4$$

Replacing x^4 in this expression with $7/y$ yields

$$x\cdot\frac{7}{y}=4$$

Multiplying both sides of this equation by y gives

$$x \cdot 7 = 4 \cdot y$$

Dividing both sides of this equation by 7 yields

$$x=\frac{4}{7}\cdot y$$

Hence, the answer is (B).

* Caution: Were you tempted to choose 1/2 for the probability because there are "twice" as many townhomes? One-half (= 50%) would be the probability if there were an equal number of townhomes and ranch-style homes. Remember the probability of selecting a townhome is not the ratio of townhomes to ranch-style homes, but the ratio of townhomes to the total number of homes. To see this more clearly, suppose there are 3 homes in the subdivision. Then 2 would be townhomes and 1 would be a ranch-style home. So the ratio of townhomes to total homes would be 2/3.

8.

- **Strategy: On hard problems, eliminate answer-choices that merely repeat numbers from the problem.**

Solution: Since we are to find the greatest value of m, we eliminate (E)—the greatest. Also, eliminate 5 because it is repeated from the problem. Now, since we are looking for the largest number, start with the greatest number remaining and work toward the smallest number. The first number that works will be the answer. To this end, let $m = 3$. Then $\frac{P}{10^m} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{10^3} = \frac{120}{1000} = \frac{3}{25}$. This is not an integer, so eliminate (C). Next, let $m = 2$. Then $\frac{P}{10^m} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{10^2} = \frac{120}{100} = \frac{6}{5}$. This still is not an integer, so eliminate (B). Hence, by process of elimination, the answer is (A).

---

9. Let the other number be y. Since the average of the two numbers is $\pi/2$, we get

$$\frac{x + y}{2} = \frac{\pi}{2}$$

Multiplying both sides of this equation by 2 yields

$$x + y = \pi$$

Subtracting x from both sides of this equation yields

$$y = \pi - x$$

The answer is (C).

10.

- **Strategy: On hard problems, eliminate answer-choices that can be derived from elementary operations.**

Solution: We can eliminate 50 (the mere average of 40 and 60) since that would be too elementary for this hard problem. Now, the average must be closer to 40 than to 60 because the car travels for a longer time at 40 mph. But 48 is the only number given that is closer to 40 than to 60. The answer is (A).

It's instructive to also calculate the answer.

$$\textit{Average Speed} = \frac{\textit{Total Distance}}{\textit{Total Time}}$$

Now, a car traveling at 40 mph will cover 120 miles in 3 hours. And a car traveling at 60 mph will cover the same 120 miles in 2 hours. So the total traveling time is 5 hours. Hence, for the round trip, the average speed is

$$\frac{120 + 120}{5} = 48$$

11.

- **Strategy: On hard problems, after you have eliminated as many answer-choices as you can, choose from the more complicated or more unusual answer-choices remaining.**

Solution: Clearly, there are more than 3 color combinations possible. This eliminates (A) and (B). We can also eliminate (C) and (E) because they are both multiples of 3, and that would be too ordinary, too easy, to be the answer to this hard problem. Hence, by process of elimination, the answer is (D).

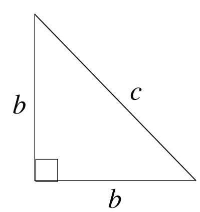
Let's also solve this problem directly. The following list displays all 27 (= 3 · 3 · 3) color combinations possible (without restriction):

RRR	BBB	GGG
RRB	BBR	GGR
RRG	BBG	GGB
RBR	BRB	GRG
RBB	BRR	GRR
RBG	BRG	GRB
RGR	BGB	GBG
RGB	BGR	GBR
RGG	BGG	GBB

If order is not considered, then there are 10 distinct color combinations in this list. You should count them.

12. Remember that the area of a square is equal to the length of its side squared. Since the area of the square is a^2, the side of the square is a. Hence, the perimeter of the square is $P = a + a + a + a = 4a$.

Now, let b represent the length of the equal sides of the right-angled isosceles triangle, and let c represent the length of the hypotenuse:



Since the hypotenuse of a right triangle is opposite the right angle, the sides labeled b are the base and height of the triangle. The area of the triangle is $\frac{1}{2}$ base × height $= \frac{1}{2}bb = \frac{1}{2}b^2$. We are given that the area of the triangle is a^2. Hence, $\frac{1}{2}b^2 = a^2$. Solving this equation for b yields $b = \sqrt{2}a$. To calculate the hypotenuse, c, of the triangle we apply the Pythagorean Theorem:

$$c^2 = b^2 + b^2$$
$$c^2 = 2b^2$$
$$c = \sqrt{2b^2}$$
$$c = \sqrt{2}b$$
$$c = \sqrt{2}\sqrt{2}a \qquad \text{(since } b = \sqrt{2}a\text{)}$$
$$c = 2a$$

The perimeter of the triangle is $P = b + b + c = 2b + c = 2\sqrt{2}a + 2a = a(2\sqrt{2} + 2)$. Recall that $\sqrt{2} \approx 1.4$. Hence, $a(2\sqrt{2} + 2) \approx a(2.8 + 2) = 4.48a > 4a$. Hence, the perimeter of the triangle is greater than the perimeter of the square, and the answer is (B).

---

13. We are told that the sum of the prime numbers x and y is odd. For a sum of two numbers to be odd, one number must be odd and another even. There is only one even prime number—2; all others are odd. Hence, either x or y must be 2. Thus, the product of x and y is a multiple of 2 and therefore is divisible by 2. The answer is (A).

---

14. This is an inverse proportion: as the number of boys increases the time required to complete the job decreases. Setting products equal yields

$$2 \times 2.5 = 5 \times t$$

$$1 = t$$

The answer is (A).

---

15. The length of the rectangle is $6m$ and the width of the rectangle is $4m$. From the standard formula for the perimeter of a rectangle, we get

$$P = 2L + 2W = 2(6m) + 2(4m) = 20m$$

Now, the formula for the perimeter of a square is $4x$, where x represents the length of a side of the square. Since we are given that the perimeter of the square is equal to that of the rectangle, we write

$$4x = 20m$$
$$x = 20m/4 = 5m$$

The answer is (C).

---

16. In the bottom chart, the bar for 1994 ends half way between 15 and 20. Thus, there were about 17.5 million cars in 1994. The answer is (E).

---

17. From the bottom chart, there were 2 million cars in 1990; and from the top chart, there were 340 thousand accidents in 1991. Forming the difference yields

$$2{,}000{,}000 - 340{,}000 = 1{,}660{,}000$$

Rounding 1.66 million off yields 1.7 million. The answer is (D).

---

18. From the charts, the number of accidents in 1993 was 360,000 and the number of cars was 11,000,000. Forming the percentage yields

$$\frac{360{,}000}{11{,}000{,}000} \approx 3\%$$

The answer is (C).

---

19. From the graphs, there is no way to predict what will happen in the future. The number of accidents could continually decrease after 1994. The answer is (E).

20. The number of cars involved in accidents will be minimized when each car has exactly 4 accidents. Now, from the top chart, there were 360,000 accidents in 1993. Dividing 360,000 by 4 yields

$$\frac{360,000}{4} = 90,000$$

The answer is (E).

Answers and Solutions Section 2:

1. A	**6. A**	**11. 21**	**16. 2**
2. E	**7. D**	**12. 30**	**17. 180**
3. E	**8. D**	**13. 0**	**18. 10**
4. C	**9. 60**	**14. 420**	
5. E	**10. 5**	**15. 27**	

1. Begin with 5/6 and 4/5. Cross-multiplying gives 25 versus 24. Hence, 5/6 > 4/5. Continuing in this manner will show that 5/6 is the greatest fraction listed. The answer is (A).

2. We are given that $p > 2$. Multiplying both sides of this inequality by 3 yields $3p > 6$. The answer is (E).

3. Triangle PQR is a right triangle with the base PR equal to 4 and height PQ. The area of Triangle PQR is $\frac{1}{2}bh = 6$. Substituting the known quantities into this formula yields $\frac{1}{2}(4)(PQ) = 6$. Solving this equation for PQ yields $PQ = 3$. Applying the Pythagorean Theorem to the triangle yields

$(PQ)^2 + (PR)^2 = (QR)^2$

$3^2 + 4^2 = (QR)^2$ by substitution

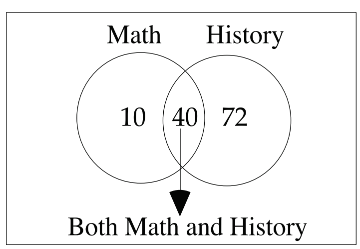
$25 = (QR)^2$

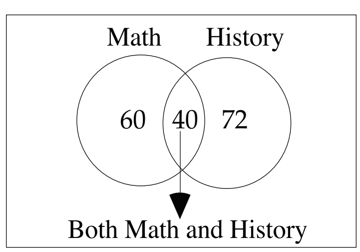
$5 = QR$ by taking the square root of both sides

The answer is (E).

4. Since the sum of the digits is 4, x must be 13, 22, 31, or 40. Further, since the difference of the digits is 4, x must be 40, 51, 15, 62, 26, 73, 37, 84, 48, 95, or 59. We see that 40 and only 40 is common to the two sets of choices for x. Hence, x must be 40. The answer is (C).

5. The given information does tell us the number of History students who are not taking Math—32; however, the statements do not tell us the number of students enrolled in Math. The following Venn diagrams show two scenarios that satisfy the given information. Yet in the first case, less than 32 students are enrolled in Math; and in the second case, more than 32 students enrolled in Math:





The answer is (E).

6. The formula for the circumference of a circle with diameter d is $C = 2\pi r = \pi(2r) = \pi d$ (since the diameter is twice the radius, $d = 2r$). Hence, the ratio of the circumference of the circle to its diameter is

$$\frac{C}{d} = \frac{\pi d}{d} = \pi$$

The answer is (A).

Note: The fact that the circumference of the circle is $4m$ was not used in solving the problem. Thus, the answer is independent of the size of the circle. In other words, the ratio of the circumference of a circle to its diameter is always π.

7. Let A, B, C stand for the three colors, and let W, X, Y, Z stand for the four luxury features. There are three ways of selecting the colors:

A	B

A	C

B	C

There are four ways of selecting the luxury features:

W	X	Y

W	Y	Z

W	X	Z

X	Y	Z

Hence, there are $3 \times 4 = 12$ ways of selecting all the features. The answer is (D).

8. Solution: $\frac{x+y}{x-y} = 3$. Multiplying both sides of this equation by $(x - y)$ yields

$$x + y = 3(x - y)$$
$$x + y = 3x - 3y$$
$$-2x = -4y$$
$$x = 2y$$

Since we have expressed x as 2 times an integer, it is even. The answer is (D).

9. We are given that $\angle A$ is 10 degrees greater than $\angle B$. Expressing this as an equation gives

$$\angle A = \angle B + 10$$

We are also given that $\angle B$ is 10 degrees greater than $\angle C$. Expressing this as an equation gives

$$\angle B = \angle C + 10$$

In a triangle, the sum of the three angles is 180 degrees. Expressing this as an equation gives

$$\angle A + \angle B + \angle C = 180$$

Solving these three equations for $\angle B$, we get $\angle B = 60$ degrees. Grid in 60.

10. The set of numbers greater than 5 and divisible by 5 is

$$\{10, 15, 20, 25, 30, 35, \ldots\}$$

Since n is odd, the possible values for n are 15, 25, 35, ... Any number in this list, when divided by 10, leaves a remainder of 5. Grid in 5.

11. Since the fence is 400 feet long and the posts are 20 feet apart, there are 400/20 = 20 sections in the fence. Now, if we ignore the first post and associate the post at the end of each section with that section, then there are 20 posts (one for each of the twenty sections). Counting the first post gives a total of 21 posts. Grid in 21.

12. To find the y-intercept of a line, we set $x = 0$:

$$y = -\frac{5}{3}(0) + 10 = 10$$

Hence, the height of the triangle is 10. To find the x-intercept of a line, we set $y = 0$:

$$-\frac{5}{3}x + 10 = 0$$

Solving this equation for x yields $x = 6$. Hence, the base of the triangle is 6. Therefore, the area of shaded portion (which is a triangle) is $\frac{1}{2} \cdot 6 \cdot 10 = 30$. Grid in 30.

13. From the equation $x \otimes y = -y$, we get

$$x\sqrt{y} - y - 2x = -y$$
$$x\sqrt{y} - 2x = 0$$
$$x\left(\sqrt{y} - 2\right) = 0$$

Now, if $x = 0$, then $x\left(\sqrt{y} - 2\right) = 0$ will be true regardless the value of y since the product of zero and any number is zero. Grid in 0.

14. We are given a formula for the sum of the first n even, positive integers. Plugging $n = 20$ into this formula yields

$$n(n + 1) = 20(20 + 1) = 20(21) = 420$$

Grid in 420.

15. We are given that one of the sides of the rectangle has length 3. This implies that either x or $x + 6$ equals 3. If $x + 6$ equals 3, then x must be –3, which is impossible since a length cannot be negative. Hence, $x = 3$ and $x + 6 = 3 + 6 = 9$. The area of the rectangle, being the product of two adjacent sides of the rectangle, is $x(x + 6) = 3(9) = 27$. Grid in 27.

16. Since the right side of the equation is positive, the left side must also be positive. Thus, $(-8)^{2n}$ is equal to

$$8^{2n}$$

This in turn can be written as

$$\left(2^3\right)^{2n}$$

Multiplying the exponents gives

$$2^{6n}$$

Plugging this into the original equation gives

$$2^{6n} = 2^{8+2n}$$

Now, since the bases are the same, the exponents must be equal:

$$6n = 8 + 2n$$

Solving this equation gives

$$n = 2$$

Grid in 2.

17. This is a direct proportion: as the time increases so does the number of steps that the sprinter takes. Setting ratios equal yields

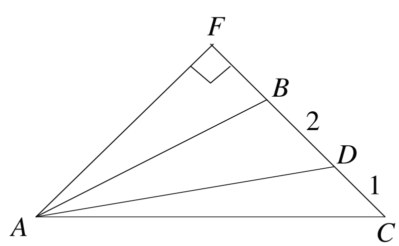
$$\frac{30}{9} = \frac{x}{54}$$

$$\frac{30 \cdot 54}{9} = x$$

$$180 = x$$

Grid in 180.

18. Let's add an altitude to Triangle *ABC* by extending side *BC* as shown in the figure below.



The formula for the area of a triangle is $A = (1/2)(\text{base})(\text{height})$. Hence, the area of Triangle *ABC* is

$(1/2)(BC)(AF) = (1/2)(2 + 1)(AF) = (3/2)(AF) = 30$ (the area of Triangle *ABC* is given to be 30)

Solving this equation for *AF* yields $AF = 20$. Now, the area of Triangle *ADC* is

$$(1/2)(DC)(AF) = (1/2)(1)(20) = 10$$

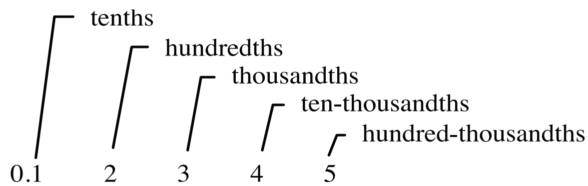
Grid in 10.

<u>Answers and Solutions Section 3:</u>

1.	**B**	**5.**	**E**	**9.**	**E**	**13.**	**D**
2.	**C**	**6.**	**C**	**10.**	**C**	**14.**	**C**
3.	**B**	**7.**	**A**	**11.**	**A**	**15.**	**C**
4.	**B**	**8.**	**A**	**12.**	**D**	**16.**	**C**

---

1. The convention used for rounding numbers is *"if the following digit is less than five, then the preceding digit is not changed. But if the following digit is greater than or equal to five, then the preceding digit is increased by one."*



Since 3 is in the thousands position and the following digit, 4, is less than 5, the digit 3 is not changed. Hence, rounded to the nearest thousandth 0.12345, is 0.123. The answer is (B).

---

2. Let x be the price before the discount. Since Stella received a 20 percent discount, she paid 80 percent of the original price. Thus, 80 percent of the original price is \$1,500. Now, translate this sentence into a mathematical equation:

80	percent	of	<u>the original price</u>	is	\$1,500
↓	↓	↓	↓	↓	↓
80	$\frac{1}{100}$	·	x	=	1500

$\frac{80}{100}x = 1500$

$\frac{100}{80}\frac{80}{100}x = \frac{100}{80}1500$ (by multiplying both sides by the reciprocal of $\frac{80}{100}$)

$x = 1875$

The answer is (C).

---

3. In the figure, $CD = x$ and AC is the hypotenuse of the right triangle ADC. Recall that in a right triangle the hypotenuse is the longest side. Hence, $AC > x$. Now, consider triangle ABC. Observe that $\angle B$ is opposite side AC and $\angle BAC$ is opposite side BC. Since, $BC = x$ and $AC > x$, we can write that $AC > BC$. Recall that in a triangle, the angle opposite the greater side is the greater angle. Hence, $\angle B > \angle BAC$. Since $\angle B = 30^\circ$, $\angle BAC$ must be less than 30°. From the exterior angle theorem,

$$\theta = \angle B + \angle BAC = 30 + \angle BAC$$

We have already derived that $\angle BAC < 30^\circ$. Adding 30 to both sides of this inequality yields $30 + \angle BAC < 60$. Replacing $30 + \angle BAC$ with θ, we get $\theta < 60$. The answer is (B).

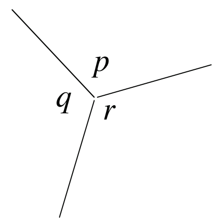
4. Note that the product of r and t is 1. The product of two numbers is positive only if both numbers are positive or both numbers are negative. Since $rt = 1$ and $r > t$, there are two possibilities:

Case I (both negative): $-1 < r < 0$ and $t < -1$

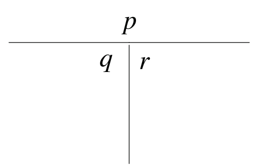
Case II (both positive): $0 < t < 1$ and $r > 1$

The second case violates the condition $r < 1$. Hence, Case I is true, and the answer is (B).

5. It is natural to make the drawing symmetric as follows:



In this case, $p = q = r = 120^\circ$, so $q + r = 240^\circ$. However, there are other drawings possible. For example:



In this case, $q + r = 180^\circ$ and therefore it cannot be determined from the information given. The answer is (E).

6. This is a hard problem, so we can eliminate (A) since it repeats the number 10 from the problem. We can also eliminate choices (B), (D), and (E) since they are derivable from elementary operations:

$$20 = 30 - 10$$

$$40 = 30 + 10$$

$$100 = 10 \cdot 10$$

This leaves choice (C) as the answer.

Let's also solve this problem directly. The clause

w is 10 percent less than *x*

translates into

$$w = x - .10x$$

Simplifying yields

1) $w = .9x$

Next, the clause

y is 30 percent less than **z**

translates into

$$y = z - .30z$$

Simplifying yields

2) $y = .7z$

Multiplying 1) and 2) gives

$$wy = (.9x)(.7z) = .63xz = xz - .37xz$$

Hence, **wy** is 37 percent less than **xz**. The answer is (C).

7. Statement I is true: From *"If p is divided by 2, the remainder is 1,"* $p = 2u + 1$; and from *"if q is divided by 6, the remainder is 1,"* $q = 6v + 1$. Hence, $pq + 1 =$

$$(2u + 1)(6v + 1) + 1 =$$

$$12uv + 2u + 6v + 1 + 1 =$$

$$12uv + 2u + 6v + 2 =$$

$$2(6uv + u + 3v + 1)$$

Since we have written $pq + 1$ as a multiple of 2, it is even.

Method II:
Since p and q each leave a remainder of 1 when divided by an even number, both are odd. Now, the product of two odd numbers is another odd number. Hence, pq is odd, and therefore $pq + 1$ is even.

Now, since $pq + 1$ is even, pq is odd. Hence, $pq/2$ is not an integer, and Statement II is not necessarily true. Next, Statement III is not necessarily true. For example, if $p = 3$ and $q = 7$, then $pq = 21$, which is not a multiple of 12. The answer is (A).

8. From the figure, observe that $\angle AOC$ and $\angle BOD$ are vertical angles between the lines *AB* and *CD*. Hence, $\angle AOC = \angle BOD = x$. Since a straight angle has 180°, we get the following equation:

$\angle EOD + \angle BOD + \angle BOF = 180$
$z + x + y = 180$ since $\angle EOD = z$, $\angle BOD = x$, $\angle BOF = y$
$z + 54 + 72 = 180$ since $x = 54°$ and $y = 72°$
$z = 180 - 54 - 72 = 54$

The answer is (A)

9. There are many different values for *w*, *x*, *y*, and *z* such that $\frac{x}{y} + \frac{w}{z} = 2$. Two particular cases are listed below:

If $x = y = w = z = 1$, then $\frac{x}{y} + \frac{w}{z} = \frac{1}{1} + \frac{1}{1} = 1 + 1 = 2$ and $\frac{y}{x} + \frac{z}{w} = \frac{1}{1} + \frac{1}{1} = 1 + 1 = 2$.

If $x = 3$, $y = 2$, $w = 1$, and $z = 2$, then $\frac{x}{y} + \frac{w}{z} = \frac{3}{2} + \frac{1}{2} = \frac{3+1}{2} = \frac{4}{2} = 2$ and $\frac{y}{x} + \frac{z}{w} = \frac{2}{3} + \frac{2}{1} = \frac{2}{3} + \frac{2}{1} \cdot \frac{3}{3} = \frac{2}{3} + \frac{6}{3} = \frac{2+6}{3} = \frac{8}{3}$

This is a double case. Hence, the answer is (E).

10. The average speed at which car X traveled is

$$\frac{\textit{Total Distance}}{30}$$

The average speed at which car Y traveled is

$$\frac{\textit{Total Distance}}{20}$$

The two fractions have the same numerators, and the denominator for car Y is smaller. Hence, the average miles per hour at which car Y traveled is greater than the average miles per hour at which car X traveled. Thus, Statement I is false and Statement III is true. As to Statement II, we do not have enough information to calculate the distance between the cities. Hence, Statement II need not be true. The answer is (C).

11. Point *A* has coordinates (0, 4), point *B* has coordinates (3, 0), and point *C* has coordinates (5, 1). Using the distance formula to calculate the distances between points *A* and *B*, *A* and *C*, and *B* and *C* yields

$$\overline{AB} = \sqrt{(0-3)^2 + (4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5$$
$$\overline{AC} = \sqrt{(0-5)^2 + (4-1)^2} = \sqrt{25+9} = \sqrt{34}$$
$$\overline{BC} = \sqrt{(5-3)^2 + (1-0)^2} = \sqrt{4+1} = \sqrt{5}$$

Adding these lengths gives the perimeter of Triangle *ABC*:

$$\overline{AB} + \overline{AC} + \overline{BC} = 5 + \sqrt{34} + \sqrt{5}$$

The answer is (A).

12. $\left(2^4\right)^2 - 1 = (16)^2 - 1 = 256 - 1 = 255$. Since the question asks for the greatest prime factor and this is a hard problem, we eliminate 19, the greatest number. Now, we start with the next largest number and work our way up the list; the first number that divides into 255 evenly will be the answer. Dividing 17 into 255 gives

$$17\overline{)255} = 15$$

Hence, 17 is the largest prime factor of $\left(2^4\right)^2 - 1$. The answer is (D).

13. This is a direct proportion: as the amount of flour increases so must the amount of shortening. First change 1/2 lb. into 8 oz., Setting ratios equal yields

$$\frac{8}{14} = \frac{x}{21}$$

$$\frac{21 \cdot 8}{14} = x$$

$$12 = x$$

The answer is (D).

14. Since $(b, 0)$ is the x-intercept of the line, it must satisfy the equation:

$$0 = pb + a$$

Subtracting a from both sides yields

$$-a = pb$$

Dividing both sides by b yields

$$-a/b = p$$

The answer is (C).

15.

$$\left(9^x\right)^3 = 9^{3x} = \qquad \text{by the rule } \left(x^a\right)^b = x^{ab}$$

$$\left(3^2\right)^{3x} = \qquad \text{since } 9 = 3^2$$

$$3^{6x} \qquad \text{again by the rule } \left(x^a\right)^b = x^{ab}$$

The answer is (C). Note, this is considered to be a hard problem.

16. Since P is in Quadrant II, its x-coordinate is negative. That is, a is negative. Since Q is in Quadrant IV, its y-coordinate is negative. That is, b is negative. Hence, (a, b) is in Quadrant III. The answer is (C).

Answers and Solutions Section 1:

1. E	**6. E**	**11. D**	**16. A**
2. D	**7. C**	**12. E**	**17. E**
3. B	**8. B**	**13. C**	**18. A**
4. E	**9. A**	**14. B**	**19. A**
5. A	**10. B**	**15. C**	**20. A**

---

1. Since the circle is centered at the origin and passes through the point (–3, 0), the radius of the circle is 3. Hence, the area is

$$A = \pi r^2 = \pi 3^2 = 9\pi$$

The answer is (E).

---

2. Let $p = 1$ and $q = 2$. Then $pq = 2$ and $p(q + 2) = 4$. This scenario has one integer, 3, greater than pq and less than $p(q + 2)$. Now, we plug $p = 1$ and $q = 2$ into the answer-choices until we find one that has the value 1. Look at choice (D): $2p - 1 = (2)(1) - 1 = 1$. Thus, the answer is (D).

---

3. $2\#3 = -\sqrt{(2+3)^2} = -\sqrt{5^2} = -\sqrt{25} = -5$. The answer is (B).

---

4. We are asked to find the value of x for which revenue is \$110. In mathematical terms, we need to solve the equation $r(x) = 110$. Since $r(x) = 50\sqrt{x} - 40$, we get

$$50\sqrt{x} - 40 = 110$$
$$50\sqrt{x} = 150$$
$$\sqrt{x} = 3$$
$$\left(\sqrt{x}\right)^2 = 3^2$$
$$|x| = 9$$
$$x = 9 \quad \text{or} \quad x = -9$$

Since $x = -9$ has no physical interpretation for this problem, we know that $x = 9$. The answer is (E).

---

5. Squaring a fraction between 0 and 1 makes it smaller, and taking the square root of it makes it larger. Therefore, Statement I is true since the top part of the fraction is larger than the bottom. This eliminates (B). Next, Statement II is false. Squaring a fraction makes it smaller only if the fraction is between 0 and 1. This eliminates (C) and (E). Finally, Statement III is false. Since $\frac{5}{6} < \sqrt{\frac{5}{6}}$, we get

$$\frac{\frac{5}{6}}{\sqrt{\frac{5}{6}}} < 1$$

Although taking the square root of this expression will make it larger, it will still be less than 1. The answer is (A).

6. Let the original number be represented by xy. (Note: Here xy does not denote multiplication, but merely the position of the digits: x first, then y.). Reversing the digits of xy gives yx. We are told that $yx > xy$. This implies that $y > x$. (For example, 73 > 69 because 7 > 6.) If $x = 9$, then the condition $y > x$ cannot be satisfied. Hence, x cannot equal 9. The answer is (E).

Method II:
Let the original number be represented by xy. In expanded form, xy can be written as $10x + y$. For example, 53 = 5(10) + 3. Similarly, $yx = 10y + x$. Since $yx > xy$, we get $10y + x > 10x + y$. Subtracting x and y from both sides of this equation yields $9y > 9x$. Dividing this equation by 9 yields $y > x$. Now, if $x = 9$, then the inequality $y > x$ cannot be satisfied. The answer is (E).

---

7. Since angles A, B, and C are the interior angles of the triangle, their angle sum is 180°. Hence, $A + B + C = 180$. Since A and y are vertical angles, they are equal. This is also true for angles B and z and angles C and x. Substituting these values into the equation yields $y + z + x = 180$. The answer is (C).

---

8. Twenty dollars is too large. The discount was only 20 percent—eliminate (A). Both (D) and (E) are impossible since they are less than the selling price—eliminate. 12 is the eye-catcher: 20% of 10 is 2 and 10 + 2 = 12. This is too easy for a hard problem—eliminate. Thus, by process of elimination, the answer is (B).

Let's also solve this problem directly. Let x stand for the origianl price. Then the discount is $20\%x$. Subtracting this discount (mark down) from the origianl price ($10) gives

$$x - 20\%x = 10$$

or

$$x - 0.2x = 10$$

or

$$0.8x = 10$$

or

$$x = 10/0.8 = 12.5$$

The answer is (B).

---

9. Adding the given inequalities $x > y > 0$ and $p > q > 0$ yields

$$x + p > y + q > 0$$

Since $y + q$ is positive, dividing the inequality by $y + q$ will not reverse the inequality:

$$\frac{x+p}{y+q} > \frac{y+q}{y+q}$$

$$\frac{x+p}{y+q} > 1$$

Hence, the answer is (A).

---

10. In a triangle, the sum of the interior angles is 180 degrees. Applying this to Triangle *ADC* yields

$$\angle DAC + \angle C + \angle CDA = 180$$
$$45 + \angle C + 90 = 180 \qquad \text{since } \angle DAC = 45^\circ \text{ and } \angle CDA = 90^\circ$$
$$\angle C = 180 - 90 - 45 = 45$$

In Triangle *ABC*, *AB* = *AC*. Recall that angles opposite equal sides of a triangle are equal. Hence, $\angle B = \angle C$. We have already derived that $\angle C = 45^\circ$. Hence, $\angle B = \angle C = 45^\circ$. Again, the sum of the interior angles of a triangle is 180 degrees. Applying this to Triangle *ABC* yields

$$\angle A + \angle B + \angle C = 180$$
$$\angle A + 45 + 45 = 180$$
$$\angle A = 90$$

This implies that Triangle *ABC* is a right triangle with right angle at *A*. Hence, the area of the triangle is

$$\frac{1}{2}\left(\text{the product of the sides containing the right angle}\right) =$$
$$\frac{1}{2}AB \cdot AC =$$
$$\frac{1}{2}10 \cdot 10 =$$
$$50$$

The answer is (B).

---

11. Let's take the first number in each pair, form its reciprocal, and then try to reduce it to the second number. Now, $1 \Rightarrow \frac{1}{1} = 1$. Hence, the pair 1 and 1 are reciprocals of each other. Next,

$$\frac{1}{11} \Rightarrow \frac{1}{\frac{1}{11}} = 1 \cdot \frac{11}{1} = 11 \neq -11$$

Hence, the pair $\frac{1}{11}$ and –11 are not reciprocals of each other. Finally,

$$\sqrt{5} \Rightarrow \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

Hence, the pair $\sqrt{5}$ and $\frac{\sqrt{5}}{5}$ are reciprocals of each other. The answer is (D).

---

12. Start with the bottom equation $3y = 9 - 6x$:

Dividing by 3 yields

$$y = 3 - 2x$$

Adding $2x$ yields

$$2x + y = 3$$

Notice that this is the top equation in the system. Hence, the system is only one equation in two different forms. Thus, there are an infinite number of solutions. For example, the pair $x = 2$, $y = -1$ is a solution as is the pair $x = 0$, $y = 3$. The answer is (E).

13. Often on the SAT you will be given numbers in different units. When this occurs, you must convert the numbers into the same units. (This is obnoxious but it does occur on the SAT, so be alert to it.) In this problem, we must convert 15 minutes into hours: $15 \cdot \frac{1}{60} = \frac{1}{4}\ hr$. Hence, the average speed is

$$\frac{\text{Total Distance}}{\text{Total Time}} = \frac{x}{y + \frac{1}{4}}$$

The answer is (C).

14. Let's write the equation of the line, using the slope-intercept form, $y = mx + b$. Since the line passes through the origin, $b = 0$. This reduces the equation to $y = mx$. Calculating the slope between (2, 1) and (0, 0) yields

$$m = \frac{1-0}{2-0} = \frac{1}{2}$$

Plugging this into the equation yields $y = \frac{1}{2}x$. Since $x = 4$, we get $y = \frac{1}{2} \cdot 4 = 2$. The answer is (B).

15. Most students struggle with this type of problem, and the SAT considers them to be difficult. However, if you can identify whether a problem is a direct proportion or an inverse proportion, then it is not so challenging. In this problem, as the number of widgets increases so does the absolute cost. This is a direct proportion, and therefore we set ratios equal:

$$\frac{w}{d} = \frac{2000}{x}$$

Cross multiplying yields

$$w \cdot x = 2000 \cdot d$$

Dividing by w yields

$$x = \frac{2000d}{w}$$

The answer is (C).

16. Plugging $x = 4$ into the expression $-2^{2\sqrt{x}} + 2$ yields

$$-2^{2\sqrt{4}} + 2 = -2^{2 \cdot 2} + 2 = -2^4 + 2 = -16 + 2 = -14$$

The answer is (A).

17. The shaded region is entirely within the third quadrant. Now, both coordinates of any point in Quadrant III are negative. The only point listed with both coordinates negative is (–1, –6). The answer is (E).

18.

$$p = \sqrt{2pq - q^2}$$

$p^2 = 2pq - q^2$ by squaring both sides

$p^2 - 2pq + q^2 = 0$ by subtracting $2pq$ and adding q^2 to both sides

$(p - q)^2 = 0$ by the formula $x^2 - 2xy + y^2 = (x - y)^2$

$p - q = 0$ by taking the square root of both sides

$p = q$ by adding q to both sides

The answer is (A).

19. $\frac{b-a}{a} = \frac{b}{a} - \frac{a}{a} = \frac{b}{a} - 1 = \frac{-3}{2} - 1 = \frac{-3}{2} - \frac{2}{2} = \frac{-3-2}{2} = \frac{-5}{2}$. The answer is (A).

20. In a triangle, the sum of any two sides is greater than the third side. Hence, $x + y > z$. We are given $y/x = 3$. Multiplying both sides of this equation by x yields $y = 3x$. Substituting this into the inequality $x + y > z$, we get $x + 3x > z$, or $4x > z$. Hence, the answer is (A).

Answers and Solutions Section 2:

1. A	**6. C**	**11. 999**	**16. 7**
2. C	**7. E**	**12. 15**	**17. 40**
3. A	**8. D**	**13. 10**	**18. 6**
4. A	**9. 30**	**14. 72**	
5. C	**10. 125**	**15. 1**	

---

1. Whatever the coordinates of *P* are, the line *OP* is the hypotenuse of a right triangle with sides being the absolute value of the *x* and *y* coordinates. Hence, *OP* is greater than the *y*-coordinate of point *P*. The answer is (A).

This problem brings up the issue of how much you can assume when viewing a diagram. We are told that *P* is a point in the coordinate system and that it appears in the second quadrant. Could *P* be on one of the axes or in another quadrant? No. Although *P* could be anywhere in Quadrant II (not necessarily where it is displayed), *P* could not be on the *y*-axis because the "position of points, angles, regions, etc. can be assumed to be in the order shown." If *P* were on the *y*-axis, then it would not be to the left of the *y*-axis, as it is in the diagram. That is, the order would be different.

2.

INTEREST PROBLEMS

These problems are based on the formula

INTEREST = AMOUNT × TIME × RATE

Often, the key to these problems is that the interest earned from one account plus the interest earned from another account equals the total interest earned:

Total Interest = (Interest from first account) + (Interest from second account)

Solution: Let x be the amount deposited at 5%. Then $1200 - x$ is the amount deposited at 7%. The interest on these investments is $.05x$ and $.07(1200 - x)$. Since the total interest is \$72, we get

$$.05x + .07(1200 - x) = 72$$
$$.05x + 84 - .07x = 72$$
$$-.02x + 84 = 72$$
$$-.02x = -12$$
$$x = 600$$

The answer is (C).

---

3. Since *"each number above the bottom row is equal to three times the number immediately below it,"*

$$x = 3(-18) = -54 \text{ and } y = 3(3) = 9$$

Hence, $x + y = -54 + 9 = -45$. The answer is (A).

---

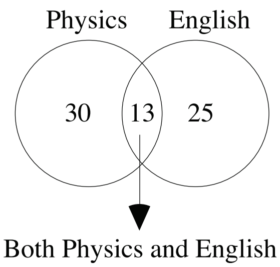
4. The shortest distance between two points is along the line joining them. So, the lengths of the arcs *PQ*, *QR*, *RS*, and *SP* are greater than the lengths of the sides *PQ*, *QR*, *RS*, and *SP*, respectively. The circumference of the circle is the sum of lengths of the arcs *PQ*, *QR*, *RS*, and *SP*, and the perimeter of the square is the sum of the sides *PQ*, *QR*, *RS*, and *SP*. Since each arc is greater than the corresponding side, the circumference of the circle must be greater than the perimeter of the square. Hence, the answer is (A).

5. Since the question asks for the number of revolutions in t seconds, we need to find the number of revolutions per second and multiply that number by t. Since the wheel is spinning at 1200 revolutions per minute and there are 60 seconds in a minute, we get

$$\frac{1200 \text{ revolutions}}{60 \text{ seconds}} = 20\,\text{rev/sec}$$

Hence, in t seconds, the wheel will make $20t$ revolutions. The answer is (C).

6. First display the information in a Venn diagram:



Adding the number of students taking physics and the number of students taking English and then subtracting the number of students taking both yields $(30 + 25) = -13 = 42$. This is the number of students enrolled in *either* physics or English or both. The total school enrollment is 90, so forming the ratio yields

$$\frac{\textit{physics or math enrollment}}{\textit{total enrollment}} = \frac{42}{90} \approx .47 = 47\%$$

The answer is (C).

7. The median in all five answer-choices is 10. By symmetry, the average in answer-choices (A), (C), and (D) is 10 as well. The average in choice (B) is larger than 10 because 13 is further away from 10 than 8 is. Similarly, the average in choice (E) is smaller than 10 because 7 is further away from 10 than 12 is. The exact average is $\frac{7+9+10+11+12}{5} = \frac{49}{5} < 10$. The answer is (E).

8. Since x and y are consecutive integers, one of them must be even. Hence, the product xy is even and Statement I is true. As to Statement II, suppose z is odd, then x must be odd as well. Now, the difference of two odd numbers is an even number. Next, suppose z is even, then x must be even as well. Now, the difference of two even numbers is again an even number. Hence, Statement II is true. As to Statement III, let $x = 1$, then $z = 3$ and $x^z = 1^3 = 1$, which is odd. Thus, Statement III is not necessarily true. The answer is (D).

9. Observe that $\angle DBA$ is an exterior angle of Triangle ABC. Applying the exterior angle theorem yields

$\angle DBA = \angle A + \angle C$	
$y + 30 = (y - 15) + (y + 15)$	
$y + 30 = 2y$	by adding like terms
$30 = y$	by subtracting y from both sides

Grid in 30.

10. Since the store would have made a profit of 20 percent on the wholesale cost, the original price P of the dress was 120 percent of the cost: $P = 1.2C$. Now, translating *"After reducing the asking price by 10 percent, the dress sold for a net profit of 10 dollars"* into an equation yields:

$$P - .1P = C + 10$$

Simplifying gives

$$.9P = C + 10$$

Solving for P yields

$$P = \frac{C + 10}{.9}$$

Plugging this expression for P into $P = 1.2C$ gives

$$\frac{C + 10}{.9} = 1.2C$$

Solving this equation for C yields $C = 125$. Grid in 125.

---

11. Since the number of integers between two integers inclusive is one more than their difference, we have $(10^3 - 2) + 1 = (1000 - 2) + 1 = 999$ integers. Grid in 999.

---

12. Since *"the capacity of glass X is 80 percent of the capacity of glass Y,"* we get

$$X = .8Y$$

Since *"glass X contains 6 ounces of punch and is half-full,"* the capacity of glass X is 12 ounces. Plugging this into the equation yields

$$12 = .8Y$$

$$12/.8 = Y$$

$$15 = Y$$

Hence, glass Y contains 15 – 6 = 9 more ounces of punch than glass X. Grin in 15.

---

13. The figure shows that the circle is located between the lines $y = 4$ and $y = -4$ and that the circle is symmetric to x-axis. From this, we make two observations:

1) The center of the circle is on the x-axis.
2) The diameter of the circle is 8.

Since the center of the circle is on the x-axis, the points (2, 0) and $(x, 0)$ must be diametrically opposite points of the circle. That is, they are end points of a diameter of the circle. Hence, the distance between the two points, $x - 2$, must equal the length of the diameter. Hence, $x - 2 = 8$. Adding 2 to both sides of this equation, we get $x = 10$. The answer is (D).

14. A number divisible by all three numbers 2, 3, and 4 is also divisible by 12. Hence, each number can be written as a multiple of 12. Let the first number be represented as $12a$ and the second number as $12b$. Assuming $a > b$, the difference between the two numbers is $12a - 12b = 12(a - b)$. Observe that this number is also a multiple of 12. Hence, the answer must also be divisible by 12. Since 72 is the only answer-choice divisible by 12, grid in 72.

15. Let x denote the number of apples bought, and let y denote the number of oranges bought. Then, translating the sentence *"Steve bought some apples at a cost of \$.60 each and some oranges at a cost of \$.50 each"* into an equation yields

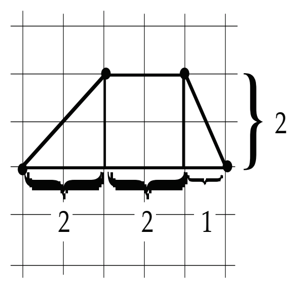
$$.60x + .50y = 4.10$$

Since there are two variables and only one equation, the key to this problem is finding a second equation that relates x and y. Since he bought a total of 8 apples and oranges, we get

$$x + y = 8$$

Solving this system yields $x = 1$. Hence, he bought one apple, so grid in 1.

16. Dividing the polygon into triangles and squares yields



The triangle furthest to the left has area

$$A = \frac{1}{2}bh = \frac{1}{2} \cdot 2 \cdot 2 = 2$$

The square has area $A = s^2 = 2^2 = 4$

The triangle furthest to the right has area

$$A = \frac{1}{2} \cdot 1 \cdot 2 = 1$$

The sum of the areas of these three figures is $2 + 4 + 1 = 7$. Grin in 7.

17. Recall the formula *Distance* = *Rate* × *Time*, or $D = R \cdot T$. From the second sentence, we get for Cyclist N:

$$D = R \cdot 2$$

Now, Cyclist M traveled at 20 miles per hour and took 4 hours. Hence, Cyclist M traveled a total distance of

$$D = R \cdot T = 20 \cdot 4 = 80 \text{ miles}$$

Since the cyclists covered the same distance at the moment they met, we can plug this value for D into the equation $D = R \cdot 2$:

$$80 = R \cdot 2$$
$$40 = R$$

Grid in 40.

18. This is a hard problem. Let x be the number of people who own both types of cars. Then the number of people who own only Fords is $16 - x$, and the number of people who own only Toyotas is $11 - x$. Adding these two expressions gives the number of people who own only one of the two types of cars, which we are are told is 15:

$$(16 - x) + (11 - x) = 15$$

Add like terms:

$$27 - 2x = 15$$

Subtract 27 from both sides of the equation:

$$-2x = -12$$

Finally, divide both sides of the equation by –2:

$$x = 6$$

Grid in 6.

Answers and Solutions Section 3:

1. B	**5. B**	**9. D**	**13. E**
2. B	**6. C**	**10. D**	**14. A**
3. C	**7. C**	**11. A**	**15. E**
4. C	**8. D**	**12. B**	**16. E**

---

1. Since we have a right triangle, the Pythagorean Theorem yields

$$y^2 + 3^2 = 6^2$$

Simplifying yields

$$y^2 + 9 = 36$$

Subtracting 9 from both sides yields

$$y^2 = 27$$

Taking the square root of both sides yields

$$y = \sqrt{27}$$

The answer is (B).

---

2. Since the store would have made a profit of 20 percent on the wholesale cost, the original price P of the dress was 120 percent of the cost: $P = 1.2C$. Now, translating *"After reducing the asking price by 10 percent, the dress sold for a net profit of 10 dollars"* into an equation yields:

$$P - .1P = C + 10$$

Simplifying gives

$$.9P = C + 10$$

Solving for P yields

$$P = \frac{C + 10}{.9}$$

Plugging this expression for P into $P = 1.2C$ gives

$$\frac{C + 10}{.9} = 1.2C$$

Solving this equation for C yields $C = 125$. The answer is (B).

3. Cross multiplying the equation $a/b = b/c$ yields

$$ac = b^2$$

Dividing by a yields

$$c = b^2/a$$

We are given that a is a perfect square. Hence, $a = k^2$, for some number k. Replacing a in the bottom equation with k^2, we get

$$c = b^2/k^2 = (b/k)^2$$

Since we have written c as the square of a number, it is a perfect square. The answer is (C).

---

4. Recall that a triangle is a right triangle if and only if the square of the longest side is equal to the sum of the squares of the shorter sides (Pythagorean Theorem). Hence, $c^2 = 6^2 + 8^2$ implies that the triangle is a right triangle. So the area of the triangle is $\frac{1}{2} \cdot 6 \cdot 8 = 24$. The answer is (C).

---

5. The first term is even, and all subsequent terms are found by multiplying a number by 2. Hence, all terms of the sequence are even. The answer is (B). Following is the sequence:

2, 10, 26, 58, . . .

---

6. The x jars have $15x$ marbles, and the $3x$ jars have $20 \cdot 3x = 60x$ marbles. Hence, there is a total of $15x + 60x = 75x$ marbles. The answer is (C).

---

7. If $x = 3$ and $y = 2$, then $x - y = 3 - 2 = 1$. This eliminates (A). If $x = 5$ and $y = 3$, then $x - y = 5 - 3 = 2$. This eliminates (B). If $x = 17$ and $y = 3$, then $x - y = 17 - 3 = 14$. This eliminates (D). If $x = 23$ and $y = 3$, then $x - y = 23 - 3 = 20$. This eliminates (E). Hence, by process of elimination, the answer is (C).

Method II (without substitution): Suppose $x - y = 13$. Now, let x and y be distinct prime numbers, both greater than 2. Then both x and y are odd numbers since the only even prime is 2. Hence, $x = 2k + 1$, and $y = 2h + 1$, for some positive integers k and h. And $x - y = (2k + 1) - (2h + 1) = 2k - 2h = 2(k - h)$. Hence, $x - y$ is even. This contradicts the assumption that $x - y = 13$, an odd number. Hence, x and y cannot both be greater than 2. Next, suppose $y = 2$, then $x - y = 13$ becomes $x - 2 = 13$. Solving yields $x = 15$. But 15 is not prime. Hence, there does not exist prime numbers x and y such that $x - y = 13$. The answer is (C).

---

8. The area of a circle is πr^2 (where r is the radius), or $\pi\left(\frac{d}{2}\right)^2$ (where d is the diameter). This formula yields

$$<4> \cdot <6> = \pi\left(\frac{4}{2}\right)^2 \cdot \pi\left(\frac{6}{2}\right)^2 = \pi 4 \cdot \pi 9 = 36\pi^2$$

Now, $\pi \cdot <12> = \pi \cdot \pi\left(\frac{12}{2}\right)^2 = \pi^2 6^2 = 36\pi^2$. Hence, the answer is (D).

9. Since the ratio of x to y is 2, we get $x/y = 2$. Solving this equation for x yields $x = 2y$. Since the sum of the angles made by a line is 180°, $y + x + y = 180$. Substituting $2y$ for x in this equation yields

$$y + 2y + y = 180$$
$$4y = 180$$
$$y = 45$$

The answer is (D).

10. Let the distance Jennifer walks be x. Then since they are 4 miles apart, Alice will walk $4 - x$ miles. The key to this problem is that when they meet each person will have walked for an equal amount of time. Solving the equation $D = R \times T$ for T yields $T = \frac{D}{R}$. Hence,

$$\frac{x}{3} = \frac{4 - x}{2}$$
$$2x = 3(4 - x)$$
$$2x = 12 - 3x$$
$$5x = 12$$
$$x = 12/5$$

Therefore, the time that Jennifer walks is $T = \frac{D}{R} = \frac{12/5}{3} = \frac{12}{5} \times \frac{1}{3} = \frac{4}{5}$ of an hour. Converting this into minutes gives $\frac{4}{5} \times 60 = 48$ minutes. The answer is (D).

11. We are told that the remainder is 7 when the number is divided by 12. Hence, we can represent the number as $12x + 7$. Now, 7 can be written as $6 + 1$. Plugging this into the expression yields

$12x + (6 + 1) =$	
$(12x + 6) + 1 =$	by regrouping
$6(2x + 1) + 1$	by factoring 6 out of the first two terms

This shows that the remainder is 1 when the expression $12x + 7$ is divided by 6. The answer is (A).

Method II (Substitution):
Choose the number 19, which gives a remainder of 7 when divided by 12. Now, divide 19 by 6:

$$\frac{19}{6} =$$
$$3\frac{1}{6}$$

This shows that 6 divides into 19 with a remainder of 1. The answer is (A).

12. Since the coordinates x and y are on the line, we know that $y = x + 2$. Hence, the difference of x and y is

$$x - y = x - (x + 2) = -2$$

The answer is (B).

13. Let's take a two-digit number whose digits add up to 9, say, 72. Adding 10 to this number gives 82. The sum of the digits of this number is 10. Now, let's choose another two-digit number whose digits add up to 9, say, 90. Then $x + 10 = 90 + 10 = 100$. The sum of the digits of this number is 1. Hence, the sum of the numbers is either 1 or 10. The answer is (E).

14. Let t be the time it takes the boys, working together, to assemble the model car. Then their combined rate is $1/t$, and their individual rates are 1/30 and 1/20. Now, their combined rate is merely the sum of their individual rates:

$$\frac{1}{t} = \frac{1}{30} + \frac{1}{20}$$

Solving this equation for t yields $t = 12$. The answer is (A).

15. Although the drawing looks to be an isosceles triangle, that cannot be assumed. We are not given the length of side AC: it could be 4 units long or 100 units long, we don't know. Hence, the answer is (E).

16. Since *"every number in the sequence* –1, 3, 2, . . . *is the sum of the two immediately preceding numbers,"* the forth term of the sequence is $5 = 3 + 2$. The first 12 terms of this sequence are

$$-1, 3, 2, 5, 7, 12, 19, 31, 50, 81, 131, 212, \ldots$$

At least four numbers in this sequence are even: 2, 12, 50, and 212. The answer is (E).

Answers and Solutions Section 1:

1.	**A**	**6.**	**D**	**11.**	**D**	**16.**	**B**
2.	**B**	**7.**	**A**	**12.**	**A**	**17.**	**B**
3.	**B**	**8.**	**C**	**13.**	**D**	**18.**	**C**
4.	**D**	**9.**	**B**	**14.**	**E**	**19.**	**E**
5.	**B**	**10.**	**A**	**15.**	**C**	**20.**	**A**

---

1. Since the diameter of circle P is 2, its radius is 1. So the area of circle P is $\pi(1)^2 = \pi$. Since the diameter of circle Q is 1, its radius is 1/2. So the area of circle Q is $\pi\left(\frac{1}{2}\right)^2 = \frac{1}{4}\pi$. The area of the shaded region is the difference between the area of circle P and the area of circle Q: $\pi - \frac{1}{4}\pi = \frac{3}{4}\pi$. The answer is (A).

---

2. Aligning the system of inequalities vertically yields

$$2x + y > m$$
$$2y + x < n$$

Multiplying both sides of the bottom inequality by –1 and flipping the direction of the inequality yields

$$-2y - x > -n$$

Adding this inequality to the top inequality yields

$$(2x + y) + (-2y - x) > m - n$$
$$(2x - x) + (-2y + y) > m - n$$
$$x - y > m - n$$

The answer is (B).

---

3. Observe that all the digits of the dividend 39693 are divisible by 3. So 3 will divide the dividend into such a number that each of its digits will be 1/3 the corresponding digit in the dividend (i.e., 39693). For example, the third digit in the dividend is 6, and hence the third digit in the quotient will be 2, which is 1/3 of 6. Applying the same process to all digits gives the quotient 13231. The answer is (B).

---

4. $\frac{1}{10^9} - \frac{1}{10^{10}} = \frac{1}{10^9} - \frac{1}{10^9} \cdot \frac{1}{10} = \frac{1}{10^9}\left(1 - \frac{1}{10}\right) = \frac{1}{10^9}\left(\frac{9}{10}\right) = \frac{9}{10^{10}}$. The answer is (D).

---

5. For a point to be within a circle, its distance from the center of the circle must be less than the radius of the circle. The distance from (6, 8) to (0, 0) is the radius of the circle:

$$R = \sqrt{(6-0)^2 + (8-0)^2} = \sqrt{36+64} = \sqrt{100} = 10$$

Now, let's calculate the distance between (–7, 7) and (0, 0)

$$D = \sqrt{(-7-0)^2 + (7-0)^2} = \sqrt{49+49} = \sqrt{98} < 10$$

Since the distance D is less than the radius, the point (–7, 7) is within the circle. The answer is (B).

6. Since we are looking for the greatest number of spaces from which not all 8 moves are possible, we can eliminate the greatest number, 56. Now, clearly not all 8 moves are possible from the outer squares, and there are 28 outer squares—not 32. Also, not all 8 moves are possible from the next to outer squares, and there are 20 of them—not 24. All 8 moves are possible from the remaining squares. Hence, the answer is 28 + 20 = 48. The answer is (D). Notice that 56, (32 + 24), is given as an answer-choice to catch those who don't add carefully.

7. Suppose $x = 1$, an integer. Then $2(x + 1) = 2(1 + 1) = 4$. The next two integers greater than 4 are 5 and 6, and their product is 30. Now, check which of the answer-choices equal 30 when $x = 1$. Begin with (A):

$$4x^2 + 14x + 12 = 4(1)^2 + 14 \cdot 1 + 12 = 30$$

No other answer-choice equals 30 when $x = 1$. Hence, the answer is (A).

8. $\underset{0,1,a}{\longleftrightarrow} = (0 - 1)a = -a$, and $\underset{1,a,0}{\longleftrightarrow} = (1 - a)0 = 0$. Setting these results equal to each other yields $-a = 0$. Multiplying by -1 yields $a = 0$. Hence, the answer is (C).

9. In the ordered set of integers from 1 through 999, every third integer is a multiple of 3. Hence, the number of integers in this set of 999 integers that are multiples of 3 is 999/3 = 333. The answer is (B).

10. Since x is the radius of the larger circle, the area of the larger circle is πx^2. Since x is the diameter of the smaller circle, the radius of the smaller circle is $x/2$. Therefore, the area of the smaller circle is $\pi\left(\frac{x}{2}\right)^2 = \pi\frac{x^2}{4}$. Subtracting the area of the smaller circle from the area of the larger circle gives

$$\pi x^2 - \pi\frac{x^2}{4} = \frac{4}{4}\pi x^2 - \pi\frac{x^2}{4} = \frac{4\pi x^2 - \pi x^2}{4} = \frac{3\pi x^2}{4}$$

The answer is (A).

11. Observe that n and $(n + 1)$ are consecutive integers. Hence, one of the numbers is even. Therefore, the 2 in the denominator divides evenly into either n or $(n + 1)$, eliminating 2 from the denominator. Thus, S can be reduced to a product of two integers. Remember, a prime number cannot be written as the product of two integers (other than itself and 1). Hence, S is not a prime number, and the answer is (D).

12. $[2] = 2^2 - 2 = 2$, and $[x] = x^2 - 2$. Substituting these values into the equation $[2] - [x] = x^2$ yields

$$2 - (x^2 - 2) = x^2$$
$$2 - x^2 + 2 = x^2$$
$$4 - x^2 = x^2$$
$$4 = 2x^2$$
$$2 = x^2$$
$$\sqrt{2} = x$$

The answer is (A).

13. Recall that $Average\ Speed = \dfrac{Total\ Distance}{Total\ Time}$. Now, the setup to the question gives the total time for the trip—30 minutes. Hence, to answer the question, we need to find the distance of the trip.

Let t equal the time for the first half of the trip. Then since the whole trip took 30 minutes (or 1/2 hour), the second half of the trip took $\dfrac{1}{2} - t$ hours. Now, from the formula $Distance = Rate \times Time$, we get for the first half of the trip:

$$\frac{d}{2} = 50 \cdot t$$

And for the second half of the trip, we get

$$\frac{d}{2} = 60\left(\frac{1}{2} - t\right)$$

Solving this system yields

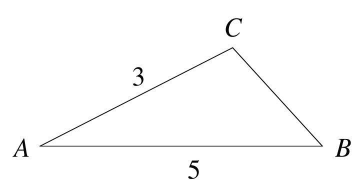
$$d = \frac{300}{11}$$

Hence,

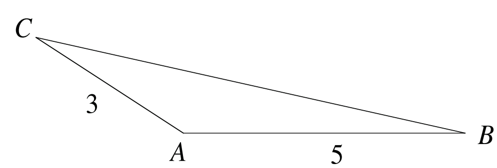
$$Average\ Speed = \frac{\text{Total Distance}}{Total\ Time} = \frac{300/11}{1/2} = \frac{600}{11}$$

The answer is (D).

14. The most natural drawing is the following:



In this case, the length of side BC is less than 7. However, there is another drawing possible, as follows:



In this case, the length of side BC is greater than 7. Hence, there is not enough information to decide, and the answer is (E).

15. Suppose $n = 1$. Then $n^3 = 1^3 = 1$, which is odd. Now, we plug this value for n into each of the answer-choices to see which ones are even. Thus, $2n^2 + 1$ becomes $2(1)^2 + 1 = 3$, which is not even. So eliminate (A). Next, $n^4 = 1^4 = 1$ is not even—eliminate (B). Next, $n^2 + 1 = 1^2 + 1 = 2$ is even, so the answer is possibly (C). Next, $n(n + 2) = 1(1 + 2) = 3$ is not even—eliminate (D). Finally, $n = 1$, which is not even—eliminate (E). Hence, by the process of elimination, the answer is (C).

16. Statement I is false. For instance, $1\Diamond 2 = 1\cdot 2 - \frac{1}{2} = \frac{3}{2}$, but $2\Diamond 1 = 2\cdot 1 - \frac{2}{1} = 0$. This eliminates (A), (D), and (E). Statement II is true: $a\Diamond a = aa - \frac{a}{a} = a^2 - 1 = (a+1)(a-1)$. This eliminates (C). Hence, by process of elimination, the answer is (B). Note: The expression $a \cdot b \neq 0$ insures that neither a nor b equals 0: if $a \cdot b = 0$, then either $a = 0$ or $b = 0$, or both.

17. If the product of the two numbers is odd, then each number in the product must be odd. Recall that the sum of two odd numbers is an even number. The answer is (B).

18. The area of square *PQRS* is $6^2 = 36$. Now, the radius of the circle is 3. (Why?) So the area of the circle is $\pi(3)^2 = 9\pi$. Subtracting the area of the circle from the area of the square yields $36 - 9\pi$. This is the combined area of the regions outside the circle and inside the square. Dividing this quantity by 2 gives $\frac{36-9\pi}{2}$. The answer is (C).

19. Let S be Scott's age and K be Kathy's age. Then translating the sentence *"If Scott is now 5 years older than Kathy, how old is Scott"* into an equation yields

$$S = K + 5$$

Now, Scott's age 7 years ago can be represented as $S = -7$, and Kathy's age can be represented as $K = -7$. Then translating the sentence *"Seven years ago, Scott was 3 times as old as Kathy was at that time"* into an equation yields $S - 7 = 3(K - 7)$.

Combining this equation with $S = K + 5$ yields the system:

$$S - 7 = 3(K - 7)$$
$$S = K + 5$$

Solving this system gives $S = 14\frac{1}{2}$. The answer is (E).

20. Since *"adding any one of the first three terms to the term immediately following it yields w/2,"* we get

$$w + x = w/2$$
$$x + y = w/2$$
$$y + 30 = w/2$$

Subtracting the last equation from the second equation yields $x - 30 = 0$. That is $x = 30$. Plugging $x = 30$ into the first equation yields

$$w + 30 = w/2$$

Multiplying both sides by 2 yields

$$2w + 60 = w$$

Subtracting w from both sides yields

$$w + 60 = 0$$

Finally, subtracting 60 from both sides yields

$$w = -60$$

The answer is (A).

Answers and Solutions Section 2:

1. E	**6. C**	**11. 1**	**16. 1**
2. E	**7. D**	**12. 90**	**17. 40**
3. B	**8. C**	**13. 2/3**	**18. 2**
4. B	**9. 500**	**14. 5**	
5. D	**10. 2**	**15. 15**	

---

1. The number 3 itself is divisible by 3 but not by 2. With this value for x, Choice (A) becomes $\frac{3+1}{2} = \frac{4}{2} = 2$, eliminate.

Choice (C) becomes $\frac{3^2}{3} = \frac{9}{3} = 3$, eliminate.

Choice (D) becomes $\frac{3^3}{3} = \frac{27}{3} = 9$, eliminate.

Next, if $x = 21$, then Choice (B) becomes 21/7 = 3, eliminate.

Hence, by process of elimination, the answer is (E).

---

2. $(x * y) * z = \left(\frac{x}{y}\right) * z = \frac{\left(\frac{x}{y}\right)}{z} = \frac{x}{y} \cdot \frac{1}{z} = \frac{x}{yz}$. Hence, the answer is (E).

---

3. The length of PR is $PR = 3 + 5 = 8$. Applying the Pythagorean Theorem to triangle PRS yields

$$8^2 + (PS)^2 = 10^2$$

Squaring yields

$$64 + (PS)^2 = 100$$

Subtracting 64 from both sides yields

$$(PS)^2 = 36$$

Taking the square root of both sides yields

$$PS = \sqrt{36} = 6$$

Now, applying the Pythagorean Theorem to triangle PQS yields

$$(QS)^2 = 5^2 + 6^2$$

Squaring and adding yields

$$(QS)^2 = 61$$

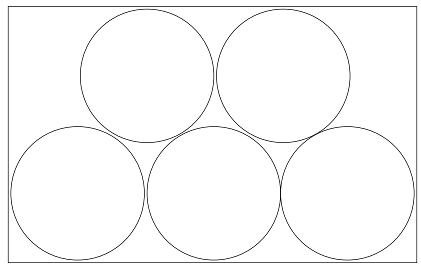
Taking the square root of both sides yields

$$QS = \sqrt{61}$$

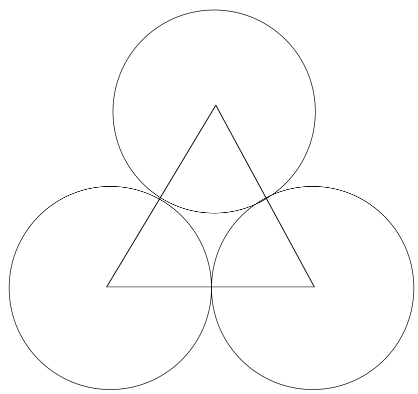
The answer is (B).

4. Since this is a hard problem, we can eliminate (E), "not enough information." And because it is too easily derived, we can eliminate (C), (8 = 4 + 4). Further, we can eliminate (A), 5, because answer-choices (B) and (D) form a more complicated set. At this stage we cannot apply any more elimination rules; so if we could not solve the problem, we would guess either (B) or (D).

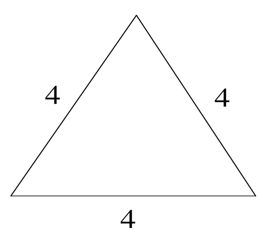
Let's solve the problem directly. The drawing below shows the position of the circles so that the paper width is a minimum.



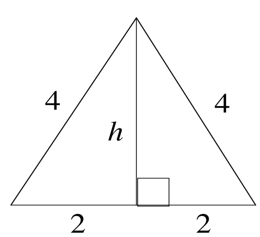
Now, take three of the circles in isolation, and connect the centers of these circles to form a triangle:



Since the triangle connects the centers of circles of diameter 4, the triangle is equilateral with sides of length 4.



Drawing an altitude gives



Applying the Pythagorean Theorem to either right triangle gives

$$h^2 + 2^2 = 4^2$$

Squaring yields

$$h^2 + 4 = 16$$

Subtracting 4 from both sides of this equation yields

$$h^2 = 12$$

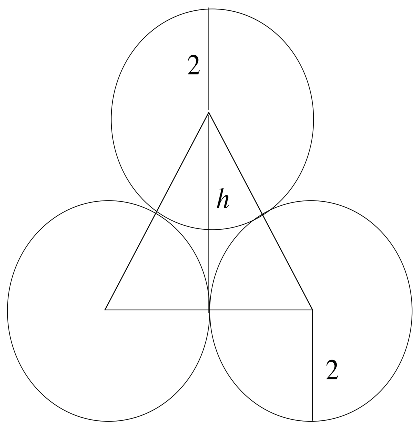
Taking the square root of both sides yields

$$h = \sqrt{12} = \sqrt{4 \cdot 3}$$

Removing the perfect square 4 from the radical yields

$$h = 2\sqrt{3}$$

Summarizing gives



Adding to the height, $h = 2\sqrt{3}$, the distance above the triangle and the distance below the triangle to the edges of the paper strip gives

$$width = (2 + 2) + 2\sqrt{3} = 4 + 2\sqrt{3}$$

The answer is (B).

---

5. Solution:

$$3^{n-1} = 3^{3n+1}$$

$$n - 1 = 3n + 1$$

$$-2n = 2$$

$$n = -1$$

Since $n = -1$,

$$m = 3^{n-1} = 3^{-1-1} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

Hence, $\frac{m}{n} = \frac{\frac{1}{9}}{-1} = -\frac{1}{9}$, and the answer is (D).

6. Let x be the amount of nuts at 80 cents a pound. Then $10 - x$ is the amount of nuts at 60 cents a pound. The cost of the 80-cent nuts is $80x$, the cost of the 60-cent nuts is $60(10 - x)$, and the cost of the mixture is 70(10) cents. Since the cost of the mixture is the sum of the costs of the 70- and 80-cent nuts, we get

$$80x + 60(10 - x) = 70(10)$$

Solving this equation for x yields $x = 5$. The answer is (C).

7. Let the three consecutive integers be x, $x + 1$, and $x + 2$. The sum of these integers is $3x + 3$. According to the question, this sum is odd. Hence $3x + 3$ is odd. Recall that if the sum of two integers is odd, then one of the integers is odd and the other one is even. Since 3 in the expression $3x + 3$ is odd, $3x$ must be even. Now, recall that the product of two numbers is odd only when one of the numbers is odd and the other is even. So x must be even. If x is an even number, then $x + 2$ is also even. Thus, the first and the last integers must both be even. The answer is (D).

8. In the figure, it appears that the small inscribed triangle divides the large triangle into four congruent triangles. Hence, the probability that a point chosen at random from the large triangle will also be from the small triangle is 1/4. (As an exercise, prove that the small inscribed triangle divides the large triangle into four congruent triangles.) The answer is (C).

9. There are 8.5 apples in the picture. Dividing the total number of apples by 8.5 yields 4250/8.5 = 500. Grid in 500.

10. We are given that l, m, and n are three positive integers such that $l < m < n$. This implies that l, m, and n are each greater than zero and not equal to each other. Since n is less than 4, the numbers l, m, and n must have the values 1, 2, and 3, respectively. Grid in 2.

11. $\left(64^*\right)^* = \left(\frac{\sqrt{64}}{2}\right)^* = \left(\frac{8}{2}\right)^* = 4^* = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$. Grid in 1.

12. **Method I:**
Recall that when a quadratic function is written in the form $y = a(x-h)^2 + k$, its vertex (in this case, the maximum height of the projectile) occurs at the point (h, k). So let's rewrite the function $h(t) = p - 10(q-t)^2$ in the form $h(t) = a(t-h)^2 + k$. Notice that we changed y to $h(t)$ and x to t.

$$\begin{aligned} h(t) &= p - 10(q-t)^2 \\ &= -10(q-t)^2 + p \\ &= -10(-[-q+t])^2 + p \\ &= -10(-[t-q])^2 + p \\ &= -10[-1]^2[t-q]^2 + p \\ &= -10([+1][t-q])^2 + p \\ &= -10(t-q)^2 + p \end{aligned}$$

In this form, we can see that the vertex (maximum) occurs at the point (q, p). We are given that the maximum height of 100 occurs when t is 3. Hence, $q = 3$ and $p = 100$. Plugging this into our function yields

$$h(t) = -10(t-q)^2 + p = -10(t-3)^2 + 100$$

We are asked to find the height of the projectile when $t = 4$. Evaluating our function at 4 yields

$$\begin{aligned} h(4) &= -10(4-3)^2 + 100 \\ &= -10(1)^2 + 100 \\ &= -10 \cdot 1 + 100 \\ &= -10 + 100 \\ &= 90 \end{aligned}$$

The answer is (E).

Method II:
In this method, we are going to solve a system of two equations in two unknowns in order to determine the values of p and q in the function $h(t) = p - 10(q-t)^2$. At time $t = 0$, the projectile had a height of 10 feet. In other words, $h(0) = 10$. At time $t = 3$, the projectile had a height of 100 feet. In other words, $h(3) = 100$. Plugging this information into the function $h(t) = p - 10(q-t)^2$ yields

$$\begin{aligned} h(0) = 10 \quad &\Rightarrow \quad 10 = p - 10(q-0)^2 \\ h(3) = 100 \quad &\Rightarrow \quad 100 = p - 10(q-3)^2 \end{aligned}$$

Now, we solve this system of equations by subtracting the bottom equation from the top equation:

$$\begin{aligned} 10 &= p - 10q^2 \\ (-) \quad 100 &= p - 10(q-3)^2 \\ \hline -90 &= -10q^2 + 10(q-3)^2 \end{aligned}$$

Solving this equation for q yields

$$\begin{aligned} -90 &= -10q^2 + 10(q-3)^2 \\ -90 &= -10q^2 + 10(q^2 - 6q + 9) \\ -90 &= -10q^2 + 10q^2 - 60q + 90 \\ -90 &= -60q + 90 \\ -180 &= -60q \\ 3 &= q \end{aligned}$$

Plugging $q = 3$ into the equation $10 = p - 10q^2$ yields

$$\begin{aligned} 10 &= p - 10 \cdot 3^2 \\ 10 &= p - 10 \cdot 9 \\ 10 &= p - 90 \\ 100 &= p \end{aligned}$$

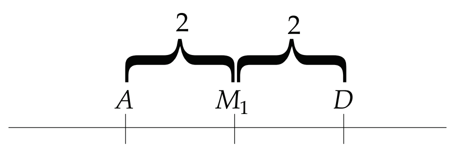
Hence, the function $h(t) = p - 10(q - t)^2$ becomes $h(t) = 100 - 10(3 - t)^2$. We are asked to find the height of the projectile when $t = 4$. Evaluating this function at 4 yields

$$\begin{aligned} h(4) &= 100 - 10(3 - 4)^2 \\ &= 100 - 10(-1)^2 \\ &= 100 - 10 \cdot 1 \\ &= 100 - 10 \\ &= 90 \end{aligned}$$

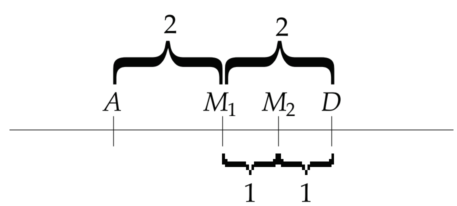
Grid in 90.

---

13. Let 4 be the length of line segment AD. Since M_1 is the midpoint of AD, this yields

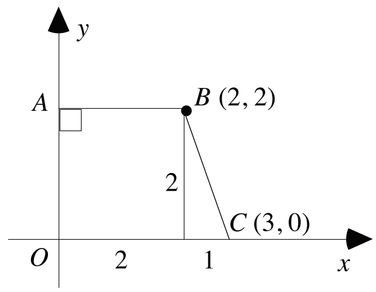


Now, since M_2 is the midpoint of M_1D, this yields



From the diagram, we see that $M_1D = 2$ and $AM_2 = 3$. Hence, $\dfrac{M_1D}{AM_2} = \dfrac{2}{3}$. Grid in 2/3.

14. Dropping a vertical line from point B perpendicular to the x-axis will form a square and a triangle:



From the figure, we see that the square has area $s^2 = 2^2 = 4$, and the triangle has area

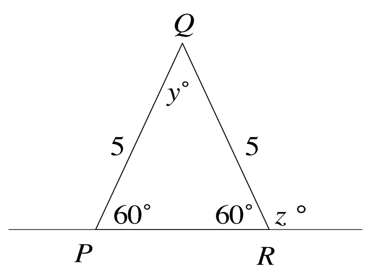
$$\frac{1}{2}bh = \frac{1}{2}\cdot 1\cdot 2 = 1$$

Hence, the area of the quadrilateral is 4 + 1 = 5. Grid in 5. Note, with this particular solution, we did not need to use the properties of the diagonal line in the original diagram.

15. Let C be Carrie's age. Then Tom's age is $C + 10$. Now, 5 years ago, Carrie's age was $C - 5$ and Tom's age was $(C + 10) - 5 = C + 5$. Since at that time, Tom was twice as old as Carrie, we get $5 + C = 2(C - 5)$. Solving this equation for C yields $C = 15$. Grid in 15.

16. $(2@8)-(3@3)=\sqrt{2\cdot 8}-\sqrt{3\cdot 3}=\sqrt{16}-\sqrt{9}=4-3=1$. Grid in 1.

17. Again since the base angles of an isosceles triangle are equal, the diagram becomes



Since x and z form a straight angle, $x + z = 180$. Hence, we have the system:

$$x + z = 180$$
$$y + z = 150$$

Subtracting these equations yields $x - y = 30$. Since there are two variables and only one equation, we need another equation in order to determine y. However, since the angle sum of a triangle is 180°, $x + x + y = 180$, or $2x + y = 180$. This yields the system:

$$x - y = 30$$
$$2x + y = 180$$

Adding the equations gives $3x = 210$. Hence, $x = 70$. Plugging this value for x back into either equation gives $y = 40$. Grid in 40.

18.

$$(2 - x)^* = (x - 2)^*$$
$$2 - (2 - x) = 2 - (x - 2)$$
$$2 - 2 + x = 2 - x + 2$$
$$x = 4 - x$$
$$2x = 4$$
$$x = 2$$

Grid in 2.

Answers and Solutions Section 3:

1.	**B**	**5.**	**E**	**9.**	**B**	**13.**	**D**
2.	**E**	**6.**	**C**	**10.**	**D**	**14.**	**A**
3.	**D**	**7.**	**E**	**11.**	**C**	**15.**	**E**
4.	**B**	**8.**	**E**	**12.**	**C**	**16.**	**A**

---

1. We could solve the equation, but it is much faster to just plug in the answer-choices. Begin with 0:

$$x^4 - 2x^2 = 0^4 - 2 \cdot 0^2 = 0 - 0 = 0$$

Hence, eliminate (A). Next, plug in 1:

$$x^4 - 2x^2 = 1^4 - 2 \cdot 1^2 = 1 - 2 = -1$$

Hence, the answer is (B).

---

2. $<x + 2> - <x - 2> = ([x + 2] + 2)[x + 2] - ([x - 2] + 2)[x - 2]$

$= (x + 4)[x + 2] - x[x - 2]$

$= x^2 + 6x + 8 - (x^2 - 2x)$

$= x^2 + 6x + 8 - x^2 + 2x$

$= 8x + 8$

$= 8(x + 1)$

The answer is (E).

---

3. Let s denote the length of a side of square $ABCD$. Since the area of the square is 16, we get $s^2 = 16$. Taking the square root of both sides of this equation yields $s = 4$. Hence, line segment AB has length 4. Since the parabola is symmetric about the y-axis, Point B is 2 units from the y-axis (as is Point A). That is, the x-coordinate of Point B is 2. Since line segment BC has length 4, the coordinates of Point B are (2, 4). Since the square and the parabola intersect at Point B, the point (2, 4) must satisfy the equation $y = a - x^2$:

$$4 = a - 2^2$$
$$4 = a - 4$$
$$8 = a$$

The answer is (D).

---

4. Since $\angle POQ = 70^\circ$, we get $x + y + 20 = 70$. Solving this equation for y yields $y = 50 - x$. Now, we are given that $x > 15$. Hence, the expression $50 - x$ must be less than 35:

$$x > 15$$
$$-x < -15$$
$$50 - x < 50 - 15$$
$$50 - x < 35$$

The answer is (B).

5. The clause "p/19 is 1 less than 3 times q/19" translates into:

$$\frac{p}{19} = 3 \cdot \frac{q}{19} - 1$$

Multiplying both sides of this equation by 19 gives

$$p = 3 \cdot q - 19$$

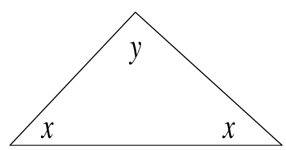
The answer is (E).

6. This is a weighted-average problem because more disks were purchased on the second day. Let x be the number of disks purchased on the first day. Then $.50x = 25$. Solving for x yields $x = 50$. Let y be the number of disks purchased on the second day. Then $.30y = 45$. Solving for y yields $y = 150$. Forming the weighted average, we get

$$Average\ \ Cost = \frac{Total\ Cost}{Total\ Number} = \frac{25+45}{50+150} = \frac{70}{200} = .35$$

The answer is (C).

7. Let x and y denote the angles:



Then $x/y = 1/3$ and since the angle sum of a triangle is 180°, $x + x + y = 180$. Solving the first equation for y and plugging it into the second equation yields

$$2x + 3x = 180$$

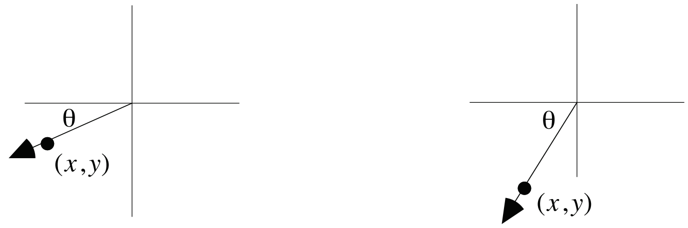
$$5x = 180$$

$$x = 36$$

Plugging this into the equation $x/y = 1/3$ yields $y = 108$. The answer is (E).

8. There are two possible drawings:

Case I | Case II



In Case I, $\theta < 45^\circ$. Whereas, in Case II, $\theta > 45^\circ$. This is a double case, and the answer therefore is (E).

9. Since x is to the left of zero on the number line, it's negative. Since y is to the right of zero, it's positive. Now, the product or quotient of a positive number and a negative number is negative. Hence, Statement I is false and Statement II is true. Regarding Statement III, since x is to the left of y on the number line, $x < y$. Subtracting y from both sides of this inequality yields $x - y < 0$. Hence, Statement III is false. Therefore, the answer is (B).

10. Since x and y are prime numbers and $x > y$, we know that $x > y > 0$. Dividing this inequality by y yields

$$x/y > y/y > 0/y$$

Reducing yields

$$x/y > 1$$

Since x and y are prime numbers, they will not have any common factors that could reduce x/y to an integer. Therefore, x/y is an irreducible fraction greater than one. The answer is (D).

11. $\sqrt{\frac{25+10x+x^2}{2}} = \sqrt{\frac{(5+x)^2}{2}} =$ since $25+10x+x^2$ factors into $(5+x)^2$

$\frac{\sqrt{(5+x)^2}}{\sqrt{2}} =$ by the rule $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

$\frac{5+x}{\sqrt{2}} =$ since $\sqrt{x^2} = x$

$\frac{5+x}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} =$ rationalizing the denominator

$\frac{\sqrt{2}(5+x)}{2}$

Hence, the answer is (C).

12. Suppose the radius of the larger circle is 2 and the radius of the smaller circle is 1. Then the area of the larger circle is $\pi r^2 = \pi(2)^2 = 4\pi$, and the area of the smaller circle is $\pi r^2 = \pi(1)^2 = \pi$. Hence, the area of the shaded region is $4\pi - \pi = 3\pi$. Now,

$$\frac{\textit{area of shaded region}}{\textit{area of smaller circle}} = \frac{3\pi}{\pi} = \frac{3}{1}$$

The answer is (C).

13.

$$\frac{7^9 + 7^8}{8} =$$

$$\frac{7^8 \cdot 7 + 7^8}{8} = \qquad \text{since } 7^9 = 7^8 \cdot 7$$

$$\frac{7^8(7+1)}{8} = \qquad \text{by factoring out the common factor } 7^8$$

$$\frac{7^8(8)}{8} =$$

$$7^8$$

Hence, the answer is (D). Note, this is considered to be a very hard problem.

14. $(x-2)^2 * x = \frac{(x-2)^2}{x} = \frac{x^2 - 4x + 4}{x} = \frac{x^2}{x} - \frac{4x}{x} + \frac{4}{x} = x - 4 + \frac{4}{x}$. The answer is (A).

15. From the distance formula, the distance between (4, 1) and Q is $\sqrt{2}$, and the distance between (4, 1) and P is

$$\sqrt{(4-1)^2 + (1-4)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{2 \cdot 3^2} = 3\sqrt{2}$$

The answer is (E).

16. Dividing both sides of the equation $p = 4q$ by 4, we get $q = p/4$. We are also given that $p < 8$. Dividing both sides of this inequality by 4 yields, $p/4 < 8/4$. Simplifying it, we get $p/4 < 2$. But $q = p/4$. Hence, $q < 2$. The only non-zero positive integer less than 2 is 1. Hence, $q = 1$. The answer is (A).

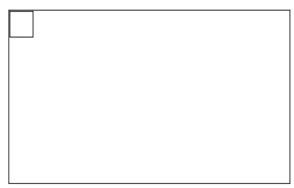
Answers and Solutions Section 1:

1. C	**6. C**	**11. A**	**16. A**
2. A	**7. E**	**12. B**	**17. B**
3. C	**8. A**	**13. C**	**18. E**
4. C	**9. A**	**14. D**	**19. D**
5. D	**10. E**	**15. A**	**20. D**

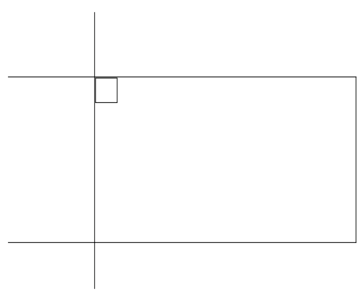
1. Answer-choice (C) consists of the product of two consecutive integers. Now, of any two consecutive integers, one of the integers must be even. Hence, their product must be even. The answer is (C).

2. If $x = y = 2$, then $y^{x-1} = 2^{2-1} = 2^1 = 2$, which is even. But $y - 1 = 2 - 1 = 1$ is odd, and $x/2 = 2/2 = 1$ is also odd. This eliminates choices (B), (C), (D), and (E). The answer is (A).

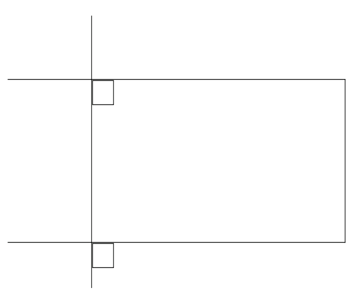
3. Note that a quadrilateral is a closed figure formed by four straight lines. Now, the given information generates the following diagram:



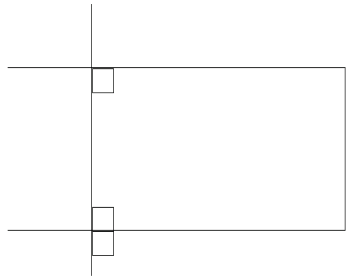
Here, our goal is to show that the other three angles are also 90 degrees. It will help to extend the sides as follows:



Since corresponding angles are congruent, we get



Or



Continuing in this manner will show that the other two angles are also 90 degrees. Hence, θ is 90˚. The answer is (C).

---

4. $q = 1* = \dfrac{\frac{1}{2}}{4\cdot 1 - 1} = \dfrac{\frac{1}{2}}{3} = \dfrac{1}{2}\cdot\dfrac{1}{3} = \dfrac{1}{6}$. Hence, $q* = \dfrac{\frac{\frac{1}{6}}{2}}{4\cdot\frac{1}{6} - 1} = \dfrac{\frac{1}{6}\cdot\frac{1}{2}}{\frac{2}{3} - 1} = \dfrac{\frac{1}{12}}{-\frac{1}{3}} = \dfrac{1}{12}\left(-\dfrac{3}{1}\right) = -\dfrac{3}{12} = -\dfrac{1}{4}$

The answer is (C).

---

5. Since the cars start at the same time, the time each has traveled is the same. Let t be the time when the cars are 210 miles apart. The equation $D = R \times T$, yields

$$210 = 45 \cdot t + 60 \cdot t$$
$$210 = 105 \cdot t$$
$$2 = t$$

The answer is (D).

---

6. From 1 @ 1 = 1, we know that @ must denote multiplication or division; and from 0 @ 0 = 0, we know that @ must denote multiplication, addition, or subtraction. The only operation common to these two groups is multiplication. Hence, the value of π @ $\sqrt{2}$ can be uniquely determined:

$$\pi \text{ @ } \sqrt{2} = \pi \cdot \sqrt{2}$$

The answer is (C).

---

7. This is considered to be a hard problem. Begin by adding the two equations:

$$x + 2y - z = 1$$
$$3x - 2y - 8z = -1$$
$$4x - 9z = 0$$
$$4x = 9z$$
$$\frac{x}{z} = \frac{9}{4}$$

The answer is (E).

8. Let the seven unknown numbers be represented by $x_1, x_2, \cdots, x_7$. Forming the average of the eight numbers yields

$$\frac{x_1 + x_2 + \cdots + x_7 + 14}{8} = A$$

Replacing 14 with 28 (= 14 + 14), and forming the average yields

$$\frac{x_1 + x_2 + \cdots + x_7 + (14 + 14)}{8}$$

Breaking up the fraction into the sum of two fractions yields

$$\frac{x_1 + x_2 + \cdots + x_7 + 14}{8} + \frac{14}{8}$$

Since $\frac{x_1 + x_2 + \cdots + x_7 + 14}{8} = A$, this becomes

$$A + 14/8$$

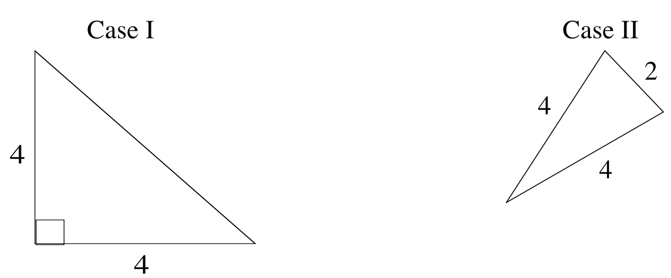
Reducing the fraction yields

$$A + 7/4$$

The answer is (A).

9. $\frac{2+\sqrt{5}}{2-\sqrt{5}} = \frac{2+\sqrt{5}}{2-\sqrt{5}} \cdot \frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{4+4\sqrt{5}+5}{4-5} = \frac{9+4\sqrt{5}}{-1} = -9-4\sqrt{5}$. Hence, the answer is (A).

10. There are many possible drawings for the triangle, two of which are listed below:



In Case I, the area is 8. In Case II, the area is $\sqrt{15}$. This is a double case and therefore the answer is (E).

11.

$$\frac{x^2+2x-10}{5}=1$$

$x^2+2x-10=5$	by multiplying both sides by 5
$x^2+2x-15=0$	by subtracting 5 from both sides
$(x+5)(x-3)=0$	since 5 · 3 =15 and 5 – 3 = 2
$x+5=0$ and $x-3=0$	by setting each factor equal to zero
$x=-5$ and $x=3$	

The answer is (A).

---

12. Since the line passes through (–4, –5) and (0, 0), its slope is

$$m=\frac{rise}{run}=\frac{-5-0}{-4-0}=\frac{5}{4}$$

Notice that the rise, 5, is larger than the run, 4. Hence, the *y*-coordinate will always be larger than the *x*-coordinate. The answer is (B).

13.

$$\left(2+\sqrt{7}\right)\left(4-\sqrt{7}\right)\left(-2x\right)=$$
$$\left(2\cdot 4-2\sqrt{7}+4\sqrt{7}-\sqrt{7}\sqrt{7}\right)\left(-2x\right)=$$
$$\left(8+2\sqrt{7}-7\right)\left(-2x\right)=$$
$$\left(1+2\sqrt{7}\right)\left(-2x\right)=$$
$$1\left(-2x\right)+2\sqrt{7}\left(-2x\right)=$$
$$-2x-4x\sqrt{7}$$

The answer is (C).

---

14. The total expense is the sum of expenses for the shirt, pants, and shoes, which is \$10 + \$20 + \$30 = \$60. Now, translate the main part of the sentence into a mathematical equation:

What	percent	of	<u>the total expense</u>	was spent for	<u>the pants</u>
↓	↓	↓	↓	↓	↓
x	$\frac{1}{100}$	$\cdot$	60	=	20

$$\frac{60}{100}x=20$$

$60x=2000$	(by multiplying both sides of the equation by 100)
$x=\frac{2000}{60}$	(by dividing both sides of the equation by 60)
$x=\frac{100}{3}=33\frac{1}{3}$	

The answer is (D).

15. The value of the x quarters is $25x$, and the value of the $x + 32$ nickels is $5(x + 32)$. Since these two quantities are equal, we get

$$\begin{aligned} 25x &= 5(x + 32) \\ 25x &= 5x + 160 \\ 20x &= 160 \\ x &= 8 \end{aligned}$$

The answer is (A).

16. The graph shows that 100 million items were exported in 2014 and 10% were autos. Hence, 10 million autos were exported. The answer is (A).

17. The chart shows that only autos and textiles exceeded 75 million total items. The answer is (B).

18. In 2014, there were 200 million items imported of which 15% were technology items. Thus, the number of technology items imported was

$$(15\%)(200 \text{ million}) = (.15)(200 \text{ million}) = 30 \text{ million}$$

In 2014, there were 100 million items exported of which 20% were textile items. Thus, the number of textile items exported was

$$(20\%)(100 \text{ million}) = (.20)(100 \text{ million}) = 20 \text{ million}$$

Forming the ratio of the above numbers yields

$$\frac{\textit{number of technology items imported}}{\textit{number of textile items exported}} = \frac{30}{20} = \frac{3}{2}$$

The answer is (E).

19. Remember, to calculate the percentage increase, find the absolute increase and divide it by the original number. Now, in 2014, the number of autos exported was 10 million (100x10%), and in 2015 it was 16 million. The absolute increase is thus:

$$16 - 10 = 6$$

Hence the percent increase in the number of autos exported is

$$\frac{\textit{absolute increase}}{\textit{original number}} = \frac{6}{10} = 60\%$$

The answer is (D).

20. If 20% of the exports broke down, then 2 million autos broke down (20%x10). Since “twice as many autos imported to Country X broke down as autos exported from Country X,” 4 million imported autos broke down. Further, Country X imported 100 million autos (50%x200). Forming the percentage yields

$$\frac{4}{100} = 0.04 = 4\%$$

The answer is (D).

Answers and Solutions Section 2:

1.	**D**	**6.**	**B**	**11.**	**1**	**16.**	**1/2**
2.	**E**	**7.**	**C**	**12.**	**4**	**17.**	**7**
3.	**D**	**8.**	**B**	**13.**	**4**	**18.**	**50**
4.	**C**	**9.**	**13.5**	**14.**	**56**		
5.	**D**	**10.**	**25**	**15.**	**20**		

---

1. The ten's digit must be twice the unit's digit. This eliminates (A), (C), and (E). Now reversing the digits in choice (B) yields 12. But $21 - 12 \neq 27$. This eliminates (B). Hence, by process of elimination, the answer is (D). $(63 - 36 = 27.)$

---

2. Statement I is true:

$a \, \phi \, 1 =$

$1 \, \phi \, a =$ [Since $a \, \phi \, b = b \, \phi \, a$]

1 [Since $1 \, \phi \, a = 1$]

This eliminates (B) and (C). Statement III is true:

$\frac{1 \, \phi \, a}{b \, \phi 1} =$

$\frac{1 \, \phi \, a}{1 \, \phi \, b} =$ [Since $a \, \phi \, b = b \, \phi \, a$]

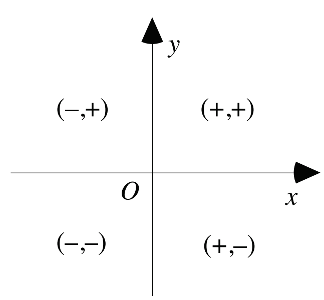
$\frac{1}{1} =$ [Since $1 \, \phi \, a = 1$]

1

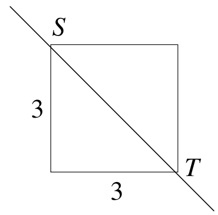
This eliminates (A) and (D). Hence, by process of elimination, the answer is (E).

---

3. If the product of two numbers is negative, the numbers must have opposite signs. Now, only the coordinates of points in quadrants II and IV have opposite signs. The diagram below illustrates the sign pattern of points for all four quadrants. The answer is (D).



4. The maximum possible distance between S and T will occur when the line intersects the square at opposite vertices:



Hence, the maximum distance is the length of the diagonal of the square. Applying the Pythagorean Theorem yields

$$ST^2 = 3^2 + 3^2$$
$$ST^2 = 18$$
$$ST = \sqrt{18} = 3\sqrt{2}$$

The answer is (C).

5. First, clear fractions by multiplying both sides by $12(s + S)$:

$$4(s + S) = 3(s - S)$$

Next, distribute the 3 and 4:

$$4s + 4S = 3s - 3S$$

Finally, subtract $3s$ and $4S$ from both sides:

$$s = -7S$$

The answer is (D).

6. $2^{12} + 2^{12} + 2^{12} + 2^{12} = 4 \cdot 2^{12} = 2^2 \cdot 2^{12} = 2^{2+12} = 2^{14}$. The answer is (B).

7. The only information we have to work with is the equation

$$x + y = 2\sqrt{xy}$$

Since radicals are awkward to work with, let's square both sides of this equation to eliminate the radical:

$$(x + y)^2 = \left(2\sqrt{xy}\right)^2$$

Applying the Perfect Square Trinomial Formula to the left side and simplifying the right side yields

$$x^2 + 2xy + y^2 = 4xy$$

Subtracting $4xy$ from both sides yields

$$x^2 - 2xy + y^2 = 0$$

Using the Perfect Square Trinomial Formula again yields

$$(x - y)^2 = 0$$

Taking the square root of both sides yields

$$\sqrt{(x - y)^2} = \pm\sqrt{0}$$

Simplifying yields

$$x - y = 0$$

Finally, adding y to both sides yields

$$x = y$$

The answer is (C).

---

8. Calculating the distance between V and the origin yields

$$\sqrt{(2 - 0)^2 + (-2 - 0)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

Since the square is rotated about the origin, the distance between the origin and V is fix. Hence, the new y-coordinate of V is $-2\sqrt{2}$. The diagram below illustrates the position of V after the rotation. The answer is (B).

---

9. Let x be the number of which the percentage is being calculated. Then 15% of the number x is $.15x$. We are told this is equal to 4.5. Hence,

$$.15x = 4.5$$

Solving this equation by dividing both sides by .15 yields

$$x = 4.5/.15 = 30$$

Now, 45% of 30 is

$$.45(30)$$

Multiplying out this expression gives 13.5. Grid in 13.5.

10. Let N stand for the number of nickels. Then the number of dimes is $N + 15$. The value of the nickels is $5N$, and the value of the dimes is $10(N + 15)$. Since the total value of the nickels and dimes is 525¢, we get

$$5N + 10(N + 15) = 525$$
$$15N + 150 = 525$$
$$15N = 375$$
$$N = 25$$

Hence, there are 25 nickels. Grid in 25.

11.

$$3^{4n-1} = 27$$
$$3^{4n-1} = 3^3$$
$$4n - 1 = 3$$
$$4n = 4$$
$$n = 1$$

Since $n = 1$, $m = 3^{n-1} = 3^{1-1} = 3^0 = 1$. Hence, $\frac{m}{n} = \frac{1}{1} = 1$. Grid in 1.

12. Begin by factoring out the common factor in the equation $5y^2 - 20y + 15 = 0$:

$$5(y^2 - 4y + 3) = 0$$

Dividing both sides of this equation by 5 yields

$$y^2 - 4y + 3 = 0$$

Since 3 + 1 = 4, the trinomial factors into

$$(y - 3)(y - 1) = 0$$

Setting each factor equal to zero yields

$$y - 3 = 0 \text{ and } y - 1 = 0$$

Solving these equations yields $y = 3$ and $y = 1$. Now, the difference of 3 and 1 is 2 and twice 2 is 4. Further, the difference of 1 and 3 is –2 and twice –2 is –4. Now, the absolute value of both 4 and –4 is 4. Grid in 4.

13. Since we are given that $p = z + 1/z$ and $q = z - 1/z$,

$$p + q = (z + 1/z) + (z - 1/z) = z + 1/z + z - 1/z = 2z.$$
$$p - q = (z + 1/z) - (z - 1/z) = z + 1/z - z + 1/z = 2/z.$$

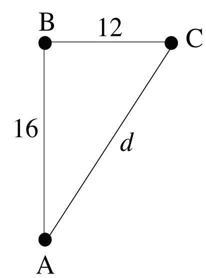
Therefore, $(p + q)(p - q) = (2z)(2/z) = 4$. Grid in 4.

14. Let t be time that Steve has been driving. Then $t + 1$ is time that Richard has been driving. Now, the distance traveled by Steve is $D = rt = 40t$, and Richard's distance is $60(t + 1)$. At the moment they cross paths, they will have traveled a combined distance of 200 miles. Hence,

$$40t + 60(t + 1) = 200$$
$$40t + 60t + 60 = 200$$
$$100t + 60 = 200$$
$$100t = 140$$
$$t = 1.4$$

Therefore, Steve will have traveled $D = rt = 40(1.4) = 56$ miles. Grid in 56.

15. Solution:



A: Initial position
B: Second position
C: Final position

The path taken by the person can be represented diagrammatically as shown. Let d be the distance between his initial location and his final location. Since a person traveling due north has to turn 90 degrees to travel due east, the Angle ABC is a right angle. Hence, we can apply the Pythagorean Theorem to the triangle, which yields

$$d^2 = 12^2 + 16^2$$
$$d^2 = 400$$
$$d = \sqrt{400}$$
$$d = 20$$

Grid in 20.

16. We are given the equations:

$$x = y/2$$
$$y = z/2$$

Solving the bottom equation for z yields $z = 2y$. Replacing x and z in the expression $\sqrt{x/z}$ with $y/2$ and $2y$, respectively, yields

$$\sqrt{x/z} = \sqrt{\frac{y/2}{2y}} = \sqrt{\frac{y}{2} \cdot \frac{1}{2y}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Grid in 1/2.

17. Since $4 < 12$, we use the bottom half of the definition of #:

$$4 \# 12 = 4 + 12/4 = 4 + 3 = 7$$

Grid in 7.

18. Let x stand for both the number of nickels and the number of quarters. Then the value of the nickels is $5x$ and the value of the quarters is $25x$. Since the total value of the coins is \$7.50, we get

$$5x + 25x = 750$$
$$30x = 750$$
$$x = 25$$

Hence, she has $x + x = 25 + 25 = 50$ coins. Grid in 50.

Answers and Solutions Section 3:

1.	**D**	**5.**	**A**	**9.**	**B**	**13.**	**C**
2.	**B**	**6.**	**B**	**10.**	**C**	**14.**	**C**
3.	**B**	**7.**	**B**	**11.**	**C**	**15.**	**E**
4.	**E**	**8.**	**C**	**12.**	**C**	**16.**	**D**

---

1. $3^x = 81 = 3^4$. Hence, $x = 4$. Replacing x with 4 in the expression $\left(3^{x+3}\right)\left(4^{x+1}\right)$ yields

$$\left(3^{4+3}\right)\left(4^{4+1}\right) =$$

$$3^7 \cdot 4^5 =$$

$$3^2 \cdot 3^5 \cdot 4^5 =$$

$$3^2 (3 \cdot 4)^5 =$$

$$9(12)^5$$

The answer is (D).

---

2. The Difference of Squares formula yields $x^2 - y^2 = (x + y)(x - y)$. Now, both x and y must be odd because 2 is the only even prime and $x > y > 2$. Remember that the sum (or difference) of two odd numbers is even. Hence, $(x + y)(x - y)$ is the product of two even numbers and therefore is divisible by 4. To show this explicitly, let $x + y = 2p$ and let $x - y = 2q$. Then

$$(x + y)(x - y) = 2p \cdot 2q = 4pq$$

Since we have written $(x + y)(x - y)$ as a multiple of 4, it is divisible by 4. The answer is (B).

Method II (substitution):
Let $x = 5$ and $y = 3$, then $x > y > 2$ and $x^2 - y^2 = 5^2 - 3^2 = 25 - 9 = 16$. Since 4 is the only number listed that divides evenly into 16, the answer is (B).

---

3. Since the area of the circle is 1.21π, we get

$$\pi r^2 = 1.21\pi$$

Dividing by π yields

$$r^2 = 1.21$$

Taking the square root of both sides gives

$$r = 1.1$$

So the diameter of the circle is

$$d = 2r = 2(1.1) = 2.2$$

Hence, a side of the square has length 2.2, and the area of the square is

$$(2.2)^2 = 4.84$$

Therefore, the area of the shaded region is

$$4.84 - 1.21\pi$$

The answer is (B).

---

4. Forming the negative reciprocal of $\frac{1}{x}+\frac{1}{y}$ yields

$$\frac{-1}{\frac{1}{x}+\frac{1}{y}}$$

Adding the fractions in the denominator yields

$$\frac{-1}{\frac{y+x}{xy}}$$

Reciprocating the denominator yields

$$-1\cdot\frac{xy}{x+y}$$

Or

$$\frac{-xy}{x+y}$$

The answer is (E).

---

5. Let t be time that Ship B has been traveling. Then $t + 3$ is time that Ship A has been traveling. The distance traveled by Ship B is $D = rt = 25t$, and Ship A's distance is $15(t + 3)$. At the moment Ship B passes Ship A, they will have traveled the same distance. Hence,

$$25t = 15(t + 3)$$
$$25t = 15t + 45$$
$$10t = 45$$
$$t = 4.5$$

Since Ship B left port at 4 PM and overtook Ship A in 4.5 hours, it passed Ship A at 8:30 PM. The answer is (A).

---

6. Solution:

$$\begin{aligned}(x*y)*z &= (xy)^2*z\\ &= \left((xy)^2z\right)^2\\ &= \left((xy)^2\right)^2z^2\\ &= (xy)^4z^2\\ &= x^4y^4z^2\end{aligned}$$

The answer is (B).

7. First, express all the numbers in the same units (inches):

The original height is

$$5\ feet = 5\ feet \times \frac{12\ inches}{1\ feet} = 60\ inches$$

The change in height is

$$(5\ feet\ 6\ inches) - (5\ feet) = 6\ inches$$

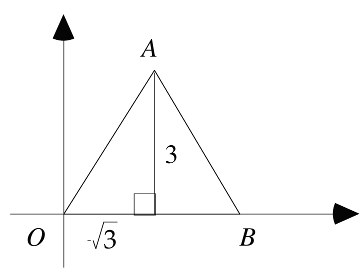
Now, use the formula for percent of change:.

Percent of change:

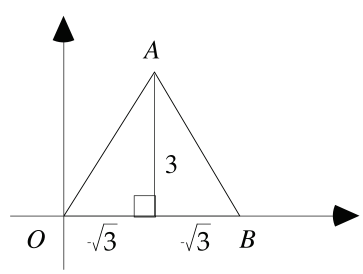
$$\frac{Amount\ of\ change}{Original\ amount} \times 100\% =$$
$$\frac{6}{60} \times 100\% =$$
$$\frac{1}{10} \times 100\% = \qquad \text{(by canceling 6)}$$
$$10\%$$

The answer is (B).

8. Since the coordinates of *A* are $(\sqrt{3}, 3)$, the diagram becomes



Further, since ΔABO is equilateral, the diagram becomes



Hence, the area is $\frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 2\sqrt{3} \cdot 3 = 3\sqrt{3}$. The answer is (C).

9. Since the time is given in mixed units, we need to change the minutes into hours. Since there are 60 minutes in an hour, y minutes is equivalent to $y/60$ hours. Hence, the car's travel time, "x hours and y minutes," is $x + \frac{y}{60}$ hours. Plugging this along with the distance traveled, z, into the formula $d = rt$ yields

$$z = r\left(x + \frac{y}{60}\right)$$
$$z = r\left(\frac{60}{60}x + \frac{y}{60}\right)$$
$$z = r\left(\frac{60x + y}{60}\right)$$
$$\frac{60z}{60x + y} = r$$

The answer is (B).

10. There are $x + y$ red and blue marbles in the bowl. Subtracting this from the total of 500 marbles yields the number of marbles that are neither red nor blue:

$$500 - (x + y) = 500 - x - y$$

Hence, the answer is (C).

11. Let the original price of the commodity be x. The reduction in price due to the 30% discount is $0.3x$. It is given that the 30% discount reduced the price of the commodity by \$90. Expressing this as an equation yields

$$0.3x = 90$$

Solving for x yields

$$x = 300$$

Hence, the original price of the commodity was \$300. The value of a 20% discount on \$300 is

$$.20(300) = 60$$

Hence, the new selling price of the commodity is

$$\$300 - \$60 = \$240$$

The answer is (C).

12. If $x = 0$, then $\frac{3 - 4x}{5}$ becomes 3/5 and the answer-choices become

(A) 5/4
(B) 10/3
(C) –10/3
(D) 3/5
(E) –3/10

Multiplying Choice (C) by 3/5, gives $\left(\frac{3}{5}\right)\left(-\frac{10}{3}\right) = -2$. The answer is (C).

13. Following is the set of all integers greater than –2.1:

$$\{-2, -1, 0, 1, 2, \ldots\}$$

The least integer in this set is –2. The answer is (C).

14. Statement I is not necessarily true. For example, if $x = 2$ and $y = 1$, then

$$\frac{x+2}{y+2} = \frac{2+2}{1+2} = \frac{4}{3} \ngtr 2 = \frac{2}{1} = \frac{x}{y}$$

This is also a counterexample to Statement II. Hence, we can eliminate (A), (B), (D), and (E). Thus, by process of elimination, the answer is (C).

However, it is instructive to prove that Statement III is true. From the expression $x > y > 0$, we get

$$x + 2 > y + 2$$

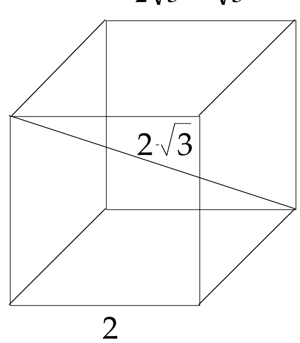
Since $y + 2 > 0$, dividing both sides of the above expression by $y + 2$ will not reverse the inequality:

$$\frac{x+2}{y+2} > 1$$

Hence, Statement III is necessarily true.

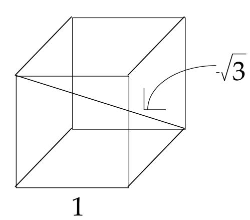
15. There is not enough information to decide since different size cubes can have the ratio $1:\sqrt{3}$:

Ratio: $\frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$



Volume: 8

Ratio: $\frac{1}{\sqrt{3}}$



Volume: 1

The answer is (E).

16. The distance traveled by the train while passing the pole is l (which is the length of the train). The train takes t seconds to pass the pole. Recall the formula velocity = distance/time. Applying this formula, we get

$$\text{velocity} = \frac{l}{t}$$

While passing the platform, the train travels a distance of $l + x$, where x is the length of the platform. The train takes $3t$ seconds at the velocity of l/t to cross the platform. Recalling the formula distance = velocity × time and substituting the values for the respective variables, we get

$$l + x = \frac{l}{t} \times 3t \qquad \text{by substitution}$$

$$l + x = 3l \qquad \text{by canceling } t$$

$$x = 2l \qquad \text{by subtracting } l \text{ from both sides}$$

Hence, the length of the platform is $2l$. The answer is (D).

Answers and Solutions Section 1:

1.	**B**	**6.**	**E**	**11.**	**B**	**16.**	**B**
2.	**B**	**7.**	**A**	**12.**	**D**	**17.**	**C**
3.	**A**	**8.**	**E**	**13.**	**E**	**18.**	**D**
4.	**B**	**9.**	**D**	**14.**	**C**	**19.**	**E**
5.	**C**	**10.**	**C**	**15.**	**C**	**20.**	**C**

1. Let the three consecutive positive integers be n, $n + 1$, and $n + 2$. The sum of these three positive integers is

$$n + (n + 1) + (n + 2) =$$
$$3n + 3 =$$
$$3(n + 1)$$

Since we have written the sum as a multiple of 3, it is divisible by 3. The answer is (B).

2. The area of a square with side s is s^2. On joining two such squares, the resulting area will be twice the area of either square: $2s^2$. The answer is (B).

3. Our goal here is to write both sides of the equation in terms of the base 2 and then equate the exponents. To that end, write 16 as 2:

$$2^{2x} = \left(2^4\right)^{x+2}$$
$$2^{2x} = 2^{4(x+2)}$$

Since we have written both sides of the equation in terms of the base 2, we now equate the exponents:

$$2x = 4(x + 2)$$
$$2x = 4x + 8$$
$$-2x = 8$$
$$x = -4$$

The answer is (A).

4. Solution:

$\dfrac{xy + x^2}{xy - x^2} =$

$\dfrac{x(y + x)}{x(y - x)} =$ by factoring out x from both the top and bottom expressions

$\dfrac{y + x}{y - x} =$ by canceling the common factor x

$\dfrac{x + y}{-(x - y)} =$ by factoring out the negative sign in the bottom and then rearranging

$-\dfrac{x + y}{x - y} =$ by recalling that a negative fraction can be written three ways: $\dfrac{a}{-b} = -\dfrac{a}{b} = \dfrac{-a}{b}$

$-\dfrac{1}{2}$ by replacing $\dfrac{x + y}{x - y}$ with $\dfrac{1}{2}$

The answer is (B).

5. Adding $2xy$ to both sides of the equation $x^2 + y^2 = xy$ yields

$$x^2 + y^2 + 2xy = 3xy$$

$$(x + y)^2 = 3xy \qquad \text{from the formula } (x + y)^2 = x^2 + 2xy + y^2$$

Squaring both sides of this equation yields

$$(x + y)^4 = (3xy)^2 = 9x^2y^2$$

The answer is (C).

6. The fraction of work done in 1 hour by the first two people working together is 1/8. The fraction of work done in 1 hour by the third person is 1/12. When the three people work together, the total amount of work done in 1 hour is 1/8 + 1/12 = 5/24. The time taken by the people working together to complete the job is

$$\frac{1}{\text{fraction of work done per unit time}} =$$

$$\frac{1}{5/24} =$$

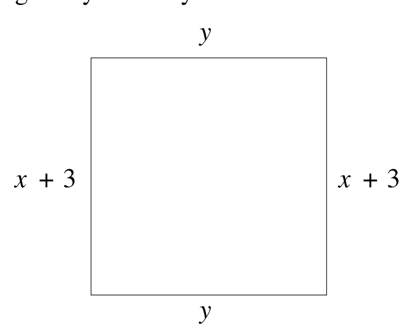
$$\frac{24}{5} =$$

$$4\frac{4}{5}$$

The answer is (E).

7. The area of the original rectangle is $A = xy$. So the goal in this problem is to find the values of x and y.

Lengthening side x of the original figure by 3 units yields



The area of this figure is $y(x + 3) = 20$. Since the resulting figure is a square, $y = x + 3$. Hence, we have the system:

$$y(x + 3) = 20$$
$$y = x + 3$$

Solving this system gives $x = \sqrt{20} - 3$ and $y = \sqrt{20}$. Hence, the area is $A = xy = \left(\sqrt{20} - 3\right)\left(\sqrt{20}\right) = 20 - 3\sqrt{20}$. The answer is (A).

8. $\left(\frac{\left(x^2y\right)^3 z}{xyz}\right)^3 = \left(\frac{\left(x^2y\right)^3}{xy}\right)^3 = \left(\frac{\left(x^2\right)^3 y^3}{xy}\right)^3 = \left(\frac{x^6y^3}{xy}\right)^3 = \left(x^5y^2\right)^3 = \left(x^5\right)^3\left(y^2\right)^3 = x^{15}y^6$

Hence, the answer is (E).

9. In choice (D), 3 + 9 = 12 and $3 = \frac{1}{3} \cdot 9$. Hence, the answer is (D).

10. Writing the system of equations vertically yields

$$x^2 + y^2 = 2ab$$
$$2xy = a^2 + b^2$$

Adding the equations yields

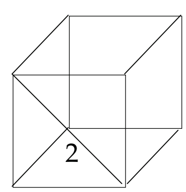
$$x^2 + 2xy + y^2 = a^2 + 2ab + b^2$$

Applying the Perfect Square Trinomial formula to both the sides of the equation yields

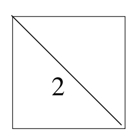
$$(x + y)^2 = (a + b)^2$$
$x + y = a + b$ by taking the square root of both sides and noting all numbers are positive

The answer is (C).

11. A diagram illustrating the situation is shown below:



Looking at the face in isolation gives



Applying the Pythagorean Theorem to this diagram gives

$$x^2 + x^2 = 2^2$$
$$2x^2 = 4$$
$$x^2 = 2$$
$$x = \sqrt{2}$$

Hence, the volume of the cube is $V = x^3 = \left(\sqrt{2}\right)^3 < 8$. The answer is (B).

12. Statement I is possible:

$$\left(-\frac{4}{5}\right) \# \left(-\frac{4}{5}\right) = -\frac{4}{5} + \frac{(-4/5)}{4} = -\frac{4}{5} - \frac{1}{5} = -\frac{5}{5} = -1$$

Statement II is not possible: since $x > y$, the top part of the definition of # applies. But a square cannot be negative (i.e., cannot equal –1). Statement III is possible: $-1 < 0$. So by the bottom half of the definition,

$$-1 \# 0 = -1 + \frac{0}{4} = -1$$

The answer is (D).

13. There is not sufficient information since the selling price is not related to any other information. Note, the phrase "initially listed" implies that there was more than one asking price. If it wasn't for that phrase, the information would be sufficient. The answer is (E).

14. Doubling the x in the expression yields

$$\frac{v+w}{2x/yz} = \frac{1}{2}\left(\frac{v+w}{x/yz}\right)$$

Since we have written the expression as 1/2 times the original expression, doubling the x halved the original expression. The answer is (C).

15. Suppose there were 8 people on the bus—choice (C). Then after the first stop, there would be 4 people left on the bus. After the second stop, there would be 2 people left on the bus. After the third stop, there would be only one person left on the bus. Hence, on the third stop the next to last person would have exited the bus. The answer is (C).

16. Point A has coordinates (0, 2), point B has coordinates (2, 0), and point C has coordinates (5, 4). Using the distance formula to calculate the distances between points A and B, A and C, and B and C yields

$$\overline{AB} = \sqrt{(0-2)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$
$$\overline{AC} = \sqrt{(0-5)^2 + (2-4)^2} = \sqrt{25+4} = \sqrt{29}$$
$$\overline{BC} = \sqrt{(2-5)^2 + (0-4)^2} = \sqrt{9+16} = 5$$

Adding these lengths gives the perimeter of Triangle ABC:

$$\overline{AB} + \overline{AC} + \overline{BC} = 2\sqrt{2} + \sqrt{29} + 5$$

The answer is (B).

17. First, let's reciprocate the expression $(x+y)^{-\frac{1}{3}}$ to eliminate the negative exponent:

$$(x-y)^{\frac{1}{3}} = \frac{1}{(x+y)^{\frac{1}{3}}}$$

Cubing both sides of this equation to eliminate the cube roots yields

$$x - y = \frac{1}{x+y}$$

Multiplying both sides of this equation by $x + y$ yields

$$(x-y)(x+y) = 1$$

Multiplying out the left side of this equation yields

$$x^2 + xy - xy - y^2 = 1$$

Reducing yields

$$x^2 - y^2 = 1$$

The answer is (C).

18. Solution:

$(x-y)^2 = x^2 + y^2$
$x^2 - 2xy + y^2 = x^2 + y^2$ by the formula $x^2 - 2xy + y^2 = (x-y)^2$
$-2xy = 0$ by subtracting x^2 and y^2 from both sides of the equation
$xy = 0$ by dividing both sides of the equation by -2

Hence, Statement III is true, which eliminates choices (A), (B), and (C). However, Statement II is false. For example, if $y = 5$ and $x = 0$, then $xy = 0 \cdot 5 = 0$. A similar analysis shows that Statement I is false. The answer is (D).

19. The area of a triangle is $\frac{1}{2}\,base \times height$. For the given triangle, this becomes

$$Area = \frac{1}{2} b \times h$$

Solving the equation $2b + h = 6$ for h gives $h = 6 - 2b$. Plugging this into the area formula gives

$$Area = \frac{1}{2} b(6 - 2b)$$

Since the value of b is not given, we cannot determine the area. Hence, there is not enough information, and the answer is (E).

20. The change in price is \$1.50 – \$1.20 = \$.30. Now, use the formula for percent of change.

$$\frac{\textit{Amount of change}}{\textit{Original amount}} \times 100\% =$$

$$\frac{.30}{1.20} \times 100\% =$$

$$\frac{1}{4} \times 100\% =$$

$$25\%$$

The answer is (C).

Answers and Solutions Section 2:

1.	**C**	**6.**	**A**	**11.**	**39**	**16.**	**4**
2.	**A**	**7.**	**C**	**12.**	**3**	**17.**	**18**
3.	**A**	**8.**	**E**	**13.**	**3**	**18.**	**18**
4.	**A**	**9.**	**2**	**14.**	**4**		
5.	**B**	**10.**	**36**	**15.**	**1**		

---

1. Dividing both sides of the equation by 3 yields

$$x + 3 = 5$$

Subtracting 1 from both sides of this equation (because we are looking for $x + 2$) yields

$$x + 2 = 4$$

The answer is (C).

2.

Within the innermost parentheses, division is performed before subtraction:

$$3 - (2^3 - 2[3 - 16 \div 2]) =$$
$$3 - (2^3 - 2[3 - 8]) =$$
$$3 - (2^3 - 2[-5]) =$$
$$3 - (8 - 2[-5]) =$$
$$3 - (8 + 10) =$$
$$3 - 18 =$$
$$-15$$

The answer is (A).

3.

$$(a + b^*)^* =$$
$$(a + [2 - b])^* =$$
$$(a + 2 - b)^* =$$
$$2 - (a + 2 - b) =$$
$$2 - a - 2 + b =$$
$$-a + b =$$
$$b - a$$

The answer is (A).

---

4. Since $pq/2$ is prime, it is an integer. Hence, either p or q must be even; otherwise, the 2 would not cancel and $pq/2$ would be a fraction. The only even prime number is 2. Hence, either p or q, but not both, must be 2. The other one is an odd prime number. Now, the sum of an even number and an odd number is an odd number. The answer is (A).

5. Using the distance formula to calculate the distance of each point from the origin yields

$$d = \sqrt{(-4)^2 + (-1)^2} = \sqrt{17}$$
$$d = \sqrt{(-3)^2 + (3)^2} = \sqrt{18}$$
$$d = \sqrt{(4)^2 + (0)^2} = \sqrt{16}$$
$$d = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$
$$d = \sqrt{(0)^2 + (4)^2} = \sqrt{16}$$

The answer is (B).

---

6. Let g be the number of girls, and b the number of boys. Calculate the number of girls in the class:

<u>Girls</u>	are	40	percent	of	<u>the class</u>
↓	↓	↓	↓	↓	↓
g	$=$	40	$\frac{1}{100}$	$\times$	200

$$g = \frac{40}{100} \times 200 = 80$$

The number of boys equals the total number of students minus the number of girls:

$$b = 200 - 80 = 120$$

Next, calculate the number of boys and girls who signed up for the tour:

25 percent of boys ($\frac{25}{100} \times 120 = 30$) and 10 percent of girls ($\frac{10}{100} \times 80 = 8$) signed up for the tour. Thus, $30 + 8 = 38$ students signed up. Now, translate the main part of the question with a little modification into a mathematical equation:

What	percent	of	<u>the class</u>	is	<u>the students who signed up for the tour</u>
↓	↓	↓	↓	↓	↓
x	$\frac{1}{100}$	$\cdot$	200	$=$	38

$$\frac{200}{100} x = 38$$

$$x = 19$$

The answer is (A).

7.

$$-2^4 - (x^2 - 1)^2 =$$
$$-16 - [(x^2)^2 - 2x^2 + 1] =$$
$$-16 - [x^4 - 2x^2 + 1] =$$
$$-16 - x^4 + 2x^2 - 1 =$$
$$-x^4 + 2x^2 - 17$$

The answer is (C).

Notice that $-2^4 = -16$, not 16. This is one of the most common mistakes on the test. To see why $-2^4 = -16$ more clearly, rewrite -2^4 as follows:

$$-2^4 = (-1)2^4$$

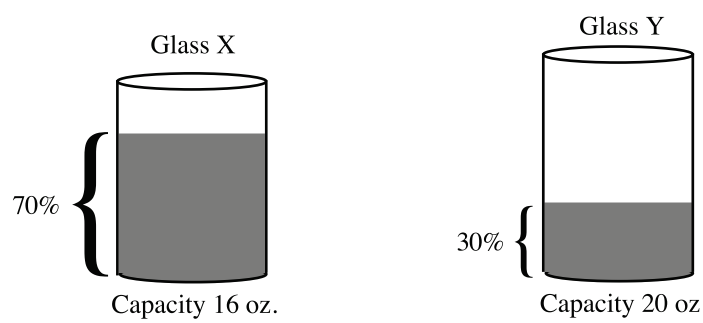
In this form, it is clearer that the exponent, 4, applies only to the number 2, not to the number -1. So $-2^4 = (-1)2^4 = (-1)16 = -16$.

To make the answer positive 16, the -2 could be placed in parentheses:

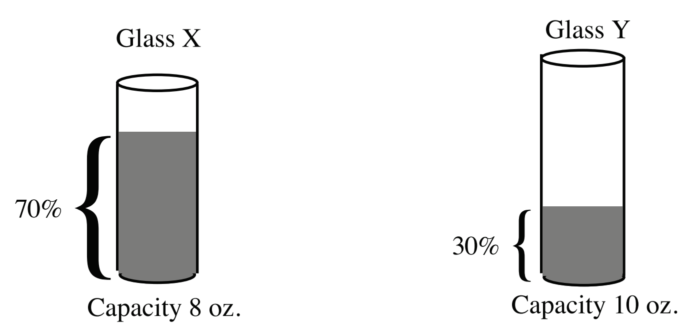
$$(-2)^4 = [(-1)2]^4 = (-1)^4\, 2^4 = (+1)16 = 16$$

8. Now, there is not sufficient information to solve the problem since it does not provide any absolute numbers. The following diagram shows two situations: one in which Glass X contains 5.2 more ounces of punch than glass Y, and one in which Glass X contains 2.6 more ounces than glass Y.

Scenario I (Glass X contains 5.2 more ounces than glass Y.)



Scenario II (Glass X contains 2.6 more ounces than glass Y.)



The answer is (E).

9.

$$\frac{a^2}{d} =$$

$$\frac{(3b)^2}{9c} = \qquad \text{since } a = 3b \text{ and } 9c = d$$

$$\frac{9b^2}{9c} =$$

$$\frac{b^2}{c} =$$

$$\frac{2c}{c} = \qquad \text{since } b^2 = 2c$$

$$2$$

Grid in 2.

10.

$$\sqrt{(42-6)(20+16)} =$$

$$\sqrt{(36)(36)} =$$

$$\sqrt{36}\sqrt{36} = \qquad \text{from the rule } \sqrt{xy} = \sqrt{x}\sqrt{y}$$

$$6 \cdot 6 =$$

$$36$$

Grid in 36.

11. The number 8 can be written as 2^3. Plugging this into the equation $8^{13} = 2^z$ yields

$$\left(2^3\right)^{13} = 2^z$$

Applying the rule $\left(x^a\right)^b = x^{ab}$ yields

$$2^{39} = 2^z$$

Since the bases are the same, the exponents must be the same. Hence, $z = 39$. Grid in 39.

12.

$$\frac{4}{\frac{1}{3}+1} =$$

$$\frac{4}{\frac{1}{3}+\frac{3}{3}} = \qquad \text{by creating a common denominator of 3}$$

$$\frac{4}{\frac{1+3}{3}} =$$

$$\frac{4}{\frac{4}{3}} =$$

$$4 \cdot \frac{3}{4} = \qquad \text{Recall: “to divide” means to invert and multiply}$$

$$3 \qquad \text{by canceling the 4's}$$

Grid in 3.

---

13. Since y is to be less than 1 and $y = -3x + 7$, we get

$-3x + 7 < 1$
$-3x < -6$ by subtracting 7 from both sides of the inequality
$x > 2$ by dividing both sides of the inequality by -3

(Note that the inequality changes direction when we divide both sides by a negative number. This is also the case if you multiply both sides of an inequality by a negative number.)

Since x is an integer and is to be as small as possible, $x = 3$. Grid in 3.

---

14. Since the triangle is isosceles, with base AC, the base angles are congruent (equal). That is, $A = C$. Since the angle sum of a triangle is 180, we get

$$A + C + x = 180$$

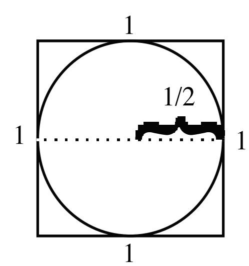
Replacing C with A and x with 60 gives

$$A + A + 60 = 180$$
$$2A + 60 = 180$$
$$2A = 120$$
$$A = 60$$

Hence, the triangle is equilateral (all three sides are congruent). Since we are given that side AB has length 4, side AC also has length 4. Grid in 4.

---

15. Since the unit square is circumscribed about the circle, the diameter of the circle is 1 and the radius of the circle is r = d/2 = 1/2. This is illustrated in the following figure:



Now, the circumference of a circle is given by the formula $2\pi r$. For this circle the formula becomes $2\pi r = 2\pi(1/2) = \pi$. We are told that the circumference of the circle is $q\pi$. Setting these two expressions equal yields

$$\pi = q\pi$$

Dividing both sides of this equation by π yields

$$1 = q$$

Grid in 1.

16. Since the average of $2x$ and $4x$ is 12, we get

$$\frac{2x+4x}{2} = 12$$
$$\frac{6x}{2} = 12$$
$$3x = 12$$
$$x = 4$$

Grid in 4.

17. Let the two numbers be x and y. Now, a ratio is simply a fraction. Forming the fraction yields $x/y = 6$, and forming the sum yields $x + y = 21$. Solving the first equation for x yields $x = 6y$. Plugging this into the second equation yields

$$6y + y = 21$$
$$7y = 21$$
$$y = 3$$

Plugging this into the equation $x = 6y$ yields

$$x = 6(3) = 18$$

Grid in 18.

18. Let x be the amount of water added. Since there is no alcohol in the water, the percent of alcohol in the water is $0\%x$. The amount of alcohol in the original solution is $40\%(30)$, and the amount of alcohol in the final solution will be $25\%(30 + x)$. Now, the concentration of alcohol in the original solution plus the concentration of alcohol in the added solution (water) must equal the concentration of alcohol in the resulting solution:

$$40\%(30) + 0\%x = 25\%(30 + x)$$

Multiplying this equation by 100 to clear the percent symbol yields

$$40(30) + 0 = 25(30 + x)$$
$$1200 = 750 + 25x$$
$$450 = 25x$$
$$18 = x$$

Grid in 18.

<u>Answers and Solutions Section 3:</u>

1.	**E**	**5.**	**E**	**9.**	**A**	**13.**	**B**
2.	**E**	**6.**	**B**	**10.**	**C**	**14.**	**B**
3.	**D**	**7.**	**D**	**11.**	**C**	**15.**	**E**
4.	**D**	**8.**	**C**	**12.**	**B**	**16.**	**C**

---

1. Multiplying (using foil multiplication) both terms in the expression yields

$$x^2 + 4x - 2x - 8 - (x^2 - x - 3x + 3) = 0$$

(Notice that parentheses are used in the second expansion but not in the first. Parentheses must be used in the second expansion because the negative sign must be distributed to *every* term within the parentheses.)

Combining like terms yields

$$x^2 + 2x - 8 - (x^2 - 4x + 3) = 0$$

Distributing the negative sign to every term within the parentheses yields

$$x^2 + 2x - 8 - x^2 + 4x - 3 = 0$$

(Note, although distributing the negative sign over the parentheses is an elementary operation, many, if not most, students will apply the negative sign to only the first term:

$$-x^2 - 4x + 3$$

The writers of the test are aware of this common mistake and structure the test so that there are many opportunities to make this mistake.)

Grouping like terms together yields

$$(x^2 - x^2) + (2x + 4x) + (-8 - 3) = 0$$

Combining the like terms yields

$$6x - 11 = 0$$
$$6x = 11$$
$$x = 11/6$$

The answer is (E).

---

2. Since the question asks for the *smallest* prime greater than 48, we start with the smallest answer-choice. Now, 49 is not prime since $49 = 7 \cdot 7$. Next, 50 is not prime since $50 = 5 \cdot 10$. Next, 51 is not prime since $51 = 3 \cdot 17$. Next, 52 is not prime since $52 = 2 \cdot 26$. Finally, 53 *is* prime since it is divisible by only itself and 1. The answer is (E).

Note, an integer is prime if it greater than 1 and divisible by only itself and 1. The number 2 is the smallest prime (and the only even prime) because the only integers that divide into it evenly are 1 and 2. The number 3 is the next larger prime. The number 4 is not prime because $4 = 2 \cdot 2$. Following is a partial list of the prime numbers. You should memorize it.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, . . .

3.

$$\left(4^x\right)^2 =$$

$4^{2x} =$ by the rule $\left(x^a\right)^b = x^{ab}$

$\left(2^2\right)^{2x} =$ by replacing 4 with 2^2

$(2)^{4x}$ by the rule $\left(x^a\right)^b = x^{ab}$

The answer is (A). Note, this is considered to be a hard problem.

As to the other answer-choices, Choice (B) wrongly adds the exponents x and 2. The exponents are added when the same bases are multiplied:

$$a^x a^y = a^{x+y}$$

For example: $2^3 2^2 = 2^{3+2} = 2^5 = 32$. Be careful not to multiply unlike bases. For example, do not add exponents in the following expression: $2^3 4^2$. The exponents cannot be added here because the bases, 2 and 4, are not the same.

Choice (C), first changes 4 into 2^2, and then correctly multiplies 2 and x: $\left(2^2\right)^x = 2^{2x}$. However, it then errs in adding $2x$ and 2: $\left(2^{2x}\right)^2 \neq 2^{2x+2}$.

Choice (D) wrongly squares the x. When a power is raised to another power, the powers are multiplied:

$$\left(x^a\right)^b = x^{ab}$$

So $\left(4^x\right)^2 = 4^{2x}$.

Choice (E) makes the same mistake as in Choice (D).

---

4. Recall that percent means to divide by 100. So .2 percent equals .2/100 = .002. (Recall that the decimal point is moved to the left one space for each zero in the denominator.) Now, as a decimal 1/2 = .5.

In percent problems, "of" means multiplication. So multiplying .5 and .002 yields

$$\begin{array}{r} .002 \\ \times \quad .5 \\ \hline .001 \end{array}$$

Hence, the answer is (D).

5.

$3x^2 + 6xy + 3y^2 =$

$3(x^2 + 2xy + y^2) =$ by factoring out the common factor 3

$3(x + y)^2 =$ by the perfect square trinomial formula $x^2 + 2xy + y^2 = (x + y)^2$

$3k^2$

Hence, the answer is (E).

6. First, replace the inequality symbol with an equal symbol:

$$x^2 = 2x$$

Subtracting $2x$ from both sides yields

$$x^2 - 2x = 0$$

Factoring by the distributive rule yields

$$x(x - 2) = 0$$

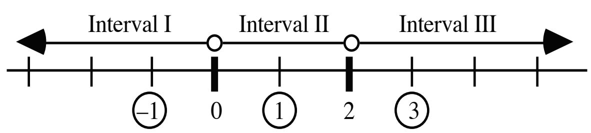
Setting each factor to 0 yields

$$x = 0 \text{ and } x - 2 = 0$$

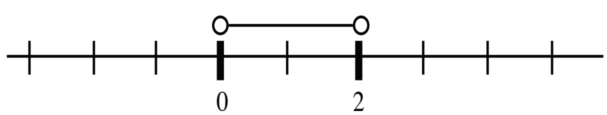
Or

$$x = 0 \text{ and } x = 2$$

Now, the only numbers at which the expression can change sign are 0 and 2. So 0 and 2 divide the number line into three intervals. Let's set up a number line and choose test points in each interval:



When $x = -1$, $x^2 < 2x$ becomes $1 < -2$. This is false. Hence, no numbers in Interval I satisfy the inequality. When $x = 1$, $x^2 < 2x$ becomes $1 < 2$. This is true. Hence, all numbers in Interval II satisfy the inequality. That is, $0 < x < 2$. When $x = 3$, $x^2 < 2x$ becomes $9 < 6$. This is false. Hence, no numbers in Interval III satisfy the inequality. The answer is (B). The graph of the solution follows:



7. Let x be the unknown side of the triangle. Applying the Pythagorean Theorem yields

$9^2 + x^2 = 15^2$	
$81 + x^2 = 225$	by squaring the terms
$x^2 = 144$	by subtracting 81 from both sides of the equation
$x = \pm\sqrt{144}$	by taking the square root of both sides of the equation
$x = 12$	since we are looking for a length, we take the positive root

In a right triangle, the legs are the base and the height of the triangle. Hence, $A = \frac{1}{2}bh = \frac{1}{2} \cdot 9 \cdot 12 = 54$. The answer is (D).

8. Recall that the average of N numbers is their sum divided by N. That is, average = sum/N. Since the average of x, y, and z is 8 and the average of y and z is 4, this formula yields

$$\frac{x+y+z}{3}=8$$
$$\frac{y+z}{2}=4$$

Solving the bottom equation for $y + z$ yields $y + z = 8$. Plugging this into the top equation gives

$$\frac{x+8}{3}=8$$
$$x+8=24$$
$$x=16$$

The answer is (C).

9. Let $z\%$ represent the unknown percent. Now, when solving percent problems, "of" means times. Translating the statement "What percent of $3x$ is $6y$" into an equation yields

$$z\%(3x) = 6y$$

Substituting $x = 4y$ into this equation yields

$$z\%(3 \cdot 4y) = 6y$$
$$z\%(12y) = 6y$$
$$z\% = 6y/12y$$
$$z\% = 1/2 = .50 = 50\%$$

The answer is (A).

10. Except for the first term, each term of the sequence is found by adding 4 to the term immediately preceding it. In other words, we are simply adding 4 to the sequence 200 times. This yields

$$4 \cdot 200 = 800$$

Adding the 2 in the first term gives 800 + 2 = 802. The answer is (C).

We can also solve this problem formally. The first term of the sequence is 2, and since each successive term is 4 more than the term immediately preceding it, the second term is 2 + 4, and the third term is (2 + 4) + 4, and the fourth term is [(2 + 4) + 4] + 4, etc. Regrouping yields (note that we rewrite the first term as 2 + 4(0). You'll see why in a moment.)

$$2 + 4(0), 2 + 4(1), 2 + 4(2), 2 + 4(3), \ldots$$

Notice that the number within each pair of parentheses is 1 less than the numerical order of the term. For instance, the *first* term has a 0 within the parentheses, the *second* term has a 1 within the parentheses, etc. Hence, the n^{th} term of the sequence is

$$2 + 4(n - 1)$$

Using this formula, the 201^{st} term is $2 + 4(201 - 1) = 2 + 4(200) = 2 + 800 = 802$.

---

11. his is considered to be a hard problem. However, it is actually quite easy. By the definition given, the function @ merely cubes the term on the left and then subtracts 1 from it (the value of the term on the right is irrelevant). The term on the left is x. Hence, x @ $1 = x^3 - 1$, and the answer is (C).

---

12. Recall that an integer is prime if it is divisible by only itself and 1. In other words, an integer is prime if it cannot be written as a product of two other integers, other than itself and 1. Now, $b^2 = bb$. Since b^2 can be written as a product of b and b, it is not prime. Statement (A) is false.

Turning to Choice (B), since a, b, and c are consecutive integers, in that order, b is one unit larger than a: $b = a + 1$, and c is one unit larger than b: $c = b + 1 = (a + 1) + 1 = a + 2$. Now, plugging this information into the expression $(a + c)/2$ yields

$$\frac{a+c}{2} =$$

$$\frac{a+(a+2)}{2} =$$

$$\frac{2a+2}{2} =$$

$$\frac{2a}{2} + \frac{2}{2} =$$

$$a + 1 =$$

$$b$$

The answer is (B).

Regarding the other answer-choices, Choice (C) is true in some cases and false in others. To show that it can be false, let's plug in some numbers satisfying the given conditions. How about $a = 1$ and $b = 2$. In this case, $a + b = 1 + 2 = 3$, which is odd, not even. This eliminates Choice (C). Notice that to show a statement is false, we need only find one exception. However, to show a statement is true by plugging in numbers, you usually have to plug in more than one set of numbers because the statement may be true for one set of numbers but not for another set. We'll discuss in detail later the conditions under which you can say that a statement is true by plugging in numbers.

Choice (D) is not necessarily true. For instance, let $a = 1$ and $b = 2$. Then $\frac{ab}{3} = \frac{1 \cdot 2}{3} = \frac{2}{3}$, which is not an integer. This eliminates Choice (D).

Finally, $c - a = b$ is not necessarily true. For instance, let $a = 2$, $b = 3$, and $c = 4$. Then $c - a = 4 - 2 = 2 \neq 3$. This eliminates Choice (E).

13.

$8x^2 - 18 =$

$2(4x^2 - 9) =$ by the distributive property $ax + ay = a(x + y)$

$2(2^2x^2 - 3^2) =$

$2([2x]^2 - 3^2) =$

$2(2x + 3)(2x - 3)$ by the difference of squares formula $x^2 - y^2 = (x + y)(x - y)$

The answer is (B).

It is common for students to wrongly apply the difference of squares formula to a perfect square:

$$(x - y)^2 \neq (x + y)(x - y)$$

The correct formulas follow. Notice that the first formula is the square of a difference, and the second formula is the difference of two squares.

Perfect square trinomial: $(x - y)^2 = x^2 - 2xy + y^2$

Difference of squares: $x^2 - y^2 = (x + y)(x - y)$

It is also common for students to wrongly distribute the 2 in a perfect square:

$$(x - y)^2 \neq x^2 - y^2$$

Note, there is no factoring formula for a sum of squares: $x^2 + y^2$. It cannot be factored.

14. The percent symbol, %, means to divide by 100. So 10% = 10/100 = .10. Hence, the expression 10% of y tranlsates into $.10y$. Since $y = 3x$, this becomes $.10y = .10(3x) = .30x$. The answer is (B).

15. For the first model, there are 5 options. So there are 5 different types of cars in this model. For the second model, there are the same number of different types of cars. Likewise, for the other two types of models. Hence, there are 5 + 5 + 5 + 5 = 20 different types of cars. The answer is (E).

This problem illustrates the *Fundamental Principle of Counting*:

> If an event occurs m times, and each of the m events is followed by a second event which occurs k times, then the first event follows the second event $m \cdot k$ times.

16. This is considered to be a hard problem. However, it is actually quite easy. By the definition given, the function @ merely cubes the term on the left and then subtracts 1 from it (the value of the term on the right is irrelevant). The term on the left is x. Hence, x @ $1 = x^3 - 1$, and the answer is (C).