Explaining Debt Covenant Amendments: A Structural Approach

Irina Luneva*

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Abstract

Financial covenants, which use financial information to determine control rights in corporate loan contracts, are frequently amended (Roberts (2015)). Despite the commonality, prior literature has not thoroughly studied the economic mechanisms behind covenant amendments. In this paper, I offer a model of debt covenant amendments and structurally estimate it using data on amendments of financial covenants in U.S. companies' loan contracts. The model highlights three forces driving financial covenant amendments: (1) the degree of contractual incompleteness is high, (2) the non-contractible post-contract information is highly informative, and (3) the costs of amendments are small relative to the costs of misallocating control rights. I find that loan contracts are indeed highly incomplete and amendments are indeed substantially cheaper than misallocation of control rights. The non-contractible post-contract information is somewhat informative. I calculate that the amendment option ex ante saves about 2.32% of control misallocation cost.

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*New York University Stern School of Business. E-mail: luneva.irina@stern.nyu.edu.

Introduction

Financial covenants are an important term in corporate loan contracts because they allocate control rights between the borrower and the lender using the borrower's financial information. Consistent with the close attention the contracting parties pay to this loan term, financial covenants are often renegotiated,[1] and the majority of these renegotiations are not merely one-time waivers to avoid a covenant violation[2] but long-term amendments to covenant levels or definitions.[3] Despite the commonality of amendments, we have a limited understanding of why contracting parties change financial covenants long-term. In this study, I offer a theory to explain debt covenant amendments, apply this theory to the novel detailed data on financial covenant amendments, measure key economic parameters describing amendment behavior, and evaluate contracting efficiency gains from the amendment option.

I start with some key facts about financial covenant amendments. I manually collect data on dynamic covenant schedules and amended financial covenants for a random sample of U.S. firms from the publicly available loan contracts filed with the SEC.[4] First, long-term amendments to financial covenants are common: about 28.9% of loan contracts amend a covenant at least once, and about 70.2% of these amendments occur within one year of loan origination. Second, long-term amendments are not simply a remedy for potential technical defaults. A large portion – 42.6% – of covenant amendments happen when a borrower has not violated any covenants, and when an amendment happens, covenants are more

[1] In a random sample of public U.S. companies, about 29% of loans get a financial covenant amended at least once before maturity.

[2] Prior empirical studies (e.g., Roberts and Sufi (2009), Denis and Wang (2014), Roberts (2015), Nikolaev (2018), Dou (2020), Caskey et al. (2022), Dyreng et al. (2023)) treated short-term waivers and long-term amendments monolithically and relied on extensive economic theories about waivers (see, among others, Berlin and Mester (1992), Rajan and Winton (1995), Gârleanu and Zwiebel (2008), Guttman and Marinovic (2018), Lu et al. (2018), Gao and Liang (2019)). As I discuss later in detail, the forward-looking nature of long-term amendments, their occurrence outside of covenant violations, and low correlation with borrowers' financial health suggest that amendments have different economic drivers than waivers, implying that theories of waivers are insufficient to explain amendments.

[3] In this paper, I use the term *amendment* to denote long-term changes to financial covenants and the term *waiver* to denote short-term covenant suspensions. For example, an amendment changes the debt-to-EBITDA ratio that the borrower must satisfy multiple years going forward until the contract's maturity date. A waiver either waives a violation of the debt-to-EBITDA ratio covenant or increases the threshold for a short period, such as a month. Empirically, manual analysis may be required to distinguish waivers from amendments. While waivers after covenant violations are legally designated as "waivers", waivers in anticipation of violations, that temporarily suspend a financial covenant, are legally designated as "amendments". In addition, some "amendments" may include both a short-term and a long-term change to financial covenants.

[4] The most commonly used loan contract database, Thomson Reuters Dealscan, does not record amendments to financial covenants and original covenants in sufficient detail. For original covenant schedules, the database only contains levels and directions for dynamic covenants, without specifying how the covenant levels change every year. For example, Dealscan would record a covenant that provides a borrower a few months grace period in the same way as a covenant that becomes stricter every year. The first covenant represents a short-term adjustment, potentially a waiver, while the second regulates an agency problem that is expected to worsen as time passes. For amendments, Dealscan indicates when amendments occur, but it does not report the content of amendments to financial covenants.

likely to get tighter, not looser. Finally, amendments do not strongly correlate with borrowers' financial health at loan origination. All these facts suggest that long-term amendments to financial covenants are an economically unique phenomenon that deserves to be studied.

To explain why financial covenants get amended, I use structural estimation approach. Many aspects of lender-borrower relationships are unobservable. Financial contracts are inherently incomplete, which by definition implies that information triggering renegotiations cannot be recorded in hard data. In addition, not even all interactions between lenders and borrowers are observable: a lot of information is exchanged through in-person encounters during banks' monitoring process. Structural estimation can use observable statistics, such as frequency and magnitudes of amendments, to infer the characteristics of unobserved information used by contracting parties.

I start by building a theoretical framework that I later estimate on the data. The model rests on the premise that the role of covenants is to optimally allocate control rights based on some financial statistics and that other information relevant to the optimal control rights allocation is not contractible (in the spirit of Aghion and Bolton (1992)). The first building block is a non-contractible state of the world and a contractible yet imperfect signal about this state. The state is binary and captures whether the lender or the borrower is the optimal party-in-control (from the total surplus perspective). The signal is also binary: whether any covenant is or is not violated. To capture various covenant menus in real-world contracts in a unifying theoretical construct, I model the contract that the parties choose as an information system – the probabilities that the covenant misallocates control in each state of the world (i.e., type-1 error, when a covenant is violated even though control should stay with the borrower, and type-2 error, when a covenant is not violated even though control should be shifted to the lender), which are limited from above by the degree of contractual completeness (i.e., the aggregate of type-1 and type-2 errors cannot be lower than a certain positive level).

The second building block of the model is efficiency losses due to various contracting frictions. When a wrong party obtains control rights, the total enterprise value may be reduced. For example, a manager can take excessive risks,[5] or a bank can limit investment and reduce the company's ultimate value.[6] I capture this first type of efficiency loss in a parameter called control misallocation cost. The

[5] Malmendier and Tate (2005) find that companies with overconfident executive managers exhibit (likely excessive) high sensitivity of their investments to cash flows. When a CEO is overconfident, the company's investment increases by about 23.4% of assets more in response to a rise in cash flows. Because overconfident CEOs overestimate returns to some investment projects, firm value can be destroyed significantly.

[6] Chava and Roberts (2008) find that a covenant violation leads to about 13% decline in capital expenditures for the violating

other type of efficiency loss occurs when contracting parties renegotiate. Renegotiations waste monetary and time resources. In addition, an optimal allocation of control rights may not be achieved because of either party's bargaining power. I capture these efficiency losses in two parameters: a waiver cost – a renegotiation about the current allocation of control rights – and an amendment cost – a renegotiation about the contract that allocates future control rights. These two efficiency loss parameters parsimoniously capture a variety of contracting frictions that may be present in the data.

The contracting parties choose an initial contract – an information system that allocates control between the lender and the borrower based on the state of the world – to minimize expected efficiency losses. The innovative part of the model is that, after signing the initial contract, when new information about future non-contractible state arrives, the parties can at a cost amend their contract.[7] For example, if the new information suggests the manager is more likely to be the total-surplus-maximizing party-in-control in the future, the parties may reduce the type-1 error in exchange for a higher type-2 error. The parties thus trade off the costs of amending the contract in the current period against the costs of misallocating control rights in the future. The option to amend ex ante helps the parties reduce future costs of control rights misallocation or waivers, improving contracting efficiency.

In equilibrium, the contracting parties amend their contract whenever post-contract non-contractible information suggests the new optimal contract is sufficiently distant from the contract the parties signed at the beginning. It is the **surprise** in the expected state of the world or the cost of misallocating control rights that drives long-term amendments, not the state of the world or the cost per se. This result is consistent with the empirical observation that the borrower's initial financial health or initial covenant strictness do not predict whether the parties will amend financial covenants. Firms with worse performance may set tighter covenants in the beginning (and are more likely to violate and waive them), but whether these covenants get amended depends on whether there is a substantial surprise in these firms' performance.

The model highlights three forces that drive financial covenant amendments. First, the degree of contractual incompleteness needs to be sufficiently high. Contractual incompleteness is the extent to

firm, suggesting considerable loss in company value if this violation is by mistake. Dyreng et al. (2022) compute that companies violating financial covenants experience between 6% and 11% lower abnormal stock returns compared to their non-violating counterparts. The numbers indicate that stockholders perceive the cost of violation, including both real and reputation costs, to be quite high.

[7] Empirically, most contracts amend financial covenants once and within one or two years after loan origination. To capture this feature of the data, I allow the parties to amend their contract only once. The main intuition remains unchanged if I consider more periods of amendments.

which the signal(s) used in a contract cannot capture the actual state of the world. When the contractible signal is imprecise, covenants are more likely in general to make an error, and thus preempting potential errors is more valuable. For example, a company whose interest coverage ratio more poorly represents the company's actual ability to pay off the loan would more likely have its interest coverage ratio covenant amended. The second force is the informativeness of the post-contract non-contractible information about the future state of the world: when this information is more predictive, the parties can better anticipate what the future optimal contract is, increasing their motivation to amend. Intuitively, if a firm is more certain about whether it will stay financially healthy based on today's non-contractible information, the firm is more likely to amend its contract. The last force driving amendments is the low cost of amendments compared to the cost of misallocating control rights. If changing the contract in the current period is substantially cheaper than efficiency losses because of the suboptimal contract later on, the parties are better off adjusting their contract in the current period. For instance, when legal fees and other frictions preventing amendments are small but potential losses due to asset substitution by the manager are large, the parties are more likely to amend financial covenants.

To evaluate the magnitudes of contractual incompleteness, informativeness of non-contractible information, and relative costs of amendments, I structurally estimate the model. The estimated parameters help me evaluate the roles of the different forces in explaining debt covenant amendments. The estimated degree of contractual incompleteness is high: if financial covenants never make, for example, type-1 errors, they will make type-2 errors about 31.3% of the time. The second force – the informativeness of post-contract non-contractible information – is of a more modest magnitude. If a company does not (does) need the lender's intervention this year, it likely will not (will) need it next year with only a 65.4% (50.1%) chance. Finally, the cost of amending the contract is small: about 1.9% of the average cost of misallocating control rights, – highlighting the importance of the third force. To summarize, the estimates suggest that firms amend financial covenants in their loan contracts primarily for two reasons: contracts are highly incomplete, and adjusting contracts in advance is considerably cheaper than later potentially granting control to the wrong party.

I also evaluate contracting efficiency gains from the amendment option. Ex ante, the option to amend financial covenants saves the contracting parties about 2.32% of control misallocation cost. Ex post, conditional on an amendment, the parties save 15.32% of the total surplus in the state when the manager is the best party-in-control and 12.62% in the state when the bank is the best party-in-control. These numbers appear economically reasonable given that control misallocation costs capture the cost of the wrong party making a suboptimal decision.

To provide some evidence on the absolute rather than relative magnitudes of control misallocation and amendment costs, I extend the estimation and include changes in firms' market values around announcements of amendments as additional moments. The monetary values of misallocating control rights appear substantial: for an average firm, the expected loss of giving control to a wrong party is about $136.4 million, or 7.44% of the market value. Amending covenants to reduce the probability of control misallocaiton in advance is a significantly cheaper option: the cost of an amendment is around $4.0 million, or only 0.22% of the market value. Even though the amendment option helps reduce contracting efficiency losses, it cannot perfectly do so. The efficiency loss due to contractual incompleteness is about $19.09 million. In other words, positive-NPV investments generating a value of at least $19.09 million are underfunded because of contractual incompleteness and costly renegotiations.

Next, I conduct counterfactual analyses to study how two aspects of contract renegotiations affect amendment behavior. The first aspect is borrowers' financial health. Simple correlation suggests that covenant amendments are not strongly related to borrowers' pre-contract financial health. I conduct a counterfactual analysis where I study two types of companies: a completely financially healthy company, that will not need the lender's intervention in the first period, and a completely financially unhealthy company, that will almost surely need the lender's intervention. Perhaps counter-intuitively, as time goes on, the healthy and unhealthy firms are similarly likely to amend their financial covenants. Common intuition for waivers would suggest that the financially unhealthy borrower is more likely to violate a covenant, and thus prompt a waiver. In contrast to waivers, amendments happen when the original contract is far enough from the optimal contract given post-contract information. Financially healthy and unhealthy companies may set different covenants in their original contracts, but then amendments are determined by new unexpected information about the firms. These findings confirm the result of Roberts (2015) that neither firms' EBITDA-to-assets ratio nor their stock returns predict that a covenant will be amended.

Second, I explore the impact of permitting more waivers on amendment behavior. In practice, waivers typically occur when a covenant is breached, and company management needs to regain control. This scenario implies a misallocation of control rights, wherein the bank unintentionally gains control, and the two parties rectify the situation by waiving the misallocation, restoring control to the manager.

The opposite case, where control rights should belong to the bank but are mistakenly granted to the manager, is not readily observable because it would not result in a technical default. Therefore, in the main estimation, I disallow waivers of this type. In an extension, I examine the scenario where both misallocation cases are eligible for waivers. The value of the amendment option decreases: ex ante gains drop by 1 percentage point, and, conditional on an amendment when the bank should have control, they decrease by 4 percentage points.

Finally, I study how corporations' and loans' characteristics are associated with amendment behavior and contractual efficiency gains from amendments. I consider three characteristics: the borrower's financial accounting quality, the past relationship between the lender and the borrower, and the lenders' coordination cost. The results suggest a substantial gain from amendments for firms with less informative earnings numbers and in close relationships with their lenders. As for coordination costs, when lenders find it difficult to negotiate, avoiding a covenant violation appears costlier, and amendments occur more often.

I contribute to the broad literature on debt contracting and renegotiations (e.g., Roberts and Sufi (2009), Gârleanu and Zwiebel (2008), Bharath et al. (2009), Christensen and Nikolaev (2012), Choi and Triantis (2012), Denis and Wang (2014), Roberts (2015), Li et al. (2016), Saavedra (2018), Armstrong et al. (2020), Dou (2020), Caskey et al. (2022), Christensen et al. (2022)). My study focuses on financial covenants and highlights an important feature: covenants are often renegotiated long-term. I study why covenant amendments occur, provide a theoretical framework and quantify key economic forces determining amendment behavior, such as contractual incompleteness, importance of non-contractible information, and control misallocation and renegotiation costs. These quantities allow me to estimate efficiency losses from contractual incompleteness (Aghion and Bolton (1992)) and various corporate frictions (Coase (1937)).

My study helps to bridge the gap between empirical observations and theoretical literature on debt contracting. I develop a general framework that captures key economic conflicts present in real-world debt contracting. First, I model agency problems in reduced form as a loss of total surplus from the firm-bank relationship. These parameters can conveniently capture multiple conflicts described by prior theories (e.g., Aghion and Bolton (1992), Rajan and Winton (1995), Sridhar and Magee (1996), Gârleanu and Zwiebel (2008), Guttman and Marinovic (2018), Gao and Liang (2019), Laux (2020), Adler (2020)), such as asset substitution, debt overhang, bargaining power, etc., in a single theoretical construct. Second, financial covenants in my model are not specific thresholds for specific financial variables (such as in Guttman and Marinovic (2018) or Gao and Liang (2019)) but a broader construct – type-1 and type-2 errors. All of these features make the model more generic and applicable to the data.

Another important stream of literature structurally estimates dynamic models of corporate financing to answer a variety of questions, about investment, debt structure, and debt contracting (see Strebulaev and Whited (2012) for a review). In the financial covenants domain, Matvos (2013) evaluates corporations' benefits from including covenants in their loan contracts. Griffin et al. (2024) use a structural model to explain the sharp decline in financial covenant violations. My paper is the first to estimate a model of within-contract renegotiations of financial covenants and their implications for contracting efficiency.

The rest of the paper is organized as follows. Section 1 presents the debt covenant amendments model and discusses the equilibrium. Section 2 describes the data collection process and main properties of my dataset. Section 3 shows the estimated parameters, analyses model fit, and provides estimates of contracting efficiency gains from amendments. Section 4 discusses counterfactual scenarios. In section 5, I estimate dollar values of contractual efficiency losses. Section 6 shows parameter estimates for different subsamples of the data. Section 7 concludes.

1 Model of debt covenant amendments

In this section, I present a model of debt covenant amendments and discuss its differences from existing debt contracting literature. Next, I discuss the equilibrium and debt contracting efficiency implications of amendments. Finally, I present theoretical moments that I use to identify model parameters.

1.1 Model setup and its difference from prior models

I study a classic debt contracting problem. A company, governed by a manager, has a positive NPV investment opportunity and has decided to borrow funds from a bank. The bank, operating in a perfectly competitive credit market, has money and can provide funding. However, the bank is worried that in some states of the world, its investment can be diluted: because the manager obtains a profit when the project is successful but the bank bears the cost if the project fails, the manager has incentives to choose actions that are too risky, destroying the value of the firm and the bank's claim. The bank addresses this issue by signing a contract that specifies when the manager stays in control of the firm and when control rights are transferred to the bank.

However, the contracting parties face the problem of non-contractibility (Aghion and Bolton (1992)). Even though the state of the world is observed by the bank and the manager, it can not be contracted upon in the debt contract, either because it is impossible to describe the state in sufficient detail or the state is not verifiable (Christensen et al. (2016)). Instead, the contract must be written on an imperfect signal about the state of the world. For example, a company's financial health is a broad concept that partially relies on difficult-to-verify soft information (e.g. growth opportunities) and cannot be described in a contract, but the company's debt-to-EBITDA and interest coverage ratios are hard metrics which are correlated with financial health and can be contracted upon.

In the classic framework (e.g. Rajan and Winton (1995), Sridhar and Magee (1996), Gârleanu and Zwiebel (2008), Guttman and Marinovic (2018), Lu et al. (2018), Gao and Liang (2019)), outlined in figure 1a, the bank-company relationship proceeds as follows. The contract, specifying control rights as a function of the contractible signal, is signed and the company makes the investment. In the next period, the bank and the manager observe the state of the world and the signal that determines which party has control rights. In cases when control rights are misallocated – or the signal does not agree with the state – the parties can renegotiate: waive the misallocation, compensate the party that got control inefficiently, and return control rights to the optimal party-in-control. The process repeats itself in the next periods until the debt contract matures.

When the classic framework is brought into the world where company-bank relationships last for more than a single period, one important possibility is missing. If the states of the world are somewhat persistent, after the parties have arrived at time $t = 1$, they have more information than they did at time $t = 0$ when they signed the initial contract. Specifically, they know the state at time $t = 1$, which is correlated with the state at time $t = 2$. As a result, the contracting parties adjust their beliefs about the expected optimal party-in-control at time $t = 2$, and according to these beliefs, the initial contract is no longer optimal. If the parties had the option to write a new contract, this contract would result in a lower expected probability and cost of misallocating control rights at time $t = 2$.

I depart from the classic framework and allow the parties to amend their contract long-term at time $t = 1$. The new timeline is shown in figure 1b. Even though the state of the world cannot be contracted upon, the parties can adjust their beliefs after observing it and amend the contract. The ability to amend the contract as the bank and the manager learn new non-contractible information allows the parties to (partially) dynamically complete the inherently incomplete debt contract.

The model has two main building blocks. The first component is the perfectly observable but non-contractible state of the world and a non-contractible signal that is imperfectly correlated with the state. The second component is the trade-off between the deadweight loss if the contract is amended at a given moment and the cost of potentially misallocating control rights in the future.

The state of the world at time $t \in \{1, 2\}$ is described by a set of two variables: $\{r_t, s_t\}$. $r_t \in \{m, b\}$ is a binary random variable representing which party should have control rights: m implies the optimal party-in-control is the manager, and b implies the optimal party-in-control is the bank. The probability that at time $t = 1$ the state of the world is $r_1 = m$ is μ_0 and that $r_1 = b$ is $(1 - \mu_0)$. Conditional on $r_1 = m$, $r_2 = m$ with probability μ and $r_2 = b$ with probability $(1 - \mu)$; conditional on $r_1 = b$, $r_2 = b$ with probability β and $r_2 = m$ with probability $(1 - \beta)$, where $\mu, \beta > \frac{1}{2}$. I assume $\mu_0 > \frac{1}{2}$ to focus on the cases when the company is more likely than not to be financially healthy and not in need of the lender's intervention. The state of the world is observed by both the manager and the bank but is non-contractible. Even though the state of the world cannot be contracted upon, there exists a contractible signal $s_t \in \{m, b\}$ about the optimal party-in-control that is imperfectly correlated with the state r_t.

The parties trade-off the cost of dealing with control rights misallocation if it occurs ex post against the cost of amending their contract in advance to reduce expected control misallocation cost. When control is already misallocated, part of the firm value is destroyed. This loss is captured by the continuous random variable c_t. c_1 is a random draw from a distribution with support $\tilde{c}_1 \sim [0, \bar{c}]$. If $r_1 = r_2$, the cost of misallocating control rights persists: $\tilde{c}_2 = \rho c_1 + (1 - \rho)\tilde{c}$, where $\tilde{c} \sim [0, \bar{c}]$. If $r_1 \neq r_2$, the cost at time $t = 2$ is a new random draw, $\tilde{c}_2 \sim [0, \bar{c}]$.[8] Sometimes the parties may be able to avoid a control misallocation. However, avoidance is not always possible because of various frictions (see, for example, Rajan and Winton (1995)). These frictions include legal fees, tax consequences (Dyreng et al. (2023)), forgone investment to avoid covenant violations (Adler (2020)), or parties' bargaining power or private benefits that prevent the transfer of control rights. I introduce variables η_w^m and η_w^b to capture the size

[8]For instance, if the manager runs the company without restrictions during financial instability, she may take excessive risk and reduce the ultimate total value of the firm. The size of this loss is likely related to the loss if the situation continues for another year. But if the next year the situation is the opposite – financial conditions are promising, but the bank steps in and limits the firm's investment – the size of the firm value loss due to forgone investment opportunities is not necessarily comparable to the loss in the case the manager takes excessive risk.

of the frictions preventing avoidance of misallocation in the states $r_t = m$ and $r_t = b$, respectively.[9] If the cost c_t is below the friction, c_t has to be borne. If the cost c_t is higher than $\eta_w^i, i \in \{m, b\}$, the value loss that exceeds the parties' private benefits can be avoided, and the total surplus only suffers the loss of η_w^i. In what follows, I call variables η_w^m and η_w^b waiver costs or waiver frictions.

A way to reduce the probability of control rights misallocation and the size of the firm value loss is an amendment – an ex ante alteration of the contract. After signing the initial contract at time $t = 0$, the parties have the option to write a new contract at time $t = 1$. Empirically, most contracts amend financial covenants once and within one or two years after loan origination. To capture this feature of the data, I allow the parties to amend their contract only once.[10] An amendment involves a fixed cost η_a, which is a deadweight loss. η_a can capture legal fees, tax consequences (Dyreng et al. (2023)), coordination costs (Becker and Ivashina (2016)), or time and effort that the manager and the bank spend writing a new contract. Importantly, to obtain any amendments in equilibrium, the amendment cost η_a has to be less than the waiver cost η_w^i. This assumption is realistic because amendments happen in advance, while waivers typically need to be completed in a short time under more pressure, implying larger direct costs of renegotiation and greater parties' bargaining power.[11] In addition, a waiver means a misallocation of control has already happened and may have been adversely noted by businesses' other counterparties.[12]

I model the contract in a general form. The financial covenants allocate the imperfection of the signal s_t between the two states, m and b. The probability that a covenant chosen at time $\tau < t$ makes a type-1 error, or is violated when should not be, at time t, is $\pi_{mt}^\tau = Pr\left[s_t = b | r_t = m\right]$. The probability that this covenant makes a type-2 error, or is not violated when should be, at time t, is $\pi_{bt}^\tau = Pr\left[s_t = m | r_t = b\right]$. The precision of the covenants in states m and b is $1 - \pi_{mt}^\tau$ and $1 - \pi_{bt}^\tau$, respectively. Note that the subscripts of the probabilities denote the realizations of the state r_t and not of the signal s_t.

The quality of the contractible signal s_t is captured by a parameter $q \in (0.5, 1]$. The possibility frontier for the precision of the contract is described by the equation $(1 - q - \pi_{mt}^\tau)^2 + (1 - q - \pi_{bt}^\tau)^2 =$

[9]Different cost of a waiver in different states allows the model to capture potentially different levels of frictions arising when control rights are misallocated in different situations in the real world. In particular, waivers of covenant violations – misallocations of control rights when the contract grants control to the bank but the manager should be in control – have been widely documented (e.g., Dichev and Skinner (2002)), while the opposite case – where no covenant was violated but the bank still demands control – has not been observed and may not be plausible as the bank could face litigation.

[10]The main intuition remains unchanged if I consider more periods of amendments.

[11]Casual conversations with lawyers in the corporate loan area suggest that legal fees paid by either the borrower or the lender are substantially higher when a deal has to be crafted in a shorter time frame. Additionally, both lenders and borrowers are typically less inclined to cooperate if approached on a short notice.

[12]For instance, rating agencies typically know about a technical default in advance and may downgrade the borrower's rating even before a covenant actually is breached.

$(1-q)^2$ and illustrated in figure 2. The parties have to trade-off the type-1 error against the type-2 error, and as one error gets closer to zero, the covenant has to sacrifice a lot of precision in another state to further reduce the error. If the covenants never make an error in one state, the maximum possible probability of not making an error in the other state is q.

Below I formally define the contract in the model.

Definition 1. *A contract, chosen at time τ, is an information system: the set of probabilities that the signal disagrees with the state at times $t = 1, 2$ in states m and b, $\{\{\pi^{\tau}_{m1}, \pi^{\tau}_{b1}\}, \{\pi^{\tau}_{m2}, \pi^{\tau}_{b2}\}\}$.*

The timeline of the events in the model is shown in figure 3. At time $t = 0$, the initial contract is chosen. I allow for dynamic covenant schedules: covenants for time $t = 1$, $\{\pi^0_{m1}, \pi^0_{b1}\}$, and time $t = 2$, $\{\pi^0_{m2}, \pi^0_{b2}\}$, which are both chosen at time $t = 0$, can be different. At time $t = 1$, the parties observe the state of the world r_1, the cost of misallocating control rights in this state, c_1, and the signal s_1. If the state disagrees with the signal, $r_1 \neq s_1$, the parties either can bear the cost c_1 or waive the violation or the non-violation and bear the cost η^i_w, $i \in \{m, b\}$. Additionally, the parties can amend the contract for time $t = 2$ from the initial one, $\{\pi^0_{m2}, \pi^0_{b2}\}$, to a new one $\{\pi^1_{m2}, \pi^1_{b2}\}$. At time $t = 2$, the parties observe the new state of the world r_2, the cost of misallocating control rights c_2, and the signal s_2. If $r_2 \neq s_2$, the parties can either incur c_2 or waive and bear η^i_w. At time $t = 3$, the contract matures.

Besides the distinction between waivers and amendments, the model has three innovations relative to the existing literature. First, the model does not have an explicit agency conflict between the borrower and the lender but focuses on the optimal choice of control rights given the information environment. This modeling choice allows me to estimate contracting efficiency and the impact of the information environment on it using real-world data. In reality, multiple agency conflicts can be present at the same time, each of them involving losses in the total value of the lender-borrower relationship. My model captures them in a reduced form through parameters c_t and η^i_w, helping me get to the aggregate estimates of efficiency losses in debt contracting. As discussed above, η^i_w can capture private benefits that either party derives from having control rights. Alternatively, it may be a measure of the parties' bargaining power: the larger the outside option, the higher amount has to be offered to a party to accept a contract or an amendment. c_t describes different moral hazard issues, such as asset substitution or the debt overhang problem. When the manager is in control suboptimally, she may take excessive risks (asset substitution); when the bank gets unjustified control, it may force the firm to underinvest and forgo profitable projects (debt overhang).

The second innovation of the model is that the contract consists solely of control rights – covenants – and does not include an interest rate. The problem of one party that offers a contract, consisting of a covenant and an interest rate, to another party, condenses into two steps. In the first step, a covenant that maximizes the total surplus from the parties' relationship is chosen. In the second step, given the optimal covenant, the interest rate is chosen to maximize the offering party's payoff conditional on the other party accepting the offer. In this setting, the interest rate just splits the total surplus between the parties and does not affect the overall contracting efficiency gain.[13] Since the focus of my study is aggregate contracting efficiency, I abstract away from the interest rate's role and limit the analysis to the choice of optimal covenant schedules. Thus, in the remainder of the paper, I use the terms contract and covenant schedule interchangeably.

Third, I abstract away from the specific structure of the signal used to determine control rights. This general notion of the signal allows me to capture diverse metrics used in real contracts (e.g., different financial ratios and amounts, upper and lower thresholds) using one theoretical construct. The covenant in the model is not a specific statistic, but broadly the probability that a signal(s) used in the contract makes a type-1 or type-2 error.

In Appendix 8.5, I demonstrate why the three simplifications described above are sufficient to describe amendment behavior and provide the mapping between my framework and classic debt contracting frameworks.

A number of caveats are worth discussing. The model assumes that both the bank and the company manager perfectly observe the non-contractible state of the world and the cost of misallocating control rights. However, extensive prior work (e.g., Gârleanu and Zwiebel (2008)) argues that information asymmetry between the lender and the borrower plays a considerable role in the contracting process: financial covenants are set stricter in the initial contract to provide the lender with greater control, and get waived later if allocating control rights to the borrower proves optimal. My model can be interpreted to allow for information asymmetry between the lender and the borrower in the following way. First, the non-contractible state of the world, r_t, and the cost of misallocating control rights, c_t, can be thought of as partial information that is disclosed by the borrower to the lender or that the lender infers from the borrower's communication. The parameters capturing the persistence of these pieces of information, μ

[13] Gigler et al. (2009) arrive at the same conclusion when examining the role of conservative accounting in debt contracts.

and β, and ρ, respectively, can be interpreted as precision and fullness of the information. For instance, if the borrower can credibly signal she is telling the truth about the state of the world, the predictive ability of her disclosure about the state, μ or β, would be higher. With this interpretation, however, the general noise in the non-contractible information and the noise induced by the party disclosing the information cannot be separated.

Second, the costs of waivers, η_w, and amendments, η_a, can be thought of as a sum of the renegotiation cost and monitoring cost. In standard debt contracting frameworks (e.g., Gârleanu and Zwiebel (2008)), provided that the cost of the lender acquiring the borrower's private information is low enough, the lender will choose to acquire it and renegotiate whenever the sum of the renegotiation cost and the monitoring cost is less than the cost of misallocating control rights (e.g., the extent of asset substitution by the manager). In my model, an amendment or a waiver occurs if the state-dependent amendment or waiver cost is less than the state-dependent cost of misallocating control rights. The disadvantage of this interpretation is that I cannot disentangle monitoring costs from renegotiation costs as two different frictions preventing renegotiations. These two frictions, however, would affect the decision to amend a contract in the same way, making the distinction between the two not critical for the purpose of my study.

Another important caveat is that in the model, the costs of control misallocation and amendments or the level of contractual incompleteness do not depend on the chosen level of financial covenants. Prior work finds that corporations may behave differently when they are close to covenant violations: for example, alter investment behavior (e.g., Adler (2020)) or manage earnings (e.g., DeFond and Jiambalvo (1994)) to avoid breaching the covenant. These behaviors can potentially affect the size of losses if the company manages to avoid a violation or make contracts more incomplete because the numbers the company reports may be of lower quality. This limitation, however, does not diminish the paper's findings. Because the model is estimated on realized debt contract choices, the reader can interpret estimated costs and degree of contractual incompleteness as averages of those achieved in equilibrium for any given level of financial covenants.

1.2 Benchmark: a model without amendments

Before stating the equilibrium of the full model, I analyze a benchmark model where the parties do not have the option to amend the contract at time $t = 1$. The benchmark is useful to understand how the dynamic covenant schedule works and why the amendment option helps the parties save the cost of control rights misallocation.

In the benchmark model, the only choice that the parties make is whether to waive control misallocation when this misallocation occurs. At time $t = 2$, if $s_2 \neq r_2$, the parties will waive if $c_2 \geq \eta_w^i$ and will not waive otherwise. Similarly, at time $t = 1$, the waiver will happen if and only if $c_1 \geq \eta_w^i$. Therefore, at any point in time, if control rights are misallocated, the deadweight loss will be $\min\{c_t, \eta_w^i\}$.

At time $t = 0$, the dynamic covenant schedule, $\{\{\pi_{m1}^0, \pi_{b1}^0\}, \{\pi_{m2}^0, \pi_{b2}^0\}\}$, is chosen. I denote the expected deadweight loss due to control misallocation for the case when the parties do not have any information about c_t by $e^i \equiv E\left[\min\{\tilde{c}_t, \eta_w^i\}\right], i \in \{m, b\}$. The expected loss of control rights misallocation equals e^i at time $t = 1$ and at time $t = 2$ when $r_2 \neq r_1$. At time $t = 2$ when $r_2 = r_1$, the parties will have some information about c_2 because they will have observed c_1. I use the random variable $\tilde{f}_j^i$ to denote the expected cost of misallocating control at time $t = 2$ when the time-1 state persists, where $j \in \{m, b\}$ denotes the state at time $t = 1$ and $i \in \{m, b\}$ denotes the state at time $t = 2$. This expected cost if given by $\tilde{f}_j^i \equiv E\left[\min\{\tilde{c}_2, \eta_w^i\} | \tilde{c}_1, r_2 = r_1\right] = E\left[\min\{\rho c_1 + (1-\rho)\tilde{c}, \eta_w\} | \tilde{c}_1, r_2 = r_1\right]$. The variable $\tilde{f}_j^i$ is a random variable because it is a function of the realization of c_1, which is unknown at time $t = 0$. Using this notation, the problem at time $t = 0$ is

$$
\begin{aligned}
\min_{\{\pi_{m1}^0, \pi_{b1}^0, \pi_{m2}^0, \pi_{b2}^0\}} \quad & \pi_{m1}^0 \mu_0 e^m + \pi_{b1}^0 (1-\mu_0) e^b + \\
\pi_{m2}^0 \left(\mu_0 \mu E[f_m^m] + (1-\mu_0)(1-\beta) e^m\right) + & \pi_{b2}^0 \left(\mu_0 (1-\mu) e^b + (1-\mu_0) \beta E[f_b^b]\right) \\
\text{s.t.} \quad & (1 - \pi_{m1}^0 - q)^2 + (1 - \pi_{b1}^0 - q)^2 = (1-q)^2 \\
& (1 - \pi_{m2}^0 - q)^2 + (1 - \pi_{b2}^0 - q)^2 = (1-q)^2 \\
& \pi_{kt}^0 \in [0, 1-q], \text{ for } k \in \{m, b\} \text{ and } t \in \{1, 2\}
\end{aligned}
$$

The optimal covenant schedule minimizes the cost of misallocating control rights at all times and in all states subject to the restriction on the total precision of the signals. The cost of misallocating control at time $t = 1$ is e^m in the state $r_1 = m$ and e^b in the state $r_1 = b$, and the probability of misallocating control is $\pi_{m1}^0 \mu_0$ in the state $r_1 = m$ and $\pi_{b1}^0 (1-\mu_0)$ in the state $r_1 = b$. At time $t = 2$, in the state $r_2 = m$ ($r_2 = b$), the covenant makes an error with probability π_{m2}^0 (π_{b2}^0). If $r_2 = m = r_1$

($r_2 = b = r_1$), which happens with probability $\mu_0\mu$ ($(1-\mu_0)\beta$), the cost of the covenant making an error is $E[f_m^m]$ $\left(E[f_b^b]\right)$. If $r_2 = m \neq r_1 = b$ ($r_2 = b \neq r_1 = m$), which is the case with the $(1-\mu_0)(1-\beta)$ $(\mu_0(1-\mu))$ chance, the cost of an error is e^m $\left(e^b\right)$.

Proposition 1. *The optimal covenant schedule chosen at time $t = 0$ in the model without amendments, $\{\{\pi_{m1}^0, \pi_{b1}^0\}, \{\pi_{m2}^0, \pi_{b2}^0\}\}$, is unique and as follows*

$$\pi_{m1}^{0*} = (1-q)\left(1 - \frac{1}{\sqrt{1+y_1^0}}\right) \tag{1}$$

$$\pi_{b1}^{0*} = (1-q)\left(1 - \frac{1}{\sqrt{1+\frac{1}{y_1^0}}}\right) \tag{2}$$

$$\pi_{m2}^{0*} = (1-q)\left(1 - \frac{1}{\sqrt{1+y_2^0}}\right) \tag{3}$$

$$\pi_{b2}^{0*} = (1-q)\left(1 - \frac{1}{\sqrt{1+\frac{1}{y_2^0}}}\right) \tag{4}$$

where $y_1^0 = \left(\frac{(1-\mu_0)e^b}{\mu_0 e^m}\right)^2$ is relative cost of misallocating control rights in state b compared to state m at time $t = 1$ and $y_2^0 = \left(\frac{\mu_0(1-\mu)e^b+(1-\mu_0)\beta E\left[f_b^b\right]}{\mu_0\mu E[f_m^m]+(1-\mu_0)(1-\beta)e^m}\right)^2$ is relative cost of misallocating control rights in state b compared to state m at time $t = 2$.

The optimal contract misallocates control rights more frequently if the degree of contractual incompleteness is higher: type-1 and type-2 errors in both periods are larger when q is lower. Given a certain level of contractual incompleteness, the optimal covenant schedule is determined by the expected relative costs of misallocating control rights in the two states of the world: y_1^0 at time $t = 1$ and y_2^0 at time $t = 2$. For example, if at time $t = 1$ the cost of misallocation in state b (e^b) is higher and the state b is more likely ($1 - \mu_0$ is higher), the relative cost of misallocating control in state b is higher, and the optimal covenant schedule for time 1 assigns a lower error to the state b (π_{b1}^{0*} is lower).

When amendments to the covenant schedule are prohibited, the contracting parties ignore valuable non-contractible information that they learn over time. At time $t = 1$, the parties have observed the state r_1 – a projection for the state at time $t = 2$ – and the cost of control misallocation c_1 – a projection for the cost at time $t = 2$ if the state of the world persists. This information can be used to adjust the contract to redistribute covenant error between the states at time $t = 2$ and reduce the cost of misallocating control rights and waivers at time $t = 2$.

In the next section, I introduce the option to amend the contract, which helps avoid the loss of non-contractible information and dynamically complete the incomplete debt contract.

1.3 Full model equilibrium

In the full model, the parties can amend the contract at time $t = 1$ at a fixed cost η_a. The timeline is shown in figure 3. I solve the model by backward induction.

At time $t = 2$, the only decision is whether to waive a potential misallocation of control rights. The parties will waive if and only if $c_2 \geq \eta_w^i$.

At time $t = 1$, two decisions are made. First, the parties choose whether to waive a misallocation of control rights if it occurs. The parties waive if and only if $c_1 \geq \eta_w^i$. Second, the parties may decide to amend the covenant schedule. In this case, they bear a deadweight loss of η_a and instead of having the initial contract, $\{\pi_{m2}^0, \pi_{b2}^0\}$, sign a new contract, $\{\pi_{m2}^1, \pi_{b2}^1\}$, for time $t = 2$. When deciding whether to amend the contract, the parties first find out the new optimal covenant schedule and then compare the cost of amendment η_a with the expected saved cost of control misallocation and waivers at time $t = 2$. The problem of choosing the new covenant schedule is state-dependent and generally resembles the choice of the contract described in the previous section. The new optimal contract minimizes the expected cost at time $t = 2$ given new non-contractible information received at time $t = 1$, and the optimal error allocated to a state decreases in the expected relative cost of misallocating control rights in this state.

Once the parties understand what the new optimal contract is, they decide whether to amend as follows:

Proposition 2. *At time $t = 1$ in the state $r_1 = j$, the parties amend the covenant schedule if and only if the expected cost of misallocating control rights at time $t = 2$ in case the time-1 state persists, f_j^j, is low enough or high enough:*

$$f_j^j \leq x_j^l \text{ or } f_j^j \geq x_j^u \tag{5}$$

x_m^l, x_m^u, x_b^l, and x_b^u are defined in Appendix 8.1.

The decision to amend is intuitive. When choosing whether to amend or not, the parties compare the cost of amendment η_a against the expected reduction in control misallocation cost under a new contract. If the state from $t = 1$ persists in $t = 2$, an estimate of this expected reduction is f_j^j. If the estimate is low enough, the optimal contract $\{\pi_{m2}^1, \pi_{b2}^1\}$ should put sufficiently more error into the state j than the initial contract $\{\pi_{m2}^{0*}, \pi_{b2}^{0*}\}$ does. Similarly, if the estimate is high enough, the optimal contract should put sufficiently less error into the state j than the initial contract does. For intermediate levels of f_j^j, the initial contract is not in sufficient disagreement with the new optimal contract to justify an amendment.

Importantly, what triggers and amendment of the contract is not the expected state of the world (e.g., borrower health) per se but the surprise in the expected state of the world. This result explains why empirically the borrower's initial financial health or initial covenant strictness do not predict whether the parties will amend financial covenants. Firms with worse performance may set tighter covenants in the beginning (and are more likely to violate and waive them), but whether these covenants get amended is determined by whether there is a substantial surprise in these firms' performance.

The model provides three key economic forces behind financial covenant amendments. First, contracts have to be incomplete in the first place. Otherwise, the signal would always agree with the state and the parties would not need to renegotiate ever. Second, the non-contractible information about future state that the parties receive (r_1) has to be predictive of the future (r_2). When this information is sufficiently informative, the parties can better anticipate future optimal allocation of control rights and thus it is more valuable to change them. Finally, it must be cheaper to amend the contract in advance than to wait and later bear control misallocation or waiver cost. The next proposition summarizes the effects of these forces on the likelihood of financial covenant amendments.

Corollary 1. *After an initial contract,* $\{\{\pi_{m1}^{0*}, \pi_{b1}^{0*}\}, \{\pi_{m2}^{0*}, \pi_{b2}^{0*}\}\}$*, is chosen, the probability of an amendment at time* $t = 1$ *is*

- *decreasing in the degree of contractual completeness, q,*
- *increasing in the informativeness of post-contract non-contractible information,* μ *and* β*,*
- *decreasing in the cost of amendments,* η_a

Finally, at time $t = 0$, the parties choose the optimal covenant schedule, anticipating that the contract will be amended in some cases. The equilibrium initial covenants for time $t = 1$, $\{\pi_{m1}^{0*}, \pi_{b1}^{0*}\}$, are the same as in the benchmark case. The covenants for time $t = 2$, $\{\pi_{m2}^{0*}, \pi_{b2}^{0*}\}$, however, are different because they play a dual role: they determine when the contract is amended and, if it is not amended, stay as the covenants at time $t = 2$. When estimating the model, I solve for $\{\pi_{m2}^{0*}, \pi_{b2}^{0*}\}$ numerically. The details are provided in Appendix 8.1.

1.4 Ex ante contracting efficiency gain

Amendments bring a weak welfare gain to the bank-firm relationship. I have demonstrated how the option to amend the debt contract allows the contracting parties to incorporate the most recent non-contractible information and, as a result, avoid bearing some cost of misallocating control rights. Because amendments occur only if they are less costly than the expected gain, and never amending the contract is still an option, the possibility to amend always weakly increases contracting efficiency.

I define contracting efficiency gain due to amendments as the difference between the total efficiency loss (from control rights misallocation, amendments, and waivers) when the parties can amend their contract and when they cannot amend their contract. In figure 4, I plot the ex ante contracting efficiency gain as a function of model parameters: the quality of contractible information, q, the persistence of the non-contractible state m, μ, the waiver friction in the state m, η_w^m, and the cost of amendments, η_a.

When the signal used in the contract becomes more precise, the total covenant error rate decreases, making the option to amend less valuable. In the extreme, if the signal perfectly represents the actual state ($q = 1$), it is unnecessary to amend it.

The importance of non-contractible information affects efficiency gain in the opposite direction. As the actual state at time $t = 1$ becomes more predictive of the state at time $t = 2$, amending at time $t = 1$ helps avoid more error and cost of control misallocation at time $t = 2$.

Finally, contracting efficiency gain is increasing in the waiver friction and decreasing in the cost of amendments. When it is more expensive for a covenant to make an error (η_w^m increases), an amendment saves more control misallocation cost. In contrast, when amendments themselves take more resources (η_a increases), the net gain from changing the contract decreases.

1.5 Theoretical moments for structural estimation

In the last part of this section, I list theoretical moments that are used to identify the model's parameters from the data. There are nine parameters. Parameters $\overline{c}$ and ρ govern the distribution and persistence of the control misallocation cost; parameters $\eta_w^i, i \in \{m, b\}$ and η_a are costs of waivers and amendments, respectively. The uncertainty about the optimal party-in-control is described by the initial probability that the firm's management should stay in control, μ_0, and by how likely the manager and the bank are to remain the optimal party, μ and β, respectively. Finally, the degree of contractual completeness is captured by the parameter q.

First, I introduce an assumption about waivers to make the model descriptive of real-world waivers. Theoretically, waivers are symmetric: the contracting parties can waive a control misallocation both when the bank and the manager get control suboptimally. In reality, however, the first situation – when a covenant is violated but the violation gets waived and control returns to the manager – is plausible and has been studied in prior work, while the symmetric situation – when the bank obtains control without any covenant violations – is hard to track and seems implausible because the bank may face litigation if it attempts to press the borrower without formal contract breaches. This disconnect between the model and the data may lead to biased estimates of the model. To avoid the potential misspecification, I manually preclude waivers when control is suboptimally granted to the manager in the model and set η_w^b prohibitively high in the estimation.

I make two additional assumptions to estimate the model. First, without observing the firms' ultimate values under different contracts, it is difficult to estimate the absolute values of the control misallocation and amendment costs. In the baseline specification, I fix the parameter $\overline{c} = 1$ and thus estimate waiver costs, η_w^m, and amendment costs, η_a, relative to the cost of having the wrong party in control. Later, in section 5, I use change in firms' market values around the announcements of covenant amendments to estimate monetary values of the control misallocation and amendment costs. Second, the parameters governing the persistence of the control misallocation cost, ρ, and of non-contractible states, μ and β, affect amendment behavior in the same way. If either control misallocation cost or the state of the world is more persistent, the parties have more motivation to amend the contract because they are more certain of what the cost of misallocation and the state will be in the future. To identify one of the parameters describing the persistence of the parties' new information, I need to fix one of the two parameters. I set

$\rho = 0.9$, and estimate the parameters μ and β. I examine the estimates' robustness to the choice of cost persistence ρ in Appendix 8.14.1.

To identify the six remaining parameters, I use the six moments listed in Appendix 8.4, describing initially set and amended covenant schedules and probabilities of amending financial covenants. Because the number of parameters equals the number of moments, the model is just identified.

The first and the second moments are the strictness of financial covenants in the first and the second periods after loan initiation. I define strictness as the probability of violating a covenant within the first period, consistent with the empirical definition of covenant strictness I use (Demerjian and Owens (2016)). Both in the data and in the model, the probability of violation is a function of, first, covenant levels and, second, the firm's financial performance: a firm is more likely to violate when its covenant levels are more restrictive and when it is less financially healthy.

The third and the fourth moments describe the unconditional probability that the covenant schedule will be amended and the probability of an amendment conditional on no covenant violation. Because the probability of amendment may be different in different states, these moments are functions of the probabilities of these states.

The fifth and the sixth moments describe the strictness of amended financial covenants, unconditionally and conditionally on no covenant violation in the first period. Similar to initial financial covenants, the strictness of amended covenants depends on the level of covenants and the company's financial performance after an amendment.

2 Data and descriptive statistics

In this section, I first describe my data sources and the process of collecting detailed financial covenant schedules and amendments to financial covenants. Second, I discuss a measure of financial covenant strictness. Finally, I present descriptive statistics.

2.1 Data sources and collection

To estimate the model, I need three series of data: (1) initial financial covenant schedules, (2) amendments of financial covenants, and (3) violations of financial covenants. Because the data on exact dynamic initial covenant schedules[14] and amendments of financial covenants are not available in common databases, I manually collect detailed covenant schedules and financial covenant amendments from loan contracts that companies are required to file with the SEC under Regulation S-K.

For loans originated before 2012, I start with the dataset created by Roberts (2015). Roberts' sample includes 114 randomly selected companies from the intersection of Compustat and Thomson Reuters Dealscan databases starting from 1984. The dataset includes all loan paths – from origination to every amendment, amendment and restatement, or rollover until maturity or refinancing – for every company in the sample. Specifically, for each event (Origination, Amendment, Amended & Restated, Rollover, Refinanced), the dataset contains binary information on whether interest rate, loan size, maturity, any covenant, investment, collateral, distribution, or financial covenant was amended, and percent changes in interest rate, loan amount, or maturity. The dataset also includes information sources: SEC filings, specifying the form, the filing date, and the exhibit where a contract can be found. In the first half of the data collection procedure, I add detailed financial covenant schedules and amendments of financial covenants to the filings recorded in Roberts' dataset.

For loans originated after 2012, I use Dealscan to find all loans for the companies in the Roberts' dataset. Next, I use the Electronic Data Gathering, Analysis, and Retrieval system (EDGAR) to search for original loan agreements and all amendments to them. The search for a given company is based on the name of the lead arranger and the loan origination date provided in Dealscan.

After the filings are collected, I record data on financial covenant schedules and amendments to them in the following way. For initial loan agreements, I manually search for financial covenants, typically found in the "Negative covenants" or "Financial covenants" sections. For each financial covenant, I record dynamic schedules on a yearly basis, starting from the contract effective date for the first five years of the loan. For example, for a debt-to-EBITDA covenant that starts with 5:1, falls to 4.5:1 in 10 months, then falls to 4:1 in another ten months, and then falls to 3:1 in another ten months and stays the same until maturity, I record the schedule as "4:1 in 1 year, 3:1 in 2 years, 3:1 in 3 years, 3:1 in 4

[14]The Dealscan database contains information on whether a financial covenant is monotone or changes over time. The latter are recorded as "Increasing from XXX to YYY" without specifying when the threshold changes exactly and how fine the steps from XXX to YYY are. For instance, Dealscan would record two covenant schedules provided in Appendix 8.6 in the same manner, while they are economically different: the first one represents a short-term adjustment with a fixed covenant for the remaining life of the loan (Li et al. (2016)), while in the second one, the covenant changes gradually over the entire period, probably to regulate a fundamental agency problem that is expected to worsen with time. My paper models dynamic covenant schedules explained by long-term fundamental considerations and not short-term adjustments, making the two cases different from this study's perspective.

years". Static covenant schedules are recorded as constant for all years. Examples of dynamic covenant schedules and how they are recorded are in Appendix 8.7.

When documenting amendments, I first check whether an amendment to a loan contract contains changes to financial covenants. These changes are typically made in two ways: the numeric level of a financial covenant is amended, or the definition of a variable used in a financial covenant is amended. For the latter, I record an amendment as containing a financial covenant amendment but do not specify the new level of a financial covenant. For the former, I record a new level of a financial covenant in the same yearly manner as for original loans, starting from the amendment effective date. Appendix 8.8 contains examples of the two types of amendments to financial covenants.

It is essential to discuss which amendments to numeric levels of financial covenants are included in the sample and which are not. In the context of this study, an amendment is a permanent change to a covenant schedule driven by new material information learned by the lender and the borrower, and a waiver is a temporary change of control rights once the parties find themselves in an inefficient situation. Some amendments to loan contracts, in reality, change financial covenants short-term to avoid an anticipated violation of these financial covenants. Such adjustments are waivers rather than amendments in my model because they are short-term changes in control rights. I define short-term changes as those that only alter financial covenant schedules for within a year after the amendment effective date but keep the schedule unchanged for the rest of time until the contract's maturity.[15] These instances are not recorded as amendments. An example of an "amendment" excluded from the sample and an amendment included in the sample are in Appendix 8.9.

The hand-collection process has limitations. One limitation is that I cannot access some loan contracts because EDGAR only contains filings up to a specific year in the past. When I collected the data for this paper, the earliest year available was 2001. To avoid throwing away all observations before this year, I make the following assumptions. First, for dynamic covenant schedules, I assume that covenants evolve linearly over time. For instance, for a covenant "Increasing from 1:1 to 1.2:1" in a 4-year loan, I assume that the covenant is "1:1" in one year, "1.1:1" in two years, and "1.2:1" in three years. Second, when an amendment contract cannot be found but is in the Roberts' dataset, I used an indicator created by Roberts (2015) for changes in financial covenants instead of using my indicators. In these cases, the

[15] Even though I choose a one-year cutoff to distinguish short-term from long-term changes to financial covenants, I have not observed instances when an amendment changes covenants for slightly less than a year. A typical short-term change amends covenants for the next month or quarter.

data on how exactly financial covenants were changed was recorded as missing.

Another limitation is that not all companies comply with the SEC's disclosure rules (Caskey et al. (2023)). For most of the original contracts recorded in Dealscan for firms in my sample, I could find the contract either in an 8-K or in a 10-K or 10-Q filing. Amendments were missing more often than original contracts. Nevertheless, for the period after 2001, I was able to find more than 70% of original contracts and more than 90% of amendments.

Lastly, I obtain original loan contract terms and borrower and lender characteristics from Thomson Reuters Dealscan and companies' financial information from Compustat.

To measure the strictness of original and amended covenants, I use the methodology proposed by Demerjian and Owens (2016), converted into an annual basis. The details are outlined in Appendix 8.10.

2.2 Descriptive statistics

The sample comprises 1,068 loans issued to 110 medium-sized companies between 1994 and 2021.

Table 1 presents the summary statistics for my sample and compares them against the summary for the whole Compustat universe. The companies in my sample are larger, have better growth opportunities, are more profitable and more levered, which is expected for companies that borrow large amounts from multiple reputable lenders. The loans in my sample are large longer-term corporate loans provided by lending syndicates. The summary statistics for my sample and the total Dealscan universe are shown in tables 2 and 3. My sample differs from the Dealscan universe in ways consistent with my firms being publicly traded companies whose data is available in the Compustat database. The loan size is larger, more loans are secured, loan syndicates have more members, and the proportion of revolvers is higher.

Summary statistics of the key data are presented in tables 5 and 6. The first two rows of table 5 show the strictness of covenant schedules in initial loan agreements: the first row of covenants for one year and the second row for two years after the contract is signed. On average, for original loan terms, financial covenants in one and in two years are of similar strictness. A median company is about 24% likely to violate one of its covenants within the first or the second year after loan origination. In the third and fourth rows of table 5, I present the distribution of strictness of amended covenants and changes in covenant strictness after amendments, respectively. Amended covenants are stricter than original more often than not, but sometimes financial covenants are relaxed when contracts are amended. As the last row suggests, a firm in the 25th percentile has about 20% looser covenants after an amendment.

Table 6 shows the frequency of financial covenant amendments in general and in cases when no covenants are violated. About 28.9% of loans in my sample have a financial covenant amended at least once, and about 20.3% have a covenant amended within the first year after signing the loan agreement. Out of these 28.9% and 20.3%, 42.6% and 42.0%, respectively, are amendments when the borrower did not violate or did not expect to violate a financial covenant. The facts that amendments often tighten covenants and many amendments occur outside of covenant violations suggest that amendments are not made simply to avoid a technical default but may have different economic drivers.

I provide correlations between variables describing financial covenants and amendment behavior and other firms' and loans' characteristics in tables 7 and 8. A few observations are worth discussing. First, the strictness of financial covenants in original contracts is associated with firm and loan characteristics in an intuitive way. Larger firms (measured by total assets) with lower leverage (measured by leverage and debt-to-EBITDA ratios) generally obtain looser covenants from their lenders, consistent with prior findings (e.g., Bradley and Roberts (2015)). Covenant strictness is positively correlated with loan spreads (all-in-drawn spread) and with collateral, consistent with riskier borrowers obtaining worse credit conditions on all dimensions (Bradley and Roberts (2015)).

Second, amendment behavior does not appear strongly correlated with the strictness of initial covenants or with the borrower's financial health. The correlation of the binary variable for amendment with the strictness of financial covenants is less than 10% and with the binary variable for being close to a covenant violation is only 24%. Similarly, amendments are not correlated with borrowers' leverage ratio and are only weakly correlated with the debt-to-EBITDA ratio. These facts suggest that amendments may not serve to resolve violations, as waivers do, but have different economic purposes.

3 Model estimation and main results

This section describes the model's estimation procedure, presents parameter estimates for the full sample, and discusses the model's fit.

3.1 Estimation procedure

I estimate the model using the Generalized Method of Moments (GMM). Intuitively, the method searches for the set of model's parameters – the quality of contractible information, q, the persistence of non-contractible information, μ and β, companies' financial health, μ_0, and waiver friction, η_w, and amendments, η_a – that minimize the distance between model-implied theoretical moments and their empirical counterparts. Since the number of moments equals the number of parameters (6), the model is just-identified. I describe the estimation procedure in detail in Appendix 8.12.

3.2 Main results

In this section, I discuss the main estimates of the model. They allow me to evaluate the roles of different forces driving financial covenant amendments: contractual incompleteness, importance of post-contract non-contractible information, and low amendment cost compared to the cost of misallocating control rights. Table 9 shows parameter estimates for the full sample.

The estimated parameters suggest that the main forces driving amendment behavior are high degree of contractual incompleteness and low cost of amendments. The estimated precision of contractible information is quite low: $q = 0.687$, or if covenants do not make any errors in one state, they will make errors in the other state about 31.3% of the time. The accounting numbers used in financial covenants do not appear to be precise descriptions of the actual state of the world.

The second force driving amendments – predictiveness of post-contract non-contractible information – is of a smaller magnitude: the estimated persistences of the non-contractible states are low. Conditional on a company being financially healthy, i.e., not needing a lender's intervention, in one year after the contract is signed, the company will stay healthy next year only with a 65.4% probability. The state when the bank needs to take control is even less persistent: if the bank should have control rights in one year, this will be the case in two years only with a probability of 50.1%. Overall, the data suggests that the non-contractible information lenders and borrowers receive post-contract is only somewhat useful and may not be the only driver of covenant amendments.

The final force in the mechanism for covenant amendments is the relative cost of amendments compared to the cost of misallocation of control rights. The empirical estimate of the amendment cost is about 1.8% of the average cost of control misallocation, while the waiver cost is around 140%,[16] suggesting that amending financial covenants in advance is indeed substantially cheaper from the total surplus perspective than misallocating control rights later on. The large difference in costs of waivers

[16]These interpretations are calculated by multiplying the respective estimates by 2 such that the percentages are not of the upper bar of the cost, $\bar{c} = 1$, but of the mean of the cost, $\frac{\bar{c}}{2} = 0.5$.

and amendments is reasonable given the model's structure and institutional evidence. In the model, an amendment can save at most one period's worth of waiver cost, while in reality, one amendment can prevent multiple waivers in the future. The large waiver cost obtained from the data may represent the present discounted value of all the waiver costs an amendment saves. In addition, institutionally, a waiver can be considerably costlier than an amendment. First, amendments are crafted in advance under little pressure, while waiver documents typically have to be compiled quickly. These differences imply larger legal fees for waivers than for amendments. Second, the reputational costs of the waiver or efficiency losses when a company forgoes investment opportunities to avoid a covenant violation may be large.

Finally, I estimate how likely a typical company is to be financially healthy in the next year after it signs a contract with a syndicate. The estimate is μ_0 =61.4%, or in 38.6% of the cases, a lender needs to intervene in some way within the first year of the lender-borrower relationship.

3.3 Contracting efficiency gain

I estimate the efficiency gain from amendments – the amount of control misallocation cost saved ex ante because the parties can amend the contract – and interpret it economically. The estimated number ranges between 2.29% and 2.36%, or the option to amend in expectation saves about 232.3 basis points of control misallocation cost. Ex post, conditional on an amendment, the parties save 15.32% of cost in the state when the manager is the best party-in-control and 12.62% in the state when the bank needs to be in control. These amounts represent economically significant savings. Recall that the cost of having a wrong party in control, parameter c, captures deadweight loss in firm value when the wrong party obtains control rights. For example, when the manager gets control during unstable financial conditions, she may take excessive risk and destroy company value. When the bank is given the right to intervene in the firm's operations without the need, it may inefficiently reduce investment and R&D (Chava and Roberts (2008), Nini et al. (2009)), fire executives (Nini et al. (2012)), or shape the company's disclosure policy (Vashishtha (2014)). These actions, if taken when they should not be, may substantially hurt the firm's long-term performance. Although the parties may waive these inefficiencies ex post when the bank gets control, the cost of doing that is also high, as the estimate of η_w^m suggests. To sum up, saving even about two percent of the loss in firm value may be a lot in monetary terms. The fact that we observe contracting parties amending their contracts implies the parties' revealed preference to save these control misallocation costs.

In section 5, I estimate the monetary value of control misallocation and renegotiation costs.

3.4 Model fit and analysis of identification

To evaluate how well the theoretical model fits empirical observations, I examine the distance between estimated theoretical and data moments. The values of moments and t-statistics of differences are shown in table 10. All of the model-generated moments are statistically indistinguishable from their counterparts at the 1% significance level.

I provide more intuition of how each parameter is identified from data moments by plotting the moments' sensitivities to parameters in figure 5. The plots show that different moments are important to identify different parameters. Generally, the moments move with parameters in the directions consistent with the theoretical discussion in section 1.5.

The precision of contractible information, q, is primarily identified from the parties' decisions to amend the contract. When the contractible information better captures the non-contractible state – q is higher – the probability of amendment unconditional and conditional on no violation decreases (figure 5a).

Companies' financial health at the time of loan origination, μ_0, mostly comes from the strictness of financial covenants in the original loan contract. Healthier firms, i.e., firms that are less likely to need the lender's intervention within the first year after loan origination, obtain less strict financial covenants (figure 5b).

Persistences of the non-contractible states, μ and β, are identified, first, from initial and amended covenant levels for the period more than one year after loan origination, and second, from the probabilities of amendment. As can be seen in figure 5c (figure 5d), if the good (bad) state is more likely to persist after it occurs in the first period, the covenants for the second period are set less (more) strict. The probability of amendment increases with the persistences of both states.

The cost of amendments primarily is obtained from the probabilities of amendments (figure 5e). As the cost of amending a contract goes up, the parties are less likely to use this option. The cost of amendment can be disentangled from the precision of contractible information, q, by using the strictness of amended financial covenants. Amended financial covenants are less strict when the precision of contractible information is higher, but are stricter when the cost of amendment is higher.

Finally, the cost of waiving a covenant violation, η_w^m, is identified from the strictness of financial covenants, mostly from amended. If waivers are more expensive, the parties try to avoid them more and set covenants looser (figure 5f).

4 Counterfactual analyses

I conduct two counterfactual analyses. In the first counterfactual, I study two companies: a very financially healthy company and a very financially unhealthy company. I compare the two companies' amendment behavior to understand to what extent frequent debt contract changes can be attributed to companies' financial health. In the second counterfactual, I introduce an option to waive control rights misallocation in the state when the manager gets control suboptimally. This analysis shows how amendment behavior changes if firms can grant control to the bank even if no financial covenants were violated.

4.1 Completely financially healthy and completely financially unhealthy companies

First, I analyze a very financially healthy company and demonstrate that this company still amends financial covenants in the contract with its lender(s). To make a firm in the model completely healthy, I set the initial probability that the company manager will be the optimal party-in-control close to one: $\mu_0 = 0.999$. The first scenario in table 11 shows that even such a stable company will still have its financial covenants amended about 12.53% of the time.

Second, I look at the opposite case – a very financially unhealthy company – and show that this company is not substantially more likely to amend its covenants than a healthy company. In the second scenario in table 11, I show the likelihood of an amendment for a company that very likely will need the lender's intervention in the near future, $\mu_0 = 0.001$. In expectation, the financially unhealthy company amends financial covenants about 15.86% of the time, similar to the healthy company.

The two counterfactual scenarios suggest that firms' general financial health does not have to be correlated with how often firms amend their debt contracts. However, this fact may be non-trivial to confirm empirically. Financial covenant amendments are determined by the amount of information learned over time about the state of the world, and this amount does not change with companies' financial health per se. In reality, firms' financial stability may be correlated with various characteristics of information systems. For example, companies that do not need lenders' intervention may also have higher accounting quality – the precision of contractible information – which would translate into fewer amendments compared to less healthy firms. A researcher who properly controls for characteristics of contractible and non-contractible information should expect to find no association between financial health and the likelihood of debt contract amendments. Roberts (2015) indeed finds that the probability of amending a covenant is largely unrelated to borrowers' financial performance at the start of the contract: neither the EBITDA-to-assets ratio nor stock performance predict the likelihood of covenant amendments.

4.2 A possibility of a waiver when no covenants are violated

The third counterfactual scenario introduces an additional waiver possibility. In reality, waivers most plausibly happen (and can be observed by a researcher) only when a covenant is violated but the company management stays in control. In the model, this situation means control rights are misallocated so that the bank gets control by mistake, and the parties waive the misallocation and return control to the manager. Theoretically, the opposite case is also possible: when the control rights are allocated to the manager by mistake but should be given to the bank. This case is hard to observe in the data because it would imply the lender intervenes in a firm without a formal technical default. In the main estimation, I disallow waivers of this type. To understand how amendment behavior would change if the parties can waive control rights misallocation in the case when the bank is the best party-in-control, I consider a counterfactual scenario where waivers are allowed in both control misallocation cases. I set the cost of giving control to the bank when no covenant was violated equal to the estimated cost of waivers of violations, $\eta_w^b = \eta_w^m = 0.700$.

When the parties can waive a misallocation of control rights in another state, they find it cheaper to misallocate control rights overall. As scenario 3 in table 11 suggests, as a result, the amendment option becomes less valuable: the ex ante contracting efficiency gain drops by 1 percentage point, and conditional on an amendment in the state b – when the state suggests the bank might need to obtain control in the future, – by about 4 percentage points. The frequency of amendments does not change.

5 Monetary estimates of amendment and control misallocation costs and of contracting efficiency

In the main specification of the model, I cannot separately estimate the cost of misallocating control rights, c, and the costs of waivers and amendments, η_w^m and η_a, and only estimate them relative to each other. However, control misallocation and negotiation frictions have an economically meaningful effect on firm value, and it may be helpful to know the absolute magnitudes of these costs. In this section, I attempt to estimate these costs using an event study.

The model is written in terms of losses to firm value whenever a control misallocation, a waiver, or an amendment occurs, and thus can speak to how the firm's valuation changes after one of these events. If in the data I can measure a change in the firm's value in a relatively short window around an event, I can argue that the change is due to the change in investors' expectations about the loss to the firm value as a result of the event. Using a short window change in the firm's value around an event as an additional moment in the estimation, I can evaluate the monetary values of the parameters capturing control misallocation and renegotiation costs.

It is hard to capture empirically when a control misallocation or a waiver occurs. However, for some amendments, I have the exact dates when the amendments become available to the public. The majority of amendments in my sample are filed in an 8-K form. To select amendments for which an amendment was a major event for the firm in a given time window, I identify amendments that were filed in an 8-K form that did not contain any other major announcements.[17] For each 8-K filing selected, I collect the date of the filing and the market value of the firm three days before and three days after the filing.

To map the change in the firms' market value into the model, I derive an additional theoretical moment – a change in the firm value as a result of an amendment. I assume that investors on the market price the firm at its expected value. When investors observe that a firm has amended its loan contract, they first update their beliefs about the state of the world the firm is in. For example, if the firm is substantially more likely to amend its contract when the state is good, investors assign a higher probability to the state being good (i.e., a firm being healthy). Second, given their updated beliefs about the state and a new contract, investors price in a new loss to the firm value. I add two new moments to the estimation: a change in the market value around an announcement of an amendment for firms that did and did not violate covenants. The new moments are derived in Appendix 8.13.

Table 14 shows the estimation results of the model with additional moments. The estimated monetary

[17] I keep 8-K forms that contain only items related to changes in or new material contracts or financial agreements. I exclude 8-K forms that additionally include items 2.01 (Completion of Acquisition or Disposition of Assets), 2.02 (Results of Operations and Financial Condition), 8.01 (Other Events), 5.02 (Departure of Directors or Certain Officers; Election of Directors; Appointment of Certain Officers; Compensatory Arrangements of Certain Officers), 7.01 (Regulation FD Disclosure), 5.01 (Changes in Control of Registrant), 3.03 (Material Modification to Rights of Security Holders), or 5.07 (Submission of Matters to a Vote of Security Holders).

values of control misallocation costs are substantial. The mean loss to firm value if control rights are given to the wrong party is about \$136.4 million, or about 7.44% of the market value for the average firm, implying that, generally, corporate borrower-lender conflicts are severe and the ability of a contract to allocate control correctly between the parties is crucial for the enterprise value. Remedying control rights misallocation is also costly: the waiver friction is about \$273.2 million, or 14.90% of the market value. The large friction implies that, if control rights are suboptimally allocated to the lender, it may be very difficult for the parties to return control to the firm management, perhaps because lenders have large bargaining power or find it very costly to acquire non-contractible information about the borrower.

Amending covenants in advance appears to be a significantly cheaper option: the cost of an amendment is about \$4.0 million, or only 0.22% of the market value for the average firm. Frictions preventing non-urgent, long-term covenant renegotiations are not as large, probably because when future (as opposed to current) control rights need to be reconsidered, the parties have more balanced bargaining power and face fewer exogenous frictions to renegotiate.

Finally, I estimate the dollar value of total efficiency losses. The expected loss due to contractual incompleteness and negotiation frictions at loan initiation is about \$19.09 million. In other words, positive-NPV investments generating a value of at least \$19.09 million are underfunded because of contractual incompleteness and costly renegotiations.

6 How does the amendment behavior and efficiency gain vary with borrowers' and loans' characteristics?

In this section, I evaluate how the economic parameters and efficiency gains from amendments vary with borrowers' and loans' characteristics. I estimate the baseline model on different subsamples of my dataset.

For subsample analysis, I identify three firms' and loans' characteristics that could affect amendment behavior: the quality of contractible information, the importance of non-contractible information, and renegotiation cost. As a proxy for the quality of borrowers' contractible information, I choose the earnings response coefficient (Lang and McNichols (1990), Imhoff and Lobo (1992)). Prior researchers have used this measure to proxy for the amount of relevant financial performance information in the companies' accounting numbers. Intuitively, if stock market investors react more strongly to earnings releases, earnings should contain less noise and more useful information. Even though earnings reported to stock market investors are rarely used in loan contracts (Dyreng et al. (2017)), measures of earnings in loan agreements and for the stock market have many components in common. I expect the quality of stock market earnings to be correlated with the quality of earnings used in loan contracts.

To measure the importance of non-contractible information in lender-borrower pairs, I use indicators for relationship loans. In contrast to the stock market investors, banks invest heavily in monitoring (Diamond (1984), Winton (1995)) and screening (Ramakrishnan and Thakor (1984), Allen (1990)) their borrowers, as a result obtaining relevant, often soft information about the company that is not available to outsiders (Bhattacharya and Chiesa (1995), Yosha (1995)). I assume that if a lead lender already issued a loan to this borrower, the lender must know more specific non-contractible information and understand how to use it to predict the borrower's future performance. Thus, for relationship loans, the degree of contractual incompleteness may be higher as more soft information is used to regulate the lender-borrower relationship.

The characteristic related to the renegotiation cost is the number of lenders in a syndicate. Literature on syndicate lending posits that one of the factors affecting how loan agreements are structured is the cost of coordination among investors (Becker and Ivashina (2016)). Coordination costs are higher for more dispersed and larger syndicates. If it is difficult to get all decision-making investors together and make them reach an agreement, the contracts will be written to reduce the need to coordinate in the first place. If there is a need to renegotiate, this action will cost a high price. In my framework, high renegotiation costs can imply both high amendment costs and high waiver friction. The empirical association between syndicate size and the likelihood of amendments will be determined by which of the costs dominates. If amendment costs are much higher but waiver frictions do not differ for larger syndicates, we would observe fewer amendments for loans with more lenders, and vice versa. There might be no association between coordination cost and amendment probability if greater amendment cost is offset by greater waiver frictions.

I construct the three variables as follows. The earnings response coefficient is estimated from regressing abnormal returns on earnings surprises for eight quarterly earnings announcements preceding the loan initiation date. Abnormal returns are computed using the market model. An indicator for relationship loans is set to one if at least one of the lead arrangers issued at least one loan to this borrower in the past. The number of lenders in a lending syndicate is directly taken from the Dealscan database.

Table 13 shows summary statistics for the proxies.

The estimated parameters, the likelihood of amendments, and contractual efficiency gains for different subsamples are presented in table 15. For companies whose earnings are perceived by the market as less informative, financial performance information used in loan contracts also seems to be of low quality, resulting in a larger gain these firms obtain when they can amend their loan contracts. The aggregate financial covenant error for firms with low ERC is about 45%, almost 30 percentage points higher than for firms with high ERC. Because low-earnings-quality borrowers' contractible information is so imprecise, they amend covenant schedules more than twice more often than their high-earnings-quality peers and obtain an ex ante 2.188% contracting efficiency gain due to the amendment option. In addition to having more precise contractible information, high-ERC companies are healthier and more stable: the initial probability that a high-ERC (low-ERC) company will not need a lender's intervention is 72% (59%), and conditional on no lender intervention in the first year, 82% (59%). Firms with better financial reporting quality also face lower renegotiation frictions than their lower-quality peers, consistent with more precise accounting numbers attracting alternative capital sources and thus improving corporate borrowers' bargaining position.

Relationship and non-relationship loans differ in the degree to which contractible information can capture the actual state of the world and in negotiation frictions that contracting parties face. First, when lenders have had a prior relationship with the borrower, contractible signals capture the borrower's non-contractible state worse. Second, in relationship loans, the costs of both amendments and waivers are higher than in non-relationship loans. This difference may be due to a hold-up problem when lenders have acquired enough information about the borrower that cannot be transferred to other parties, thus reducing the bargaining power of the borrower.

Syndicates with more lenders seem to face higher waiver frictions but not amendment costs. One explanation might be that for amendments a lower share of syndicate members needs to approve a contract change than for waivers. Because harder waivers dominate, larger syndicates are slightly more likely to amend their contracts. In addition, loans by larger syndicates seem to use more precise contractible information, perhaps because they are typically provided to larger and more established borrowers.

7 Conclusion

This study examines why financial covenants in corporate loan contracts are amended. The option to adjust a loan agreement as contracting parties learn new information allows lenders and borrowers to (partially) dynamically complete the inherently incomplete debt contract. I show that contracting efficiency, and thus incentives to amend, are more significant for companies with less precise contractible information, more important non-contractible information, and when amending the contract in advance is considerably cheaper than resolving control rights misallocation ex post. In the U.S., the first and the last explanations appear to be most supported by the data.

My paper advances the literature's understanding of why and how corporations and their investors change the contracts regulating their relationship. Many dimensions of contract renegotiations remain understudied. For example, in this paper, I treat companies' information systems as fixed, while in reality, they are not static. Properties of firms' hard and soft information and contracting parties' learning what this information represents evolve over the life of the lender-borrower relationship, which can potentially affect contract structure.

Another limitation of my framework is its sole focus on financial covenants. Even though financial covenants are the most common feature that gets amended, this is not the only term determining control rights,[18] nor are control rights the sole dimension that loan contracts regulate. Other features are also not static and get amended before contracts mature (Roberts (2015)). Studying why these are changed and why sometimes the parties amend multiple features at a time can uncover new trade-offs that lenders and borrowers face.

Overall, corporate loan contract amendments are an intriguing phenomenon that invites a lot of future research.

[18] Control rights in loan agreements can be regulated by other "action" covenants, such as negative pledge clauses or payout covenants, cov-lite financial covenants (Bräuning et al. (2022)), sweep provisions, collateral, and other terms.

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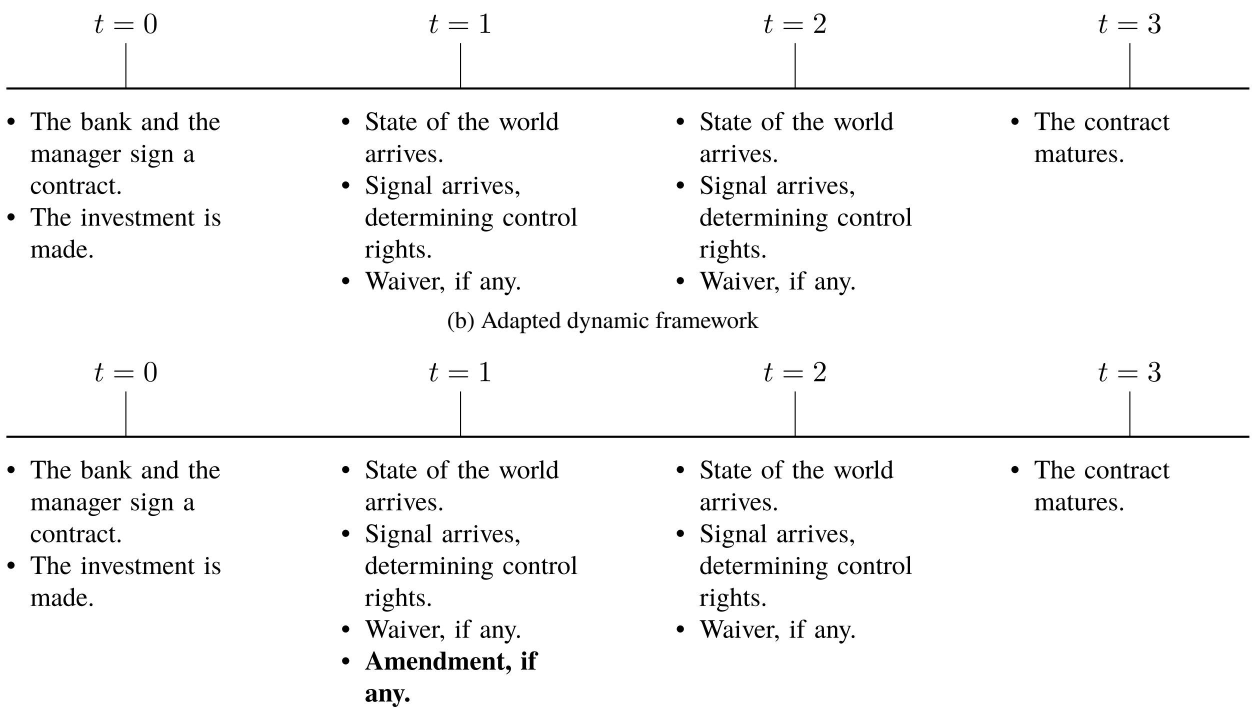
Figures and tables

Figure 1: Timelines in the classic and adapted debt contracting frameworks

(a) Classic framework



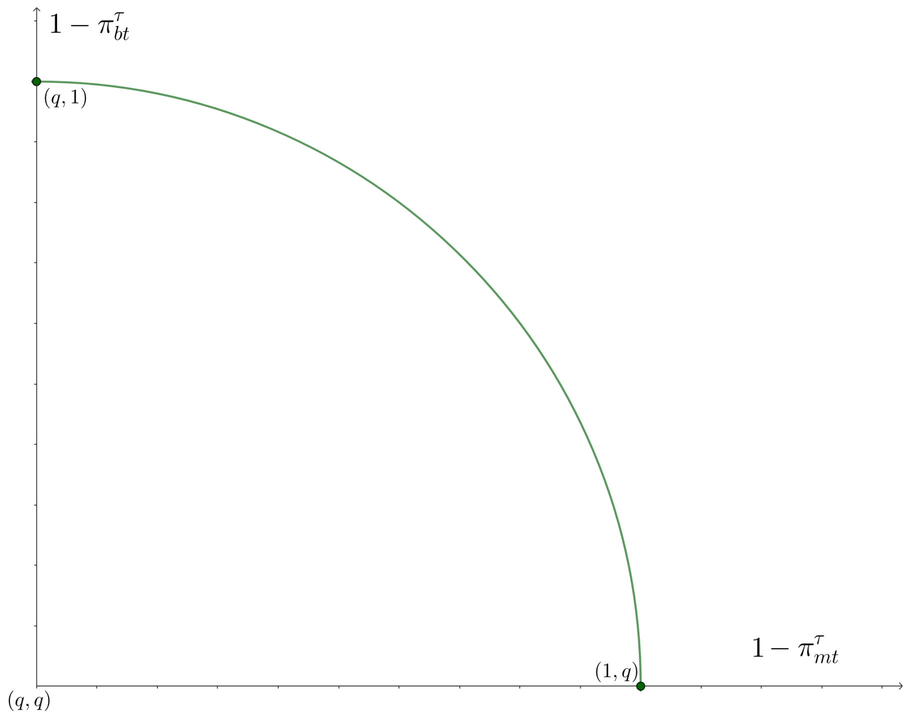


Figure 2: Possibility frontier for the precision of the covenants in states m and b. The precision in the state m is on the horizontal axis, the precision in the state b is on the vertical axis. $q = 0.8$.

Figure 3: Timeline of events in the model

$t = 0$

- The initial contract, $\{\{\pi^0_{m1}, \pi^0_{b1}\}, \{\pi^0_{m2}, \pi^0_{b2}\}\}$, is chosen.

$t = 1$

- The manager and the bank observe $\{r_1, s_1, c_1\}$.
- Waiver, if any, takes place.
- Amendment of the contract to a new one, $\{\pi^1_{m2}, \pi^1_{b2}\}$, if any, takes place.

$t = 2$

- The manager and the bank observe $\{r_2, s_2, c_2\}$.
- Waiver, if any, takes place.

$t = 3$

- The contract matures.

Figure 4: Saved cost of control rights misallocation as a function of model parameters. $q = 0.6, \eta_a = 0.02, \eta_w^m = 0.95, \eta_w^b = 1000, \bar{c} = 1, \mu = 0.65, \beta = 0.6, \rho = 0.9, \mu_0 = 0.6$.

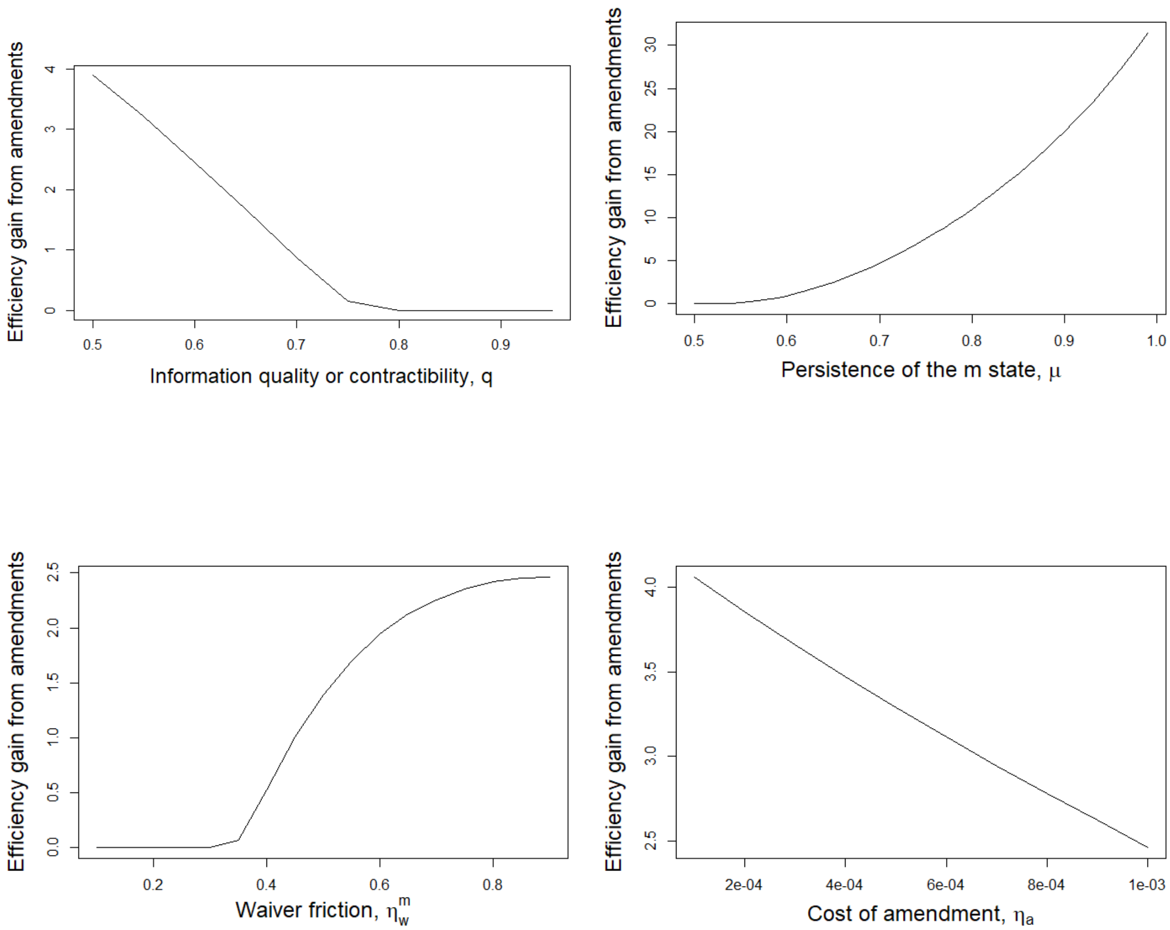
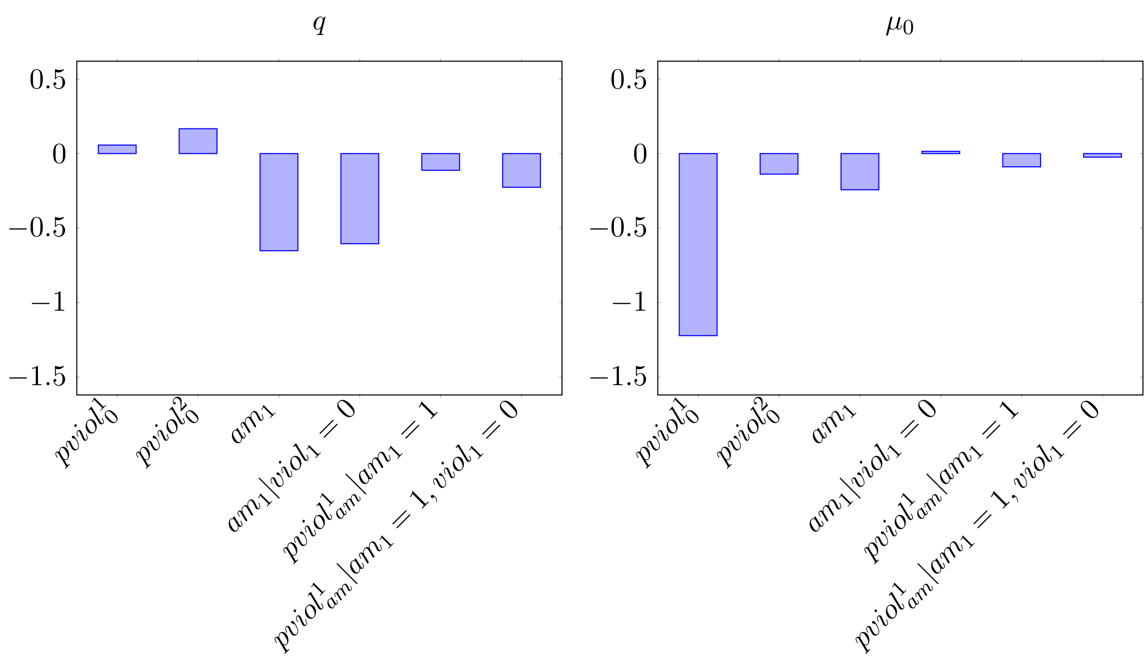
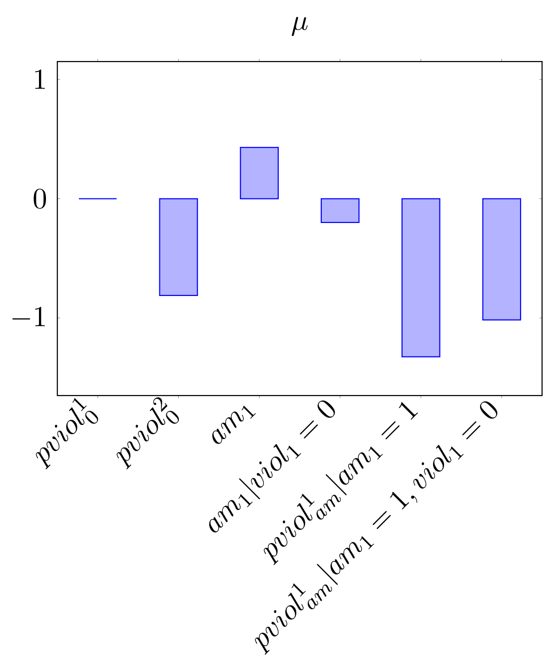


Figure 5: Sensitivities of moments to the model's parameters, at the estimated parameters. $pviol_0^1$ is the strictness of initial financial covenants for the first year after loan origination. $pviol_0^2$ is the strictness of initial financial covenants for the second year after loan origination. am_1 is the probability of amending a covenant within the first year after loan origination. $am_1|viol_1 = 0$ is the probability of amending a covenant within the first year after loan origination, conditional on no violation within the first year after origination. $pviol_{am}^1|am_1 = 1$ is the strictness of amended financial covenants for the first year after an amendment. $pviol_{am}^1|am_1 = 1, viol_1 = 0$ is the strictness of amended financial covenants for the first year after an amendment, conditional on no violation within the first year after origination. The sensitivity of a moment with respect to a parameter is computed by varying the parameter 1% up and down (keeping other parameters constant) and dividing the difference between the new value of the moment at the 1% higher and 1% lower parameter by 2% of the parameter value.

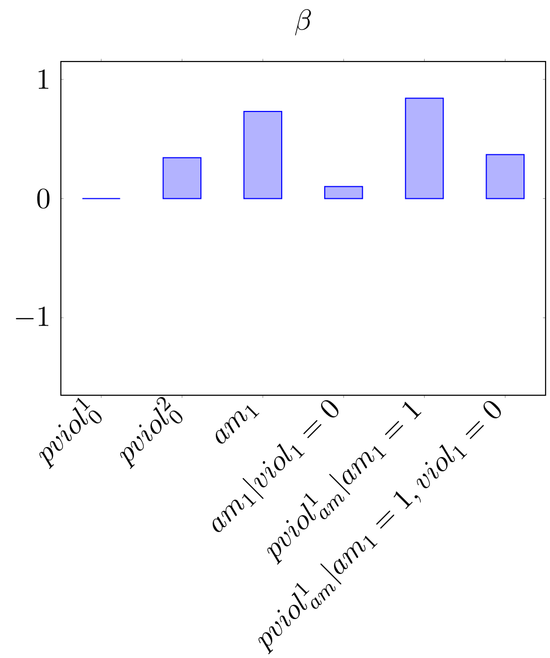


(a) Sensitivities of the model parameters to the precision of contractible information, q.

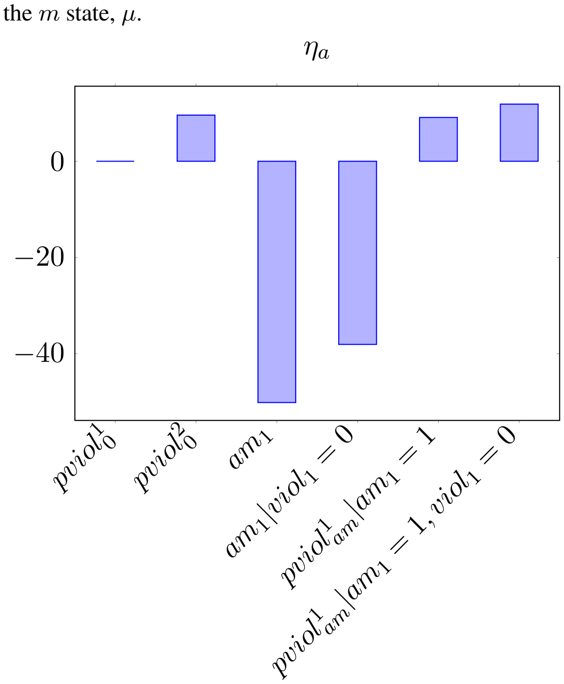
(b) Sensitivities of the model parameters to the initial probability of the manager being the optimal party-in-control, μ_0.



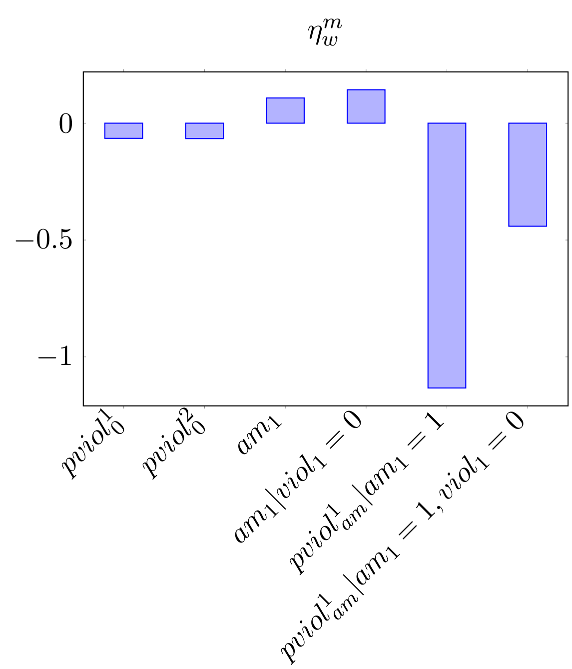
(c) Sensitivities of the model parameters to the persistence of the m state, μ.



(d) Sensitivities of the model parameters to the persistence of the b state, β.



(e) Sensitivities of the model parameters to the amendment cost, η_a.



(f) Sensitivities of the model parameters to the waiver cost, η_w^m.

Table 1: Summary statistics: companies' characteristics in my sample and in the Compustat universe. For the Compustat data, I truncate all the variables at 1% from above and below and leave only observations that have no missing data on any of the variables. The market-to-book ratio is the close price multiplied by the number of common shares outstanding and divided by the total value of common/ordinary equity. The return on assets is income before extraordinary items divided by total assets. The leverage ratio is the sum of the total long-term debt and the total debt in current liabilities, divided by the sum of the total long-term debt, the total debt in current liabilities, and the total value of common/ordinary equity. The debt-to-EBITDA ratio is the sum of the total long-term debt and the total debt in current liabilities, divided by the operating income before depreciation.

	This paper's data						Compustat data					
	N	Mean	SD	P25	P50	P75	N	Mean	SD	P25	P50	P75
Total assets ($mil)	1,003	3,496.74	9,108.32	401.01	1,049.30	2,803.15	208,668	3,899.90	15,353.75	37.64	236.41	1,365.63
Market-to-book ratio	952	3.42	11.25	1.43	2.14	3.20	208,668	2.69	5.74	0.96	1.71	3.23
Return on assets	1,002	0.03	0.09	0.01	0.04	0.07	208,668	-0.19	0.97	-0.10	0.01	0.05
Leverage ratio	1,003	0.50	0.30	0.30	0.48	0.65	208,668	0.31	0.41	0.01	0.26	0.52
Debt-to-EBITDA ratio	998	3.34	3.82	1.28	2.66	4.34	208,668	1.97	5.10	0.00	0.84	3.20

Table 2: Summary statistics: loans' characteristics in my sample and in the Dealscan universe.

	This paper's data						Dealscan data					
	N	Mean	SD	P25	P50	P75	N	Mean	SD	P25	P50	P75
Loan amount ($mil)	956	365.74	599.38	75.00	200.00	400.00	332,110	5,076.11	208,182.05	40.00	130.00	570.00
Maturity (months)	940	52.52	21.23	36.00	59.00	60.00	302,546	58.52	47.23	35.00	59.00	71.00
All-in-drawn spread (bps)	904	226.77	167.06	125.00	200.00	300.00	179,894	242.50	176.97	112.50	220.00	325.00
Secured	957	0.62	0.49	0.00	1.00	1.00	332,786	0.35	0.48	0.00	0.00	1.00
Number of lenders	957	9.09	7.87	4.00	7.00	12.00	332,786	5.62	6.25	2.00	4.00	7.00

Table 3: Summary statistics: types of loans in my sample and in the Dealscan universe.

Loan type	% in this paper's sample	% in Dealscan data
Revolver/Line >= 1 year	50.797	30.422
Term Loan	29.709	35.860
Other	19.494	33.718

Table 4: Average number of loans per borrower in a given year.

Year	Number of deals an average borrower has	Number of tranches an average borrower has
1994	1.00	1.00
1995	1.00	1.33
1996	1.19	1.94
1997	1.45	2.15
1998	1.84	2.92
1999	2.01	3.37
2000	2.04	3.35
2001	2.11	3.42
2002	2.00	3.11
2003	2.05	3.10
2004	1.98	2.96
2005	2.11	3.17
2006	2.08	2.84
2007	1.99	2.77
2008	1.71	2.36
2009	1.60	2.06
2010	1.58	2.24
2011	1.59	2.30
2012	1.67	2.54
2013	1.91	3.06
2014	1.91	3.11
2015	1.84	2.95
2016	1.95	2.98
2017	1.78	2.83
2018	1.83	2.98
2019	1.70	2.70
2020	1.69	2.59
2021	1.81	2.58
2022	1.63	2.50

Table 5: Summary statistics: financial covenants. The definitions of the variables are provided in Appendix 8.11.

Variable	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Probability of violating a covenant during the 1st year after origination	0.369	0.355	0.042	0.245	0.692
Probability of violating a covenant during the 2nd year after origination	0.366	0.356	0.035	0.237	0.682
Probability of violating a covenant within the 1st year after an amendment	0.549	0.366	0.174	0.718	0.875
Change in covenant strictness after an amendment, %	NA	NA	-20.067	2.340	60.056

Table 6: Summary statistics: financial covenants amendments. The definitions of the variables are provided in Appendix 8.11.

Event	Frequency
Amend a covenant at least once	0.289
of them not in violation	0.426
Amend a covenant within the 1st year	0.203
of them not in violation	0.420

Table 7: Correlation matrix of variables used in the estimation. The definitions of the variables are provided in Appendix 8.11.

	Probability of violating a covenant during the 1^{st} year after origination	Probability of violating a covenant during the 2^{nd} year after origination	A financial covenant got amended during the 1^{st} year after origination	A financial covenant was violated during the 1^{st} year after origination
Probability of violating a covenant during the 1^{st} year after origination	1	0.926	0.097	0.394
Probability of violating a covenant during the 2^{nd} year after origination	0.926	1	0.111	0.390
A financial covenant got amended during the 1^{st} year after origination	0.097	0.111	1	0.241
A financial covenant was violated during the 1^{st} year after origination	0.394	0.390	0.241	1

Table 8: Correlation of variables used in the estimation with firms' and loans' characteristics. The definitions of the variables used in the estimation are provided in Appendix 8.11. The market-to-book ratio is the close price multiplied by the number of common shares outstanding and divided by the total value of common/ordinary equity. The return on assets is income before extraordinary items divided by total assets. The leverage ratio is the sum of the total long-term debt and the total debt in current liabilities, divided by the sum of the total long-term debt, the total debt in current liabilities, and the total value of common/ordinary equity. The debt-to-EBITDA ratio is the sum of the total long-term debt and the total debt in current liabilities, divided by the operating income before depreciation.

	Probability of violating a covenant during the 1^{st} year after origination	Probability of violating a covenant during the 2^{nd} year after origination	A financial covenant got amended during the 1^{st} year after origination	A financial covenant was violated during the 1^{st} year after origination	Change in covenant strictness after the amendment
Total assets	-0.063	-0.070	-0.050	-0.104	0.020
Market-to-book ratio	-0.054	-0.057	0.089	-0.040	-0.040
Return on assets	-0.238	-0.188	-0.122	-0.071	0.148
Leverage ratio	0.182	0.250	0.020	0.029	-0.316
Debt-to-EBITDA ratio	0.335	0.371	-0.003	0.128	-0.344
Loan amount	-0.180	-0.167	-0.072	-0.159	-0.150
Maturity	-0.047	-0.031	-0.063	-0.061	0.170
All-in-drawn spread	0.110	0.113	0.075	0.003	-0.052
Secured	0.149	0.169	0.032	0.114	0.083
Number of lenders	-0.031	0.001	-0.034	0.028	-0.108

Table 9: Parameter estimates for the full sample. Bootstrapped standard errors are in parentheses. In estimation, the parameters are set $\bar{c} = 1$, $\rho = 0.9$. The estimation procedure and calculation of standard errors are described in Appendix.

Parameter	Estimate
Contractible information precision, q	0.687 (0.002)
Initial probability of the state m, μ_0	0.614 (0.000)
Persistence of the state m, μ	0.654 (0.001)
Persistence of the state b, β	0.501 (0.001)
Amendment cost, η_a	0.009 (0.000)
Waiver friction, η_w	0.700 (0.001)
Contracting efficiency gain, %	2.323 (0.033)

Table 10: Model fit: theoretical and empirical moments. Calculations of the moments are presented in Appendix 8.12.1.

Moment	Theoretical value	Empirical value	t-statistic [p-value]
Strictness of initial financial covenants for the first year after loan origination	0.368	0.370	−0.170 [0.865]
Strictness of initial financial covenants for the second year after loan origination	0.366	0.367	−0.088 [0.930]
Probability of amending a covenant within the first year after loan origination	0.212	0.194	1.475 [0.141]
Probability of amending a covenant within the first year after loan origination, conditional on no violation within the first year after origination	0.124	0.125	−0.015 [0.988]
Strictness of amended financial covenants for the first year after an amendment	0.547	0.558	−0.722 [0.471]
Strictness of amended financial covenants for the first year after an amendment, conditional on no violation within the first year after origination	0.501	0.487	0.611 [0.543]

Table 11: Likelihood of amendments and contracting efficiency gain from amendments in counterfactual scenarios

Scenario	Likelihood of amendments, %	Contracting efficiency gain, %	Contracting efficiency gain in state m conditional on amendments, %	Contracting efficiency gain in state b conditional on amendments, %
Baseline estimates	21.303	2.323	15.383	12.702
1. A completely financially healthy company, $\mu_0 = 0.999$.	12.532	1.393	17.067	15.149
2. A completely financially unhealthy company, $\mu_0 = 0.001$.	15.856	1.434	14.581	17.843
3. An economy where waivers without violations are allowed, $\eta_w^b = \eta_w^m = 0.700$.	21.833	1.363	14.365	8.772

Table 12: Summary statistics for firms' market values and changes in firms' market values within a $[-3, 3]$ day window around the release of the 8-K form which contains an amended contract.

(a) Full sample

Statistic	N	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Market value, $ million	24	1833.93	2525.34	271.61	1089.75	2290.34
Change in market value around the 8-K filing, $ million	24	-21.46	111.83	-56.93	-9.98	18.88

(b) Subsample where a firm has not violated any covenants in the period preceding an amendment

Statistic	N	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Market value, $ million	5	3014.0	1952.1	2999.8	2999.8	2999.8
Change in market value around the 8-K filing, $ million	5	28.26	196.74	-17.67	152.36	152.36

(c) Subsample where a firm has violated a covenant in the period preceding an amendment

Statistic	N	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Market value, $ million	19	1081.9	786.2	284.8	829.0	1446.4
Change in market value around the 8-K filing, $ million	19	-31.06	89.96	-63.59	-18.42	7.97

Table 13: Summary statistics for accounting quality, relationship loans, and syndicate size.

Statistic	N	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Earnings response coefficient[19]	562	0.704	1.218	0.082	0.704	0.953
Relationship loan	1,067	0.228	0.420	0	0	0
Number of lenders	957	9.088	7.867	4	7	12

[19] For this subsample analysis, I remove observations with estimated earnings response coefficients below -0.01, and compute the median for the remaining sample.

Table 14: Parameter estimates with additional moments. Bootstrapped standard errors are in parentheses. In estimation, the control misallocation cost persistence parameter is set to $\rho = 0.9$. The estimation procedure and calculation of standard errors are described in Appendix.

Parameter	Estimate
Contractible information precision, q	0.508 (0.005)
Initial probability of the state m, μ_0	0.577 (0.001)
Persistence of the state m, μ	0.597 (0.000)
Persistence of the state b, β	0.500 (0.001)
Amendment cost, \$ million, η_a	3.982 (0.028)
Waiver friction, \$ million, η_w	273.174 (237.239)
Average cost of misallocating control rights, \$ million, $\frac{\bar{c}}{2}$	136.361 (0.610)

Table 15: Parameter estimates for different data subsamples. Bootstrapped standard errors are in parentheses. In estimation, the parameters are set $\bar{c} = 1$, $\rho = 0.9$. The estimation procedure and calculation of standard errors are described in Appendix.). Amendment frequency is computed as the right-hand side of the equation (3).

Parameter	Quality of contractible info		Importance of non-contractible info		Renegotiation cost	
	ERC above median	ERC below median	Relationship loan	Non-relationship loan	Number of lenders above the 3rd quartile	Number of lenders below the 1st quartile
Contractible information precision, q	0.861	0.548	0.507	0.666	0.623	0.536
	(0.000)	(0.009)	(0.067)	(0.001)	(0.003)	(0.003)
Initial probability of the state m, μ_0	0.722	0.594	0.607	0.603	0.619	0.610
	(0.001)	(0.001)	(0.011)	(0.000)	(0.000)	(0.001)
Persistence of the state m, μ	0.820	0.592	0.650	0.637	0.641	0.648
	(0.000)	(0.002)	(0.001)	(0.000)	(0.000)	(0.000)
Persistence of the state b, β	0.500	0.500	0.500	0.502	0.500	0.500
	(0.012)	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)
Amendment cost, η_a	0.0003	0.009	0.016	0.011	0.012	0.014
	(0.000)	(0.000)	(0.005)	(0.000)	(0.000)	(0.000)
Waiver friction, η_w	0.024	0.590	0.991	0.774	0.743	0.522
	(0.005)	(0.007)	(3.115)	(0.003)	(0.001)	(0.009)
Amendment frequency, %	11.021	26.368	19.564	19.307	20.488	19.157
Efficiency gain, %	0.063	2.188	1.812	1.590	1.858	1.725

8 Appendix

8.1 Solution of the full model

At time 2, if $r_2 = m$ ($r_2 = b$) and $s_2 = b$ ($s_2 = m$), the parties waive and m (b) regains the $t = 2$ control rights if and only if $\eta_w^b \leq c_2$ ($\eta_w^m \leq c_2$).

At time 1, if $r_1 = m$ ($r_1 = b$) and $s_1 = b$ ($s_1 = m$), the parties waive and m (b) regains the $t = 1$ control rights if and only if $\eta_w^b \leq c_2$ ($\eta_w^m \leq c_2$). To determine whether the covenant schedule is amended, first consider the optimal covenant schedule $\{\pi_{m2}^1, \pi_{b2}^1\}$ had the parties amend. When $r_1 = m$, the problem is

$$\begin{aligned} \min_{\{\pi_{m2}^1, \pi_{b2}^1\}} \quad & \mu f_m^m \pi_{m2}^1 + (1-\mu) e^b \pi_{b2}^1 \\ \text{s.t.} \quad (1 - \pi_{m2}^1 - q)^2 + (1 - \pi_{b2}^1 - q)^2 &= (1-q)^2 \\ \pi_{mt}^\tau &\in [0, 1-q] \\ \pi_{bt}^\tau &\in [0, 1-q] \end{aligned}$$

where

$$\begin{aligned} f_m^m &= E[\min\{c_2, \eta_w^m\} | c_1, r_2 = r_1 = m] \\ e^b = E[\min\{c_2, \eta_w^b\} | c_1, r_2 = b \neq r_1] &= E[\min\{c_2, \eta_w^b\}] \end{aligned}$$

f_m^m is a random variable with a pdf $h_m(f)$ and a cdf $H_m(f)$ and the support $[0, \eta_w^m]$, and e^b is a constant. Lagrangian for the problem with only the first constraint is

$$\mathcal{L} = \mu e^b \pi_{m2}^1 + (1-\mu) f_m^m \pi_{b2}^1 + \lambda \left((1-q)^2 - (1 - \pi_{m2}^1 - q)^2 - (1 - \pi_{b2}^1 - q)^2 \right)$$

The derivatives with respect to π_{m2}^1, π_{b2}^1, and λ are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \pi_{m2}^1} &= \mu e^b + 2\lambda(1 - \pi_{m2}^1 - q) \\ \frac{\partial \mathcal{L}}{\partial \pi_{b2}^1} &= (1-\mu) f_m^m + 2\lambda(1 - \pi_{b2}^1 - q) \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= (1-q)^2 - (1 - \pi_{m2}^1 - q)^2 - (1 - \pi_{b2}^1 - q)^2 \end{aligned}$$

Equating the derivatives above to zero and solving the system yields:

$$\pi_{m2}^{1m*} = 1 - q - \frac{\mu f_m^m (1-q)}{\sqrt{(\mu f_m^m)^2 + ((1-\mu)e^b)^2}}$$
$$\pi_{b2}^{1m*} = 1 - q - \frac{(1-\mu) e^b (1-q)}{\sqrt{(\mu f_m^m)^2 + ((1-\mu)e^b)^2}}$$

The determinant of the matrix of second derivatives of the Largangian is

$$\begin{vmatrix} -2\lambda & 0 \\ 0 & -2\lambda \end{vmatrix} = 4\lambda^2 > 0,$$

suggesting that the solution found is a minimum.

The parties will amend the contract if and only if

$$\begin{aligned}
\eta_a + \mu f_m^m \pi_{m2}^{1*} + (1-\mu)e^b \pi_{b2}^{1*} &\le \mu f_m^m \pi_{m2}^0 + (1-\mu)e^b \pi_{b2}^0 \\
\eta_a + (1-q)(\mu f_m^m + (1-\mu)e^b) - (1-q)\sqrt{(\mu f_m^m)^2 + ((1-\mu)e^b)^2} &\le \mu f_m^m \pi_{m2}^0 + (1-\mu)e^b \pi_{b2}^0 \\
(f_m^m)^2 \left[\mu^2((1-q)^2 - (1-q-\pi_{m2}^0)^2)\right] - f_m^m \left[2(1-q-\pi_{m2}^0)\mu(\eta_a + (1-q-\pi_{b2}^0)(1-\mu)e^b)\right] & \\
+ \left((1-q)(1-\mu)e^b\right)^2 - (\eta_a + (1-q-\pi_{b2}^0)(1-\mu)e^b)^2 &\ge 0
\end{aligned}$$

The quadratic expression of f_m^m on the left-hand side has two roots. Therefore, the parties will not amend the contract if and only if $f_m^m \in (x_m^l, x_m^u)$, and amend otherwise, where

$$x_m^l = \frac{(1-q-\pi_{m2}^0)(\eta_a + (1-q-\pi_{b2}^0)(1-\mu)e^b) - (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}}{\mu((1-q)^2 - (1-q-\pi_{m2}^0)^2)}$$
$$x_m^u = \frac{(1-q-\pi_{m2}^0)(\eta_a + (1-q-\pi_{b2}^0)(1-\mu)e^b) + (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}}{\mu((1-q)^2 - (1-q-\pi_{m2}^0)^2)}$$

When $r_1 = b$,

$$\pi_{m2}^{1b*} = 1 - q - \frac{(1-\beta) e^m (1-q)}{\sqrt{\left(\beta f_b^b\right)^2 + ((1-\beta)e^m)^2}}$$
$$\pi_{b2}^{1b*} = 1 - q - \frac{\beta f_b^b (1-q)}{\sqrt{\left(\beta f_b^b\right)^2 + ((1-\beta)e^m)^2}}$$

and the parties will amend the covenant schedule if and only if $f_b^b \in (x_b^l, x_b^u)$, where

$$x_b^l = \frac{(1-q-\pi_{b2}^0)(\eta_a + (1-q-\pi_{m2}^0)(1-\beta)e^m) - (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{m2}^0)(1-\beta)e^m}}{\beta((1-q)^2 - (1-q-\pi_{b2}^0)^2)}$$
$$x_b^u = \frac{(1-q-\pi_{b2}^0)(\eta_a + (1-q-\pi_{m2}^0)(1-\beta)e^m) + (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{m2}^0)(1-\beta)e^m}}{\beta((1-q)^2 - (1-q-\pi_{b2}^0)^2)}$$

At time 0, the problem is

$$\begin{aligned}
\min_{\{\pi_{m1}^0,\pi_{b1}^0,\pi_{m2}^0,\pi_{b2}^0\}} \quad & \mu_0 \times \big(\pi_{m1}^0 e^m \\
+E\Big[\min\{\mu f_m^m \pi_{m2}^0 + (1-\mu)e^b\pi_{b2}^0, \eta_a + (1-q)(\mu f_m^m + (1-\mu)e^b) - (1-q)\sqrt{(\mu f_m^m)^2 + ((1-\mu)e^b)^2}\}\Big]\Big) \\
& +(1-\mu_0) \times \big(\pi_{b1}^0 e^b \\
+E\Big[\min\{\beta f_b^b \pi_{b2}^0 + (1-\beta)e^m\pi_{m2}^0, \eta_a + (1-q)(\beta f_b^b + (1-\beta)e^m) - (1-q)\sqrt{(\beta f_b^b)^2 + ((1-\beta)e^m)^2}\}\Big]\Big) \\
\text{s.t.} \quad & (1-\pi_{m1}^0 - q)^2 + (1-\pi_{b1}^0 - q)^2 = (1-q)^2 \\
& (1-\pi_{m2}^0 - q)^2 + (1-\pi_{b2}^0 - q)^2 = (1-q)^2
\end{aligned}$$

The problems of finding $\{\pi_{m1}^0, \pi_{b1}^0\}$ and $\{\pi_{m2}^0, \pi_{b2}^0\}$ are separable. First, we can find $\{\pi_{m1}^0, \pi_{b1}^0\}$:

$$\pi_{m1}^{0*} = 1-q-\frac{(1-q)\mu_0 e^m}{\sqrt{(\mu_0 e^m)^2 + ((1-\mu_0)e^b)^2}}$$
$$\pi_{b1}^{0*} = 1-q-\frac{(1-q)(1-\mu_0) e^b}{\sqrt{(\mu_0 e^m)^2 + ((1-\mu_0)e^b)^2}}$$

$\{\pi_{m2}^0, \pi_{b2}^0\}$ are computed numerically.

8.2 Proof of Proposition 1

In the model where amendments are prohibited, the problem at time 0 is

$$\begin{aligned}
\min_{\{\pi_{m1}^0,\pi_{b1}^0,\pi_{m2}^0,\pi_{b2}^0\}} \quad & \mu_0 \left(\pi_{m1}^0 e^m + \mu\pi_{m2}^0 E[f_m^m] + (1-\mu)\pi_{b2}^0 e^b\right) + (1-\mu_0)\left(\pi_{b1}^0 e^b + \beta\pi_{b2}^0 E[f_b^b] + (1-\beta)\pi_{m2}^0 e^b\right) \\
\text{s.t.} \quad & (1-\pi_{m1}^0 - q)^2 + (1-\pi_{b1}^0 - q)^2 = (1-q)^2 \\
& (1-\pi_{m2}^0 - q)^2 + (1-\pi_{b2}^0 - q)^2 = (1-q)^2
\end{aligned}$$

π_{m1}^{0*} and π_{b1}^{0*} are the same as in the main model. The optimal π_{m2}^0 and π_{b2}^0 are

$$\pi_{m2}^{0*} = 1 - q - \frac{(1-q)(\mu_0 \mu E[f_m^m] + (1-\mu_0)(1-\beta)e^m)}{\sqrt{(\mu_0 \mu E[f_m^m] + (1-\mu_0)(1-\beta)e^m)^2 + (\mu_0(1-\mu)e^b + (1-\mu_0)\beta E[f_b^b])^2}}$$
$$\pi_{b2}^{0*} = 1 - q - \frac{(1-q)(\mu_0(1-\mu)e^b + (1-\mu_0)\beta E[f_b^b])}{\sqrt{(\mu_0 \mu E[f_m^m] + (1-\mu_0)(1-\beta)e^m)^2 + (\mu_0(1-\mu)e^b + (1-\mu_0)\beta E[f_b^b])^2}}$$

8.3 Proof of Corollary 1

In this section, I present proofs for the case $r_1 = m$. The case $r_1 = b$ can be proven analogously.

$$\frac{\partial x_m^l}{\partial \eta_a} \propto (1-q-\pi_{m2}^0) - (1-q)\frac{\eta_a + (1-q-\pi_{b2}^0)(1-\mu)e^b}{\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}} \propto$$
$$(1-q-\pi_{m2}^0)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b} - (1-q)(1-q-\pi_{b2}^0)(1-\mu)e^b - (1-q)\eta_a <$$
$$-\pi_{m2}^0(1-q-\pi_{b2}^0)(1-\mu)e^b - \pi_{m2}^0\eta_a < 0$$

$$\frac{\partial x_m^u}{\partial \eta_a} \propto (1-q-\pi_{m2}^0) + (1-q)\frac{\eta_a + (1-q-\pi_{b2}^0)(1-\mu)e^b}{\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}} \propto$$
$$(1-q-\pi_{m2}^0)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b} + (1-q)(1-q-\pi_{b2}^0)(1-\mu)e^b + (1-q)\eta_a > 0$$

Since x_m^l is decreasing in η_a and x_m^u is increasing in η_a, the probability of amendment ($f_m^m \leq x_m^l$ or $f_m^m \geq x_m^u$) is decreasing in η_a.

The no-amendment region is

$$x_m^u - x_m^l = \frac{2(1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}}{\mu(1-q-\pi_{b2}^0)^2}$$

$$\frac{\partial\left(x_m^u - x_m^l\right)}{\partial \mu} \propto -\frac{2(1-q)\eta_a(1-q-\pi_{b2}^0)e^b\mu}{\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}} - 2(1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b} \propto$$
$$-(1-q)\eta_a(1-q-\pi_{b2}^0)e^b\mu - (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b} < 0$$

The no-amendment region is decreasing in μ, thus the probability of amendment is increasing in μ.

$$\frac{\partial\left(x_m^u - x_m^l\right)}{\partial q} \propto \left(-2\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b} - \frac{(1-q)2\eta_a(1-\mu)e^b}{\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}}\right) \times$$
$$\mu(1-q-\pi_{b2}^0)^2 + 4\mu(1-q-\pi_{b2}^0)(1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b} \propto$$
$$2\eta_a^2\mu(1-q-\pi_{b2}^0)\left[2(1-q)-(1-q-\pi_{b2}^0)\right] + 2\eta_a(1-\mu)\mu e^b(1-q-\pi_{b2}^0)^2\left[(1-q)-(1-q-\pi_{b2}^0)\right] > 0$$

The no-amendment region is increasing in q, thus the probability of amendment is decreasing in q.

8.4 Theoretical moments

The first set of moments describes the initially set covenant schedule:

1. Ex ante probability of violating a covenant at time $t = 1$, as set at time $t = 0$:

$$\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=1})] =$$
$$\mu_0 \times \pi_{m1}^{0*} + (1-\mu_0) \times (1-\pi_{b1}^{0*}) \tag{6}$$

2. Ex ante probability of violating a covenant at time $t = 2$, as set at time $t = 0$:

$$\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=2})] =$$
$$\mu_0 \times \left(\mu \times \pi_{m2}^{0*} + (1-\mu) \times (1-\pi_{b2}^{0*})\right) + (1-\mu_0) \times \left(\beta \times (1-\pi_{b2}^{0*}) + (1-\beta) \times \pi_{m2}^{0*}\right) \tag{7}$$

The second set of moments is about the frequency of amendments to the covenant schedule:

3. Ex ante unconditional probability of amending the covenant schedule:

$$\mathbf{E}_0[\mathbf{1}(\text{amendment}_{t=1})] =$$
$$\mu_0 \times (1-H_m) + (1-\mu_0) \times (1-H_b) \tag{8}$$

4. Ex ante probability of amending the contract at time $t = 1$, conditional on no covenant violation:

$$\mathbf{E}_0[\mathbf{1}(\text{amendment}_{t=1})|\mathbf{1}(\text{violation}_{t=1}) = 0] = \frac{\mu_0 \times (1 - \pi_{m1}^{0*}) \times (1 - H_m) + (1 - \mu_0) \times \pi_{b1}^{0*} \times (1 - H_b)}{\mu_0 \times (1 - \pi_{m1}^{0*}) + (1 - \mu_0) \times \pi_{b1}^{0*}} \tag{9}$$

The last set of moments captures the distribution of amended covenant schedules:

5. The amended probability of violating a covenant at time $t = 2$, conditional on amendment at time $t = 1$:

$$\begin{aligned}\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=2})|\mathbf{1}(\text{amendment}_{t=1})] &= \frac{1}{\mu_0 \times (1 - H_m) + (1 - \mu_0) \times (1 - H_b)} \\ \times \left(\mu_0 \times (1 - H_m) \times \left(\mu \times E\left[\pi_{m2}^{1m*}|f_m^m \notin \left(x_m^l, x_m^u\right)\right] + (1 - \mu) \times \left(1 - E\left[\pi_{b2}^{1m*}|f_m^m \notin \left(x_m^l, x_m^u\right)\right]\right)\right)\right. \\ \left. +(1 - \mu_0) \times (1 - H_b) \times \left(\beta \times \left(1 - E\left[\pi_{b2}^{1b*}|f_b^b \notin \left(x_b^l, x_b^u\right)\right]\right) + (1 - \beta) \times E\left[\pi_{m2}^{1b*}|f_b^b \notin \left(x_b^l, x_b^u\right)\right]\right)\right)\end{aligned} \tag{10}$$

6. The amended probability of violating a covenant at time $t = 2$, conditional on no covenant violation and amendment at time $t = 1$:

$$\begin{aligned}\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=2})|\mathbf{1}(\text{amendment}_{t=1}) \times (1 - \mathbf{1}(\text{violation}_{t=1}))] &= \frac{1}{\mu_0 \times (1 - \pi_{m1}^{0*}) \times (1 - H_m) + (1 - \mu_0) \times \pi_{b1}^{0*} \times (1 - H_b)} \\ \times \left(\mu_0 \times (1 - H_m) \times (1 - \pi_{m1}^{0*}) \times \left(\mu \times E\left[\pi_{m2}^{1m*}|f_m^m \notin \left(x_m^l, x_m^u\right)\right] + (1 - \mu) \times \left(1 - E\left[\pi_{b2}^{1m*}|f_m^m \notin \left(x_m^l, x_m^u\right)\right]\right)\right)\right. \\ \left. +(1 - \mu_0) \times (1 - H_b) \times \pi_{b1}^{0*} \times \left(\beta \times \left(1 - E\left[\pi_{b2}^{1b*}|f_b^b \notin \left(x_b^l, x_b^u\right)\right]\right) + (1 - \beta) \times E\left[\pi_{m2}^{1b*}|f_b^b \notin \left(x_b^l, x_b^u\right)\right]\right)\right)\end{aligned} \tag{11}$$

$\mathbf{1}()$ above denotes the indicator function, and H_j denotes the probability that the random variable f_j^j falls into the corresponding non-amendment region.

8.5 Analogy between classic debt contracting models and the model in this paper

This section extends the model in Guttman and Marinovic (2018).

For the classic debt contracting model, assume the manager needs to invest the amount I in the project that generates income $\tilde{G} = \tilde{g}_1 + \tilde{g}_2$ in three periods. The investment is made in period 0, $\tilde{g}_1$ is realized in period 1, $\tilde{g}_2$ is realized in period 2, and the payoff can be accessed in period 3. $\tilde{g}_1$ is distributed with the pdf $f(g)$ and the CDF $F(g)$, $\tilde{g}_2 = \rho\tilde{g}_1 + \tilde{\varepsilon}_g$, where $\tilde{\varepsilon}_g$ is mean-zero error term, implying that

g_2 can be predicted when g_1 is relized. The project is NPV-positive: $I < 2E[\tilde{g}]$. The parties have an option to liquidate the project early for a liquidation value of L. If a liquidation occurs, the bank gets L and the manager gets 0.

The parties cannot contract on the outcomes, g_1 and g_2, however, they can contract on the signals about these outcomes: $\tilde{h}_t = \tilde{g}_t + \tilde{\varepsilon}_h$, where $\tilde{\varepsilon}_h$ is a mean-zero noise term, capturing contractual incompleteness.

The contract chosen by the parties consists of (1) the interest payment K^τ - the amount the manager pays to the bank when the project cash flow materializes (τ denotes the time when the interest payment is chosen), and (2) the covenant threshold z_t^τ such that realization of h_t below it results in the transfer of control rights to the bank.

The cost of waiving a control rights misallocation is n_w, and the cost of amending the contract is n_a. In period t, all the waiver and amendment costs incurred before and not including that period are denoted as N^t.

The timeline is summarized below.

Figure 6: Timeline of events in the classic model

$t = 0$

- The initial contract, $\{K^0, z_1^0, z_2^0\}$, is chosen.

$t = 1$

- The manager and the bank observe $\{g_1, h_1\}$.
- Waiver, if any, takes place for a cost towards the ultimate payoff G of n_w. If a waiver occurs, the interest rate may be renegotiated to K^1.
- Amendment of the contract to a new one, $\{K^1, z_2^1\}$, if any, takes place for a cost towards the ultimate payoff G of n_a.

$t = 2$

- The manager and the bank observe $\{g_2, h_2\}$.
- Waiver, if any, takes place for a cost towards the ultimate payoff of n_w. If a waiver occurs, the interest rate may be renegotiated to K^2.

$t = 3$

- The contract matures. The manager receives $\max\{G - N - K, 0\}$; the bank receives $\min\{K, G - N\}$.[a]

[a] K is either K^0, K^1, or K^2, depending on whether any renegotiations took place. N is the sum of all the waiver and amendment costs incurred. For example, if the parties amended the contract at time 1 and waived a control rights misallocation at time 2, $N = n_a + n_w$.

I start solving the model from time 2. Let us consider various possible cases.

- Case $h_2 \geq z_2$ and $g_1 + g_2 - N^1 \geq L - N^1 - n_w$, or covenant was not violated and the ultimate cash flows are above the value were the parties to liquidate. No waiver is needed in this case.
- Case $h_2 \geq z_2$ and $g_1 + g_2 - N^1 < L - N^1 - n_w$, or covenant was not violated and the ultimate cash flows is below the value were the parties to liquidate. A waiver is needed to maximize the total surplus.

  Since the manager has control rights by default and receives 0 in case of liquidation, in order to achieve the maximum total surplus, the bank offers a transfer T^2 to the manager. I assume the parties use Nash bargaining. The bank maximizes:

$$\max_{T^2} L - N^1 - n_w - T^2$$
$$\text{s.t. } T^2 \geq \max\{g_1 + g_2 - N^1 - K^1, 0\}$$

  The optimal value of the transfer is $T^2 = \max\{g_1 + g_2 - N^1 - K^1, 0\}$. Because a waiver is welfare-improving $\left(g_1 + g_2 - N^1 < L - N^1 - n_w\right)$, such a value of transfer always exists.

  Based on the derivation above, the decision whether to waive or not is determined by whether the gain from correcting control rights misallocation, $L - g_1 - g_2$ is greater than the waiver cost n_w.

- Case $h_2 < z_2$ and $g_1 + g_2 - N^1 - n_w < L - N^1$, or covenant was violated and the ultimate cash flows are below the value were the parties to liquidate. The control is with the bank, who optimally liquidates the firm. No waiver is necessary in this case.
- Case $h_2 < z_2$ and $g_1 + g_2 - N^1 - n_w \geq L - N^1$, or covenant was violated and the ultimate cash flows are above the value were the parties to liquidate. The control is with the bank, who will liquidate the firm suboptimally. A waiver is needed.

  In this case, the party that is offering the contract is the manager. She offers a new contract K^2 to the bank.

$$\max_{K^2} \max\{g_1 + g_2 - N^1 - n_w - K^2, 0\}$$
$$\text{s.t. } \min\{K^2, g_1 + g_2 - N^1 - n_w\} \geq L - N^1$$

  The new contract K^2 can always be found because $g_2 + g_2 - N^1 - n_w \geq L - N^1$. Whether a waiver happens is again determined by whether the gain from correcting control rights misallocation, $-L+g_1+g_2$ is greater than the waiver cost n_w.

Next, consider period 1. I focus on only one case: $h_1 \geq z_1^0$ and $g_1 + E\left[g_2|g_1\right] \geq L - n_w$, or the covenant was not violated and the expected payoff from continuation is above the payoff from liquidation.

If an amendment were to happen, the manager would offer the bank a contract:

$$\begin{aligned}
\max_{\{z_2^1, K^1\}} Pr\left[h_2 \geq z_2^1 \;\&\; g_1+g_2-n_a \geq L-n_a-n_w\right] \times \max\{g_1+E[g_2|g_1]-n_a-K^1, 0\}\\
+Pr\left[h_2 \geq z_2^1 \;\&\; g_1+g_2-n_a < L-n_a-n_w\right] \times 0\\
+Pr\left[h_2 < z_2^1 \;\&\; g_1+g_2-n_a-n_w < L-n_a\right] \times 0\\
+Pr\left[h_2 < z_2^1 \;\&\; g_1+g_2-n_a-n_w \geq L-n_a\right] \times \max\{g_1+E[g_2|g_1]-n_a-n_w-E[K^2], 0\}\\
\text{s.t. } Pr\left[h_2 \geq z_2^1 \;\&\; g_1+g_2-n_a \geq L-n_a-n_w\right] \times \min\{K^1, g_1+E[g_2|g_1]-n_a\}\\
+Pr\left[h_2 \geq z_2^1 \;\&\; g_1+g_2-n_a < L-n_a-n_w\right] \times (L-n_a-n_w)\\
+Pr\left[h_2 < z_2^1 \;\&\; g_1+g_2-n_a-n_w < L-n_a\right] \times (L-n_a)\\
+Pr\left[h_2 < z_2^1 \;\&\; g_1+g_2-n_a-n_w \geq L-n_a\right] \times \min\{E[K^2], g_1+E[g_2|g_1]-n_a-n_w\}\\
\geq Pr\left[h_2 \geq z_2^0 \;\&\; g_1+g_2-n_a \geq L-n_a-n_w\right] \times \min\{K^0, g_1+E[g_2|g_1]-n_a\}\\
+Pr\left[h_2 \geq z_0^1 \;\&\; g_1+g_2-n_a < L-n_a-n_w\right] \times (L-n_a-n_w)\\
+Pr\left[h_2 < z_0^1 \;\&\; g_1+g_2-n_a-n_w < L-n_a\right] \times (L-n_a)\\
+Pr\left[h_2 < z_0^1 \;\&\; g_1+g_2-n_a-n_w \geq L-n_a\right] \times \min\{E[K^2], g_1+E[g_2|g_1]-n_a-n_w\}
\end{aligned}$$

This problem can be re-written if I assume the lender's participation constraint binds and substitute for K^1:

$$\begin{aligned}
\max_{z_2^1} Pr\left[h_2 \geq z_2^1 \;\&\; g_1+g_2-n_a \geq L-n_a-n_w\right] \times (g_1+E[g_2|g_1]-n_a)\\
+Pr\left[h_2 \geq z_2^1 \;\&\; g_1+g_2-n_a < L-n_a-n_w\right] \times (L-n_a-n_w)\\
+Pr\left[h_2 < z_2^1 \;\&\; g_1+g_2-n_a-n_w < L-n_a\right] \times (L-n_a)\\
+Pr\left[h_2 < z_2^1 \;\&\; g_1+g_2-n_a-n_w \geq L-n_a\right] \times (g_1+E[g_2|g_1]-n_a-n_w)
\end{aligned}$$

In the problem above, the manager essentially chooses type-1 and type-2 errors:

$Pr\left[h_2 \geq z_2^1 \text{ \& } g_1 + g_2 - n_a \geq L - n_a - n_w\right]$, $Pr\left[h_2 \geq z_2^1 \text{ \& } g_1 + g_2 - n_a < L - n_a - n_w\right]$, $Pr\left[h_2 < z_2^1 \text{ \& } g_1 + g_2 - n_a - n_w < L - n_a\right]$, and $Pr\left[h_2 < z_2^1 \text{ \& } g_1 + g_2 - n_a - n_w \geq L - n_a\right]$ are the only terms that depend on the covenant threshold z. The interest rate, K^1, simply follows after the covenant z_2^1 is chosen to maximize the total surplus.

To simplify the notation, denote $Pr\left[h_2 \geq z_2^1 \text{ \& } g_1 + g_2 - n_a \geq L - n_a - n_w\right] = Pr[nv \text{ \& } nw]$, $Pr\left[h_2 \geq z_2^1 \text{ \& } g_1 + g_2 - n_a < L - n_a - n_w\right] = Pr[nv \text{ \& } w]$, $Pr\left[h_2 < z_2^1 \text{ \& } g_1 + g_2 - n_a - n_w < L - n_a\right] = Pr[v \text{ \& } nw]$, and $Pr\left[h_2 < z_2^1 \text{ \& } g_1 + g_2 - n_a - n_w \geq L - n_a\right] = Pr[v \text{ \& } w]$, where v and nv mean violation and non-violation, and w and nw mean a waiver does and does not go through.

I can change the notation: $Pr[m \text{ \& } nv \text{ \& } nw] = \mu\left(1 - \pi_{m_2}^1\right)$, $Pr[m \text{ \& } v \text{ \& } nw] = \mu\pi_{m_2}^1 Pr\left[g_1 + g_2 - L < n_w\right]$, $Pr[m \text{ \& } v \text{ \& } w] = \mu\pi_{m_2}^1 Pr\left[g_1 + g_2 - L \geq n_w\right]$, $Pr[b \text{ \& } nv \text{ \& } nw] = \beta\pi_{b_2}^1 Pr\left[L - g_1 - g_2 < n_w\right]$, $Pr[b \text{ \& } nv \text{ \& } w] = \beta\pi_{b_2}^1 Pr\left[L - g_1 - g_2 \geq n_w\right]$, $Pr[b \text{ \& } v \text{ \& } nw] = \beta\left(1 - \pi_{b_2}^1\right)$. In addition, I set $V \equiv E\left[\max\{G, L\}\right]$ the total-surplus-maximizing value.

One can re-write the problem from above as

$$\begin{aligned}
\min -V + Pr[m \text{ \& } nv \text{ \& } nw]n_a \\
+Pr[b \text{ \& } nv \text{ \& } nw]\left(L - g_1 - E[g_2|g_1] + n_a\right) \\
+Pr[b \text{ \& } nv \text{ \& } w]\left(n_w + n_a\right) \\
+Pr[m \text{ \& } v \text{ \& } nw]\left(g_1 + E[g_2|g_1] - L + n_a\right) \\
+Pr[m \text{ \& } v \text{ \& } nw]n_a \\
+Pr[m \text{ \& } v \text{ \& } w]\left(n_a + n_w\right)
\end{aligned}$$

The term V does not depend on the level of covenant. Denoting $n_a \equiv \eta_a$, $n_w \equiv \eta_w$, $L - g_1 - E[g_2|g_1] \equiv c_b$, and $g_1 + E[g_2|g_1] - L \equiv c_m$ makes the problem equivalent to the problem in the paper.

8.6 An example of economically different financial covenant schedules that would be recorded the same way by Dealscan

(c) Leverage Ratio. Permit the Leverage Ratio at any time during any period of four fiscal quarters of Limited set forth below to be greater than the ratio set forth opposite such period:

Four Fiscal Quarters Ending:	
November 30, 2005	4.00 to 1.00
Each fiscal quarter thereafter	3.50 to 1.00

Figure 7: The first example of a covenant that would be recorded by Dealscan as "Decreasing from 4:1 to 3.5:1". The contract effective date is November 15, 2005. The screenshot is taken from the Third Amendment to a Credit Agreement filed by Helen of Troy Limited in Exhibit 10.2 of the 10-Q on January 9, 2006.

SECTION 6.08. Maximum Consolidated Total Leverage Ratio. The Borrower will cause the Consolidated Total Leverage Ratio to be less than (a) 4.00 to 1.00 at all times during the period from the Effective Date to and including December 30, 2009, (b) 3.75 to 1.00 at all times during the period from December 31, 2009 to and including December 30, 2010 and (c) less than 3.50 to 1.00 at all times thereafter.

Figure 8: The second example of a covenant that would be recorded by Dealscan as "Decreasing from 4:1 to 3.5:1". The contract effective date is August 25, 2008. The screenshot is taken from the Amended and Restated Credit Agreement filed by The Manitowoc Company, Inc. in Exhibit 4.1 of the 10-Q on November 10, 2008.

8.7 Examples of financial covenant schedules and how they are recorded

7.6 Financial Covenants.

A. Minimum Interest Coverage Ratio. Company shall not permit the ratio of (i) Consolidated EBITDA to (ii) Consolidated Interest Expense for any four-Fiscal Quarter period to be less than 4.00:1.00.

Figure 9: An example of a static financial covenant schedule. This excerpt is taken from the Credit Agreement between Hexcel Corporation and a lending syndicate with Bank of America Securities LLC as a Lead Arranger, dated as of July 9, 2010. This financial covenant schedule will be recorded as "4:1 in 1 year, 4:1 in 2 years, 4:1 in 3 years, 4:1 in 4 years".

8.09 Consolidated Leverage Ratio.

The Borrower will not permit the Consolidated Leverage Ratio as of the end of any fiscal quarter of the Borrower ending during a period set forth below to exceed the ratio set forth below opposite such period:

Period	Consolidated Leverage Ratio
Effective Date through December 31, 2011	5.00:1.00
January 1, 2012 through September 30, 2012	4.50:1.00
October 1, 2012 through September 30, 2013	4.00:1.00
October 1, 2013 and thereafter	3.75:1.00

Figure 10: An example of a dynamic financial covenant schedule. This excerpt is taken from the Credit Agreement between Hanger Orthopedic Group, Inc. and a lending syndicate with Merrill Lynch, Pierce, Fenner & Smith Incorporated and Jefferies Finance LLC as Joint Lead Arrangers, dated as of December 1, 2010. This financial covenant schedule will be recorded as "5:1 in 1 year, 4:1 in 2 years, 3.75:1 in 3 years, 3.75:1 in 4 years".

(b) Fixed Charge Coverage Ratio. Permit the Fixed Charge Coverage Ratio to be less than (i) 1.75 to 1.00 as of April 30, 2003 or July 31, 2003 for the period of four fiscal quarters most recently then ended, (ii) 1.50 to 1.00 as of October 31, 2003 or January 31, 2004 for the period of four fiscal quarters most recently then ended, (iii) 1.75 to 1.00 as of April 30, 2004 for the period of four fiscal quarters most recently then ended, (iv) 1.50 to 1.00 as of July 31, 2004, October 31, 2004, or January 31, 2005 for the period of four fiscal quarters most recently then ended, or (v) 1.75 to 1.00 as of April 30, 2005 or any fiscal quarter ending thereafter, for the period of four fiscal quarters most recently then ended.

Figure 11: An example of a dynamic financial covenant schedule. This excerpt is taken from the Credit Agreement between Gerber Scientific, Inc., and a lending syndicate with Ableco Finance LLC as Agent, dated as of May 9, 2003. This financial covenant schedule will be recorded as "1.5:1 in 1 year, 1.75:1 in 2 years, 1.75:1 in 3 years, 1.75:1 in 4 years".

8.8 Types of amendments to financial covenants

(x) Section 1.10 of the Credit Agreement is hereby amended by deleting the text of such Section in its entirety and replacing such deleted text with the following:

> Notwithstanding anything to the contrary herein, the Total Leverage Ratio and the Total Senior Secured Leverage Ratio shall be calculated (including, but not limited to, for purposes of determining the Maximum Incremental Amount with respect to Sections 2.14(a), 2.16(a), and 2.17(a), on a Pro Forma Basis with respect to each Specified Transaction occurring during the applicable four quarter period to which such calculation relates, or subsequent to the end of such four-quarter period but not later than the date of such calculation; provided that notwithstanding the foregoing, when calculating the Total Leverage Ratio and the Total Senior Secured Leverage Ratio for purposes of (i) determining the applicable percentage of Excess Cash Flow set forth in Section 2.05 and (ii) determining actual compliance (and not Pro Forma Compliance or compliance on a Pro Forma Basis) with any applicable covenant pursuant to Section 7.11, any Specified Transaction and any related adjustment contemplated in the definition of Pro Forma Basis (and corresponding provisions of the definition of Consolidated EBITDA) that occurred subsequent to the end of the applicable four quarter period shall not be given pro forma effect.

Figure 12: An amendment to the definition of Leverage Ratio used in financial covenants. The excerpt is taken from the First Amendment and Refinancing Agreement dated as of March 1, 2011 amending the Credit Agreement between NBTY, Inc., a Delaware Corporation and Barclays Capital, Merrill Lynch, Pierce, Fenner & Smith Incorporated, and Credit Suisse Securities (USA) LLC as Joint Lead Arrangers dated as of October 1, 2010.

1. Amendment to Credit Agreement – Section 1. The definition of EBITDAR in Section 1 of the Credit Agreement is hereby restated as follows:

> **EBITDAR** means, as determined, on a rolling twelve month basis and in respect of any Person the sum of (a) the Net Income of such Person, plus (b) the interest expense of such Person for such period as determined in accordance with GAAP and as such item is reported on such Person's financial statements, plus (c) the income tax expense of such Person for such period, plus (d) the amount reported as the depreciation of the assets of such Person for such period, computed in accordance with GAAP, and as such item is used in the computation of such Person's Net Income for such period, plus (e) the amount reported as the amortization of intangibles for such Person for such period, computed in accordance with GAAP, and as such item is used in the computation of such Person's Net Income for such period, minus (f) Rental Payments related to Capitalized Leases, plus (g) Rental Payments.

Figure 13: An amendment to the definition of EBITDAR used in financial covenants. The excerpt is taken from the Amendment No. 1 dated as of August 26, 2016 to Credit Agreement between Monro Muffler Brake, Inc. and Citizens Bank, N.A. as Administrative Agent dated as of January 25, 2016.

6. Amendment to Section 5.20. Section 5.20 of the Credit Agreement hereby is amended by deleting it in its entirety, and substituting the following therefor:

SECTION 5.20. Minimum Consolidated Net Worth. Consolidated Net Worth will at no time be less than $240,000,000, less the amount of any non-cash loss from the sale of Gerber Coburn in an Approved Gerber Coburn Sale and less the amount of restructuring charges recorded in the fourth Fiscal Quarter of the 2001 Fiscal Year, not to exceed $30,000,000, and excluding any change in the "accumulated other comprehensive income/(loss)" component of shareholders' equity after January 31, 2001, plus the sum of: (i) 75% of the cumulative Consolidated Net Income of the Borrower and its Consolidated Subsidiaries during any period after the Closing Date (taken as one accounting period), calculated quarterly at the end of each Fiscal Quarter but excluding from such calculations of Consolidated Net Income for purposes of this clause (i), any Fiscal Quarter in which the Consolidated Net Income of the Borrower and its Consolidated Subsidiaries is negative; and (ii) 100% of the cumulative Net Proceeds of Capital Stock received during any period after the Closing Date, calculated quarterly at the end of each Fiscal Quarter.

Figure 14: An amendment to the Minimum Net Worth Covenant. The excerpt is taken from the Second Amendment dated as of January 31, 2002 amending the Credit Agreement between Gerber Scientific, Inc. and Wachovia Bank, N.A. as Agent dated as of March 14, 2001.

8.09 Consolidated Leverage Ratio.

The Borrower will not permit the Consolidated Leverage Ratio as of the end of any fiscal quarter of the Borrower ending during a period set forth below to exceed the ratio set forth below opposite such period:

Period	**Consolidated Leverage Ratio**
Effective Date through December 31, 2011	5.00:1.00
January 1, 2012 through September 30, 2012	4.50:1.00
~~**October 1, 2012 through September 30, 2013**~~	~~4.00:1.00~~
October 1, ~~2013~~2012 and thereafter	~~3.75~~4.00:1.00

Figure 15: An amendment to the Leverage Ratio Covenant. The excerpt is taken from Amendment No. 1 dated as of March 11, 2011 amending the Credit Agreement between Hanger Orthopedic Group, Inc., a Delaware corporation and Bank of America, N.A. as Administrative Agent dated as of December 1, 2010.

8.9 An example of an amendment and waivers in the data

c. Leverage Ratio. Section 6.10 of the Credit Agreement is amended in its entirety to read as follows:

"Section 6.10 Leverage Ratio. The Company will not permit its Leverage Ratio at the end of any fiscal quarter to be greater than the levels indicated below for the corresponding periods:

Period	Ratio
January 1, 2002 through March 31, 2002	2.95 to 1.00
April 1, 2002 through June 30, 2002	2.25 to 1.00
July 1, 2002 through September 30, 2002	2.00 to 1.00
October 1, 2002 through March 31, 2003	1.75 to 1.00
April 1, 2003 through September 30, 2003	2.50 to 1.00
October 1, 2003 through December 31, 2003	4.50 to 1.00
January 1, 2004 through March 31, 2004	2.50 to 1.00
April 1, 2004 through June 30, 2004	2.25 to 1.00
July 1, 2004 and thereafter	2.00 to 1.00"

Figure 16: An example of a precautionary waiver. The level of leverage ratio is lifted up for three months (October-December 2003) and then goes back to its original level. This excerpt is taken from the Amendment No. 5, dated as of December 29, 2003, amending the Second Amended and Restated Credit Agreement, dated as of April 30, 2002, between Global Industries, Ltd., a Louisiana Corporation, and Bank One, NA, as administrative agent.

a. Leverage Ratio.

Period Ratio

January 1, 2002 through March 31, 2002 2.95 to 1.00

April 1, 2002 through June 30, 2002 2.25 to 1.00

July 1, 2002 through September 30, 2002 2.00 to 1.00

October 1, 2002 through March 31, 2003 1.75 to 1.00

April 1, 2003 through March 31, 2004 2.50 to 1.00

April 1, 2004 through June 30, 2004 2.25 to 1.00

July 1, 2004 and thereafter 2.00 to 1.00

Figure 17: The leverage covenant in the original Credit Agreement, dated as of April 30, 2002, between Global Industries, Ltd., a Louisiana Corporation, and Bank One, NA, as administrative agent.

2.1. WAIVER. As of the Effective Date and solely for the period through January 31, 2002 (the "Waiver Period"), the Majority Lenders signatory hereto waive any Default or Event of Default under Section 15(c) of the Credit Agreement resulting from the Borrower's failure to comply with any one or more of the covenants set forth in Section 14.1 of the Credit Agreement for the periods ending on December 31, 2001 and as of December 31, 2001, as the case may be.

Figure 18: An example of an ex post waiver. The violation of a financial covenant by the borrower is waived. This excerpt is taken from the Waiver, dated as of December 31, 2001, to the Second Amended and Restated Credit Agreement, dated as of September 15, 1998, between Hexcel Corporation and Citibank, N.A., as Documentation Agent, and Credit Suisse First Boston, as Administrative Agent.

6. Amendment to Section 5.20. Section 5.20 of the Credit Agreement hereby is amended by deleting it in its entirety, and substituting the following therefor:

SECTION 5.20. Minimum Consolidated Net Worth. Consolidated Net Worth will at no time be less than $240,000,000, less the amount of any non-cash loss from the sale of Gerber Coburn in an Approved Gerber Coburn Sale and less the amount of restructuring charges recorded in the fourth Fiscal Quarter of the 2001 Fiscal Year, not to exceed $30,000,000, and excluding any change in the "accumulated other comprehensive income/(loss)" component of shareholders' equity after January 31, 2001, plus the sum of: (i) 75% of the cumulative Consolidated Net Income of the Borrower and its Consolidated Subsidiaries during any period after the Closing Date (taken as one accounting period), calculated quarterly at the end of each Fiscal Quarter but excluding from such calculations of Consolidated Net Income for purposes of this clause (i), any Fiscal Quarter in which the Consolidated Net Income of the Borrower and its Consolidated Subsidiaries is negative; and (ii) 100% of the cumulative Net Proceeds of Capital Stock received during any period after the Closing Date, calculated quarterly at the end of each Fiscal Quarter.

Figure 19: An example of an amendment. An amendment to the Minimum Net Worth Covenant. The excerpt is taken from the Second Amendment, dated as of January 31, 2002, amending the Credit Agreement between Gerber Scientific, Inc. and Wachovia Bank, N.A. as Agent, dated as of March 14, 2001.

8.10 Measure of financial covenant strictness

There exists a disconnect between the notion of financial covenant strictness in the model and the data. I model financial covenants as an aggregate signal, while real-world loan contracts often use more than one financial covenant. To bridge the gap between theory and empirical setting, I use the measure of financial covenant strictness proposed by Demerjian and Owens (2016).

Demerjian and Owens (2016) develop a non-parametric measure of how likely a company is to violate any of its financial covenants in the next quarter after the contract's effective date. Because my study concentrates on longer-term financial covenant schedules and amendments, I adapt the Demerjian and Owens (2016) measure to an annual basis. My measure captures the probability of violation in the next year after the contract is signed or amended.

The measure is computed as follows. First, all companies available in Compustat are matched into twelve groups based on size and profitability. For a given year, the firms are sorted into size quartiles based on average total assets, and then within each size quartile, they are sorted into terciles based on their return-on-assets (ROA).

Next, to compute financial covenant strictness for a company i, another company j is randomly drawn from the bins matched to the company i one and two years before the loan initiation date. For company j, a rate of change in its financial variables is calculated and company i's financial variables are multiplied by these rates. Finally, company i's projected changes are compared with its financial covenants, and an indicator for whether any covenant is violated is recorded. In other words, I check whether the company i would have violated at least one of its covenants if it evolved exactly as the company j. The procedure is repeated 1000 times. The resulting probability of violating a covenant within a year for the company i is the fraction of simulations that are recorded as violations:

$$PVIOL_i = \frac{\sum_{j \neq i} VIOL_j}{1000} \tag{12}$$

Both empirical measure and theoretical construct of financial covenant strictness are functions of two variables: the level of financial covenants and the company's financial performance. Demerjian and Owens (2016) measure would increase if either financial covenant levels were more restrictive or if companies similar to the company i performed worse financially. As can be seen from theoretical moments 1-2 and 5-6, the probability of violating a covenant is a product of covenant levels and probabilities of states where the borrower or the lender are the optimal party-in-control.

8.11 List of variables used in estimation

Variable	Notation	Definition
Probability of violating a covenant within the 1st year after origination	$pviol_0^1$	Covenant strictness, adapted from Demerjian and Owens (2016), measuring the probability of violating a covenant within 1 year after the loan origination date.
Probability of violating a covenant within the 2nd year after origination	$pviol_0^2$	Covenant strictness, adapted from Demerjian and Owens (2016), measuring the probability of violating a covenant in the period between the end of the 1st and the end of the 2nd years after the loan origination date.
Probability of violating a covenant within the 1st year after an amendment	$pviol_{am}^1$	Covenant strictness, adapted from Demerjian and Owens (2016), measuring the probability of violating a covenant within 1 year after the amendment effective date.
Covenant violation within the 1st year after origination	$viol_1$	An indicator variable equal to 1 if at least one of the company's financial ratios or characteristics for the 1st year violate a financial covenant effective for this year. Definitions of financial ratios and characteristics for financial covenants are from Demerjian and Owens (2016).
Covenant violation at any time	$viol_{ever}$	An indicator variable equal to 1 if at least one of the company's financial ratios or characteristics for a year violate a financial covenant effective for this year, for all years before the contract maturity date. Definitions of financial ratios and characteristics for financial covenants are from Demerjian and Owens (2016).
Covenant amendment within the 1st year after origination	am_1	An indicator variable equal to 1 if financial covenants are amended at least once within 1 year after the loan origination. Amendments are defined as described in section 2.1.
Covenant amendment at any time	am_{ever}	An indicator variable equal to 1 if financial covenants are amended at least once at any time before the contract maturity date. Amendments are defined as described in section 2.1.

8.12 Estimation procedure

8.12.1 Calculation of differences between empirical and theoretical moments

In this Appendix, I explain how the empirical moments used to fit the model are computed. The paper uses 6 moments, listed in Appendix 8.4: ex ante probabilities of violating a covenant (i.e., covenant strictness) within the first and the second year after the contract origination, unconditional probability of amendment and probability of amendment conditional on no covenant violation within the first year after loan origination, probabilities of violating a covenant (i.e., covenant strictness) within the first year after an amendment unconditional and conditional on no violation within the first year after loan origination.

The data series used in estimation are initial financial covenant levels from the Dealscan database, hand-collected financial covenant schedules, hand-collected amended financial covenants, and firms' financial characteristics from the Compustat database. The variables are described in Appendix 8.11.

I treat my data as cross-sectional, i.e. every loan is an independent draw from a population of loans described by one distribution function. For each loan i, I have 5 columns:

1. Strictness of initial financial covenants for the first year after loan origination, $pviol^1_{0_i}$.
2. Strictness of initial financial covenants for the second year after loan origination, $pviol^2_{0_i}$.
3. Binary variable for whether a covenant was violated within the first year after loan origination, $viol_{1_i}$.
4. Binary variable for whether a covenant was amended within the first year after loan origination, am_{1_i}.
5. Strictness of amended financial covenants (if there was an amendment) for the first year after an amendment, $pviol^1_{am_i}$.

In the table 17 below, I provide formulas used to calculate differences between empirical and theoretical moments.

Table 17: Formulas to calculate differences between empirical and theoretical moments. N denotes the number of loans in the sample. Description of variables is provided in Appendix 8.11.

Moment	**Formula for difference between empirical and theoretical moments**
Strictness of initial financial covenants for the first year after loan origination	$\frac{1}{N}\sum_{i=1}^{N} pviol^1_{0_i} - \left[\mu_0 \times \pi^{0*}_{m1} + (1-\mu_0)\times(1-\pi^{0*}_{b1})\right]$
Strictness of initial financial covenants for the second year after loan origination	$\frac{1}{N}\sum_{i=1}^{N} pviol^2_{0_i} - \left[\mu_0 \times \left(\mu \times \pi^{0*}_{m2} + (1-\mu)\times(1-\pi^{0*}_{b2})\right) + (1-\mu_0)\times\left(\beta\times(1-\pi^{0*}_{b2}) + (1-\beta)\times\pi^{0*}_{m2}\right)\right]$
Probability of amending a covenant within the first year after loan origination	$\frac{1}{N}\sum_{i=1}^{N} am_{1_i} - \left[\mu_0 \times (1-H_m) + (1-\mu_0)\times(1-H_b)\right]$
Probability of amending a covenant within the first year after loan origination, conditional on no violation within the first year after origination	$\frac{\sum_{i=1}^{N} am_{1_i}\left(1-viol_{1_i}\right)}{\sum_{i=1}^{N}\left(1-viol_{1_i}\right)} - \left[\frac{\mu_0\times(1-\pi^{0*}_{m1})\times(1-H_m)+(1-\mu_0)\times\pi^{0*}_{b1}\times(1-H_b)}{\mu_0\times(1-\pi^{0*}_{m1})+(1-\mu_0)\times\pi^{0*}_{b1}}\right]$
Strictness of amended financial covenants for the first year after an amendment	$\frac{\sum_{i=1}^{N} pviol^1_{am} am_{1_i}}{\sum_{i=1}^{N} am_{1_i}} - \Big[\frac{1}{\mu_0\times(1-H_m)+(1-\mu_0)\times(1-H_b)}\times$ $\left(\mu_0\times(1-H_m)\times\left(\mu\times E\left[\pi^{1m*}_{m2}\middle|f^m_m\notin\left(x^l_m,x^u_m\right)\right]+(1-\mu)\times\left(1-E\left[\pi^{1m*}_{b2}\middle|f^m_m\notin\left(x^l_m,x^u_m\right)\right]\right)\right)+\right.$ $\left.(1-\mu_0)\times(1-H_b)\times\left(\beta\times\left(1-E\left[\pi^{1b*}_{b2}\middle|f^b_b\notin\left(x^l_b,x^u_b\right)\right]\right)+(1-\beta)\times E\left[\pi^{1b*}_{m2}\middle|f^b_b\notin\left(x^l_b,x^u_b\right)\right]\right)\right)\Big]$
Strictness of amended financial covenants for the first year after an amendment, conditional on no violation within the first year after origination	$\frac{\sum_{i=1}^{N} pviol^1_{am} am_{1_i}\left(1-viol_{1_i}\right)}{\sum_{i=1}^{N} am_{1_i}\left(1-viol_{1_i}\right)} - \Big[\frac{1}{\mu_0\times(1-\pi^{0*}_{m1})\times(1-H_m)+(1-\mu_0)\times\pi^{0*}_{b1}\times(1-H_b)}\times$ $\left(\mu_0\times(1-H_m)\times(1-\pi^{0*}_{m1})\times\left(\mu\times E\left[\pi^{1m*}_{m2}\middle|f^m_m\notin\left(x^l_m,x^u_m\right)\right]+(1-\mu)\times\left(1-E\left[\pi^{1m*}_{b2}\middle|f^m_m\notin\left(x^l_m,x^u_m\right)\right]\right)\right)+\right.$ $\left.(1-\mu_0)\times(1-H_b)\times\pi^{0*}_{b1}\times\left(\beta\times\left(1-E\left[\pi^{1b*}_{b2}\middle|f^b_b\notin\left(x^l_b,x^u_b\right)\right]\right)+(1-\beta)\times E\left[\pi^{1b*}_{m2}\middle|f^b_b\notin\left(x^l_b,x^u_b\right)\right]\right)\right)\Big]$

8.12.2 Parameter search algorithm

The objective of the GMM procedure is to minimize the distance between the theoretical moments, which are functions of the model parameters, and empirical moments, which are calculated from the data. In other words, the goal is to find a set of parameters $\hat{\theta}$ such that

$$\hat{\theta} = \text{argmin}_{\theta \in \boldsymbol{\Theta}} \quad (G(Y_i, \theta))^T \hat{W} (G(Y_i, \theta)), \tag{13}$$

where $G(Y_i, \theta) = m(d) - \hat{m}(\theta)$ is the vector of differences between moments computed from the data $m(d)$ – a function of data d – and their counterparts computed from the model $\hat{m}(\theta)$ the model – a function of the model's parameters θ. I show how each element of this vector is calculated in table 17. The matrix W is the weighting matrix.

The estimation is conducted in two steps. In the first step, the algorithm searches for $\hat{\theta}_1$ that minimizes (13) with an identity matrix as the weighting matrix $\hat{W}_1 = E$. Next, I take the obtained estimates $\hat{\theta}_1$, plug them into the vector $G(Y_i, \theta)$, and calculate the covariance matrix of this vector, $\hat{\Omega} \equiv G(Y_i, \theta) G(Y_i, \theta)'$, using bootstrap. I create 1,000 randomly drawn subsamples of size 500 from my original dataset and calculate vectors of moment differences $G(Y_i^k, \hat{\theta}_1), k = 1, 2, ..., 1,000$ for each of these subsamples. Next, I calculate the covariance matrix of moments based on these 1,000 observations, $\hat{\Omega}$.

In the second step, the algorithm searches for $\hat{\theta}_2$ that minimizes (13) where the weighting matrix is the inverse of the covariance matrix: $\hat{W}_2 = \hat{\Omega}^{-1}$. The parameter estimates obtained in the second step $\hat{\theta}_2$ are the ultimate estimates. I use the Nelder-Mead optimization method (Nelder and Mead (1965)) using R NLOPTR function (Johnson (2007)) to search for $\hat{\theta}$ in both steps.

I calculate standard errors of the estimates using the formula for the asymptotic covariance matrix of estimates:

$$\mathbf{V} \equiv \frac{1}{N} \left[\hat{G} \hat{\Omega}^{-1} \hat{G}^T \right]^{-1}, \tag{14}$$

where $\hat{G} \equiv \frac{\partial \left(\frac{1}{N} \sum_{i=1}^{N} g(Y_i, \theta) \right)}{\partial \theta}$ is the Jacobian matrix, evaluated at $\hat{\theta}_2$. The derivative of moment k with respect to parameter p, $\frac{\partial \left(\frac{1}{N} \sum_{i=1}^{N} g(Y_i, \theta) \right)_k}{\partial \theta_p}$, is calculated by varying the parameter $\hat{\theta}_p$ by 1% up and down (keeping other parameters constant) and dividing the difference between the new value of the moment at the 1% higher parameter and the new value of the moment at the 1% lower parameter $\hat{\theta}_p$,

$\left(\frac{1}{N}\sum_{i=1}^{N} g(Y_i,\theta)\right)_k \left(1.01\hat{\theta}_p\right) - \left(\frac{1}{N}\sum_{i=1}^{N} g(Y_i,\theta)\right)_k \left(0.99\hat{\theta}_p\right)$ by 2% of $\hat{\theta}_p$.

Because the model is just-identified, I can not calculate the J-statistic.

8.13 Moments for the estimation of the monetary value of costs

I add two moments to the moments used in the baseline estimation.

Let us consider the case when, at time $t = 1$, investors observe an amendment and no violation of covenants. First, investors update their beliefs about the probabilities of states m and b:

$$Pr\left[r_1 = m|\text{amendment \& no viol}\right] = \frac{\mu_0\left(1-\pi^0_{m_1}\right)(1-H_m)}{\mu_0\left(1-\pi^0_{m_1}\right)(1-H_m) + (1-\mu_0)\pi^0_{b_1}(1-H_b)}$$
$$Pr\left[r_1 = b|\text{amendment \& no viol}\right] = \frac{(1-\mu_0)\pi^0_{b_1}(1-H_b)}{\mu_0\left(1-\pi^0_{m_1}\right)(1-H_m) + (1-\mu_0)\pi^0_{b_1}(1-H_b)}$$

Second, investors update their beliefs about the losses to the enterprise value given the new contract. If $r_1 = m$, this loss is

$$-\left(\eta_a + E\left[(1-q)\left(\mu f^m_m + (1-\mu)e^b\right) - (1-q)\sqrt{(\mu f^m_m)^2 + ((1-\mu)e^b)^2}|\text{amendment}\right]\right).$$

If $r_1 = b$, this loss is

$$-\left(e^b + \eta_a + E\left[(1-q)\left(\beta f^b_b + (1-\beta)e^m\right) - (1-q)\sqrt{\left(\beta f^b_b\right)^2 + ((1-\beta)e^m)^2}|\text{amendment}\right]\right).$$

The change in the market value after observing amendment and no violation is

$$\begin{aligned}
&-\frac{\mu_0\left(1-\pi^0_{m_1}\right)(1-H_m)}{\mu_0\left(1-\pi^0_{m_1}\right)(1-H_m) + (1-\mu_0)\pi^0_{b_1}(1-H_b)} \\
&\times\left(\eta_a + E\left[(1-q)\left(\mu f^m_m + (1-\mu)e^b\right) - (1-q)\sqrt{(\mu f^m_m)^2 + ((1-\mu)e^b)^2}|\text{amendment}\right]\right) \\
&-\frac{(1-\mu_0)\pi^0_{b_1}(1-H_b)}{\mu_0\left(1-\pi^0_{m_1}\right)(1-H_m) + (1-\mu_0)\pi^0_{b_1}(1-H_b)} \\
&\times\left(e^b + \eta_a + E\left[(1-q)\left(\beta f^b_b + (1-\beta)e^m\right) - (1-q)\sqrt{\left(\beta f^b_b\right)^2 + ((1-\beta)e^m)^2}|\text{amendment}\right]\right) \\
&-V,
\end{aligned}$$

where V is the value of the objective function at the solution to the problem at time $t = 0$ outlined in section 1.3.

The derivation of the change in market value after an amendment and a violation is derived analogously:

$$
\begin{aligned}
&-\frac{\mu_0 \pi^0_{m_1}(1-H_m)}{\mu_0 \pi^0_{m_1}(1-H_m)+(1-\mu_0)(1-\pi^0_{b_1})(1-H_b)} \\
&\times\left(e^m+\eta_a+E\left[(1-q)\left(\mu f^m_m+(1-\mu)e^b\right)-(1-q)\sqrt{(\mu f^m_m)^2+((1-\mu)e^b)^2}|\text{amendment}\right]\right) \\
&-\frac{(1-\mu_0)(1-\pi^0_{b_1})(1-H_b)}{\mu_0 \pi^0_{m_1}(1-H_m)+(1-\mu_0)(1-\pi^0_{b_1})(1-H_b)} \\
&\times\left(\eta_a+E\left[(1-q)\left(\beta f^b_b+(1-\beta)e^m\right)-(1-q)\sqrt{\left(\beta f^b_b\right)^2+((1-\beta)e^m)^2}|\text{amendment}\right]\right) \\
&-V
\end{aligned}
$$

8.14 Alternative specifications

8.14.1 Alternative cost persistence parameters

Because the persistence of the cost of leaving the "wrong" party in control (ρ) and the waiver friction (η_w) are difficult to separate, in the estimation, I assume $\rho = 0.9$. In this section, I investigate how robust the estimation results are to the choice of the persistence parameter. I estimate two alternative versions of the model: in one, ρ is set close to one, 0.999, and in the other, ρ is smaller than in the main specification, 0.8.

The estimates are shown in table 18. The conclusion based on estimated parameters does not change with different assumed cost persistence: the quality of contractible information is low, persistences of the non-contractible states are low, and the waiver friction considerably exceeds amendment cost. However, the waiver friction estimate goes up by about 40%.

8.14.2 Alternative data specification

The data allows flexibility in constructing financial covenant amendment variables. In particular, since the model is not about amendment frequencies but about a single decision to amend, I can look either at whether financial covenant schedules are amended within a year or whether they are ever amended in the entire life of a contract.

In the main body of the paper, I choose to only analyze amendment behavior within one year after contract origination. This section investigates how robust the parameter estimates are to specifications of amendment data. The results are presented in table 19.

Most of the parameters are close for the two data specifications. The most sensitive to the data is the estimated quality of contractible information, q. When I consider amendments at any time until a loan's maturity, the frequency of amendments in the data goes up, and the parameter search algorithm attributes it to a lower precision of contractible information.

Table 18: Parameter estimates for different cost persistence parameters

Parameter	Main specification, $\rho = 0.9$	$\rho = 0.999$	$\rho = 0.8$
Contractible information quality, q	0.687 (0.002)	0.699 (0.005)	0.615 (0.004)
Initial probability of the state m, μ_0	0.614 (0.000)	0.605 (0.000)	0.600 (0.000)
Persistence of the state m, μ	0.654 (0.000)	0.657 (0.004)	0.637 (0.001)
Persistence of the state b, β	0.501 (0.001)	0.521 (0.001)	0.500 (0.000)
Amendment cost, η_a	0.009 (0.000)	0.013 (0.001)	0.010 (0.000)
Waiver friction, η_w	0.700 (0.001)	0.997 (0.150)	0.998 (0.013)
Efficiency gain, %	2.323	1.094	2.707

Standard errors are in parentheses. In estimation, $\bar{c} = 1$.

Table 19: Parameter estimates for an alternative data specification

Parameter	Main specification, amendments within the 1st year	Amendments at all times
Contractible information quality, q	0.687 (0.002)	0.582 (0.004)
Initial probability of the state m, μ_0	0.614 (0.000)	0.607 (0.002)
Persistence of the state m, μ	0.654 (0.001)	0.618 (0.001)
Persistence of the state b, β	0.501 (0.001)	0.508 (0.002)
Amendment cost, η_a	0.009 (0.000)	0.007 (0.000)
Waiver friction, η_w	0.700 (0.001)	0.793 (0.034)
Contracting efficiency gain, %	2.323	4.039

Standard errors are in parentheses. In estimation, the parameters are set $\bar{c} = 1$, $\rho = 0.9$.