Explaining Debt Covenant Amendments: A Structural Approach

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Abstract

Financial covenants, which allocate control rights in corporate loan contracts based on financial information, are often renegotiated (Roberts (2015)), frequently resulting in long-term amendments. In this paper, I study the economic mechanisms behind these amendments. I develop and estimate a structural model of covenant amendments using U.S. loan contract data. The model highlights three forces driving amendments: high contractual incompleteness, highly informative non-contractible information arriving after the initial contract is signed, and low amendment costs compared to the cost of misallocating control rights. I find that, on average, frequent amendments occur mostly because of high contractual incompleteness and low amendment costs. Ex ante, amendments save about 2.32% in control misallocation costs.

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Introduction

Financial covenants in corporate loan contracts allocate control rights between borrowers and lenders based on the borrower's financial information. Consistent with their importance, covenants are renegotiated frequently and are the most commonly renegotiated loan term (Roberts (2015)). Most renegotiations take the form of long-term amendments rather than short-term waivers. Unlike waivers, which temporarily suspend covenant violations, amendments permanently alter contractual thresholds or definitions going forward. This distinction is economically important because amendments allow contracting parties to revise the contract in response to evolving conditions, rather than merely provide temporary relief from a realized or anticipated violation.

Despite the prevalence of covenant amendments, their underlying economic mechanisms remain underexplored. Existing theories focus on waivers of covenant violations and do not accommodate forward-looking changes to contractual terms. Empirically, it is also difficult to disentangle the relative importance of different forces driving amendments. For example, amendments may arise because a contract cannot specify actions in all possible situations, because borrower and lender incentives diverge enough that adjusting the contract becomes worthwhile, or because unanticipated information arrives after contract initiation. In this study, I develop and structurally estimate a model of covenant amendments that allows me to quantify the roles of contractual incompleteness, the frictions involved in revising contracts, and the informativeness of post-contract non-contractible information in driving amendment behavior.

Addressing these challenges requires a structural estimation approach for three main reasons. First, key aspects of lender-borrower relationships are unobservable. Lenders monitor borrowers through private information exchange, so the information that triggers renegotiation does not appear in the data. Structural estimation addresses this issue by using observable outcomes, such as the frequency and magnitude of amendments, to infer key latent economic parameters. Second, the underlying forces driving amendments may be correlated in the cross-section, so reduced-form proxies are unlikely to isolate their individual effects. The structural approach instead models these forces in a unified framework and applies the resulting mathematical structure to the data to identify their relative magnitudes. Finally, the structural approach delivers economically interpretable parameter estimates, enabling me to quantify contracting efficiency gains from renegotiation and losses due to contractual incompleteness.

The model rests on the premise that financial covenants allocate control rights between the lender and the borrower based on contractible financial statistics, while other information relevant for optimal control allocation remains non-contractible (in the spirit of Aghion and Bolton (1992)). The first building block is a non-contractible state of the world and a contractible but imperfect signal about this state. The state is a time-varying binary variable that determines whether, to maximize total surplus, control rights should be assigned to the lender or the borrower. The state can be interpreted as summarizing the borrower's performance: when performance is strong, the borrower is more likely to be the efficient holder of control rights, and when it deteriorates, lender intervention may become optimal. The signal, arriving in each time period, is also binary and captures whether a covenant is violated.

I model the contract as an information system that maps the underlying state into control allocation with two kinds of errors. Contractual incompleteness manifests as these errors, each of which represents a misallocation of control rights: a type-1 error grants control to the lender when it should stay with the borrower, and a type-2 error leaves control with the borrower when it should shift to the lender. Although the signal is binary, this formulation captures rich covenant menus in practice: different combinations of accounting-based covenants correspond to different type-1 and type-2 error probabilities, that is, different mappings from borrower performance into control allocation.

The second building block is a set of reduced-form costs: (1) the cost of the contract making an error, (2) the cost of correcting an error that has already occurred, and (3) the cost of changing the contract to reduce future errors. I refer to these as the *control misallocation cost*, the *waiver cost*, and the *amendment cost*, respectively. The first is a composite parameter to capture losses arising from the incentives misalignment between the borrower and the lender. When the signal leads to control being assigned to the wrong party, total surplus is directly reduced. This total surplus loss reflects, for example, inefficient investment decisions when managers retain control in states where lenders should intervene, or overly restrictive policies when lenders take control in states where firms should continue operating freely.

The contracting parties can mitigate this inefficiency using two (non-mutually exclusive) means. First, once an error has occurred, they can correct it: if a covenant is violated, they can waive the violation and renegotiate the allocation of control rights in the current period, incurring a *waiver cost*. Second, they can amend the contract to reduce future errors. Unlike waivers, which affect only the current allocation of control, amendments modify the contract itself – and hence the future signal-contingent allocation of control rights – and involve an *amendment cost*.

Both waiver and amendment costs are composite parameters that capture direct costs as well as bargaining frictions that may prevent efficient renegotiation. Direct costs include legal fees, tax consequences, time and effort spent, or coordination costs (e.g., Rajan and Winton (1995); Becker and Ivashina (2016); Dyreng et al. (2025)). Bargaining frictions arise from unequal bargaining positions, where one party can demand excessive compensation as a condition for agreeing to renegotiate, thereby blocking value-improving adjustments. These frictions may be meaningfully different for waivers and amendments. Waivers occur when the borrower is already in technical default, which grants the lender a strong negotiating position. For instance, if the borrower lacks alternative financing sources, the lender may extract a control allocation that is privately beneficial but inefficient from a total surplus perspective. Amendments, by contrast, are often negotiated outside of default, when the borrower's position is comparatively stronger.

At the beginning, the parties choose an initial contract to minimize expected total costs. The key innovation of the model is that the parties can update the contract by incurring an amendment cost after the initial contract is signed and new information about future states arrives. This creates a trade-off between incurring an amendment cost today to reshape how future states map into control allocation, and continuing under the existing contract, which may lead to inefficient control allocation and costly waivers. For example, suppose new information indicates that the borrower is more likely to be the efficient holder of control going forward. The parties may then optimally loosen the contract's triggers, accepting more cases in which control fails to transfer to the lender when it should (a higher type-2 error rate) in exchange for fewer cases in which control is taken from a borrower who should have retained it (a lower type-1 error rate). The option to amend the contract thus allows the parties to trade off current amendment costs against future misallocation and waiver costs by adjusting how control rights are allocated across states of the world.

In equilibrium, the contracting parties amend their contract when new information leads to a sufficiently large revision in the expected cost of misallocating control rights, relative to what was anticipated at contract initiation. This expected cost is a function of the borrower's expected performance. Amendments are therefore driven by the surprise in that performance, or equivalently by the gap between the newly optimal covenant tightness and the tightness written into the existing contract, not by the level of performance or tightness per se. An outcome that was already anticipated, however weak, is already priced into the initial contract.

The model highlights three forces that govern the likelihood of covenant amendments. Amendments are more likely when contractual incompleteness is greater, when the cost of adjusting the contract through an amendment is low relative to the expected future costs of misallocation and waivers under the existing contract, and when new non-contractible information about future performance is more informative. Intuitively, amendments are most valuable when the initial contract is more prone to error, adjusting it is relatively inexpensive, and new information is more predictive.

To estimate the model, I hand-collect detailed information on covenant schedules and amendments for a random sample of U.S. firms from loan contracts filed with the SEC.

Estimating the model on loan-level data yields economically meaningful estimates of each of the three forces, which I take in turn. The first force is contractual incompleteness, and the estimates suggest it is large. On average, financial covenants appear to misallocate control rights approximately 31% of the time. This incompleteness varies substantially across firms. Firms with high financial reporting quality have much more precise contracts, with misallocation rates around 14%, whereas firms with worse reporting face substantially higher error rates, around 45%. Similarly, contracts are relatively accurate for investment-grade firms but much less so for speculative-grade and especially unrated firms, for whom misallocation rates are about 50%. These patterns are consistent with more opaque and financially weaker firms contracting on noisier signals (Costello and Wittenberg-Moerman (2010); Ball et al. (2015); Donelson et al. (2017)).

The second force is the cost of adjusting the contract, where the estimates show a sharp difference between amendments and waivers. Amending a contract is inexpensive, at about 2% of the average cost of misallocating control, and is particularly cheap for firms with strong reporting quality and investment-grade credit ratings. In contrast, waiving a covenant violation is costly, at approximately 140% of the average misallocation cost. Waiver cost varies systematically with proxies for bargaining power and coordination frictions: it is higher for firms with low reporting quality and weak or missing credit ratings, in relationship lending, and in loans with many syndicate participants, consistent with lenders having greater bargaining power and with higher coordination costs in renegotiation (Sharpe (1990); Petersen and Rajan (1994); Becker and Ivashina (2016); Chu (2021)). Consistent with this interpretation, I also find that the cost of amendments is about 80% higher when the amendment occurs after a covenant violation, indicating that lenders gain substantial bargaining power following violations. These patterns align with prior evidence that covenant violations shift bargaining power to lenders and lead to economically significant concessions, such as reduced investment and increased management turnover (Chava and Roberts (2008); Nini et al. (2012); Vashishtha (2014)).

The third force is the informativeness of new non-contractible information about firm performance, which also varies across firms. In the full sample, firm performance is only moderately persistent: if a firm is performing well today (implying it does not need the lender's intervention), it is likely to continue doing well next year about 65% of the time, while poor performance persists only about 50% of the time. This persistence differs sharply across firms. For investment-grade firms, strong performance is highly persistent – above 90% – whereas for speculative-grade or unrated firms, performance is much less stable, with persistence closer to 60%. These patterns are consistent with prior evidence that financially stronger firms have more predictable performance, while weaker firms are more volatile and harder to forecast (Sufi (2007); Wittenberg-Moerman (2008); Armstrong et al. (2010)). As a result, new non-contractible information is substantially more informative about future performance for financially strong firms than for weaker or more opaque firms.

Taken together, the relative magnitudes of the three estimated forces point to what drives amendment activity. Contractual incompleteness is economically large, and amendment costs are very low relative to the cost of misallocation, whereas the informativeness of new non-contractible information is only moderate in the full sample. These magnitudes suggest that amendments are driven primarily by the first two forces: covenants are imprecise enough to misallocate control frequently, and amendments provide a low-cost way to correct these errors. The importance of the forces nonetheless varies across firms. For opaque firms, amendments mainly reflect the need to correct frequent misallocation arising from noisy contracts. For financially strong firms, where new information is more informative, amendments are more closely tied to its arrival, allowing contracting parties to redesign contracts to better align future control allocation.

Finally, the estimates imply economically significant gains from allowing contracts to be amended. Ex ante, the option to amend increases total surplus by approximately 2.3%. Conditional on an amendment, the gains are substantially larger – about 15.3% of total surplus when the borrower is the efficient party in control and 12.6% when the lender is. These magnitudes are economically meaningful given that the underlying inefficiencies reflect losses in firm value from misallocated control and costly renegotiation. The gains are largest for firms with low reporting quality and missing credit ratings, where contractual incompleteness is highest and the potential for improvement is greatest.

To provide evidence on the absolute magnitudes of control misallocation and renegotiation costs, I extend the estimation by incorporating changes in firms' market values around amendment announcements. The estimated costs of misallocating control arising from the incentives misalignment between the borrower and the lender are economically large: for an average firm, assigning control to the wrong party leads to an expected total surplus loss of about \$136.4 million, or 7.44% of the average firm's market value. These magnitudes are comparable to losses documented in prior studies of specific agency conflicts. For example, Chava and Roberts (2008) show that covenant violations lead to reductions in investment, Malmendier and Tate (2005) document overinvestment by overconfident managers, and Griffin et al. (2026) show that delayed lender intervention is associated with lower recovery rates. While these studies quantify economically meaningful losses, each focuses on a specific margin and a particular party. In contrast, my estimates capture the total surplus loss from misallocating control, which may or may not be fully reflected in observed monetary outcomes. In addition to investment distortions or lower recovery rates, these losses may include firm value reductions due to inappropriate management decisions, reputation costs, or the loss of customers and suppliers.

Amending contracts to reduce the probability of control misallocation in advance is a significantly cheaper option: the cost of an amendment is about \$4.0 million, or 0.22% of market value. The losses described above represent expected or potential inefficiencies that contracting parties trade off ex ante. Because contracts more often than not allocate control correctly and are frequently adjusted through amendments, the realized efficiency losses are much smaller. I estimate that the actual surplus loss due to contractual incompleteness and renegotiation frictions is about \$19.09 million for the average firm. This magnitude is economically meaningful – implying that positive-NPV investments of this size may be forgone – while at the same time substantially smaller than the potential losses, highlighting the role of amendments in mitigating, but not eliminating, inefficiencies.

I contribute to the broad literature on debt contracting and renegotiations (e.g., Roberts and Sufi (2009b), Gârleanu and Zwiebel (2008), Bharath et al. (2009), Christensen and Nikolaev (2012), Choi and Triantis (2012), Denis and Wang (2014), Roberts (2015), Li et al. (2016), Saavedra (2018), Armstrong et al. (2020), Dou (2020), Christensen et al. (2022), Caskey et al. (2025)). My study focuses on renegotiations of the most commonly renegotiated term – financial covenants – and highlights an important feature: covenants are often renegotiated long-term. I provide a theoretical framework for contract amendments and use a structural approach to assess the relative economic importance of the key forces driving amendment behavior: contractual incompleteness, costs of amendments relative to control misallocation costs, and the informativeness of non-contractible information. The estimated parameter values allow me to estimate total surplus losses from contractual incompleteness (Aghion and Bolton (1992)) and various corporate frictions (Coase (1937)).

On the empirical front, my study helps to understand some existing empirical facts about covenant amendments and provides some novel facts. Prior studies (e.g., Roberts and Sufi (2009b), Roberts (2015), Nikolaev (2018)) have already documented that renegotiations of financial covenants are common. These studies, however, did not distinguish long-term covenant amendments from short-term covenant waivers and based their empirical tests on theories about covenant waivers (e.g., Berlin and Mester (1992), Rajan and Winton (1995), Gârleanu and Zwiebel (2008), Guttman and Marinovic (2018), Lu et al. (2018), Gao and Liang (2019)). I document that many covenant renegotiations do not appear to be simply a remedy for a technical default and instead are long-term amendments. A large portion of covenant amendments happen when a borrower has not violated any covenants, and when an amendment happens, covenants are more likely to get tighter, not looser. Amendments also do not strongly correlate with borrowers' financial health at loan origination. My theory helps reconcile these facts and explains why theories about waivers cannot be used to explain most covenant renegotiations in the data. Long-term amendments, as opposed to waivers, happen if there is a sufficient surprise in the borrower's performance, irrespective of whether this surprise is positive or negative. This result implies that the borrower's initial financial health or initial covenant strictness should not predict whether the parties will amend financial covenants. Firms with worse performance may set tighter covenants in the beginning (and are more likely to violate and waive them), but whether these covenants get amended depends on whether there is a substantial surprise in these firms' performance. My study suggests caution when using prior theories about waivers to explain amendments of financial covenants in the data.

On the theoretical front, my study helps to bridge the gap between empirical observations and theoretical literature on debt contracting. I develop a general framework that captures key economic conflicts present in real-world debt contracting. First, I model agency problems in reduced form as a loss of total surplus from the firm-bank relationship. These parameters can conveniently capture multiple conflicts described by prior theories (e.g., Aghion and Bolton (1992), Rajan and Winton (1995), Sridhar and Magee (1996), Gârleanu and Zwiebel (2008), Guttman and Marinovic (2018), Gao and Liang (2019), Laux (2020)), such as asset substitution, debt overhang, bargaining power, etc., in a single theoretical construct. Second, financial covenants in my model are not specific thresholds for specific financial variables (such as in Guttman and Marinovic (2018) or Gao and Liang (2019)) but a broader construct – type-1 and type-2 errors. All of these features make the model more generic and amenable to a more direct mapping into the data.

A closely related paper is Griffin et al. (2026), which also structurally estimates a model where covenants serve to minimize the expected costs of type-1 and type-2 errors. The research questions and approaches of the two papers differ. Griffin et al. (2026) aim to explain the stark decline in covenant violations over the past two decades. I instead quantify the roles of contractual incompleteness, incentive misalignment, and the informativeness of post-contract information in driving amendment behavior. The approaches differ in two main ways. First, the two papers treat amendments differently, and, as a result, estimate contractual incompleteness differently. Griffin et al. (2026) do not treat incompleteness as a latent variable. They infer it directly from observed outcomes, defining a type-2 error as a bankruptcy without a prior covenant violation and a type-1 error as a covenant violation not followed by an amendment. This approach treats amendments as responses to violations. I instead treat amendments more broadly, as forward-looking adjustments to contract design that can occur with or without a violation, and I use observed amendment behavior to identify latent contractual incompleteness. Second, Griffin et al. (2026) do not estimate control misallocation or renegotiation costs in detail. They provide reduced-form evidence on the implications of type-2 errors for recovery rates at bankruptcy. Because I aim to quantify the relative forces driving amendments, I use detailed data, such as amendments with and without a covenant violation, and can estimate the costs of misallocating control, waivers, and amendments separately. My estimates are total surplus losses, which may incorporate the reduced-form estimate of Griffin et al. (2026).

The rest of the paper is organized as follows. Section 1 presents the debt covenant amendments model and describes the estimation procedure. Section 2 describes the data and collection process. Section 3 discusses estimated parameters, and Section 4 discusses model fit. Section 5 shows two extensions of the model to estimate the monetary values of costs and how amendment costs differ with and without covenant violations. Section 6 discusses counterfactual scenarios. Section 7 concludes.

1 Model of debt covenant amendments

In this section, I present a parsimonious theoretical framework to study when and why contracting parties amend financial covenants as new information arrives over the life of a loan. The model captures three key features of debt contracting: (i) control rights are allocated based on imperfect but contractible signals, (ii) misallocation of control is costly, and (iii) contracting parties can respond to these costs either ex post through waivers or ex ante by amending the contract. The parsimonious structure allows me to isolate the role of contract amendments and to take the model to the data using structural estimation.

1.1 Model setup

At time $t = 0$, a firm (the borrower) and a bank (the lender) enter a debt contracting relationship. Their relationship lasts three periods, until time $t = 3$, when the contract matures.

The first essential feature of debt contracting is that the borrower's and the lender's incentives are not perfectly aligned in the sense that, depending on the state of the world, it is best from the total surplus perspective to give control rights to one party or another. For example, the borrower may have incentives to invest in risky assets, and in financially unstable times, it is best if the bank takes control and limits the firm's investment. Alternatively, the lender is not skilled to run the firm properly, and in good times, control should stay in the firm manager's hands. This notion of who is the best party-in-control at a given time in the model is captured by the binary state variable $r_t, r_t \in \{m, b\}$, where m implies that the best party-in-control is the firm's manager, and b implies that the best party-in-control is the bank. The probability that $r_1 = m$, or that at time $t = 1$ the best party-in-control is the manager, is μ_0. Conditional on $r_1 = m$, $r_2 = m$ with probability μ and $r_2 = b$ with probability $(1 - \mu)$; conditional on $r_1 = b$, $r_2 = b$ with probability β and $r_2 = m$ with probability $(1 - \beta)$. Figure 1 illustrates probabilities of states and how states transition from one period to the next.

The incentives misalignment is costly: whenever the party that is not the best party-in-control receives control rights, the total surplus from the contracting relationship is reduced. I capture this surplus loss by the continuous random variable c_t. At time $t = 1$, c_1 is a random draw from a uniform distribution with bounded support, $\tilde{c}_1 \sim U[0, \bar{c}]$. If the state persists, i.e., $r_1 = r_2$, the cost of misallocating control rights persists: $\tilde{c}_2 = \rho c_1 + (1 - \rho)\tilde{c}$, where $\rho \in (0, 1)$ is the degree of persistence in the cost and $\tilde{c} \sim U[0, \bar{c}]$ is an innovation to the cost. If the state does not persist, i.e., $r_1 \neq r_2$, the cost at time

$t = 2$ is a new random draw, $\tilde{c}_2 \sim U[0, \bar{c}]$. For instance, if the manager runs the company without restrictions during financial instability, she may take excessive risk and reduce the ultimate total value of the firm.[1] The size of this loss is likely related to the loss if the situation continues for another year. But if the next year the situation is the opposite, i.e., if financial stability improves but the bank steps in and limits the firm's investment,[2] the firm value lost due to forgone investment opportunities is not necessarily comparable to the loss in the case the manager takes excessive risk.[3]

The second essential feature is incomplete contracting (Aghion and Bolton (1992); Christensen et al. (2016)). If the contracting parties could write a contract specifying who gets control based on the state r_t, the total surplus would always be at its maximum level. However, the state r_t is non-contractible. Instead, the parties can contract on a contractible signal that is correlated with the state, but does not perfectly reflect it. For example, the parties can specify whether the lender obtains a right to step in based on whether the borrower has exceeded a certain value of debt-to-EBITDA ratio. While the debt-to-EBITDA ratio captures the borrower's financial health, it does so imperfectly. I capture this imperfect contractible signal in the model by the binary variable s_t, $s_t \in \{m, b\}$. If $s_t = r_t$, the signal has correctly captured the actual state, if $s_t \neq r_t$, the signal has made an error. The degree to which the signal s_t captures the actual state r_t, i.e., the degree of contractual completeness, is $q, q \in (0.5, 1]$.

The role of the contract between the parties is effectively to allocate, at a given time t, the imperfection of the signal s_t between type-1 and type-2 errors to minimize the overall costs. I model the contract as an information system: the combination of a type-1 and type-2 error. The probability that a covenant makes a type-1 error, or is violated when it should not be, at time t, is $\pi_{mt} = Pr\left[s_t = b | r_t = m\right] \in [0, 1-q]$. The probability that a covenant makes a type-2 error, or is not violated when it should be, at time t, is $\pi_{bt} = Pr\left[s_t = m | r_t = b\right] \in [0, 1-q]$. The possibility frontier for the precision of the contract is described by the equation $(1-q-\pi_{mt})^2 + (1-q-\pi_{bt})^2 = (1-q)^2$ and illustrated in Figure 2. The parties choose the contract $\{\pi_{mt}, \pi_{bt}\}$ trading off the type-1 error against the type-2 error. Note that the first subscript (m or b) of the probabilities corresponds to the state, r_t, and not to the signal. In what

[1]For example, Malmendier and Tate (2005) find that companies with overconfident executive managers exhibit (likely excessive) high sensitivity of their investments to cash flows. When a CEO is overconfident, the company's investment increases by about 23.4% of assets more in response to a rise in cash flows. Because overconfident CEOs overestimate returns to some investment projects, firm value can be destroyed significantly.

[2]Chava and Roberts (2008) find that a covenant violation leads to about 13% decline in capital expenditures for the violating firm, suggesting considerable loss in company value if this violation is suboptimal.

[3]In Internet Appendix 9.8.1, I estimate an alternative model without persistence in the cost of misallocating control rights. The major conclusions from the model remain unchanged. The only difference is that the estimated persistence of the b state, β, is substantially higher.

follows, the superscript τ in π^τ denotes the time τ at which the contract is chosen.

The final key feature is the parties' ability to renegotiate after they sign the initial contract. The renegotiations can be of two types: (1) a waiver – a renegotiation to change **current** allocation of control rights and (2) an amendment – a renegotiation to change the contract specifying **future** signal-contingent allocation of control rights. For example, a waiver occurs when a firm has just violated (or is about to violate) a financial covenant due to an error but the bank is willing to ignore the violation and give the control back to firm management. In contrast, an amendment occurs when (with or without a covenant violation) the parties understand that financial covenants they have in a contract are not suitable anymore and change them. These changes will affect how control is allocated at some point in the future. The parties can use waivers to avoid the costs of misallocating control in the current period and use amendments to reduce expected costs in future periods.

Neither waivers nor amendments can occur all the time. First, they involve direct costs like legal fees, tax consequences, time and effort spent, or coordination costs (e.g., Rajan and Winton (1995); Becker and Ivashina (2016); Dyreng et al. (2025)). Second, parties' unequal bargaining positions can sometimes prevent renegotiations: one party can demand excessive compensation as a condition for agreeing to renegotiate, thereby blocking value-improving adjustments. These bargaining frictions can differ meaningfully for waivers and amendments. Specifically, waivers occur when the borrower is in technical default, and thus the lender may have large bargaining power, while amendments may occur outside of default states when the lender's position is not so strong. I introduce composite variables to capture these costs of waivers and amendments.

For waivers, variables η_w^m and η_w^b capture the costs of waivers in the states $r_t = m$ and $r_t = b$, respectively.[4] Because of these costs, a waiver will occur only if its cost $\eta_w^i, i \in \{m, b\}$ is smaller than the cost this waiver saves. If c_t is less than η_w^i, c_t has to be borne. If c_t is greater than η_w^i, the total surplus loss that exceeds the waiver cost can be avoided, and the total surplus only suffers the loss of η_w^i. In what follows, I call η_w^i waiver cost or waiver friction.

The amendment cost is captured in the composite variable η_a.[5] An amendment can only go through

[4]Different costs of waivers in different states allow the model to capture potentially different levels of frictions arising when control rights are misallocated in different situations in the real world. In particular, waivers of covenant violations – misallocations of control rights when the contract grants control to the bank but the manager should be in control – have been widely documented (e.g., Dichev and Skinner (2002)), while the opposite case – where no covenant was violated but the bank still demands control – has not been documented and is implausible.

[5]In Section 5.2, I estimate an extended model where the cost of amendment η_a is allowed to be different for cases when a firm did and did not violate a covenant.

if its cost is smaller than the expected saved cost from changing the contract. Importantly, an amendment can reduce the need for waivers in the future and thus the expected costs of waivers.

Figure 3 shows the timeline of the model. At time $t = 0$, the initial contract, regulating signal-contingent allocation of control for times $t = 1$ and $t = 2$, $\{\{\pi^0_{m1}, \pi^0_{b1}\}, \{\pi^0_{m2}, \pi^0_{b2}\}\}$, is chosen. At time $t = 1$, the parties observe the realized state (i.e., the best party-in-control) r_1, the cost of misallocating control rights in this state c_1, and the signal (i.e., which party actually obtains control) s_1. A waiver can take place. The parties can also amend their contract, i.e., change their contract for time $t = 2$ from $\{\pi^0_{m2}, \pi^0_{b2}\}$ to $\{\pi^1_{m2}, \pi^1_{b2}\}$. At time $t = 2$, the parties again observe the realized state r_2, the cost of misallocating control rights in this state c_2, and the signal s_2. A waiver can take place. Since at time $t = 3$ the contract matures, no amendments take place at time $t = 2$.[6,7]

1.2 Benchmark: a model without amendments

To highlight the role of amendments, I first consider a benchmark model in which contracts cannot be revised after origination. In this model, the parties choose their contract $\{\{\pi^0_{m1}, \pi^0_{b1}\}, \{\pi^0_{m2}, \pi^0_{b2}\}\}$ once at time $t = 0$ and cannot change it afterwards.

I solve the model by backward induction. At time $t = 2$, if the contract makes no errors, i.e., $r_2 = s_2$, the parties do not need to do anything. If a type-1 or type-2 error occurs, i.e., if $r_2 \neq s_2$, the parties either let it stay and lose surplus because of incentives misalignment, c_2, or waive the error and lose surplus because of the waiver, η^i_w. To sum up, the surplus loss from misallocating control is zero if $r_2 = s_2$, $\min\{c_2, \eta^m_w\}$ if there is a type-1 error, i.e., $r_2 = m \neq s_2 = b$, and $\min\{c_2, \eta^b_w\}$ if there is a type-2 error, i.e., $r_2 = b \neq s_2 = m$.

At time $t = 1$, the problem is the same. The parties cannot amend the contract, so the only choice they make is whether to waive if needed. The surplus loss is zero if $r_1 = s_1$ and $\min\{c_1, \eta^i_w\}$ otherwise.

At time $t = 0$, the dynamic contract, $\{\{\pi^0_{m1}, \pi^0_{b1}\}, \{\pi^0_{m2}, \pi^0_{b2}\}\}$, is chosen. The role of the contract

[6]Empirically, most contracts amend financial covenants once and within one or two years after loan origination. To capture this feature of the data, I allow the parties to amend their contract only once. The main intuition remains unchanged if I consider more periods of amendments.

[7]My stylized framework differs from classic debt contracting models (see, for example, Rajan and Winton (1995); Sridhar and Magee (1996); Gârleanu and Zwiebel (2008); Guttman and Marinovic (2018); Lu et al. (2018); Gao and Liang (2019)) in multiple directions. First, I do not explicitly model agency conflicts between the borrower and the lender and model them in reduced form as a loss to total surplus. Second, the contract in my model does not have an interest rate. Third, the covenant is not modeled as a threshold for a signal but as type-1 and type-2 errors. I depart from traditional debt contracting models to keep tractability that allows me to consider a dynamic problem and structurally estimate the model. In Internet Appendix 9.1, I explain in detail why a model with these three simplifications is sufficient to describe amendment behavior and provide the mapping between my framework and classic debt contracting frameworks.

is to reduce the expected costs of type-1 and type-2 errors at time $t = 1$ and $t = 2$.

To simplify the notation, I introduce new definitions. Let $L_i \equiv E\left[\min\{\tilde{c}_t, \eta_w^i\}\right], i \in \{m, b\}$ denote the ex-ante total surplus loss due to a type-1 (for $i = m$) or type-2 (for $i = b$) error. Recall that the cost due to incentives misalignment, $\tilde{c}_t$, persists from one period to the next if the state r_t persists. Then, once the parties observe c_1 and the state persists, $r_1 = r_2$, the expected total surplus loss at time $t = 2$ becomes dependent on the value c_1. Let $\tilde{l}_i \equiv E\left[\min\{\tilde{c}_2, \eta_w^i\}|\tilde{c}_1, r_2 = r_1\right] = E\left[\min\{\rho c_1 + (1-\rho)\tilde{c}, \eta_w^i\}|\tilde{c}_1, r_2 = r_1\right], i \in \{m, b\}$ denote the expected total surplus loss due to type-1 and type-2 errors at time $t = 2$, which is a function of the realization of $\tilde{c}_1$. At time $t = 0$, the realization of $\tilde{c}_1$ is unknown, and thus $\tilde{l}_i$ is a random variable. Finally, let $\bar{l}_i \equiv E\left[\tilde{l}_i\right] = E\left[l_i(c_1)\right], i \in \{m, b\}$ denote the expectation of the variable $\tilde{l}_i$ across all possible realizations of $\tilde{c}_1$.[8]

Using this notation, the problem at time $t = 0$ is

$$
\begin{aligned}
\min_{\pi^0_{m1}, \pi^0_{b1}, \pi^0_{m2}, \pi^0_{b2}} \quad & \underbrace{\pi^0_{m1}\mu_0 L_m}_{\substack{\text{type-1 error cost} \\ \text{at } t=1}} + \underbrace{\pi^0_{b1}(1-\mu_0)L_b}_{\substack{\text{type-2 error cost} \\ \text{at } t=1}} \\
& + \underbrace{\pi^0_{m2}\big(\mu_0\mu\bar{l}_m + (1-\mu_0)(1-\beta)L_m\big)}_{\substack{\text{type-1 error cost} \\ \text{at } t=2}} + \underbrace{\pi^0_{b2}\big(\mu_0(1-\mu)L_b + (1-\mu_0)\beta\bar{l}_b\big)}_{\substack{\text{type-2 error cost} \\ \text{at } t=2}} \\
\text{s.t.} \quad & (1-\pi^0_{m1}-q)^2 + (1-\pi^0_{b1}-q)^2 = (1-q)^2, \\
& (1-\pi^0_{m2}-q)^2 + (1-\pi^0_{b2}-q)^2 = (1-q)^2, \\
& \pi^0_{kt} \in [0,\, 1-q] \quad \forall\, k \in \{m, b\},\ t \in \{1, 2\}.
\end{aligned}
$$

The following Proposition describes the optimal covenant schedule.

Proposition 1. *The optimal contract chosen at time $t = 0$ in the model without amendments,*

[8] Recall that $\tilde{c}_1 \sim U[0, \bar{c}]$, and $\tilde{c}_2 = \rho c_1 + (1-\rho)\tilde{c}, \tilde{c} \sim U[0, \bar{c}]$ if $r_2 = r_1$ and $\tilde{c}_2 \sim U[0, \bar{c}]$ if $r_2 \neq r_1$. L_i is a constant that is computed using the fact that $\tilde{c}_t$ is uniformly distributed. $\tilde{l}_i(c_1) = \begin{cases} \eta_w^i, & \text{if } \eta_w^i \leq \rho c_1 \\ \frac{\eta_w^i\left(\rho c_1 + (1-\rho)\bar{c} - \eta_w^i\right)}{(1-\rho)\bar{c}} + \frac{\eta_w^{i2} - \rho^2 c_1^2}{2(1-\rho)\bar{c}}, & \text{if } \rho c_1 < \eta_w^i < \rho c_1 + (1-\rho)\bar{c} \\ \rho c_1 + \frac{(1-\rho)\bar{c}}{2}, & \text{if } \eta_w^i \geq \rho c_1 + (1-\rho)\bar{c} \end{cases}$, is a random variable whose pdf and cdf for the estimation are computed numerically given the fact that c_1 is uniformly distributed. Finally, $\bar{l}_i$ is a constant computed as the expectation of $\tilde{l}_i$.

$\{\{\pi^0_{m1}, \pi^0_{b1}\}, \{\pi^0_{m2}, \pi^0_{b2}\}\}$, *is unique and as follows:*

$$\pi^{0*}_{m1} = (1-q)\left(1 - \frac{1}{\sqrt{1+y_1}}\right), \quad \pi^{0*}_{b1} = (1-q)\left(1 - \frac{1}{\sqrt{1+\frac{1}{y_1}}}\right)$$
$$\pi^{0*}_{m2} = (1-q)\left(1 - \frac{1}{\sqrt{1+y_2}}\right), \quad \pi^{0*}_{b2} = (1-q)\left(1 - \frac{1}{\sqrt{1+\frac{1}{y_2}}}\right)$$

where $y_1 = \left(\frac{(1-\mu_0)L_b}{\mu_0 L_m}\right)^2$ *is the cost of type-2 error relative to type-1 error at time* $t = 1$ *and* $y_2 = \left(\frac{\mu_0(1-\mu)L_b + (1-\mu_0)\beta \bar{l}_b}{\mu_0 \mu \bar{l}_m + (1-\mu_0)(1-\beta)L_m}\right)^2$ *is the cost of type-2 error relative to type-1 error at time* $t = 2$.

When the parties can choose the contract only at time $t = 0$, the optimal contract is simply the one that minimizes the expected costs of type-1 and type-2 errors in each period. When the cost of type-2 error relative to type-1 error is higher, the optimal contract allows for a less likely type-2 error $\left(\pi^{0*}_{b1} \text{ and } \pi^{0*}_{b2}\right)$ in exchange for a more likely type-1 error $\left(\pi^{0*}_{m1} \text{ and } \pi^{0*}_{m2}\right)$.

The parties could reduce their total surplus losses even more if they could amend their contract at time $t = 1$. In the setup where amendments are not allowed, the parties ignore valuable non-contractible information they learn later in the relationship, at time $t = 1$. Specifically, at time $t = 1$, the parties observe the state of the world r_1, which helps predict the state of the world r_2. For example, learning that the best party-in-control at time $t = 1$ is the manager (which happens with probability μ_0) implies the best party-in-control will be the manager again at time $t = 2$ with probability μ. The state r_1 was not known at time $t = 0$ when parties designed the contract. If the parties could amend the contract at time $t = 1$ with the new non-contractible information in mind, they could further reduce expected total surplus losses.

In the next section, I consider the full model with the option to amend the contract at time $t = 1$.

1.3 Full model equilibrium

In the full model, the parties can amend the contract at time $t = 1$ at a fixed cost η_a. The timeline is shown in Figure 3. I solve the model by backward induction.

At times $t = 2$ and $t = 1$, the decisions whether to waive or not are the same as in the benchmark model.

A new decision that the parties make is whether to amend their contract at time $t = 1$. The first step of the decision is to understand, if the contract is amended, what the new optimal contract would be, $\{\pi^1_{m2}, \pi^1_{b2}\}$. The new optimal contract minimizes the expected costs of type-1 and type-2 errors at time $t = 2$ given the new information – the state of the world r_1.

The second step is to compute the expected costs of type-1 and type-2 errors under the old contract $\left(\{\pi^0_{m2}, \pi^0_{b2}\}\right)$ and under the new contract $\left(\{\pi^1_{m2}, \pi^1_{b2}\}\right)$, and to check whether the reduction exceeds the cost of the amendment η_a. If the reduction in the expected cost of errors is greater than the amendment cost, the parties amend the contract.

The Proposition below summarizes the optimal decision to amend the contract at time $t = 1$.

Proposition 2. *At time $t = 1$ in the state $r_1 = i$, the parties amend the covenant schedule if and only if the expected cost of misallocating control rights at time $t = 2$ in case the time-1 state persists, l_i, is sufficiently low or sufficiently high:*

$$l_i(c_1) \in \left(-\infty, x^l_i\right] \cup [x^u_i, \infty)\,, \tag{1}$$

where x^l_m, x^u_m, x^l_b, and x^u_b are defined in Appendix 8.1.1.

Intuitively, it is optimal to amend the contract at time $t = 1$ when new information about the state of the world and the cost of misallocating control in this state is sufficiently surprising. The expected cost $l_i(c_1)$ being within the bounds $\left(x^l_i, x^u_i\right)$ implies that the parties' estimated cost when they wrote the contract at time $t = 0$, $\bar{l}_i$, is not substantially different from their updated estimate of the cost at time $t = 1$, $l_i(c_1)$. If that is the case, the cost of the amendment η_a is too large compared to the saved cost of misallocating control, and an amendment is not optimal. In contrast, the very high or low expected cost $l_i(c_1)$ implies that the parties' estimate of the cost changed substantially, and an amendment is justified.

Importantly, what triggers an amendment of the contract is not the expected state of the world (e.g., borrower health) per se but the surprise in the expected state of the world. This result explains why, empirically, the borrower's initial financial health or initial covenant strictness do not predict whether the parties will amend financial covenants. Firms with worse performance may set tighter covenants in the beginning (and are more likely to violate and waive them), but whether these covenants get amended is determined by whether there is a substantial surprise in these firms' performance.

The model highlights three key economic forces behind financial covenant amendments. First, contracts have to be incomplete. Otherwise, no type-1 or type-2 error would occur and the parties would not need to renegotiate ever. Second, the non-contractible information parties receive after signing the contract (r_1) has to be predictive about the future (r_2). When this information is sufficiently predictive, the parties can amend a contract to a sufficiently better one. Finally, it must be cheaper to amend the contract in advance than to wait and later bear control misallocation cost. The next proposition summarizes the effects of these forces on the likelihood of financial covenant amendments.

Corollary 1. *After an initial contract is chosen, the probability of an amendment at time $t = 1$ is*

- *decreasing in the degree of contractual completeness, q,*
- *increasing in the informativeness of the post-contract non-contractible information, μ and β,*
- *decreasing in the cost of amendments, η_a.*

Finally, at time $t = 0$, the parties choose the optimal contract, anticipating that the contract might be amended in some cases. The equilibrium initial contract for time $t = 1$, $\{\pi^{0*}_{m1}, \pi^{0*}_{b1}\}$, is the same as in the benchmark case. The contract for time $t = 2$, $\{\pi^{0*}_{m2}, \pi^{0*}_{b2}\}$, however, is different because it plays a dual role: it determines whether it is amended at time $t = 1$ and, if not amended, stays until time $t = 2$. When estimating the model, I solve for $\{\pi^{0*}_{m2}, \pi^{0*}_{b2}\}$ numerically. The details are provided in Appendix 8.1.1.

1.4 Ex ante contracting efficiency gain

My framework demonstrates how an option to amend a contract helps improve contracting efficiency by allowing the parties to write a better contract once they learn new non-contractible information. I define contracting efficiency gain due to amendments as the difference between the total surplus loss (from giving control to wrong parties, amendments, and waivers) when the parties can amend the contract and when they cannot amend the contract. In Figure 4, I plot the ex ante contracting efficiency gain as a function of model parameters: the degree of contractual completeness, q, the persistence of the non-contractible state m, μ, the waiver friction in the state m, η^m_w, and the cost of amendments, η_a.

When the contractible signal becomes more precise, the overall probability of type-1 and type-2 errors decreases, making the option to amend less valuable. In the extreme, if the signal perfectly represents the actual state ($q = 1$), amendments add no value.

The informativeness of non-contractible information affects efficiency gains in the opposite direction. When the new information at time $t = 1$ is more predictive about the future, amending the contract at time $t = 1$ reduces the expected cost of misallocating control more.

Finally, contracting efficiency gain increases in the waiver friction and decreases in the amendment cost. When it is more costly to waive a type-1 error (η_w^m increases), an amendment in advance saves more control misallocation costs. In contrast, when amendments themselves are costly (η_a increases), the net gain from them decreases.

1.5 Theoretical moments for structural estimation

My ultimate goal is to estimate the model on the data to evaluate the magnitudes of the key forces driving covenant amendments: contractual incompleteness, informativeness of non-contractible information, and costs of amendments relative to the costs of misallocating control rights. There are nine model parameters. The degree of contractual completeness is captured by the parameter q. The informativeness of non-contractible information is described by parameters μ and β – the persistences of the state of the world. The parameter μ_0 captures the firm's general financial health – the probability that the lender will need to intervene within the first year after loan origination. Finally, the cost of amendments is η_a, the cost of waivers is $\eta_w^i, i \in \{m, b\}$, and the distribution of the cost of having a wrong party in control is described by $\bar{c}$ and ρ.

Before the estimation, I make some assumptions about the model parameters to minimize misspecification and identify the parameters. First, theoretically, the contracting parties can waive a control misallocation both when the bank and the manager get control suboptimally. In reality, however, the first situation – when a covenant is violated but the violation gets waived and control returns to the manager – is plausible, while the symmetric situation – when the bank obtains control without any covenant violations – is unobservable and likely implausible because the bank may face litigation if it attempts to take control without formal contract breaches. This disconnect between the model and the data may lead to biased estimates of the model. To avoid potential misspecification, I preclude waivers in the case of type-2 error and set η_w^b prohibitively high. To simplify notation, in what follows, η_w implies η_w^m.

Second, because total surplus can be challenging to measure, I cannot estimate the absolute values of the control misallocation and amendment costs. In the baseline specification, I fix the parameter $\bar{c} = 1$ and thus can only estimate costs of waivers and amendments relative to the cost of having the wrong party in control. Later, in Section 5.1, I use changes in firms' market values around announcements of covenant amendments to estimate monetary values of the control misallocation and amendment costs.

Finally, the parameters governing the persistence of the control misallocation cost, ρ, and of non-contractible states, μ and β, affect amendment behavior in the same way. The parties are more likely to amend the contract because they are more certain of either the future cost of control misallocation or the future state of the world. To identify one of the parameters describing the persistence of the parties' new information, I need to fix one of the two parameters. I set $\rho = 0.9$, and estimate the parameters μ and β. I examine the estimates' robustness to the choice of cost persistence ρ in Internet Appendix 9.8.1.

To identify the six remaining parameters, I use the six moments listed in Appendix 8.1.4, describing the initially set and amended covenant schedules and the probabilities of amending financial covenants. Because the number of parameters equals the number of moments, the model is just identified.

The first and the second moments are the strictness of financial covenants in the first and the second periods after loan initiation. I define strictness as the probability of violating a covenant within the first period, consistent with the empirical definition of covenant strictness I use (Demerjian and Owens (2016)). Both in the data and in the model, the probability of violation is a function of (1) covenant levels and (2) the firm's financial performance: a firm is more likely to violate when its covenant levels are more restrictive and when it is less financially healthy.

The third and the fourth moments describe the unconditional probability of an amendment and the probability of an amendment conditional on no covenant violation. Because the probability of an amendment may be different in different states, these moments are functions of the probabilities of these states.

The fifth and the sixth moments describe the strictness of amended financial covenants, unconditional and conditional on no covenant violation in the first period. Similar to initial financial covenants, the strictness of amended covenants depends on covenant strictness and the company's financial performance after an amendment.

1.6 Estimation procedure

I estimate the model using the Generalized Method of Moments (GMM). Intuitively, the method searches for the set of the model parameters – the degree of contractual completeness, q, the persistence of non-contractible information, μ and β, companies' financial health, μ_0, costs of waivers, η_w, and costs of amendments, η_a – that minimize the distance between model-implied theoretical moments and their empirical counterparts. I describe the estimation procedure in detail in Internet Appendix 9.7. The next section describes how I construct the empirical moments.

2 Data and descriptive statistics

To estimate my model, I need the following six empirical moments from the data: (1)-(2) strictness of financial covenants in the first and second years[9] after loan origination, (3) unconditional probability of a covenant amendment, (4) probability of a covenant amendment conditional on no covenant violation, and (5)-(6) strictness of amended covenants unconditional and conditional on no covenant violation at the time of the amendment. In this section, I describe how I collect this data and present summary statistics for firms and loans in my sample.

2.1 Data sources and collection

The most commonly used database, Dealscan, does not contain data on financial covenants and amendments to them in sufficient detail.[10] Because of this limitation, in this paper, I combine Dealscan data with hand-collected data from publicly available loan contracts filed under Regulation S-K. In addition, to ensure my hand-collection process is accurate, I begin with the dataset of loan originations and amendments created by Roberts (2015). The Roberts (2015) dataset contains data on loans for 1984-2012 for 114 randomly selected companies from the intersection of Compustat and Dealscan databases. I add data on financial covenants to the Roberts (2015) dataset and extend the dataset until 2021.

For the strictness of financial covenants in original loans (moments 1 and 2), I need to collect the levels of financial covenants that the borrower needs to satisfy in one and two years after the loan origination date. In each original loan document on EDGAR, I manually search for financial covenants, typically found in the "Negative covenants" or "Financial covenants" sections. For each financial covenant, I

[9]Because, empirically, the majority of amendments occur within one year after loan origination, I say that a "year" in the data is a "period" in the model.

[10]For the purposes of my study, the exact dynamic schedule of financial covenants is needed. The Dealscan database contains information on whether a financial covenant is monotone or changes over time. The latter are recorded as “Increasing from XXX to YYY” without specifying when the threshold changes exactly and how fine the steps from XXX to YYY are. For instance, Dealscan would record two covenant schedules provided in Internet Appendix 9.2 in the same manner, while they are economically different: the first one represents a short-term adjustment with a fixed covenant for the remaining life of the loan (Li et al. (2016)), while in the second one, the covenant changes gradually over the entire period, probably to regulate a fundamental agency problem that is expected to worsen with time. My paper focuses on dynamic covenant schedules explained by long-term fundamental considerations and not short-term adjustments, making the two cases different from this study’s perspective.

record dynamic schedules on a yearly basis, starting from the contract effective date for the first five years of the loan. For example, for a debt-to-EBITDA covenant that starts with 5:1, falls to 4.5:1 in 10 months, then falls to 4:1 in another ten months, and then falls to 3:1 in another ten months and stays the same until maturity, I record the schedule as "4:1 in 1 year, 3:1 in 2 years, 3:1 in 3 years, 3:1 in 4 years". If details on how covenant levels change over time are missing, I assume that covenants evolve linearly over time. Static covenant schedules are recorded as constant for all years. Examples of dynamic covenant schedules and how they are recorded are in Internet Appendix 9.3. To measure covenant strictness, I use the methodology proposed by Demerjian and Owens (2016) who come up with a measure of ex ante covenant strictness that can incorporate potential multiple financial covenants in a contract. The details are outlined in Internet Appendix 9.5.

Next, I identify instances of covenant amendments (moments 3 and 4).[11] I use instances of amendments to loan contracts recorded in Dealscan to search for amendment documents on EDGAR. I first check whether an amendment to a loan contract contains changes to financial covenants. If a covenant is amended, I include it in the sample if the change to the covenant is long-term. Recall that in this study, an amendment is a permanent change to a covenant schedule, and a waiver is a transitory change. Some changes in loan contracts legally designated as "amendments" change financial covenants short-term to avoid an anticipated violation of these financial covenants. Such adjustments are waivers rather than amendments in the context of this study because they are short-term changes in control rights. I define short-term changes as those that only alter financial covenant schedules for less than a year after the amendment effective date but keep the schedule unchanged for the rest of time until the contract's maturity.[12] These instances are not recorded as amendments. An example of an "amendment" excluded from the sample and an amendment included in the sample are in Appendix 8.2.

To separate instances of amendments without covenant violations (moment 4), I identify whether firms violated at least one financial covenant at the time of the amendment in the following way. Using the Demerjian and Owens (2016) error-minimizing definition of financial covenant variables, I compute

[11] Most of the loan contracts amend a financial covenant only once; and a majority of these amendments happen within one year of loan origination. For these reasons, I model only a single amendment period and use the probability of an amendment within the first year as a data moment. Alternatively, I could have used the probability of an amendment at any point in time as the data moment. In Internet Appendix 9.8.1, I present estimates with this alternative data specification and find generally similar results.

[12] Even though I choose a one-year cutoff to distinguish short-term from long-term changes to financial covenants, I have not observed instances when an amendment changes covenants for slightly less than a year. A typical short-term change amends covenants for the next month or quarter.

values of all financial covenant variables for the firm in the relevant year. If any of these variables violate levels specified in covenants, I record a covenant violation.

Finally, I record the strictness of amended covenants (moments 5 and 6). Covenant amendments are typically made in two ways: either the numeric level of a financial covenant is amended, or the definition of a variable used in a financial covenant is amended. For the latter, I record the instance of a financial covenant amendment but record strictness of the amended covenant as missing. For the former, I record a new level of a financial covenant in the same yearly manner as for original loans, starting from the amendment effective date, and use the Demerjian and Owens (2016) measure for strictness. Internet Appendix 9.4 contains examples of the two types of amendments to financial covenants.

Some firms may not comply with the SEC's rules and may not disclose their loan contracts (Caskey et al. (2023)). For most of the original contracts recorded in Dealscan for firms in my sample, I was able to find the contract in either an 8-K, 10-K, or 10-Q filing. Amendments were missing more often than original contracts. Nevertheless, for the period after 2001, I was able to find more than 70% of original contracts and more than 90% of amendments.

For general characteristics of firms, I use Compustat database, and for other characteristics of loans, I use Dealscan.

2.2 Descriptive statistics

The final sample comprises 1,068 loans issued to 110 medium-sized companies between 1994 and 2021.

Table 1 shows summary statistics for firms in my sample. A median firm has total assets of about $1,049 million and a market-to-book ratio of 2.14. Because I focus on firms that have bank loans, naturally, my firms have a higher leverage ratio (about 0.48) than a typical firm in the Compustat universe. As for financial health, about 16.7% of my firms have an investment-grade S&P credit rating, about 39.5% have a speculative-grade rating, and about 39.7% are missing a credit rating in the S&P ratings database.

Table 2 shows summary statistics at the loan level; panel (a) uses the full sample, and panel (b) uses a subsample of loans that amend a financial covenant within the first year after origination. In the full sample, the median loan amount is $200 million, the maturity is 59 months, and the all-in-drawn spread is 200 bps. About 56.6% of loans in my sample are revolvers longer than one year, and about 30.6% are term loans.

When looking at statistics describing financial covenants and amendments, a few important conclusions emerge. First, original financial covenants are reasonably strict: the median probability of violating a financial covenant within the first year after loan origination is 0.25. Second, financial covenant amendments are common. About 20.3% of loans in my sample amend a financial covenant within a year after origination, and about 28.9% amend a financial covenant at least once during the entire life of the loan. Third, many instances of covenant amendments occur without prior covenant violations. Out of the amendments within the first year of origination, in 42.0% of the cases, the borrower did not violate any financial covenants (untabulated). Fourth, when financial covenants are amended, they are more likely to get stricter than looser. The average probability of a violation after an amendment is 0.538, which is higher than the average probability of a violation of the original covenant at 0.366. The facts that amendments often tighten covenants and many amendments occur outside of covenant violations suggest that amendments are not made simply to avoid a technical default but may have different economic drivers.

I provide some additional descriptive evidence on covenant amendments. First, financial covenant amendments do not appear to be primarily related to contract maturity extensions: only 16.8% of financial covenant amendments are coupled with amendments to contract maturity. To minimize the possibility that financial covenant amendments in my sample are not primarily amending control rights but rather are due to maturity extensions, I estimate my model on financial covenant amendments that occur within one year of contract origination. In addition, in Internet Appendix 9.8.3, I completely remove all financial covenant amendments coupled with maturity extensions and show that the estimates of my model do not change.

Second, in Tables 12 and 13 in Internet Appendix 9.6, I provide correlations between variables describing financial covenants and amendment behavior and other firms' and loans' characteristics. The strictness of financial covenants in original contracts is associated with firm and loan characteristics in an intuitive way. Larger firms with lower leverage generally obtain looser covenants from their lenders, consistent with the literature (e.g., Bradley and Roberts (2015)). Covenant strictness is positively correlated with loan spreads (all-in-drawn spread) and with collateral, consistent with riskier borrowers obtaining worse credit conditions on all dimensions (Bradley and Roberts (2015)). In addition, amendment behavior is not strongly correlated with the strictness of initial covenants or with the borrower's financial health. The correlation of the binary variable for amendment with the strictness of financial covenants is less than 10% and with the binary variable for being close to a covenant violation is only 24%. Similarly, amendments are not correlated with borrowers' leverage ratios and are only weakly correlated with debt-to-EBITDA ratios. These facts suggest that amendments may not serve to resolve violations, as waivers do, but have different economic purposes.

3 Estimation results

In this section, I discuss the main results obtained on the full sample and on subsamples of firms, and implications for contracting efficiency.

3.1 Parameter estimates

I first estimate the model on the full sample of loans. The estimated parameters are in Table 3. Next, because the parameters can be substantially different for firms with different economic characteristics and to provide the intuition behind parameter values, I estimate the model on different subsamples of firms. The four subsamples are based on: firms' financial reporting quality (measured using the ERC (Lang and McNichols (1990); Imhoff and Lobo (1992))), relationship with the lender (measured using prior history with the lead arranger of the loan), lender coordination cost (measured using syndicate size), and firms' financial health (measured using the firm's credit rating). The estimates for different subsamples are in Table 4.

The first key parameter is the degree of contractual completeness. For the full sample, the estimated contractual completeness is low: q=0.687. This number implies that, for a typical contract in my sample, financial covenants make either a type-1 or type-2 error 31.3% of the time. The typical contract appears to be highly incomplete, and the subsample estimates help shed light on which firms have these highly incomplete contracts. First, firms with high ERCs have type-1 or type-2 errors only about 13.9% of the time, while firms with low ERCs have them as often as 45.2% of the time. Consistent with the documented link between financial information quality and debt contracting (Costello and Wittenberg-Moerman (2010); Ball et al. (2015); Donelson et al. (2017)), firms with high financial reporting quality contract on more reliable metrics, improving the alignment between the firm's actual state and the contract's allocation of control rights. Second, the estimated degree of contractual incompleteness increases meaningfully as firms' credit ratings worsen. Firms with investment-grade ratings have type-1 or type-2 errors 15.8% of the time, and firms with speculative-grade ratings 22.7% of the time, consistent with less financially healthy firms being more opaque and having worse accounting quality (e.g., Sufi (2007); Wittenberg-Moerman (2008); Armstrong et al. (2010)). Further, firms that do not have a credit rating have errors 49.7% of the time.

My estimates of contractual incompleteness contrast with the estimates of type-1 and type-2 error probabilities in Griffin et al. (2026): my aggregate error rate is 31.3%, while Griffin et al. (2026) find a False Positive rate of 2.9% and a False Negative rate of 1.1%. The approaches in the two papers differ in two related ways. The methods are different: I treat contractual incompleteness as a latent parameter and estimate it with a structural model, while Griffin et al. (2026) assume that type-1 and type-2 errors have certain features in the data and collect these statistics directly. The assumptions behind the methods are also different: my structural model defines type-1 and type-2 errors differently than Griffin et al. (2026) do. Griffin et al. (2026) define a False Negative as a firm going bankrupt without a prior covenant violation. In their view, the role of a violation is to give the lender control when the firm is close to bankruptcy. My view differs. I treat violations as tripwires (Berlin and Mester (1992); Rajan and Winton (1995); Gorton and Kahn (2000); Gârleanu and Zwiebel (2008); Christensen and Nikolaev (2012)) that give lenders the option to extract concessions from borrowers who may be far from a violation but whose financial health is starting to deteriorate. Lenders rarely take full control of the borrower. Instead, they may demand changes to investment and borrowing policies or to the management team. This view implies that lenders should hold these rights more often than only near bankruptcy, so the False Negative rate should be higher under my approach. For False Positives, Griffin et al. (2026) define them as covenant violations without a subsequent contract amendment. I do not view amendments this way. My study treats amendments as general changes to loan contracts. My model suggests that they occur whenever the new information (which may or may not be associated with a covenant violation) implies the current contract is sufficiently far from the optimal one. The data are consistent with this: a large proportion of amendments (42%) occur outside of covenant violations. I use the frequency of amendments, unconditional and conditional on no covenant violation, as a moment to identify the degree of contractual incompleteness. Using my approach, the False Positive rate is a latent statistic that is a part of the aggregate contractual incompleteness estimate.

The next parameter captures firms' general financial health. For the full sample, μ_0=0.614, or, within the first year of loan origination, a typical firm benefits from retaining full control 61.4% of the time, and is better off giving some power to the lender 38.6% of the time. Recall that giving power to the lender does not imply giving full control of the firm but rather it being worth it to trip a covenant so that the lender gains a right to check and impose some restrictions on the firm's actions (for examples of such actions, see Chava and Roberts (2008); Nini et al. (2009, 2012); Vashishtha (2014)). Predictably, firms with investment grade ratings need lenders' intervention less often and benefit from retaining control 75.2% of the time. Firms with speculative grade ratings (no ratings) benefit from retaining control fully only 58.6% (57.0%) of the time. Perhaps because financial health is correlated with accounting information quality, the general financial health parameter also differs for firms with different reporting quality: μ_0=0.722 for firms with high ERCs and μ_0=0.594 for firms with low ERCs.

The third and fourth parameters, μ and β, describe the persistence of the non-contractible state of the world, or the importance of the non-contractible information. Conditional on the firm not needing the lender's intervention within the first year, the firm will not (will) need the intervention again next year with probability μ $(1-\mu)$. Conditional on the firm needing the lender's intervention within the first year, the firm will (will not) need the intervention again next year with probability β $(1-\beta)$. In the full sample, the estimates are μ=0.654 and β=0.501, suggesting that, on average, the non-contractible state is not highly persistent and the non-contractible information is not highly important. Again, the parameters vary predictably with firms' financial health. For investment-grade firms, μ=0.905 and β=0.143. If an investment-grade firm does not need the lender's intervention at the moment, it will not need it in a year with a 90.5% probability. If it does need an intervention at the moment, it will need it again with only 14.3% probability. For financially healthy firms, if a lender observes that the firm performs well for the first year, this good information is highly predictive and thus important. In contrast, financially unhealthy firms appear to have highly unpredictable performance: for speculative-grade (missing) rating firms, μ=0.622 (0.639) and β=0.554 (0.560). This stark difference suggests that the importance of non-contractible information is a primary reason for amendments for high-credit-rating firms but not for firms with low or missing credit ratings.

The fifth parameter, η_a, is the amendment cost. Recall that this parameter captures the total surplus loss from amending the contract and also the magnitude of frictions that sometimes prevent amendments. The amendment cost is estimated relative to the cost of granting control rights to the wrong party. For the full sample, $\eta_a = 0.009$, or the cost of amending a contract equals about 1.8% of the average cost of granting control rights to the wrong party.[13] In other words, if a lender and a borrower renegotiate and amend a contract, the total surplus is damaged very little compared to the potential damages from the incentives misalignment between the lender and the borrower. Another way to interpret this estimate is that the frictions preventing amendments are small, and it is generally easy for contracting parties to agree and implement an amendment. The magnitude of the amendment cost varies with firms' and loans' characteristics in a manner consistent with the amount of bargaining power lenders and borrowers have. Firms with high ERCs face lower amendment costs (0.06%) than firms with low ERCs (1.8%), perhaps because high accounting quality firms have more sources of capital and thus their lenders have smaller bargaining power (Mansi et al. (2004); Lambert et al. (2007); Bharath et al. (2008); Li et al. (2021)). Likely for similar reasons, firms with investment grade credit ratings face lower amendment costs (0.1%) than firms with speculative grade ratings (1.6%) and firms with missing ratings (3.6%). In relationship loans, amendment costs (3.2%) are higher than in non-relationship loans (2.2%), consistent with lenders having greater negotiating power in relationship loans (Sharpe (1990); Petersen and Rajan (1994); Dass and Massa (2011)).

The final parameter is the waiver cost or waiver friction, η_w. The waiver cost captures the total surplus loss from waiving a covenant violation, or also the magnitude of frictions preventing waivers. Like the amendment cost, the waiver cost is estimated relative to the cost of granting control rights to the wrong party. For the full sample, $\eta_w = 0.7$, which implies the total surplus loss from waiving a violation is as large as 140% of the average cost of granting control rights to the wrong party. This large estimate suggests that renegotiations to avoid the lender enforcing a violation cause a substantial reduction in the total surplus, perhaps manifesting in reduced firm value. As another interpretation, lenders obtain substantial bargaining power if a borrower violates a covenant, making it very hard to persuade the lender not to use any of their rights. Only if the lender exercising their rights will substantially reduce the total surplus (i.e., c above $\eta_w = 0.7$, or c in the 70$^{\text{th}}$ quantile of its distribution) will the lender agree not to exercise them.[14]

The magnitude of the waiver cost is much larger than the magnitude of the amendment cost, highlighting how economically different the two types of renegotiations are. Amendments are renegotia-

[13] The number 1.8% is calculated by multiplying the respective estimate by 2 such that the percentage is not of the upper bar of the cost, $\bar{c} = 1$, but of the mean of the cost, $\frac{\bar{c}}{2} = 0.5$.

[14] The large estimate of the waiver cost parameter is not an outcome of one particular approach but is robust to alternative data construction processes, sets of moment conditions, and estimation algorithms.

tions about the contract, and many of them occur not when a borrower violates a covenant. Waivers are renegotiations to prevent the lender from enforcing their rights after a violation. The literature has highlighted that, if a borrower violates a covenant, lenders gain substantial power and extract substantial concessions from the borrower. For example, lenders force reductions in investment (Chava and Roberts (2008)), management turnover (Nini et al. (2012)), and different disclosure policies (Vashishtha (2014)), all of which can reduce firm value and thus the total surplus. Griffin et al. (2026) also acknowledge that lenders obtain large bargaining power if a borrower violates a covenant, which can make negotiations about waivers difficult. Institutionally, the direct cost of a waiver can be considerably higher than that of an amendment. Amendments are crafted in advance under little pressure, while waiver documents typically have to be compiled quickly. These differences imply, first, larger legal fees for waivers and, second, the possibility of drafting a less efficient contract upon a waiver. Finally, a technical reason for the large waiver cost is that, in the model, an amendment can only prevent one instance of a waiver, while in the data, an amendment likely prevents multiple waivers. The large waiver cost that I obtain may be a discounted sum of multiple waiver costs in reality. In Section 5.2, I examine whether the cost of an amendment is different depending on whether the borrower violated a covenant or not and find that amendments are substantially costlier upon a covenant violation.

The estimated cost of waivers varies with firm and loan characteristics in an intuitive way. Similar to amendment costs, waiver costs are higher for firms with poor accounting quality (i.e., low ERC) and worse or missing credit ratings. Waiver costs are also higher in relationship loans, highlighting lenders' greater bargaining power. In addition, waivers are substantially costlier for syndicated loans with a large number of lender participants: $\eta_w = 0.743$ for loans with many lenders and $\eta_w = 0.522$ for loans with few lenders. This variation is consistent with lender coordination costs (Becker and Ivashina (2016); Chu (2021); Amiraslani et al. (2023); Dyreng et al. (2025)) being larger in larger syndicates.

In the main estimation, I can only obtain the costs of waivers and amendments relative to the average costs of having the wrong party in control. Later in Section 5.1, I use market reactions to announcements of contract amendments to estimate the dollar value of these costs and the costs of having the wrong party in control.

3.2 Key drivers of amendments

The model in Section 1 highlights three economic forces necessary for long-term contract amendments: contractual incompleteness, the persistence or informativeness of non-contractible information, and a low cost of amendments relative to the costs of waivers and giving control to the wrong party. The structural estimation of the model provides magnitudes of parameters describing these forces. In this section, I use these estimates to infer which of these forces are most important in explaining why contracts get amended.

For the full sample, the estimates suggest that contracts are highly incomplete (the degree of contractual completeness is $q = 0.687$), the non-contractible information is not very persistent ($\mu = 0.654$ and $\beta = 0.501$), and the cost of amendments is very low ($\eta_a = 0.009$, or only 1.8% of the average cost of having the wrong party in control). To sum up, the primary reasons for amendments are highly incomplete contracts and very low costs of amendments. The informativeness of non-contractible information does not play a big role.

The forces that are most important in explaining amendments vary substantially with firms' and loans' characteristics. High contractual incompleteness seems to be the most important for firms with low financial reporting quality and firms with a missing credit rating; these firms amend covenants because the financial statistics used in these covenants poorly capture the actual state of the firms.

In contrast, for firms with high financial reporting quality and investment grade credit ratings, the primary reason behind amendments is the informativeness of non-contractible information – the persistence of the non-contractible state of the world. For these healthy firms, once the lender observes that the borrower is doing well in the first year, it is highly likely that the borrower will remain well. This precise knowledge about the future allows the parties to draft a contract that will be highly efficient in future periods.

The low relative cost of amendments is an important driver for all firms, but to a different extent. It is more important for firms with high financial reporting quality and high credit ratings because, while these firms have lower costs of both amendments and waivers, amendments are meaningfully cheaper for them. Interestingly, low costs of amendments appear an important reason for amendments for loans where lenders have large bargaining power or high coordination costs: relationship loans and loans with many syndicate members. The estimates suggest that amendments are not significantly costlier for these loans, but waivers are a lot costlier, making amendments a very valuable tool.

3.3 Implications for contracting efficiency

Next, I use my estimates to evaluate the contracting efficiency gains from having the option to amend a contract. The efficiency gain from amendments represents the amount of control misallocation and waiver costs saved because the parties can amend the contract. For the full sample, ex ante, the contracting efficiency gain ranges between 2.29% and 2.36%, or the option to amend ex ante saves about 232.3 basis points of the total surplus. Ex post, conditional on an amendment, the parties save 15.32% of the total surplus in the state when the manager is the best party-in-control and 12.62% in the state when the bank is the best party-in-control. These amounts represent economically significant savings. Recall that the cost of having the wrong party in control, parameter c, captures the loss in firm value when the wrong party obtains control rights. For example, when the manager gets control during unstable financial conditions, she may take excessive risks and destroy company value. When the bank is given the right to intervene in the firm's operations without the need, it may inefficiently reduce investment and R&D (Chava and Roberts (2008), Nini et al. (2009)), fire executives (Nini et al. (2012)), or shape the company's disclosure policy (Vashishtha (2014)). These actions, if taken when they should not be, may substantially hurt the firm's long-term performance. Although the parties may waive these inefficiencies ex post when the bank gets control, the cost of doing that is also high, as the estimate of η_w suggests. To sum up, saving even about two percent of the loss in firm value may be a lot in monetary terms. The fact that we observe contracting parties amending their contracts implies the parties' revealed preference to save these control misallocation costs.

The contracting efficiency gains from amendments are highest for firms with missing credit ratings and low financial reporting quality.

4 Model fit and analysis of identification

To evaluate how well the theoretical model fits the data, I examine the distance between estimated theoretical and data moments. The values of moments and t-statistics of differences are shown in Table 5. All of the model-generated moments are statistically indistinguishable from their counterparts at the 1% significance level.

I provide more intuition of how each parameter is identified from data moments by plotting the moments' sensitivities to parameters in Figure 5. The plots show that different moments are important to identify different parameters. Generally, the moments move with parameters in the directions consistent with the theoretical discussion in Section 1.5.

The degree of contractual completeness, q, is primarily identified from the parties' decisions to amend the contract. When the contractible information better captures the non-contractible state – q is higher – the probability of an amendment unconditionally and conditional on no violation decreases (Figure 5a).

Companies' financial health at the time of loan origination, μ_0, mostly comes from the strictness of financial covenants in the original loan contract. Healthier firms, i.e., firms that are less likely to need the lender's intervention within the first year after loan origination, obtain less strict financial covenants (Figure 5b).

Persistences of the non-contractible states, μ and β, are identified, first, from initial and amended covenants for the period more than one year after loan origination, and, second, from the probabilities of amendment. As can be seen in Figure 5c (Figure 5d), if the good (bad) state is more likely to persist after it occurs in the first period, the covenants for the second period are less (more) strict. The probability of an amendment increases with the persistences of both states.

The cost of amendments is primarily obtained from the probabilities of amendments (Figure 5e). As the cost of amending a contract goes up, the parties are less likely to amend. The cost of an amendment can be disentangled from the contractual completeness, q, by using the strictness of amended financial covenants. Amended financial covenants are less strict when the contractual completeness is higher but are stricter when the costs of amendments are higher.

Finally, the cost of waiving a covenant violation, η_w, is identified from the strictness of financial covenants, mostly from amended covenants. If waivers are more expensive, the parties try to avoid them more and set looser covenants (Figure 5f).

5 Extensions of the main estimation

In this section, I present estimates of two extensions of my model. The first extension uses market reactions to announcements of loan amendments to estimate dollar values of control misallocation and renegotiation costs. The second extension allows the cost of amendments do be different depending on whether the borrower violated a covenant or not.

5.1 Monetary estimates of costs

In the main specification of the model, I cannot separately estimate the cost of giving control rights to the wrong party, c, and the costs of waivers and amendments, η_w and η_a, and only estimate them relative to each other. These costs likely have an economically meaningful effect on firm value, and it is important to know the absolute magnitudes of these costs. In this section, to estimate these magnitudes, I incorporate an event-study approach into my structural estimation.

The model is written in terms of losses to firm value whenever a control misallocation, a waiver, or an amendment occurs, and thus can speak to how the firm's valuation changes after one of these events. If in the data I can measure a change in the firm's value in a relatively short window around an event, I can argue that the change is due to the change in investors' expectations about the loss to the firm value as a result of the event. Using a short window change in the firm's value around an event as an additional moment in the estimation, I can evaluate the monetary values of control misallocation and renegotiation costs.

It is hard to capture empirically when a control misallocation or a waiver occurs. However, for some amendments, I have the exact dates when the amendments become available to the public. The majority of amendments in my sample are filed in an 8-K form. To select amendments for which an amendment was a major event for the firm in a given time window, I identify amendments that were filed in an 8-K form that did not contain any other major announcements.[15] For each 8-K filing selected, I collect the date of the filing and the market value of the firm three days before and three days after the filing.

Table 6 shows summary statistics. Changes in firm market values around announcements of contract amendments are, on average, negative, about -1.2%. When split by whether an amendment was preceded by a covenant violation, the market value drops by about 2.9% on average for amendments preceded by a violation and increases by about 0.9% on average for amendments not preceded by a vio-

[15] I keep 8-K forms that contain only items related to changes in or new material contracts or financial agreements. I exclude 8-K forms that additionally include items 2.01 (Completion of Acquisition or Disposition of Assets), 2.02 (Results of Operations and Financial Condition), 8.01 (Other Events), 5.02 (Departure of Directors or Certain Officers; Election of Directors; Appointment of Certain Officers; Compensatory Arrangements of Certain Officers), 7.01 (Regulation FD Disclosure), 5.01 (Changes in Control of Registrant), 3.03 (Material Modification to Rights of Security Holders), or 5.07 (Submission of Matters to a Vote of Security Holders).

lation. An increase rather than a decrease in firm value upon an amendment not preceded by a violation highlights that amendments are viewed by investors as value-improving. While it is hard to obtain data on covenant violations (Stice (2018)), my estimated market reaction to amendments coupled with implied covenant violations (-2.9%) is similar to market reactions to violations documented in prior work. For instance, Beneish and Press (1995) report that the average abnormal return in their sample of 87 first-time disclosures of technical default is -3.52%.

To map changes in firms' market values into the model, I derive an additional theoretical moment – a change in the firm value as a result of an amendment. I assume that investors price the firm at its expected value. When investors observe that a firm has amended its loan contract, two things change. First, investors update their beliefs about the state of the world the firm is in. For example, if the firm is substantially more likely to amend its contract when the state is m (i.e., a firm is healthy), investors assign a higher probability to the state being m. Second, given their updated beliefs about the state and a new contract, investors price in a new expected loss to the firm value. I add two new moments to the estimation: a change in the market value around an announcement of an amendment for firms that did and did not violate covenants. The new moments are derived in Internet Appendix 9.9.

Table 7 shows the estimation results of the model with additional moments. First, the estimated average costs of giving control to the wrong party (i.e., allowing the manager to run the firm when the bank should intervene or allowing the bank to intervene when the company does not need a lender's intervention) are about \$136.4 million, or about 7.44% of the market value for the average firm. Because I assume that these costs are uniformly distributed, for a given firm, the cost can vary between \$0 and \$272.7 million. These values reflect the potential total surplus losses caused by misaligned incentives between lenders and borrowers when control is assigned to the wrong party. My estimated values are comparable to the estimated losses from prior studies that focus on specific agency conflicts. For example, Chava and Roberts (2008) find that, upon a covenant violation, lenders force borrowers to reduce investment (CapEx) by about 13%. For an average firm in my sample, the 13% CapEx reduction is about a \$21.43 million reduction in CapEx. If some of the covenant violations are type-1 errors, this reduction in investment can result in a meaningfully lower firm value in the long run. For the case where firm managers gain control suboptimally, Malmendier and Tate (2005) find that overconfident CEOs choose investment volumes that are about 23.4% of assets greater than the optimal volumes. For an average firm in my sample, the 23.4% of assets implies roughly an \$818.24 million greater investment level.

If managers are sometimes given rights when they should not be (type-2 errors), this large investment implies a sizable loss in firm value. Griffin et al. (2026) estimate lender losses due to type-2 errors. They find that, if there is no covenant violation prior to a default, lender recovery rates are about 10 percentage points lower. For an average loan in my sample, this statistic implies lenders lose about $36.57 million in recovery amounts. My estimate is larger than most of those in prior studies. While all these studies estimate a specific monetary loss by one party, my approach aims to capture the total surplus loss, which may or may not be measurable in monetary terms. In addition to suboptimal investment or lower recovery rates, my estimate may include firm value losses due to inappropriate management, reputation costs, or costs of losing customers and suppliers.

Second, the costs of amendments are relatively small, about $4.0 million or only 0.22% of the average firm market value, while the costs of waivers are very large, about $273.2 million or 14.90% of the average firm market value. As with control misallocation costs, my waiver costs cannot be directly found in prior studies because these studies may estimate firm value losses upon a covenant violation, which commingle efficient violations with type-1 errors. The waiver cost is not incurred in efficient violations and is sometimes incurred in the case of type-1 errors. Nevertheless, to provide a reference, recall that Beneish and Press (1995) find about -3.52% abnormal market reaction to a disclosed covenant violation. This number translates into about $169.2 million loss in firm market value for an average firm in my sample. Some of these losses may capture investors' perceived firm value loss due to waiving the violation. Again, my estimates of renegotiation costs measure the total surplus loss, monetary or non-monetary, and may include, in addition to firm value reductions, lender and borrower reputation costs or legal fees. More importantly, as discussed in Sections 1 and 3, these costs may capture the difficulty that parties have in negotiating. Large waiver costs may arise due to lenders obtaining large bargaining power when a borrower violates a financial covenant.

Finally, I estimate the dollar value of total efficiency losses. The estimated total surplus losses from giving control to the wrong party or from renegotiations reported above are estimates of **potential** losses the parties trade off. Because the contract allocates control rights correctly most of the time and because the parties amend their contracts often, the **actual** costs the parties incur are much lower. I estimate that the expected total surplus loss due to contractual incompleteness and negotiation frictions is, on average, $19.09 million. In other words, positive-NPV investments generating a value of at least $19.09 million are underfunded because of contractual incompleteness and costly renegotiations. The actual surplus loss is meaningfully smaller than potential costs and is closer to losses estimated in studies presented above.

5.2 Amendment costs with and without covenant violation

The main specification of the model assumes that the cost of amending a contract, which represents direct costs as well as frictions potentially preventing amendments, is the same in all scenarios. While this specification significantly simplifies the analysis, it may not be representative of reality. Extensive work (e.g., Chava and Roberts (2008); Roberts and Sufi (2009a); Nini et al. (2012); Chodorow-Reich and Falato (2022)) suggests that lenders gain substantial bargaining power when a borrower violates a financial covenant. This bargaining power can, among other things, affect how hard it is for the bank and the borrower to amend a contract. In this section, I extend my model to allow for different amendment costs in cases when the borrower did and did not violate a covenant.

Internet Appendix 9.10 describes in detail how the model's solution and theoretical moments change, and also which moments I add for the extended version of the model. The model extension with different amendment costs is significantly more challenging analytically, and I solve it numerically.

Table 8 presents estimates of the extended model. The results reveal that amendment negotiation frictions indeed increase substantially if a borrower violates a financial covenant: the estimated cost of amendments is about 80% higher for amendments upon a violation than for amendments without any covenant violation. The cost of an amendment is about 2.0% of the average cost of misallocating control rights if a borrower did not violate any covenants and increases to about 3.6% if a borrower violated a covenant. These estimates highlight that lender bargaining power is an important determinant of amendment costs and can change drastically depending on the borrower's compliance with contractual restrictions.

6 Counterfactual analyses

My estimates allow me to conduct counterfactual analyses to study how firms' and lenders' behavior changes in hypothetical scenarios. In the first counterfactual, I study two firms: a very financially healthy firm and a very financially unhealthy firm. I compare the two firms' amendment behavior to understand to what extent frequent debt contract changes can be attributed to firms' financial health. In the second counterfactual, I introduce an option to waive in the state when the manager gets control suboptimally. This analysis shows how amendment behavior changes if firms can grant control to the bank even if no financial covenants are violated.

6.1 Completely financially healthy and completely financially unhealthy firms

First, I analyze a very financially healthy firm and demonstrate that this firm still amends financial covenants in the contract with its lender(s). To make a firm in the model completely healthy, I set the initial probability that the firm manager will be the optimal party-in-control close to one: $\mu_0 = 0.999$. The first scenario in Table 9 shows that this firm will still have its financial covenants amended about 12.53% of the time.

Second, I look at the opposite case – a very financially unhealthy firm – and show that this firm is not substantially more likely to amend its covenants than a healthy firm. In the second scenario in Table 9, I show the likelihood of an amendment for a firm that very likely will need the lender's intervention in the near future, $\mu_0 = 0.001$. In expectation, the financially unhealthy firm amends financial covenants about 15.86% of the time, similar to the healthy firm.

The two counterfactual scenarios suggest that firms' general financial health does not have to be correlated with how often firms amend their debt contracts. However, this fact may be non-trivial to confirm empirically. Financial covenant amendments are determined by the amount of information learned over time about the state of the world, and this amount does not change with firms' financial health per se. In reality, firms' financial stability may be correlated with various characteristics of information systems. For example, firms that do not need lenders' intervention may also have higher accounting quality, which would translate into fewer amendments compared to less healthy firms. A researcher who properly controls for characteristics of contractible and non-contractible information should expect to find no association between financial health and the likelihood of debt contract amendments. Roberts (2015) indeed finds that the probability of amending a covenant is largely unrelated to borrowers' financial performance at the start of the contract: neither the EBITDA-to-assets ratio nor stock performance predict the likelihood of covenant amendments.

6.2 A possibility of a waiving a type-2 error

The next counterfactual scenario introduces an additional waiver possibility. In reality, waivers most plausibly happen (and can be observed by a researcher) only when a covenant is violated but the firm management stays in control. In the model, this situation means control rights are misallocated so that the bank gets control by mistake, and the parties waive the misallocation and return control to the manager. Theoretically, the opposite case is also possible: when the control rights are allocated to the manager by mistake but should be given to the bank. This case is hard to observe in the data because it would imply the lender intervenes in a firm without a formal technical default. In the main estimation, I disallow waivers of this type. To understand how amendment behavior would change if the parties could waive control rights misallocation in the case when the bank is the best party-in-control, I consider a counterfactual scenario where waivers are allowed in both control misallocation cases. I set the cost of giving control to the bank when no covenant was violated equal to the estimated cost of waivers of violations, $\eta_w^b = \eta_w^m = 0.700$.

When the parties can waive a misallocation of control rights in another state, they find it cheaper to misallocate control rights overall. As scenario 3 in Table 9 suggests, as a result, the amendment option becomes less valuable: the ex ante contracting efficiency gain drops by 1 percentage point, and conditional on an amendment in the state b – when the state suggests the bank might need to obtain control in the future – by about 4 percentage points. The frequency of amendments does not change.

7 Conclusion

This study examines why financial covenants in corporate loan contracts are amended. The option to adjust a loan agreement as contracting parties learn new information allows lenders and borrowers to (partially) dynamically complete the inherently incomplete debt contract. I show that contracting efficiency, and thus incentives to amend, are more significant for companies with less precise contractible information, more important non-contractible information, and when amending the contract in advance is considerably cheaper than resolving control rights misallocation ex post. In the U.S., the first and the last explanations appear to be most supported by the data.

My paper advances the literature's understanding of why and how corporations and their investors change the contracts regulating their relationship. Many dimensions of contract renegotiations remain understudied. For example, in this paper, I treat companies' information systems as fixed, while in reality, they are not static. Properties of firms' hard and soft information and contracting parties' learning what this information represents evolve over the life of the lender-borrower relationship, which can potentially affect contract structure.

Another limitation of my framework is its sole focus on financial covenants. Even though financial covenants are the most common feature that gets amended, this is not the only term determining control rights,[16] nor are control rights the sole dimension that loan contracts regulate. Other features are also not static and get amended before contracts mature (Roberts (2015)). Studying why these are changed and why sometimes the parties amend multiple features at a time can uncover new trade-offs that lenders and borrowers face.

Overall, corporate loan contract amendments are an intriguing phenomenon that invites future research.

[16] Control rights in loan agreements can be regulated by other "action" covenants, such as negative pledge clauses or payout covenants, cov-lite financial covenants (Bräuning et al. (2022)), sweep provisions, collateral, and other terms.

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Figures and tables

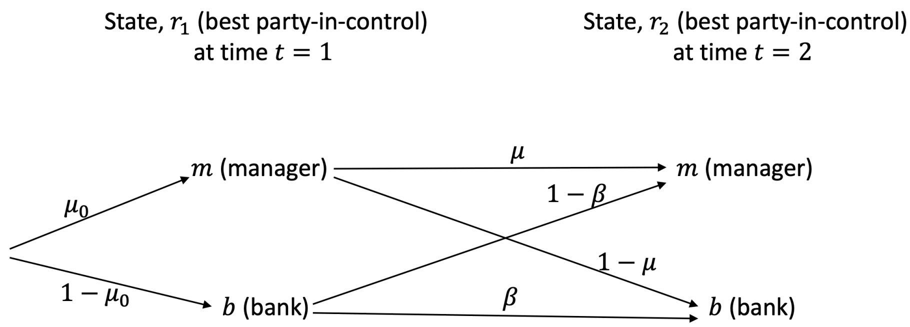


Figure 1: Tree chart of the evolution of states.

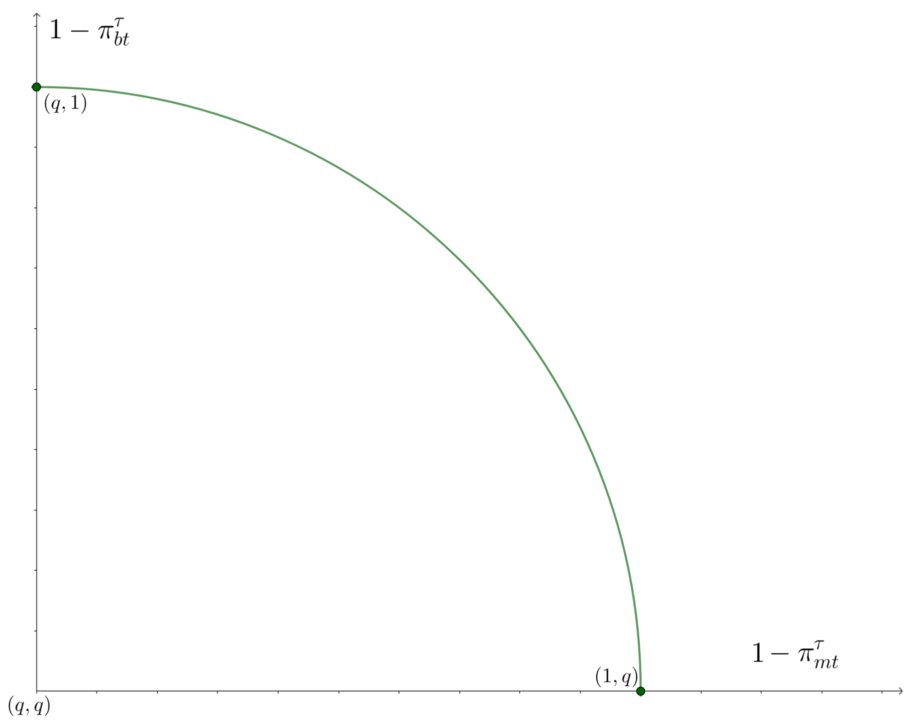


Figure 2: Possibility frontier for the precision of the covenants in states m and b. The precision in the state m is on the horizontal axis, the precision in the state b is on the vertical axis. $q = 0.8$.

Figure 3: Timeline of events in the model.

$t = 0$	$t = 1$	$t = 2$	$t = 3$
• The initial contract, $\{\{\pi^0_{m1}, \pi^0_{b1}\}, \{\pi^0_{m2}, \pi^0_{b2}\}\}$, is chosen.	• The manager and the bank observe $\{r_1, s_1, c_1\}$. • Waiver, if any, takes place. • Amendment of the contract to a new one, $\{\pi^1_{m2}, \pi^1_{b2}\}$, if any, takes place.	• The manager and the bank observe $\{r_2, s_2, c_2\}$. • Waiver, if any, takes place.	• The contract matures.

Figure 4: Ex ante contracting efficiency gain from amendments as a function of model parameters. $q = 0.6, \eta_a = 0.02, \eta_w^m = 0.95, \eta_w^b = 1000, \bar{c} = 1, \mu = 0.65, \beta = 0.6, \rho = 0.9, \mu_0 = 0.6$.

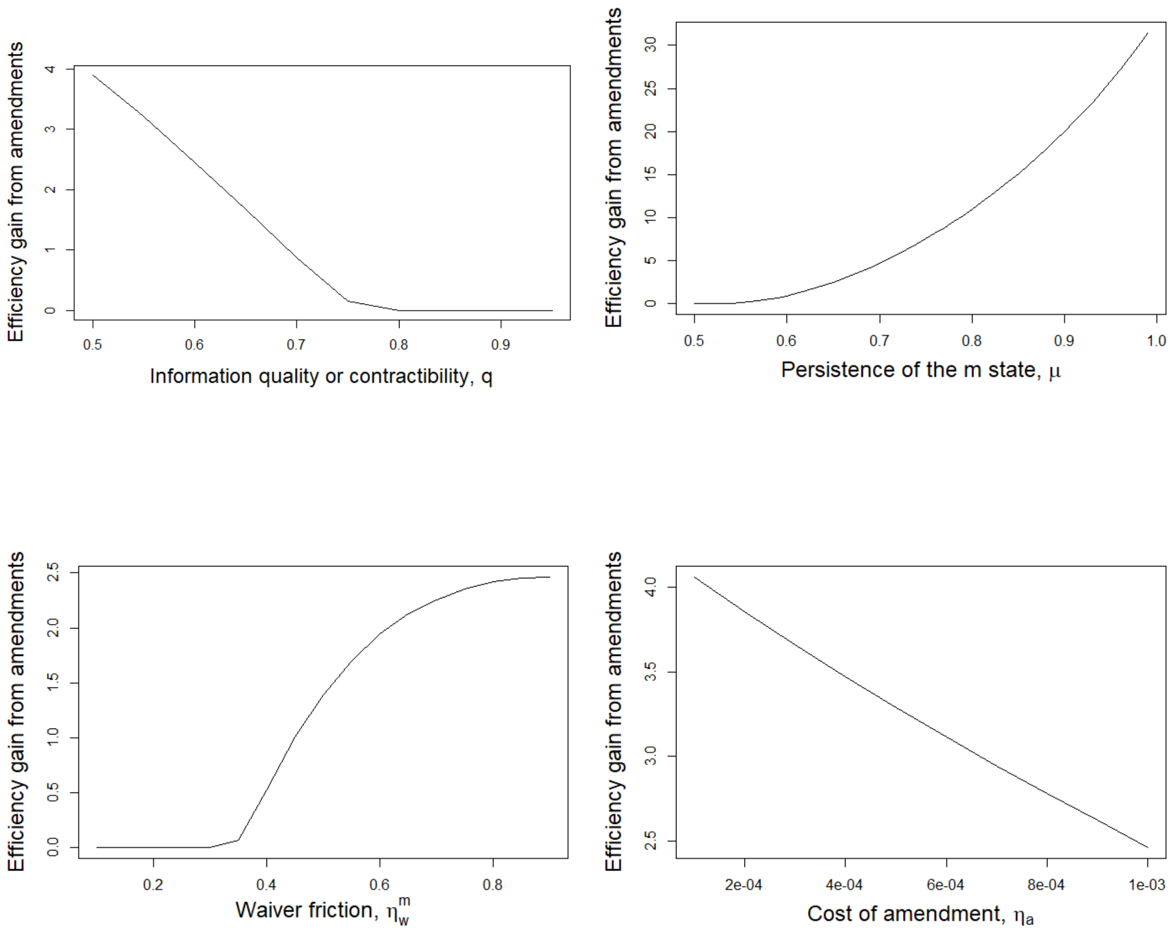
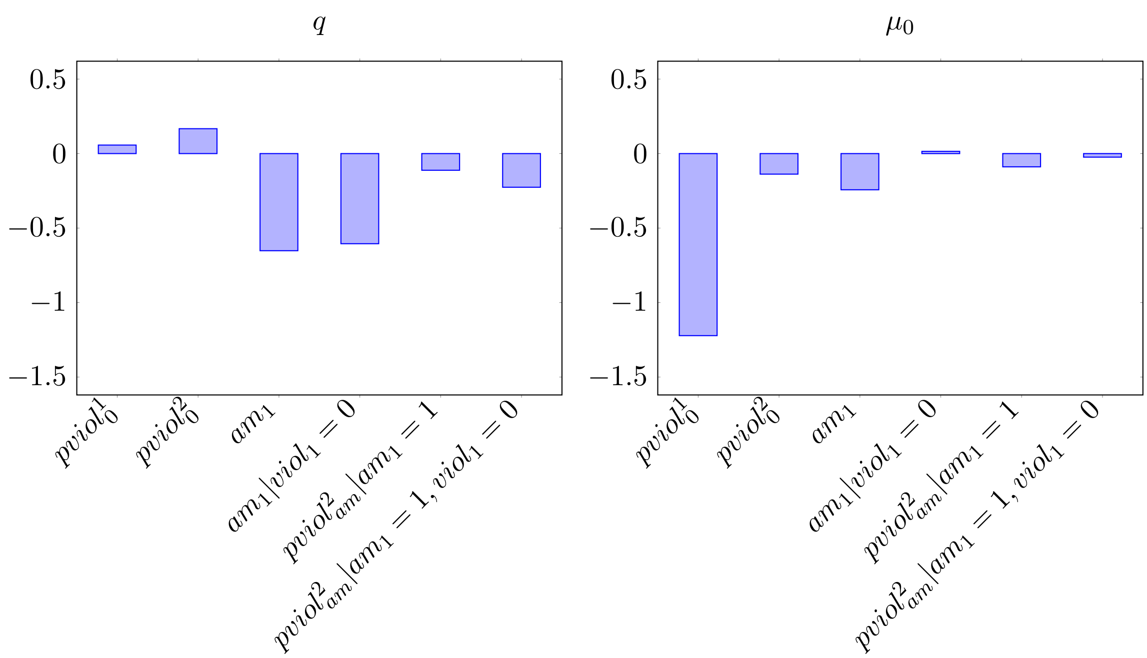
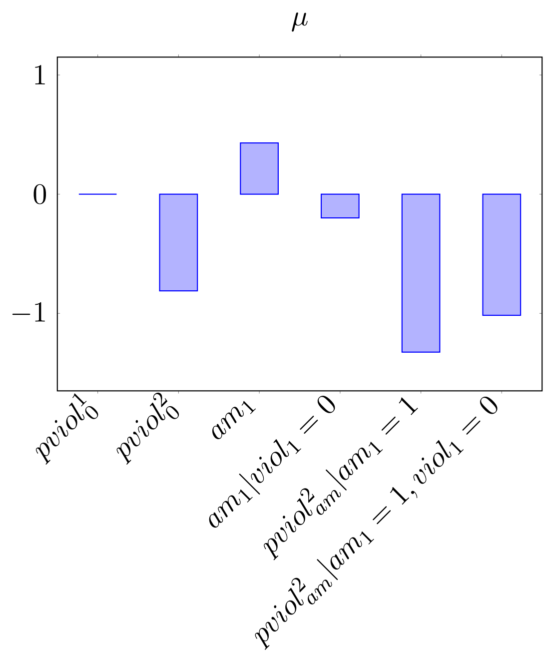


Figure 5: Sensitivities of moments to the model's parameters, at the estimated parameters. $pviol_0^1$ is the strictness of initial financial covenants for the first year after loan origination. $pviol_0^2$ is the strictness of initial financial covenants for the second year after loan origination. am_1 is the probability of amending a covenant within the first year after loan origination. $am_1|viol_1 = 0$ is the probability of amending a covenant within the first year after loan origination, conditional on no violation within the first year after origination. $pviol_{am}^2|am_1 = 1$ is the strictness of amended financial covenants for the first year after an amendment. $pviol_{am}^2|am_1 = 1, viol_1 = 0$ is the strictness of amended financial covenants for the first year after an amendment, conditional on no violation within the first year after origination. The sensitivity of a moment with respect to a parameter is computed by varying the parameter 1% up and down (keeping other parameters constant) and dividing the difference between the new value of the moment at the 1% higher and 1% lower parameter by 2% of the parameter value.

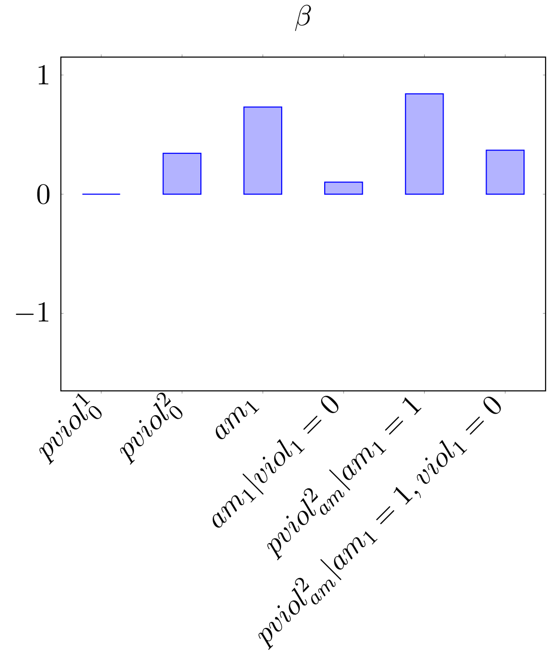


(a) Sensitivities of the model moments to the degree of contractual completeness, q.

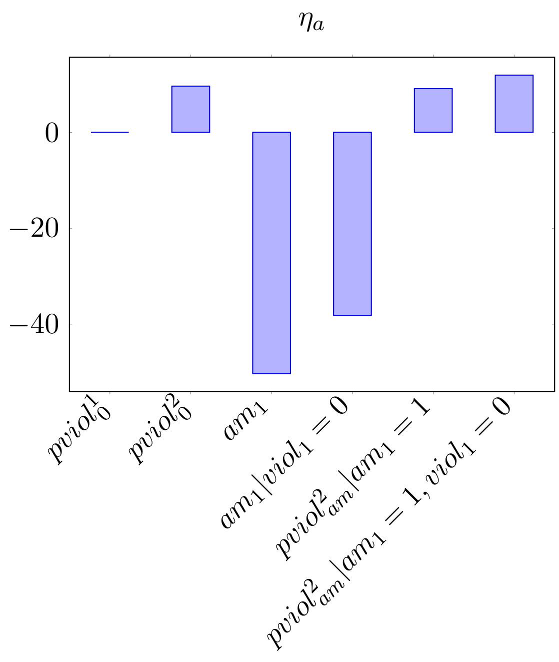
(b) Sensitivities of the model moments to the initial probability of the manager being the optimal party-in-control, μ_0.



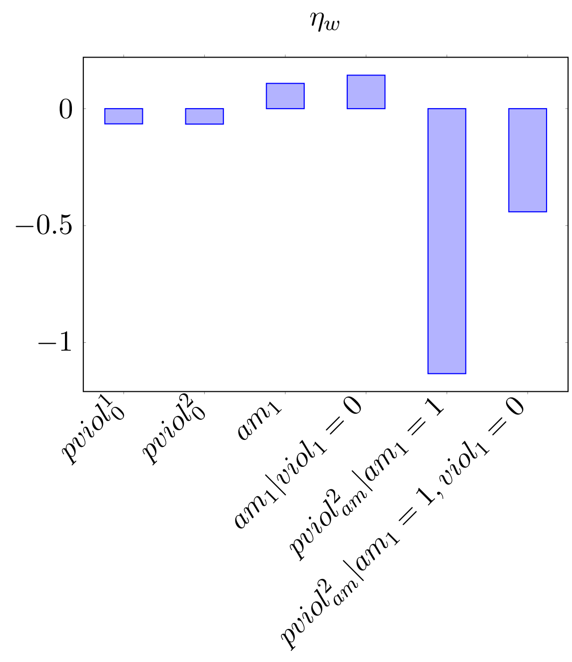
(c) Sensitivities of the model moments to the persistence of the m state, μ.



(d) Sensitivities of the model moments to the persistence of the b state, β.



(e) Sensitivities of the model moments to the amendment cost, η_a.



(f) Sensitivities of the model moments to the waiver cost, η_w.

Table 1: Summary statistics for firm characteristics. The market-to-book ratio is the close price multiplied by the number of common shares outstanding and divided by the total value of common/ordinary equity. The return on assets is income before extraordinary items divided by total assets. The leverage ratio is the sum of the total long-term debt and the total debt in current liabilities, divided by the sum of the total long-term debt, the total debt in current liabilities, and the total value of common/ordinary equity. The debt-to-EBITDA ratio is the sum of the total long-term debt and the total debt in current liabilities, divided by the operating income before depreciation. The earnings response coefficient is estimated from regressing abnormal returns on earnings surprises for eight quarterly earnings announcements preceding the loan initiation date. Abnormal returns are computed using the market model. An indicator for relationship loans equals one if at least one of the lead arrangers issued at least one loan to this borrower in the past. The number of lenders in a lending syndicate is directly taken from the Dealscan database. The earnings response coefficient is estimated from regressing abnormal returns on earnings surprises for eight quarterly earnings announcements preceding the loan initiation date. Abnormal returns are computed using the market model. The investment-grade rating is an indicator equal to one if the firm's S&P credit rating is BBB+, BBB-, BBB, A+, A-, A, AA, or AA-. The speculative-grade rating is an indicator equal to one if the firm's S&P credit rating is B+, B, BB, BB-, B-, BB+, CC, CCC+, or CCC. No rating data equals one if the firm does not have a rating in the S&P Credit Ratings database.

Statistic	N	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Total assets ($mil)	1,003	3,496.74	9,108.32	401.01	1,049.30	2,803.15
Market-to-book ratio	952	3.42	11.25	1.43	2.14	3.20
Return on assets	1,002	0.03	0.09	0.01	0.04	0.07
Leverage ratio	1,003	0.50	0.30	0.30	0.48	0.65
Debt-to-EBITDA ratio	998	3.34	3.82	1.28	2.66	4.34
Earnings response coefficient[17]	562	0.704	1.218	0.082	0.704	0.953
Investment-grade rating	1,067	0.167	0.373	0	0	0
Speculative-grade rating	1,067	0.395	0.489	0	0	1
No rating data	1,067	0.397	0.490	0	0	1

[17]For this subsample analysis, I remove observations with estimated earnings response coefficients below -0.01, and compute the median for the remaining sample.

Table 2: Summary statistics for loan characteristics. *Type: Revolver (maturity $\geq$ 1 year)* (*Type: Term loan*, *Type: Other*) is an indicator equal to one if the loan's type is described in Dealscan as Revolver (Maturity $\geq$ 1 year) (Term Loan, none of these). *Probability of covenant violation during the 1^{st} year after origination* (*Probability of covenant violation during the 2^{nd} year after origination*, *Probability of covenant violation during the 1^{st} year after an amendment*) is the probability of covenant violation within the first year after origination (the second year after origination, the first year after the amendment) computed as of the origination date as in Demerjian and Owens (2016) and described in Internet Appendix 9.5. *Covenant amended at least once* is an indicator equal to one if a financial covenant is amended at least once in the lifetime of the loan. *Covenant amended within the 1^{st} year* is an indicator equal to one if a financial covenant is amended within the first year after loan origination. *Change in covenant strictness after an amendment* is a percentage change in probability of violating a covenant computed as $100 \times \left(\frac{\textit{Probability of covenant violation during the } 1^{st} \textit{ year after an amendment}}{\textit{Probability of covenant violation during the } 2^{nd} \textit{ year after origination}} - 1\right)$.

(a) Full sample

Statistic	N	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Loan amount ($mil)	956	365.74	599.38	75.00	200.00	400.00
Maturity (months)	940	52.52	21.23	36.00	59.00	60.00
All-in-drawn spread (bps)	904	226.77	167.06	125.00	200.00	300.00
Secured	957	0.62	0.49	0	1	1
Number of lenders	957	9.09	7.87	4	7	12
Type: Revolver (maturity $\geq$ 1 year)	957	0.566	0.496	0	1	1
Type: Term loan	957	0.306	0.461	0	0	1
Type: Other	957	0.092	0.289	0	0	0
Probability of covenant violation during the 1^{st} year after origination	935	0.369	0.355	0.042	0.245	0.692
Probability of covenant violation during the 2^{nd} year after origination	861	0.366	0.356	0.035	0.237	0.682
Covenant amended at least once	1,068	0.289	0.454	0	0	1
Covenant amended within the 1^{st} year	1,068	0.203	0.403	0	0	0

(b) Loans that amended a covenant within the 1^{st} year after origination

Variable	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Probability of covenant violation during the 1^{st} year after an amendment	0.538	0.369	0.164	0.716	0.869
Change in covenant strictness after an amendment, %	NA	NA	-20.792	0	47.009

Table 3: Parameter estimates for the full sample. Bootstrapped standard errors are in parentheses. In estimation, the parameters are set $\bar{c} = 1$, $\rho = 0.9$. The estimation procedure and calculation of standard errors are described in Internet Appendix.

Parameter	Estimate
Degree of contractual completeness, q	0.687 (0.002)
Initial probability of the state m, μ_0	0.614 (0.000)
Persistence of the state m, μ	0.654 (0.001)
Persistence of the state b, β	0.501 (0.001)
Amendment cost, η_a	0.009 (0.000)
Waiver friction, η_w	0.700 (0.001)
Contracting efficiency gain, %	2.323 (0.033)

Table 4: Parameter estimates for different data subsamples. Bootstrapped standard errors are in parentheses. In estimation, the parameters are set $\bar{c} = 1$, $\rho = 0.9$. The estimation procedure and calculation of standard errors are described in Internet Appendix.

Parameter	Financial reporting quality		Relationship with the lender		Lender coordination cost		Credit rating		
	ERC above median	ERC below median	Relationship loan	Non-relationship loan	Number of lenders above the 3rd quartile	Number of lenders below the 1st quartile	Investment grade	Speculative grade	Missing rating
Degree of contractual completeness, q	0.861	0.548	0.507	0.666	0.623	0.536	0.842	0.773	0.503
	(0.000)	(0.009)	(0.067)	(0.001)	(0.003)	(0.003)	(0.009)	(0.003)	(0.001)
Initial probability of the state m, μ_0	0.722	0.594	0.607	0.603	0.619	0.610	0.752	0.586	0.570
	(0.001)	(0.001)	(0.011)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)
Persistence of the state m, μ	0.820	0.592	0.650	0.637	0.641	0.648	0.905	0.622	0.639
	(0.000)	(0.002)	(0.001)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.002)
Persistence of the state b, β	0.500	0.500	0.500	0.502	0.500	0.500	0.143	0.554	0.560
	(0.012)	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)
Amendment cost, η_a	0.0003	0.009	0.016	0.011	0.012	0.014	0.0005	0.008	0.018
	(0.000)	(0.000)	(0.005)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Waiver friction, η_w	0.024	0.590	0.991	0.774	0.743	0.522	0.801	0.836	0.935
	(0.005)	(0.007)	(3.115)	(0.003)	(0.001)	(0.009)	(0.002)	(0.002)	(0.001)
Contracting efficiency gain, %	0.063	2.188	1.812	1.590	1.858	1.725	0.873	1.669	2.551

Table 5: Model fit: theoretical and empirical moments. Calculations of the moments are presented in Internet Appendix 9.7.1.

Moment	Theoretical value	Empirical value	t-statistic [p-value]
Strictness of initial financial covenants for the first year after loan origination	0.368	0.370	−0.170 [0.865]
Strictness of initial financial covenants for the second year after loan origination	0.366	0.367	−0.088 [0.930]
Probability of amending a covenant within the first year after loan origination	0.212	0.194	1.475 [0.141]
Probability of amending a covenant within the first year after loan origination, conditional on no violation within the first year after origination	0.124	0.125	−0.015 [0.988]
Strictness of amended financial covenants for the first year after an amendment	0.547	0.558	−0.722 [0.471]
Strictness of amended financial covenants for the first year after an amendment, conditional on no violation within the first year after origination	0.501	0.487	0.611 [0.543]

Table 6: Summary statistics for firms' market values and changes in firms' market values within a $[-3, 3]$ day window around the release of the 8-K form which contains an amended contract.

(a) Full sample of amendments

Statistic	N	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Market value, $ million	24	1833.93	2525.34	271.61	1089.75	2290.34
Change in market value around the 8-K filing, $ million	24	-21.46	111.83	-56.93	-9.98	18.88

(b) Subsample where a firm has not violated any covenants in the period preceding an amendment

Statistic	N	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Market value, $ million	5	3014.0	1952.1	2999.8	2999.8	2999.8
Change in market value around the 8-K filing, $ million	5	28.26	196.74	-17.67	152.36	152.36

(c) Subsample where a firm has violated a covenant in the period preceding an amendment

Statistic	N	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Market value, $ million	19	1081.9	786.2	284.8	829.0	1446.4
Change in market value around the 8-K filing, $ million	19	-31.06	89.96	-63.59	-18.42	7.97

Table 7: Parameter estimates of the extension with monetary estimates of costs. Bootstrapped standard errors are in parentheses. In estimation, the control misallocation cost persistence parameter is set to $\rho = 0.9$. The estimation procedure and calculation of standard errors are described in Internet Appendix.

Parameter	Estimate
Degree of contractual completeness, q	0.508 (0.005)
Initial probability of the state m, μ_0	0.577 (0.001)
Persistence of the state m, μ	0.597 (0.000)
Persistence of the state b, β	0.500 (0.001)
Amendment cost, \$ million, η_a	3.982 (0.028)
Waiver friction, \$ million, η_w	273.174 (237.239)
Average cost of misallocating control rights, \$ million, $\frac{\bar{c}}{2}$	136.361 (0.610)

Table 8: Parameter estimates of the extension with different costs of amendment for violation and non-violation cases. Bootstrapped standard errors are in parentheses. In estimation, the control misallocation cost persistence parameter is set to $\rho = 0.9$. The estimation procedure and calculation of standard errors are described in Internet Appendix.

Parameter	Estimate
Degree of contractual completeness, q	0.672 (0.005)
Initial probability of the state m, μ_0	0.610 (0.006)
Persistence of the state m, μ	0.675 (0.002)
Persistence of the state b, β	0.544 (0.001)
Waiver friction, η_w	0.837 (0.008)
Amendment cost upon a covenant violation, η_a^v	0.018 (0.0003)
Amendment cost upon no covenant violation, η_a^{nv}	0.010 (0.0001)

Table 9: Likelihood of amendments and contracting efficiency gain from amendments in counterfactual scenarios.

Scenario	Likelihood of amendments, %	Contracting efficiency gain, %	Contracting efficiency gain in state m conditional on amendments, %	Contracting efficiency gain in state b conditional on amendments, %
Baseline estimates	21.303	2.323	15.383	12.702
1. A completely financially healthy company, $\mu_0 = 0.999$.	12.532	1.393	17.067	15.149
2. A completely financially unhealthy company, $\mu_0 = 0.001$.	15.856	1.434	14.581	17.843
3. An economy where waivers without violations are allowed, $\eta_w^b = \eta_w^m = 0.700$.	21.833	1.363	14.365	8.772

8 Appendix

8.1 Solution of the model and theoretical moments

8.1.1 Solution of the full model

At time 2, if $r_2 = m$ ($r_2 = b$) and $s_2 = b$ ($s_2 = m$), the parties waive and m (b) regains the $t = 2$ control rights if and only if $\eta_w^m \leq c_2$ $\left(\eta_w^b \leq c_2\right)$.

At time 1, if $r_1 = m$ ($r_1 = b$) and $s_1 = b$ ($s_1 = m$), the parties waive and m (b) regains the $t = 1$ control rights if and only if $\eta_w^b \leq c_1$ ($\eta_w^m \leq c_1$). To determine whether the covenant schedule is amended, first consider the optimal covenant schedule $\{\pi_{m2}^1, \pi_{b2}^1\}$ had the parties amend. When $r_1 = m$, the problem is

$$\begin{aligned}
&\min_{\{\pi_{m2}^1,\pi_{b2}^1\}} \quad \mu l_m \pi_{m2}^1 + (1-\mu) L_b \pi_{b2}^1 \\
\text{s.t.} \quad & (1-\pi_{m2}^1 - q)^2 + (1-\pi_{b2}^1 - q)^2 = (1-q)^2 \\
& \pi_{mt}^\tau \in [0, 1-q] \\
& \pi_{bt}^\tau \in [0, 1-q]
\end{aligned}$$

where

$$l_m = E[\min\{c_2, \eta_w^m\} | c_1, r_2 = r_1 = m]$$

$$L_b = E[\min\{c_2, \eta_w^b\} | c_1, r_2 = b \neq r_1] = E[\min\{c_2, \eta_w^b\}]$$

$l_m \equiv l_m(c_1) = \begin{cases} \eta_w^m, & \text{if } \eta_w^m \leq \rho c_1 \\ \frac{\eta_w^m(\rho c_1 + (1-\rho)\bar{c} - \eta_w^m)}{(1-\rho)\bar{c}} + \frac{{\eta_w^m}^2 - \rho^2 c_1^2}{2(1-\rho)\bar{c}}, & \text{if } \rho c_1 < \eta_w^m < \rho c_1 + (1-\rho)\bar{c} \\ \rho c_1 + \frac{(1-\rho)\bar{c}}{2}, & \text{if } \eta_w^m \geq \rho c_1 + (1-\rho)\bar{c} \end{cases}$ is a random variable with the support $[0, \eta_w^m]$, and L_b is a constant. Lagrangian for the problem with only the first constraint is

$$\mathcal{L} = \mu l_m \pi_{m2}^1 + (1-\mu) L_b \pi_{b2}^1 + \lambda\left((1-q)^2 - (1-\pi_{m2}^1 - q)^2 - (1-\pi_{b2}^1 - q)^2\right)$$

The derivatives with respect to π^1_{m2}, π^1_{b2}, and λ are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \pi^1_{m2}} &= \mu l_m + 2\lambda(1-\pi^1_{m2}-q) \\
\frac{\partial \mathcal{L}}{\partial \pi^1_{b2}} &= (1-\mu)L_b + 2\lambda(1-\pi^1_{b2}-q) \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= (1-q)^2 - (1-\pi^1_{m2}-q)^2 - (1-\pi^1_{b2}-q)^2
\end{aligned}$$

Equating the derivatives above to zero and solving the system yields one solution within the boundaries $[0, 1-q]$:

$$\begin{aligned}
\pi^{1m*}_{m2} &= 1-q-\frac{\mu l_m(1-q)}{\sqrt{(\mu l_m)^2+((1-\mu)L_b)^2}} \\
\pi^{1m*}_{b2} &= 1-q-\frac{(1-\mu)L_b(1-q)}{\sqrt{(\mu l_m)^2+((1-\mu)L_b)^2}}
\end{aligned}$$

The determinant of the matrix of second derivatives of the Lagrangian is

$$\begin{vmatrix} -2\lambda & 0 \\ 0 & -2\lambda \end{vmatrix} = 4\lambda^2 > 0,$$

suggesting that the solution found is a minimum.

The parties will amend the contract if and only if

$$\begin{aligned}
\eta_a + \mu l_m \pi^{1*}_{m2} + (1-\mu)L_b\pi^{1*}_{b2} &\le \mu l_m \pi^0_{m2} + (1-\mu)L_b\pi^0_{b2} \\
\eta_a + (1-q)(\mu l_m + (1-\mu)L_b) - (1-q)\sqrt{(\mu l_m)^2+((1-\mu)L_b)^2} &\le \mu l_m \pi^0_{m2} + (1-\mu)L_b\pi^0_{b2} \\
(l_m)^2\left[\mu^2((1-q)^2-(1-q-\pi^0_{m2})^2)\right] - l_m\left[2(1-q-\pi^0_{m2})\mu(\eta_a+(1-q-\pi^0_{b2})(1-\mu)L_b)\right] & \\
+((1-q)(1-\mu)L_b)^2 - (\eta_a+(1-q-\pi^0_{b2})(1-\mu)L_b)^2 &\ge 0 \quad (2)
\end{aligned}$$

The quadratic expression of l_m on the left-hand side has two roots. Therefore, the parties will not amend the contract if and only if $l_m \in (x_m^l, x_m^u)$, and amend otherwise, where

$$x_m^l = \frac{(1-q-\pi_{m2}^0)(\eta_a + (1-q-\pi_{b2}^0)(1-\mu)L_b) - (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)L_b}}{\mu((1-q)^2 - (1-q-\pi_{m2}^0)^2)}$$
$$x_m^u = \frac{(1-q-\pi_{m2}^0)(\eta_a + (1-q-\pi_{b2}^0)(1-\mu)L_b) + (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)L_b}}{\mu((1-q)^2 - (1-q-\pi_{m2}^0)^2)}$$

When $r_1 = b$,

$$\pi_{m2}^{1b*} = 1 - q - \frac{(1-\beta)L_m(1-q)}{\sqrt{(\beta l_b)^2 + ((1-\beta)L_m)^2}}$$
$$\pi_{b2}^{1b*} = 1 - q - \frac{\beta l_b(1-q)}{\sqrt{(\beta l_b)^2 + ((1-\beta)L_m)^2}}$$

and the parties will amend the covenant schedule if and only if $l_b \in (x_b^l, x_b^u)$, where

$$x_b^l = \frac{(1-q-\pi_{b2}^0)(\eta_a + (1-q-\pi_{m2}^0)(1-\beta)L_m) - (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{m2}^0)(1-\beta)L_m}}{\beta((1-q)^2 - (1-q-\pi_{b2}^0)^2)}$$
$$x_b^u = \frac{(1-q-\pi_{b2}^0)(\eta_a + (1-q-\pi_{m2}^0)(1-\beta)L_m) + (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{m2}^0)(1-\beta)L_m}}{\beta((1-q)^2 - (1-q-\pi_{b2}^0)^2)}$$

At time 0, the problem is

$$\begin{aligned}
\min_{\{\pi_{m1}^0,\pi_{b1}^0,\pi_{m2}^0,\pi_{b2}^0\}} \quad & \mu_0 \times \big(\pi_{m1}^0 L_m \\
&+E\Big[\min\{\mu l_m \pi_{m2}^0 + (1-\mu)L_b\pi_{b2}^0, \eta_a + (1-q)(\mu l_m + (1-\mu)L_b) - (1-q)\sqrt{(\mu l_m)^2 + ((1-\mu)L_b)^2}\}\Big]\big) \\
&+(1-\mu_0) \times \big(\pi_{b1}^0 L_b \\
&+E\Big[\min\{\beta l_b \pi_{b2}^0 + (1-\beta)e^m\pi_{m2}^0, \eta_a + (1-q)(\beta l_b + (1-\beta)L_m) - (1-q)\sqrt{(\beta l_b)^2 + ((1-\beta)L_m)^2}\}\Big]\big) \\
\text{s.t.} \quad & (1-\pi_{m1}^0-q)^2 + (1-\pi_{b1}^0-q)^2 = (1-q)^2 \\
& (1-\pi_{m2}^0-q)^2 + (1-\pi_{b2}^0-q)^2 = (1-q)^2
\end{aligned}$$

The problems of finding $\{\pi^0_{m1}, \pi^0_{b1}\}$ and $\{\pi^0_{m2}, \pi^0_{b2}\}$ are separable. First, we can find $\{\pi^0_{m1}, \pi^0_{b1}\}$:

$$\pi^{0*}_{m1} = 1 - q - \frac{(1-q)\mu_0 L_m}{\sqrt{(\mu_0 L_m)^2 + ((1-\mu_0)L_b)^2}}$$
$$\pi^{0*}_{b1} = 1 - q - \frac{(1-q)(1-\mu_0) L_b}{\sqrt{(\mu_0 L_m)^2 + ((1-\mu_0)L_b)^2}}$$

$\{\pi^0_{m2}, \pi^0_{b2}\}$ are computed numerically.

8.1.2 Proof of Proposition 1

In the model where amendments are prohibited, the problem at time 0 is

$$\begin{aligned} \min_{\{\pi^0_{m1},\pi^0_{b1},\pi^0_{m2},\pi^0_{b2}\}} \quad & \mu_0\left(\pi^0_{m1}L_m + \mu\pi^0_{m2}\bar{l}_m + (1-\mu)\pi^0_{b2}L_b\right) + (1-\mu_0)\left(\pi^0_{b1}L_b + \beta\pi^0_{b2}\bar{l}_b + (1-\beta)\pi^0_{m2}L_b\right) \\ \text{s.t.} \quad & (1-\pi^0_{m1}-q)^2 + (1-\pi^0_{b1}-q)^2 = (1-q)^2 \\ & (1-\pi^0_{m2}-q)^2 + (1-\pi^0_{b2}-q)^2 = (1-q)^2 \end{aligned}$$

π^{0*}_{m1} and π^{0*}_{b1} are the same as in the main model. The optimal π^0_{m2} and π^0_{b2} are

$$\pi^{0*}_{m2} = 1 - q - \frac{(1-q)(\mu_0\mu\bar{l}_m + (1-\mu_0)(1-\beta)L_m)}{\sqrt{(\mu_0\mu\bar{l}_m + (1-\mu_0)(1-\beta)L_m)^2 + (\mu_0(1-\mu)L_b + (1-\mu_0)\beta\bar{l}_b)^2}}$$
$$\pi^{0*}_{b2} = 1 - q - \frac{(1-q)(\mu_0(1-\mu)L_b + (1-\mu_0)\beta\bar{l}_b)}{\sqrt{(\mu_0\mu\bar{l}_m + (1-\mu_0)(1-\beta)L_m)^2 + (\mu_0(1-\mu)L_b + (1-\mu_0)\beta\bar{l}_b)^2}}$$

8.1.3 Proof of Corollary 1

In this section, I present proofs for the case $r_1 = m$. The case $r_1 = b$ can be proved analogously.

$$\frac{\partial x^l_m}{\partial \eta_a} \propto (1-q-\pi^0_{m2}) - (1-q)\frac{\eta_a + (1-q-\pi^0_{b2})(1-\mu)L_b}{\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi^0_{b2})(1-\mu)L_b}} \propto$$
$$(1-q-\pi^0_{m2})\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi^0_{b2})(1-\mu)L_b} - (1-q)(1-q-\pi^0_{b2})(1-\mu)L_b - (1-q)\eta_a <$$
$$-\pi^0_{m2}(1-q-\pi^0_{b2})(1-\mu)L_b - \pi^0_{m2}\eta_a < 0$$

$$\frac{\partial x_m^u}{\partial \eta_a} \propto (1-q-\pi_{m2}^0) + (1-q)\frac{\eta_a + (1-q-\pi_{b2}^0)(1-\mu)e^b}{\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b}} \propto$$

$$(1-q-\pi_{m2}^0)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)e^b} + (1-q)(1-q-\pi_{b2}^0)(1-\mu)e^b + (1-q)\eta_a > 0$$

Since x_m^l is decreasing in η_a and x_m^u is increasing in η_a, the probability of amendment ($l_m \leq x_m^l$ or $l_m \geq x_m^u$) is decreasing in η_a.

The no-amendment region is

$$x_m^u - x_m^l = \frac{2(1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)L_b}}{\mu(1-q-\pi_{b2}^0)^2}$$

$$\frac{\partial\left(x_m^u - x_m^l\right)}{\partial \mu} \propto -\frac{2(1-q)\eta_a(1-q-\pi_{b2}^0)L_b\mu}{\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)L_b}} - 2(1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)L_b} \propto$$

$$-(1-q)\eta_a(1-q-\pi_{b2}^0)L_b\mu - (1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)L_b} < 0$$

The no-amendment region is decreasing in μ, thus the probability of amendment is increasing in μ.

$$\frac{\partial\left(x_m^u - x_m^l\right)}{\partial q} \propto \left(-2\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)L_b} - \frac{(1-q)2\eta_a(1-\mu)L_b}{\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)L_b}}\right) \times$$

$$\mu(1-q-\pi_{b2}^0)^2 + 4\mu(1-q-\pi_{b2}^0)(1-q)\sqrt{\eta_a^2 + 2\eta_a(1-q-\pi_{b2}^0)(1-\mu)L_b} \propto$$

$$2\eta_a^2\mu(1-q-\pi_{b2}^0)\left[2(1-q)-(1-q-\pi_{b2}^0)\right] + 2\eta_a(1-\mu)\mu L_b(1-q-\pi_{b2}^0)^2\left[(1-q)-(1-q-\pi_{b2}^0)\right] > 0$$

The no-amendment region is increasing in q, thus the probability of amendment is decreasing in q.

8.1.4 Theoretical moments

Let $\mathbf{1}()$ denote the indicator function. Let H_i denote the probability that the random variable l_i falls into the non-amendment region: $H_i = \Pr\left[x_i^l < \tilde{l}_i < x_i^u\right]$. This probability is computed numerically given the fact that the variable $\tilde{l}_i(c_1)$ depends on the variable c_1, which is uniformly distributed between 0 and $\bar{c}$.

The first set of moments describes the initially set covenant schedule:

1. Ex ante probability of violating a covenant at time $t = 1$, as set at time $t = 0$:

$$\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=1})] =$$
$$\mu_0 \times \pi_{m1}^{0*} + (1 - \mu_0) \times (1 - \pi_{b1}^{0*}) \tag{3}$$

2. Ex ante probability of violating a covenant at time $t = 2$, as set at time $t = 0$:

$$\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=2})] =$$
$$\mu_0 \times \left(\mu \times \pi_{m2}^{0*} + (1 - \mu) \times (1 - \pi_{b2}^{0*})\right) + (1 - \mu_0) \times \left(\beta \times (1 - \pi_{b2}^{0*}) + (1 - \beta) \times \pi_{m2}^{0*}\right) \tag{4}$$

The second set of moments is about the frequency of amendments to the covenant schedule:

3. Ex ante unconditional probability of amending the covenant schedule:

$$\mathbf{E}_0[\mathbf{1}(\text{amendment}_{t=1})] =$$
$$\mu_0 \times (1 - H_m) + (1 - \mu_0) \times (1 - H_b) \tag{5}$$

4. Ex ante probability of amending the contract at time $t = 1$, conditional on no covenant violation:

$$\mathbf{E}_0[\mathbf{1}(\text{amendment}_{t=1})|\mathbf{1}(\text{violation}_{t=1}) = 0] =$$
$$\frac{\mu_0 \times (1 - \pi_{m1}^{0*}) \times (1 - H_m) + (1 - \mu_0) \times \pi_{b1}^{0*} \times (1 - H_b)}{\mu_0 \times (1 - \pi_{m1}^{0*}) + (1 - \mu_0) \times \pi_{b1}^{0*}} \tag{6}$$

The last set of moments captures the distribution of amended covenant schedules:

5. The amended probability of violating a covenant at time $t = 2$, conditional on amendment at time $t = 1$:

$$\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=2})|\mathbf{1}(\text{amendment}_{t=1})] =$$
$$\frac{1}{\mu_0 \times (1 - H_m) + (1 - \mu_0) \times (1 - H_b)}$$
$$\times \left(\mu_0 \times (1 - H_m) \times \left(\mu \times E\left[\pi_{m2}^{1m*}|l_m \notin \left(x_m^l, x_m^u\right)\right] + (1 - \mu) \times \left(1 - E\left[\pi_{b2}^{1m*}|l_m \notin \left(x_m^l, x_m^u\right)\right]\right)\right)\right.$$
$$\left. + (1 - \mu_0) \times (1 - H_b) \times \left(\beta \times \left(1 - E\left[\pi_{b2}^{1b*}|l_b \notin \left(x_b^l, x_b^u\right)\right]\right) + (1 - \beta) \times E\left[\pi_{m2}^{1b*}|l_b \notin \left(x_b^l, x_b^u\right)\right]\right)\right) \tag{7}$$

6. The amended probability of violating a covenant at time $t = 2$, conditional on no covenant violation and amendment at time $t = 1$:

$$\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=2})|\mathbf{1}(\text{amendment}_{t=1}) \times (1 - \mathbf{1}(\text{violation}_{t=1}))] = \frac{1}{\mu_0 \times (1-\pi_{m1}^{0*}) \times (1-H_m) + (1-\mu_0) \times \pi_{b1}^{0*} \times (1-H_b)}$$
$$\times \left(\mu_0 \times (1-H_m) \times (1-\pi_{m1}^{0*}) \times \left(\mu \times E\left[\pi_{m2}^{1m*}|l_m \notin \left(x_m^l, x_m^u\right)\right] + (1-\mu) \times \left(1 - E\left[\pi_{b2}^{1m*}|l_m \notin \left(x_m^l, x_m^u\right)\right]\right)\right)\right.$$
$$\left. + (1-\mu_0) \times (1-H_b) \times \pi_{b1}^{0*} \times \left(\beta \times \left(1 - E\left[\pi_{b2}^{1b*}|l_b \notin \left(x_b^l, x_b^u\right)\right]\right) + (1-\beta) \times E\left[\pi_{m2}^{1b*}|l_b \notin \left(x_b^l, x_b^u\right)\right]\right)\right) \quad (8)$$

8.2 Examples of amendments and waivers in the data

c. Leverage Ratio. Section 6.10 of the Credit Agreement is amended in its entirety to read as follows:

"Section 6.10 Leverage Ratio. The Company will not permit its Leverage Ratio at the end of any fiscal quarter to be greater than the levels indicated below for the corresponding periods:

Period	Ratio
January 1, 2002 through March 31, 2002	2.95 to 1.00
April 1, 2002 through June 30, 2002	2.25 to 1.00
July 1, 2002 through September 30, 2002	2.00 to 1.00
October 1, 2002 through March 31, 2003	1.75 to 1.00
April 1, 2003 through September 30, 2003	2.50 to 1.00
October 1, 2003 through December 31, 2003	4.50 to 1.00
January 1, 2004 through March 31, 2004	2.50 to 1.00
April 1, 2004 through June 30, 2004	2.25 to 1.00
July 1, 2004 and thereafter	2.00 to 1.00"

Figure 6: An example of a precautionary waiver. The level of leverage ratio is lifted up for three months (October-December 2003) and then goes back to its original level. This excerpt is taken from the Amendment No. 5, dated as of December 29, 2003, amending the Second Amended and Restated Credit Agreement, dated as of April 30, 2002, between Global Industries, Ltd., a Louisiana Corporation, and Bank One, NA, as administrative agent.

a. Leverage Ratio.

Period Ratio

January 1, 2002 through March 31, 2002 2.95 to 1.00

April 1, 2002 through June 30, 2002 2.25 to 1.00

July 1, 2002 through September 30, 2002 2.00 to 1.00

October 1, 2002 through March 31, 2003 1.75 to 1.00

April 1, 2003 through March 31, 2004 2.50 to 1.00

April 1, 2004 through June 30, 2004 2.25 to 1.00

July 1, 2004 and thereafter 2.00 to 1.00

Figure 7: The leverage covenant in the original Credit Agreement, dated as of April 30, 2002, between Global Industries, Ltd., a Louisiana Corporation, and Bank One, NA, as administrative agent.

2.1. WAIVER. As of the Effective Date and solely for the period through January 31, 2002 (the "Waiver Period"), the Majority Lenders signatory hereto waive any Default or Event of Default under Section 15(c) of the Credit Agreement resulting from the Borrower's failure to comply with any one or more of the covenants set forth in Section 14.1 of the Credit Agreement for the periods ending on December 31, 2001 and as of December 31, 2001, as the case may be.

Figure 8: An example of an ex post waiver. The violation of a financial covenant by the borrower is waived. This excerpt is taken from the Waiver, dated as of December 31, 2001, to the Second Amended and Restated Credit Agreement, dated as of September 15, 1998, between Hexcel Corporation and Citibank, N.A., as Documentation Agent, and Credit Suisse First Boston, as Administrative Agent.

8.09 Consolidated Leverage Ratio.

The Borrower will not permit the Consolidated Leverage Ratio as of the end of any fiscal quarter of the Borrower ending during a period set forth below to exceed the ratio set forth below opposite such period:

Period	Consolidated Leverage Ratio
Effective Date through December 31, 2011	5.00:1.00
January 1, 2012 through September 30, 2012	4.50:1.00
~~**October 1, 2012 through September 30, 2013**~~	~~4.00:1.00~~
October 1, ~~2013~~2012 and thereafter	~~3.75~~4.00:1.00

Figure 9: An example of an amendment. An amendment to the Leverage Ratio Covenant. The excerpt is taken from Amendment No. 1 dated as of March 11, 2011 amending the Credit Agreement between Hanger Orthopedic Group, Inc., a Delaware corporation and Bank of America, N.A. as Administrative Agent dated as of December 1, 2010.

8.3 List of variables used in estimation

Variable	Notation	Definition
Probability of violating a covenant within the 1st year after origination	$pviol_0^1$	Covenant strictness, adapted from Demerjian and Owens (2016), measuring the probability of violating a covenant within one year after the loan origination date.
Probability of violating a covenant within the 2nd year after origination	$pviol_0^2$	Covenant strictness, adapted from Demerjian and Owens (2016), measuring the probability of violating a covenant in the period between the end of the first and the end of the second years after the loan origination date.
Probability of violating a covenant within the 1st year after an amendment	$pviol_{am}^1$	Covenant strictness, adapted from Demerjian and Owens (2016), measuring the probability of violating a covenant within one year after the amendment effective date.
Covenant violation within the 1st year after origination	$viol_1$	An indicator variable equal to 1 if at least one of the company's financial ratios or characteristics for the 1st year violate a financial covenant effective for this year. Definitions of financial ratios and characteristics for financial covenants are from Demerjian and Owens (2016).
Covenant amendment within the 1st year after origination	am_1	An indicator variable equal to one if financial covenants are amended at least once within one year after the loan origination. Amendments are defined as described in section 2.1.

9 Internet Appendix

9.1 Analogy between classic debt contracting models and the model in this paper

This section extends the model in Guttman and Marinovic (2018).

For the classic debt contracting model, assume the manager needs to invest the amount I in the project that generates income $\tilde{G} = \tilde{g}_1 + \tilde{g}_2$ in three periods. The investment is made in period 0, $\tilde{g}_1$ is realized in period 1, $\tilde{g}_2$ is realized in period 2, and the payoff can be accessed in period 3. $\tilde{g}_1$ is distributed with the pdf $f(g)$ and the CDF $F(g)$, $\tilde{g}_2 = \rho\tilde{g}_1 + \tilde{\varepsilon}_g$, where $\tilde{\varepsilon}_g$ is a mean-zero error term, implying that g_2 can be predicted when g_1 is realized. The project is NPV-positive: $I < 2E[\tilde{g}]$. The parties have an option to liquidate the project early for a liquidation value of L. If a liquidation occurs, the bank gets L and the manager gets 0.

The parties cannot contract on the outcomes, g_1 and g_2, however, they can contract on the signals about these outcomes: $\tilde{h}_t = \tilde{g}_t + \tilde{\varepsilon}_h$, where $\tilde{\varepsilon}_h$ is a mean-zero noise term, capturing contractual incompleteness.

The contract chosen by the parties consists of (1) the interest payment K^τ - the amount the manager pays to the bank when the project cash flow materializes (τ denotes the time when the interest payment is chosen), and (2) the covenant threshold z_t^τ such that realization of h_t below it results in the transfer of control rights to the bank.

The cost of waiving a control rights misallocation is n_w, and the cost of amending the contract is n_a. In period t, all the waiver and amendment costs incurred before and not including that period are denoted as N^t.

The timeline is summarized below.

I start solving the model from time 2. Let us consider various possible cases.

- Case $h_2 \geq z_2$ and $g_1 + g_2 - N^1 \geq L - N^1 - n_w$, or covenant was not violated and the ultimate cash flows are above the value were the parties to liquidate. No waiver is needed in this case.

- Case $h_2 \geq z_2$ and $g_1 + g_2 - N^1 < L - N^1 - n_w$, or covenant was not violated and the ultimate cash flows are below the value were the parties to liquidate. A waiver is needed to maximize the total surplus.

  Since the manager has control rights by default and receives 0 in case of liquidation, in order to

Figure 10: Timeline of events in the classic model

$t = 0$

- The initial contract, $\{K^0, z_1^0, z_2^0\}$, is chosen.

$t = 1$

- The manager and the bank observe $\{g_1, h_1\}$.
- Waiver, if any, takes place for a cost towards the ultimate payoff G of n_w. If a waiver occurs, the interest rate may be renegotiated to K^1.
- Amendment of the contract to a new one, $\{K^1, z_2^1\}$, if any, takes place for a cost towards the ultimate payoff G of n_a.

$t = 2$

- The manager and the bank observe $\{g_2, h_2\}$.
- Waiver, if any, takes place for a cost towards the ultimate payoff of n_w. If a waiver occurs, the interest rate may be renegotiated to K^2.

$t = 3$

- The contract matures. The manager receives $\max\{G - N - K, 0\}$; the bank receives $\min\{K, G - N\}$.[a]

[a] K is either K^0, K^1, or K^2, depending on whether any renegotiations took place. N is the sum of all the waiver and amendment costs incurred. For example, if the parties amended the contract at time 1 and waived a control rights misallocation at time 2, $N = n_a + n_w$.

achieve the maximum total surplus, the bank offers a transfer T^2 to the manager. I assume the parties use Nash bargaining. The bank maximizes:

$$\max_{T^2} L - N^1 - n_w - T^2$$
$$\text{s.t. } T^2 \geq \max\{g_1 + g_2 - N^1 - K^1, 0\}$$

The optimal value of the transfer is $T^2 = \max\{g_1 + g_2 - N^1 - K^1, 0\}$. Because a waiver is welfare-improving $\left(g_1 + g_2 - N^1 < L - N^1 - n_w\right)$, such a value of transfer always exists.

Based on the derivation above, the decision whether to waive or not is determined by whether the gain from correcting control rights misallocation, $L - g_1 - g_2$ is greater than the waiver cost n_w.

- Case $h_2 < z_2$ and $g_1 + g_2 - N^1 - n_w < L - N^1$, or covenant was violated and the ultimate cash flows are below the value were the parties to liquidate. The control is with the bank, who optimally liquidates the firm. No waiver is necessary in this case.

- Case $h_2 < z_2$ and $g_1 + g_2 - N^1 - n_w \geq L - N^1$, or covenant was violated and the ultimate cash flows are above the value were the parties to liquidate. The control is with the bank, who will liquidate the firm suboptimally. A waiver is needed.

In this case, the party that is offering the contract is the manager. She offers a new contract K^2 to the bank.

$$\begin{aligned} &\max_{K^2} \max\{g_1 + g_2 - N^1 - n_w - K^2, 0\} \\ \text{s.t. } &\min\{K^2, g_1 + g_2 - N^1 - n_w\} \geq L - N^1 \end{aligned}$$

The new contract K^2 can always be found because $g_2 + g_2 - N^1 - n_w \geq L - N^1$. Whether a waiver happens is again determined by whether the gain from correcting control rights misallocation, $-L + g_1 + g_2$ is greater than the waiver cost n_w.

Next, consider period 1. I focus on only one case: $h_1 \geq z_1^0$ and $g_1 + E\left[g_2|g_1\right] \geq L - n_w$, or the covenant was not violated and the expected payoff from continuation is above the payoff from liquidation.

If an amendment were to happen, the manager would offer the bank a contract:

$$\begin{aligned} \max_{\{z_2^1, K^1\}} &Pr\left[h_2 \geq z_2^1 \,\&\, g_1 + g_2 - n_a \geq L - n_a - n_w\right] \times \max\{g_1 + E[g_2|g_1] - n_a - K^1, 0\} \\ &+Pr\left[h_2 \geq z_2^1 \,\&\, g_1 + g_2 - n_a < L - n_a - n_w\right] \times 0 \\ &+Pr\left[h_2 < z_2^1 \,\&\, g_1 + g_2 - n_a - n_w < L - n_a\right] \times 0 \\ +&Pr\left[h_2 < z_2^1 \,\&\, g_1 + g_2 - n_a - n_w \geq L - n_a\right] \times \max\{g_1 + E[g_2|g_1] - n_a - n_w - E[K^2], 0\} \\ \text{s.t. } &Pr\left[h_2 \geq z_2^1 \,\&\, g_1 + g_2 - n_a \geq L - n_a - n_w\right] \times \min\{K^1, g_1 + E[g_2|g_1] - n_a\} \\ &+Pr\left[h_2 \geq z_2^1 \,\&\, g_1 + g_2 - n_a < L - n_a - n_w\right] \times (L - n_a - n_w) \\ &+Pr\left[h_2 < z_2^1 \,\&\, g_1 + g_2 - n_a - n_w < L - n_a\right] \times (L - n_a) \\ &+Pr\left[h_2 < z_2^1 \,\&\, g_1 + g_2 - n_a - n_w \geq L - n_a\right] \times \min\{E[K^2], g_1 + E[g_2|g_1] - n_a - n_w\} \\ &\geq Pr\left[h_2 \geq z_2^0 \,\&\, g_1 + g_2 - n_a \geq L - n_a - n_w\right] \times \min\{K^0, g_1 + E[g_2|g_1] - n_a\} \\ &+Pr\left[h_2 \geq z_0^1 \,\&\, g_1 + g_2 - n_a < L - n_a - n_w\right] \times (L - n_a - n_w) \\ &+Pr\left[h_2 < z_0^1 \,\&\, g_1 + g_2 - n_a - n_w < L - n_a\right] \times (L - n_a) \\ &+Pr\left[h_2 < z_0^1 \,\&\, g_1 + g_2 - n_a - n_w \geq L - n_a\right] \times \min\{E[K^2], g_1 + E[g_2|g_1] - n_a - n_w\} \end{aligned}$$

This problem can be re-written if I assume the lender's participation constraint binds and substitute for K^1:

$$\begin{aligned}
\max_{z_2^1} Pr\left[h_2 \geq z_2^1 \;\&\; g_1+g_2-n_a \geq L-n_a-n_w\right] \times \left(g_1+E[g_2|g_1]-n_a\right) \\
+Pr\left[h_2 \geq z_2^1 \;\&\; g_1+g_2-n_a < L-n_a-n_w\right] \times \left(L-n_a-n_w\right) \\
+Pr\left[h_2 < z_2^1 \;\&\; g_1+g_2-n_a-n_w < L-n_a\right] \times \left(L-n_a\right) \\
+Pr\left[h_2 < z_2^1 \;\&\; g_1+g_2-n_a-n_w \geq L-n_a\right] \times \left(g_1+E[g_2|g_1]-n_a-n_w\right)
\end{aligned}$$

In the problem above, the manager essentially chooses type-1 and type-2 errors: $Pr\left[h_2 \geq z_2^1 \;\&\; g_1+g_2-n_a \geq L-n_a-n_w\right]$, $Pr\left[h_2 \geq z_2^1 \;\&\; g_1+g_2-n_a < L-n_a-n_w\right]$, $Pr\left[h_2 < z_2^1 \;\&\; g_1+g_2-n_a-n_w < L-n_a\right]$, and $Pr\left[h_2 < z_2^1 \;\&\; g_1+g_2-n_a-n_w \geq L-n_a\right]$ are the only terms that depend on the covenant threshold z. The interest rate, K^1, simply follows after the covenant z_2^1 is chosen to maximize the total surplus.

To simplify the notation, denote $Pr\left[h_2 \geq z_2^1 \;\&\; g_1+g_2-n_a \geq L-n_a-n_w\right] = Pr[nv \;\&\; nw]$, $Pr\left[h_2 \geq z_2^1 \;\&\; g_1+g_2-n_a < L-n_a-n_w\right] = Pr[nv \;\&\; w]$, $Pr\left[h_2 < z_2^1 \;\&\; g_1+g_2-n_a-n_w < L-n_a\right] = Pr[v \;\&\; nw]$, and $Pr\left[h_2 < z_2^1 \;\&\; g_1+g_2-n_a-n_w \geq L-n_a\right] = Pr[v \;\&\; w]$, where v and nv mean violation and non-violation, and w and nw mean a waiver does and does not go through.

I can change the notation: $Pr[m \;\&\; nv \;\&\; nw] = \mu\left(1-\pi_{m2}^1\right)$, $Pr[m \;\&\; v \;\&\; nw] = \mu\pi_{m_2}^1 Pr\left[g_1+g_2-L < n_w\right]$, $Pr[m \;\&\; v \;\&\; w] = \mu\pi_{m_2}^1 Pr\left[g_1+g_2-L \geq n_w\right]$, $Pr[b \;\&\; nv \;\&\; nw] = \beta\pi_{b_2}^1 Pr\left[L-g_1-g_2 < n_w\right]$, $Pr[b \;\&\; nv \;\&\; w] = \beta\pi_{b_2}^1 Pr\left[L-g_1-g_2 \geq n_w\right]$, $Pr[b \;\&\; v \;\&\; nw] = \beta\left(1-\pi_{b_2}^1\right)$. In addition, I set $V \equiv E\left[\max\{G,L\}\right]$ the total-surplus-maximizing value, which does not depend on the level of the covenant. Denote $n_a \equiv \eta_a$, $n_w \equiv \eta_w$, $L-g_1-E[g_2|g_1] \equiv c_b$, and $g_1+E[g_2|g_1]-L \equiv c_m$.

I can re-write the problem as

$$
\begin{aligned}
\min -V + \mu(1-\pi^1_{m2})\eta_a \\
+\beta\pi^1_{b2}Pr[L-g_1-g_2<\eta_w]\,(c_b+\eta_a) \\
+\beta\pi^1_{b2}Pr[L-g_1-g_2\geq n_w]\,(\eta_w+\eta_a) \\
+\mu\pi^1_{m2}Pr[g_1+g_2-L<n_w]\,(c_m+\eta_a) \\
+Pr[m\ \&\ v\ \&\ nw]\eta_a \\
+Pr[m\ \&\ v\ \&\ w]\,(\eta_a+\eta_w)
\end{aligned}
$$

In addition, I need to account for two facts. First, $\left(1-\pi^1_{m2}\right)$ and $\left(1-\pi^1_{b2}\right)$ are related through the covenant threshold z^1_2. Given that a single covenant threshold z^1_2 regulates not making both type-1 and type-2 errors, any improvement in the type-1 error must imply that the type-2 error increases, creating a trade-off in the error rates. Second, cases in which a covenant is more likely to misclassify than to classify correctly are unrealistic, implying that some lower bounds need to be imposed on $\left(1-\pi^1_{m2}\right)$ and $\left(1-\pi^1_{b2}\right)$. I approximate this trade-off and lower bounds using the following constraints: $(1-\pi_m-q)^2+(1-\pi_b-q)^2=(1-q)^2$, $\pi_m,\pi_b\in[0,1-q]$. The parameter q captures the level of noise in the signal h. My specification is a tractable reduced-form approximation of the relationship between the two error rates. While my model abstracts away from explicitly estimating the underlying signal distribution, it approximates the trade-off that arises in threshold-based models such as Griffin et al. (2026), where $\pi_{mt}(z)=1-F_m(z)$ and $\pi_{bt}(z)=F_b(z)$ for overlapping normal distributions, in the empirically relevant region.

9.2 An example of economically different financial covenant schedules that would be recorded the same way by Dealscan

(c) Leverage Ratio. Permit the Leverage Ratio at any time during any period of four fiscal quarters of Limited set forth below to be greater than the ratio set forth opposite such period:

Four Fiscal Quarters Ending:	
November 30, 2005	4.00 to 1.00
Each fiscal quarter thereafter	3.50 to 1.00

Figure 11: The first example of a covenant that would be recorded by Dealscan as "Decreasing from 4:1 to 3.5:1". The contract effective date is November 15, 2005. The screenshot is taken from the Third Amendment to a Credit Agreement filed by Helen of Troy Limited in Exhibit 10.2 of the 10-Q on January 9, 2006.

SECTION 6.08. Maximum Consolidated Total Leverage Ratio. The Borrower will cause the Consolidated Total Leverage Ratio to be less than (a) 4.00 to 1.00 at all times during the period from the Effective Date to and including December 30, 2009, (b) 3.75 to 1.00 at all times during the period from December 31, 2009 to and including December 30, 2010 and (c) less than 3.50 to 1.00 at all times thereafter.

Figure 12: The second example of a covenant that would be recorded by Dealscan as "Decreasing from 4:1 to 3.5:1". The contract effective date is August 25, 2008. The screenshot is taken from the Amended and Restated Credit Agreement filed by The Manitowoc Company, Inc. in Exhibit 4.1 of the 10-Q on November 10, 2008.

9.3 Examples of financial covenant schedules and how they are recorded

7.6 <u>**Financial Covenants**</u>.

A. **Minimum Interest Coverage Ratio**. Company shall not permit the ratio of (i) Consolidated EBITDA to (ii) Consolidated Interest Expense for any four-Fiscal Quarter period to be less than 4.00:1.00.

Figure 13: An example of a static financial covenant schedule. This excerpt is taken from the Credit Agreement between Hexcel Corporation and a lending syndicate with Bank of America Securities LLC as a Lead Arranger, dated as of July 9, 2010. This financial covenant schedule will be recorded as "4:1 in 1 year, 4:1 in 2 years, 4:1 in 3 years, 4:1 in 4 years".

8.09 <u>Consolidated Leverage Ratio</u>.

The Borrower will not permit the Consolidated Leverage Ratio as of the end of any fiscal quarter of the Borrower ending during a period set forth below to exceed the ratio set forth below opposite such period:

Period	Consolidated Leverage Ratio
Effective Date through December 31, 2011	5.00:1.00
January 1, 2012 through September 30, 2012	4.50:1.00
October 1, 2012 through September 30, 2013	4.00:1.00
October 1, 2013 and thereafter	3.75:1.00

Figure 14: An example of a dynamic financial covenant schedule. This excerpt is taken from the Credit Agreement between Hanger Orthopedic Group, Inc. and a lending syndicate with Merrill Lynch, Pierce, Fenner & Smith Incorporated and Jefferies Finance LLC as Joint Lead Arrangers, dated as of December 1, 2010. This financial covenant schedule will be recorded as "5:1 in 1 year, 4:1 in 2 years, 3.75:1 in 3 years, 3.75:1 in 4 years".

(b) <u>Fixed Charge Coverage Ratio</u>. Permit the Fixed Charge Coverage Ratio to be less than (i) 1.75 to 1.00 as of April 30, 2003 or July 31, 2003 for the period of four fiscal quarters most recently then ended, (ii) 1.50 to 1.00 as of October 31, 2003 or January 31, 2004 for the period of four fiscal quarters most recently then ended, (iii) 1.75 to 1.00 as of April 30, 2004 for the period of four fiscal quarters most recently then ended, (iv) 1.50 to 1.00 as of July 31, 2004, October 31, 2004, or January 31, 2005 for the period of four fiscal quarters most recently then ended, or (v) 1.75 to 1.00 as of April 30, 2005 or any fiscal quarter ending thereafter, for the period of four fiscal quarters most recently then ended.

Figure 15: An example of a dynamic financial covenant schedule. This excerpt is taken from the Credit Agreement between Gerber Scientific, Inc., and a lending syndicate with Ableco Finance LLC as Agent, dated as of May 9, 2003. This financial covenant schedule will be recorded as "1.5:1 in 1 year, 1.75:1 in 2 years, 1.75:1 in 3 years, 1.75:1 in 4 years".

9.4 Types of amendments to financial covenants

(x) Section 1.10 of the Credit Agreement is hereby amended by deleting the text of such Section in its entirety and replacing such deleted text with the following:

> Notwithstanding anything to the contrary herein, the Total Leverage Ratio and the Total Senior Secured Leverage Ratio shall be calculated (including, but not limited to, for purposes of determining the Maximum Incremental Amount with respect to Sections 2.14(a), 2.16(a), and 2.17(a), on a Pro Forma Basis with respect to each Specified Transaction occurring during the applicable four quarter period to which such calculation relates, or subsequent to the end of such four-quarter period but not later than the date of such calculation; provided that notwithstanding the foregoing, when calculating the Total Leverage Ratio and the Total Senior Secured Leverage Ratio for purposes of (i) determining the applicable percentage of Excess Cash Flow set forth in Section 2.05 and (ii) determining actual compliance (and not Pro Forma Compliance or compliance on a Pro Forma Basis) with any applicable covenant pursuant to Section 7.11, any Specified Transaction and any related adjustment contemplated in the definition of Pro Forma Basis (and corresponding provisions of the definition of Consolidated EBITDA) that occurred subsequent to the end of the applicable four quarter period shall not be given pro forma effect.

Figure 16: An amendment to the definition of Leverage Ratio used in financial covenants. The excerpt is taken from the First Amendment and Refinancing Agreement dated as of March 1, 2011 amending the Credit Agreement between NBTY, Inc., a Delaware Corporation and Barclays Capital, Merrill Lynch, Pierce, Fenner & Smith Incorporated, and Credit Suisse Securities (USA) LLC as Joint Lead Arrangers dated as of October 1, 2010.

1. Amendment to Credit Agreement – Section 1. The definition of EBITDAR in Section 1 of the Credit Agreement is hereby restated as follows:

> **EBITDAR** means, as determined, on a rolling twelve month basis and in respect of any Person the sum of (a) the Net Income of such Person, plus (b) the interest expense of such Person for such period as determined in accordance with GAAP and as such item is reported on such Person's financial statements, plus (c) the income tax expense of such Person for such period, plus (d) the amount reported as the depreciation of the assets of such Person for such period, computed in accordance with GAAP, and as such item is used in the computation of such Person's Net Income for such period, plus (e) the amount reported as the amortization of intangibles for such Person for such period, computed in accordance with GAAP, and as such item is used in the computation of such Person's Net Income for such period, minus (f) Rental Payments related to Capitalized Leases, plus (g) Rental Payments.

Figure 17: An amendment to the definition of EBITDAR used in financial covenants. The excerpt is taken from the Amendment No. 1 dated as of August 26, 2016 to Credit Agreement between Monro Muffler Brake, Inc. and Citizens Bank, N.A. as Administrative Agent dated as of January 25, 2016.

6. Amendment to Section 5.20. Section 5.20 of the Credit Agreement hereby is amended by deleting it in its entirety, and substituting the following therefor:

SECTION 5.20. Minimum Consolidated Net Worth. Consolidated Net Worth will at no time be less than $240,000,000, less the amount of any non-cash loss from the sale of Gerber Coburn in an Approved Gerber Coburn Sale and less the amount of restructuring charges recorded in the fourth Fiscal Quarter of the 2001 Fiscal Year, not to exceed $30,000,000, and excluding any change in the "accumulated other comprehensive income/(loss)" component of shareholders' equity after January 31, 2001, plus the sum of: (i) 75% of the cumulative Consolidated Net Income of the Borrower and its Consolidated Subsidiaries during any period after the Closing Date (taken as one accounting period), calculated quarterly at the end of each Fiscal Quarter but excluding from such calculations of Consolidated Net Income for purposes of this clause (i), any Fiscal Quarter in which the Consolidated Net Income of the Borrower and its Consolidated Subsidiaries is negative; and (ii) 100% of the cumulative Net Proceeds of Capital Stock received during any period after the Closing Date, calculated quarterly at the end of each Fiscal Quarter.

Figure 18: An amendment to the Minimum Net Worth Covenant. The excerpt is taken from the Second Amendment dated as of January 31, 2002 amending the Credit Agreement between Gerber Scientific, Inc. and Wachovia Bank, N.A. as Agent dated as of March 14, 2001.

8.09 Consolidated Leverage Ratio.

The Borrower will not permit the Consolidated Leverage Ratio as of the end of any fiscal quarter of the Borrower ending during a period set forth below to exceed the ratio set forth below opposite such period:

Period	**Consolidated Leverage Ratio**
Effective Date through December 31, 2011	5.00:1.00
January 1, 2012 through September 30, 2012	4.50:1.00
~~**October 1, 2012 through September 30, 2013**~~	~~4.00:1.00~~
October 1, ~~2013~~2012 and thereafter	~~3.75~~4.00:1.00

Figure 19: An amendment to the Leverage Ratio Covenant. The excerpt is taken from Amendment No. 1 dated as of March 11, 2011 amending the Credit Agreement between Hanger Orthopedic Group, Inc., a Delaware corporation and Bank of America, N.A. as Administrative Agent dated as of December 1, 2010.

9.5 Measure of financial covenant strictness

There exists a disconnect between the notion of financial covenant strictness in the model and the data. I model financial covenants as an aggregate signal, while real-world loan contracts often use more than one financial covenant. To bridge the gap between theory and empirical setting, I use the measure of financial covenant strictness proposed by Demerjian and Owens (2016).

Demerjian and Owens (2016) develop a non-parametric measure of how likely a company is to violate any of its financial covenants in the next quarter after the contract's effective date. Because my study concentrates on longer-term financial covenant schedules and amendments, I adapt the Demerjian and Owens (2016) measure to an annual basis. My measure captures the probability of violation in the next year after the contract is signed or amended.

The measure is computed as follows. First, all companies available in Compustat are matched into twelve groups based on size and profitability. For a given year, the firms are sorted into size quartiles based on average total assets, and then within each size quartile, they are sorted into terciles based on their return-on-assets (ROA).

Next, to compute financial covenant strictness for a company i, another company j is randomly drawn from the bins matched to the company i one and two years before the loan initiation date. For company j, a rate of change in its financial variables is calculated and company i's financial variables are multiplied by these rates. Finally, company i's projected changes are compared with its financial covenants, and an indicator for whether any covenant is violated is recorded. In other words, I check whether the company i would have violated at least one of its covenants if it evolved exactly as the company j. The procedure is repeated 1000 times. The resulting probability of violating a covenant within a year for the company i is the fraction of simulations that are recorded as violations:

$$PVIOL_i = \frac{\sum_{j \neq i} VIOL_j}{1000} \tag{9}$$

Both the empirical measure and the theoretical construct of financial covenant strictness are functions of two variables: the level of financial covenants and the company's financial performance. Demerjian and Owens (2016) measure would increase if either financial covenant levels were more restrictive or if companies similar to the company i performed worse financially. As can be seen from theoretical moments 1-2 and 5-6, the probability of violating a covenant is a product of covenant levels and probabilities of states where the borrower or the lender is the optimal party-in-control.

9.6 Additional summary statistics

Table 11: Average number of loans per borrower in a given year.

Year	Number of deals an average borrower has	Number of tranches an average borrower has
1994	1.00	1.00
1995	1.00	1.33
1996	1.19	1.94
1997	1.45	2.15
1998	1.84	2.92
1999	2.01	3.37
2000	2.04	3.35
2001	2.11	3.42
2002	2.00	3.11
2003	2.05	3.10
2004	1.98	2.96
2005	2.11	3.17
2006	2.08	2.84
2007	1.99	2.77
2008	1.71	2.36
2009	1.60	2.06
2010	1.58	2.24
2011	1.59	2.30
2012	1.67	2.54
2013	1.91	3.06
2014	1.91	3.11
2015	1.84	2.95
2016	1.95	2.98
2017	1.78	2.83
2018	1.83	2.98
2019	1.70	2.70
2020	1.69	2.59
2021	1.81	2.58
2022	1.63	2.50

Table 12: Correlation matrix of variables used in the estimation. The definitions of the variables are provided in Appendix 8.3.

	Probability of violating a covenant during the 1^{st} year after origination	Probability of violating a covenant during the 2^{nd} year after origination	A financial covenant was amended during the 1^{st} year after origination	A financial covenant was violated during the 1^{st} year after origination
Probability of violating a covenant during the 1^{st} year after origination	1	0.926	0.097	0.394
Probability of violating a covenant during the 2^{nd} year after origination	0.926	1	0.111	0.390
A financial covenant was amended during the 1^{st} year after origination	0.097	0.111	1	0.241
A financial covenant was violated during the 1^{st} year after origination	0.394	0.390	0.241	1

Table 13: Correlation of variables used in the estimation with firms' and loans' characteristics. The definitions of the variables used in the estimation are provided in Appendix 8.3. The market-to-book ratio is the close price multiplied by the number of common shares outstanding and divided by the total value of common/ordinary equity. The return on assets is income before extraordinary items divided by total assets. The leverage ratio is the sum of the total long-term debt and the total debt in current liabilities, divided by the sum of the total long-term debt, the total debt in current liabilities, and the total value of common/ordinary equity. The debt-to-EBITDA ratio is the sum of the total long-term debt and the total debt in current liabilities, divided by the operating income before depreciation.

	Probability of violating a covenant during the 1st year after origination	Probability of violating a covenant during the 2nd year after origination	A financial covenant got amended during the 1st year after origination	A financial covenant was violated during the 1st year after origination	Change in covenant strictness after the amendment
Total assets	-0.063	-0.070	-0.050	-0.104	0.020
Market-to-book ratio	-0.054	-0.057	0.089	-0.040	-0.040
Return on assets	-0.238	-0.188	-0.122	-0.071	0.148
Leverage ratio	0.182	0.250	0.020	0.029	-0.316
Debt-to-EBITDA ratio	0.335	0.371	-0.003	0.128	-0.344
Loan amount	-0.180	-0.167	-0.072	-0.159	-0.150
Maturity	-0.047	-0.031	-0.063	-0.061	0.170
All-in-drawn spread	0.110	0.113	0.075	0.003	-0.052
Secured	0.149	0.169	0.032	0.114	0.083
Number of lenders	-0.031	0.001	-0.034	0.028	-0.108

9.7 Estimation procedure

9.7.1 Calculation of differences between empirical and theoretical moments

In this section, I explain how the empirical moments used to fit the model are computed. The paper uses 6 moments, listed in Appendix 8.1.4: ex ante probabilities of violating a covenant (i.e., covenant strictness) within the first and the second year after the contract origination, unconditional probability of amendment and probability of amendment conditional on no covenant violation within the first year after loan origination, probabilities of violating a covenant (i.e., covenant strictness) within the first year after an amendment unconditional and conditional on no violation within the first year after loan origination.

The data series used in estimation are initial financial covenant levels from the Dealscan database, hand-collected financial covenant schedules, hand-collected amended financial covenants, and firms' financial characteristics from the Compustat database. The variables are described in Appendix 8.3.

I treat my data as cross-sectional, i.e. every loan is an independent draw from a population of loans described by one distribution function. For each loan i, I have 5 columns:

1. Strictness of initial financial covenants for the first year after loan origination, $pviol^1_{0_i}$.
2. Strictness of initial financial covenants for the second year after loan origination, $pviol^2_{0_i}$.
3. Binary variable for whether a covenant was violated within the first year after loan origination, $viol_{1_i}$.
4. Binary variable for whether a covenant was amended within the first year after loan origination, am_{1_i}.
5. Strictness of amended financial covenants (if there was an amendment) for the first year after an amendment, $pviol^1_{am_i}$.

In the Table 14 below, I provide formulas used to calculate differences between empirical and theoretical moments.

9.7.2 Parameter search algorithm

The objective of the GMM procedure is to minimize the distance between the theoretical moments, which are functions of the model parameters, and empirical moments, which are calculated from the

Table 14: Formulas to calculate differences between empirical and theoretical moments. N denotes the number of loans in the sample. Description of variables is provided in Appendix 8.3.

Moment	**Formula for difference between empirical and theoretical moments**
Strictness of initial financial covenants for the first year after loan origination	$\frac{1}{N}\sum_{i=1}^{N} pviol_{0_i}^{1} - \left[\mu_0 \times \pi_{m1}^{0*} + (1-\mu_0) \times (1-\pi_{b1}^{0*})\right]$
Strictness of initial financial covenants for the second year after loan origination	$\frac{1}{N}\sum_{i=1}^{N} pviol_{0_i}^{2} -$ $\left[\mu_0 \times \left(\mu \times \pi_{m2}^{0*} + (1-\mu) \times (1-\pi_{b2}^{0*})\right) + (1-\mu_0) \times \left(\beta \times (1-\pi_{b2}^{0*}) + (1-\beta) \times \pi_{m2}^{0*}\right)\right]$
Probability of amending a covenant within the first year after loan origination	$\frac{1}{N}\sum_{i=1}^{N} am_{1_i} - \left[\mu_0 \times (1-H_m) + (1-\mu_0) \times (1-H_b)\right]$
Probability of amending a covenant within the first year after loan origination, conditional on no violation within the first year after origination	$\frac{\sum_{i=1}^{N} am_{1_i}\left(1-viol_{1_i}\right)}{\sum_{i=1}^{N}\left(1-viol_{1_i}\right)} - \left[\frac{\mu_0\times(1-\pi_{m1}^{0*})\times(1-H_m)+(1-\mu_0)\times\pi_{b1}^{0*}\times(1-H_b)}{\mu_0\times(1-\pi_{m1}^{0*})+(1-\mu_0)\times\pi_{b1}^{0*}}\right]$
Strictness of amended financial covenants for the first year after an amendment	$\frac{\sum_{i=1}^{N} pviol_{am}^{1} am_{1_i}}{\sum_{i=1}^{N} am_{1_i}} - \left[\frac{1}{\mu_0\times(1-H_m)+(1-\mu_0)\times(1-H_b)} \times\right.$ $\left(\mu_0 \times (1-H_m) \times \left(\mu \times E\left[\pi_{m2}^{1m*} \mid f_m^m \notin \left(x_m^l, x_m^u\right)\right] + (1-\mu) \times \left(1 - E\left[\pi_{b2}^{1m*} \mid f_m^m \notin \left(x_m^l, x_m^u\right)\right]\right)\right) + \right.$ $\left.\left.(1-\mu_0) \times (1-H_b) \times \left(\beta \times \left(1 - E\left[\pi_{b2}^{1b*} \mid f_b^b \notin \left(x_b^l, x_b^u\right)\right]\right) + (1-\beta) \times E\left[\pi_{m2}^{1b*} \mid f_b^b \notin \left(x_b^l, x_b^u\right)\right]\right)\right)\right]$
Strictness of amended financial covenants for the first year after an amendment, conditional on no violation within the first year after origination	$\frac{\sum_{i=1}^{N} pviol_{am}^{1} am_{1_i}\left(1-viol_{1_i}\right)}{\sum_{i=1}^{N} am_{1_i}\left(1-viol_{1_i}\right)} - \left[\frac{1}{\mu_0\times(1-\pi_{m1}^{0*})\times(1-H_m)+(1-\mu_0)\times\pi_{b1}^{0*}\times(1-H_b)} \times\right.$ $\left(\mu_0 \times (1-H_m) \times (1-\pi_{m1}^{0*}) \times \left(\mu \times E\left[\pi_{m2}^{1m*} \mid f_m^m \notin \left(x_m^l, x_m^u\right)\right] + (1-\mu) \times \left(1 - E\left[\pi_{b2}^{1m*} \mid f_m^m \notin \left(x_m^l, x_m^u\right)\right]\right)\right) + \right.$ $\left.\left.(1-\mu_0) \times (1-H_b) \times \pi_{b1}^{0*} \times \left(\beta \times \left(1 - E\left[\pi_{b2}^{1b*} \mid f_b^b \notin \left(x_b^l, x_b^u\right)\right]\right) + (1-\beta) \times E\left[\pi_{m2}^{1b*} \mid f_b^b \notin \left(x_b^l, x_b^u\right)\right]\right)\right)\right]$

data. In other words, the goal is to find a set of parameters $\hat{\theta}$ such that

$$\hat{\theta} = \text{argmin}_{\theta \in \mathbf{\Theta}} \quad (G(Y_i, \theta))^T \, \hat{W} \, (G(Y_i, \theta)) \,, \tag{10}$$

where $G(Y_i, \theta) = m(d) - \hat{m}(\theta)$ is the vector of differences between moments computed from the data – $m(d)$ a function of data d – and their counterparts computed from the model – $\hat{m}(\theta)$, a function of the model's parameters θ. I show how each element of this vector is calculated in Table 14. The matrix W is the weighting matrix.

The estimation is conducted in two steps. In the first step, the algorithm searches for $\hat{\theta}_1$ that minimizes (10) with an identity matrix as the weighting matrix $\hat{W}_1 = E$. Next, I take the obtained estimates $\hat{\theta}_1$, plug them into the vector $G(Y_i, \theta)$, and calculate the covariance matrix of this vector, $\hat{\Omega} \equiv G(Y_i, \theta) G(Y_i, \theta)'$, using bootstrap. I create 1,000 randomly drawn subsamples of size 500 from my original dataset and calculate vectors of moment differences $G(Y_i^k, \hat{\theta}_1), k = 1, 2, ..., 1,000$ for each of these subsamples. Next, I calculate the covariance matrix of moments based on these 1,000 observations, $\hat{\Omega}$.

In the second step, the algorithm searches for $\hat{\theta}_2$ that minimizes (10) where the weighting matrix is the inverse of the covariance matrix: $\hat{W}_2 = \hat{\Omega}^{-1}$. The parameter estimates obtained in the second step $\hat{\theta}_2$ are the ultimate estimates. I use the Nelder-Mead optimization method (Nelder and Mead (1965)) using R NLOPTR function (Johnson (2007)) to search for $\hat{\theta}$ in both steps.

I calculate standard errors of the estimates using the formula for the asymptotic covariance matrix of estimates:

$$\mathbf{V} \equiv \frac{1}{N} \left[\hat{G} \hat{\Omega}^{-1} \hat{G}^T \right]^{-1} , \tag{11}$$

where $\hat{G} \equiv \frac{\partial \left(\frac{1}{N} \sum_{i=1}^N g(Y_i, \theta) \right)}{\partial \theta}$ is the Jacobian matrix, evaluated at $\hat{\theta}_2$. The derivative of moment k with respect to parameter p, $\frac{\partial \left(\frac{1}{N} \sum_{i=1}^N g(Y_i, \theta) \right)_k}{\partial \theta_p}$, is calculated by varying the parameter $\hat{\theta}_p$ by 1% up and down (keeping other parameters constant) and dividing the difference between the new value of the moment at the 1% higher parameter and the new value of the moment at the 1% lower parameter $\hat{\theta}_p$, $\left(\frac{1}{N} \sum_{i=1}^N g(Y_i, \theta) \right)_k \left(1.01 \hat{\theta}_p \right) - \left(\frac{1}{N} \sum_{i=1}^N g(Y_i, \theta) \right)_k \left(0.99 \hat{\theta}_p \right)$ by 2% of $\hat{\theta}_p$.

Because the model is just-identified, I cannot calculate the J-statistic.

9.8 Alternative specifications

9.8.1 Alternative cost persistence parameters

Because the persistence of the cost of leaving the "wrong" party in control (ρ) and the waiver friction (η_w) are difficult to separate, in the estimation, I assume $\rho = 0.9$. In this section, I investigate how robust the estimation results are to the choice of the persistence parameter. I estimate three alternative versions of the model: in the first one ρ is set close to one, 0.999, in the second one, ρ is smaller than in the main specification, 0.8, and in the third one, ρ is set close to zero, 0.001.

The estimates are shown in Table 15. When some level of persistence in the cost is assumed (i.e., $\rho = 0.999$ or $\rho = 0.8$), the conclusion based on estimated parameters does not change: the degree of contractual completeness is low, persistences of the non-contractible states are low, and the waiver friction considerably exceeds amendment cost. However, the waiver friction estimate goes up by about 40%.

When I assume away persistence in the cost (i.e., $\rho = 0.001$), the two estimates that change substantially are of the persistence of the bad state, β and of the cost of waivers. In the model without persistent costs, the bad state appears very persistent and the waivers appear cheaper than in the main specification, although the cost of waivers is still substantially higher than the amendment cost.

9.8.2 Alternative data specification

The data allows flexibility in constructing financial covenant amendment variables. In particular, since the model is not about amendment frequencies but about a single decision to amend, I can look either at whether financial covenant schedules are amended within a year or whether they are ever amended in the entire life of a contract.

In the main body of the paper, I choose to only analyze amendment behavior within one year after contract origination. This section investigates how robust the parameter estimates are to specifications of amendment data. The results are presented in Table 16.

Most of the parameters are close for the two data specifications. The most sensitive to the data is the estimated degree of contractual completeness, q. When I consider amendments at any time until a loan's maturity, the frequency of amendments in the data goes up, and the parameter search algorithm attributes it to a lower degree of contractual completeness.

9.8.3 Removing maturity extensions

A potential concern might arise that financial covenants are amended when a contract's maturity is renegotiated. In this case, the need to change control rights might not necessarily be the primary reason for amendments and my model thus might omit an important force. Even though I investigate financial covenant amendments in the first year after origination (when opportunistic refinancing is unlikely) and few financial covenant amendments appear to be paired with maturity extensions, in this section, I investigate whether removing financial covenant amendments which are paired with maturity extensions changes my results.

I code financial covenant amendments that are paired with maturity extensions as no financial covenant amendment and re-estimate my model. Table 17 shows the results. All of the estimates are close to the estimates of the main specification. The only difference is that, with some financial covenant amendments removed, amendment costs and waiver costs are estimated to be slightly higher.

Table 15: Parameter estimates for different cost persistence parameters

Parameter	Main specification, $\rho = 0.9$	$\rho = 0.999$	$\rho = 0.8$	$\rho = 0.001$
Degree of contractual completeness, q	0.687	0.699	0.615	0.619
	(0.002)	(0.005)	(0.004)	(0.000)
Initial probability of the state m, μ_0	0.614	0.605	0.600	0.585
	(0.000)	(0.000)	(0.000)	(0.000)
Persistence of the state m, μ	0.654	0.657	0.637	0.673
	(0.000)	(0.004)	(0.001)	(0.000)
Persistence of the state b, β	0.501	0.521	0.500	0.775
	(0.001)	(0.001)	(0.000)	(0.000)
Amendment cost, η_a	0.009	0.013	0.010	0.010
	(0.000)	(0.001)	(0.000)	(0.000)
Waiver friction, η_w	0.700	0.997	0.998	0.403
	(0.001)	(0.150)	(0.013)	(0.000)

Standard errors are in parentheses. In estimation, $\bar{c} = 1$.

Table 16: Parameter estimates for an alternative data specification

Parameter	Main specification, amendments within the 1st year	Amendments at all times
Degree of contractual completeness, q	0.687 (0.002)	0.582 (0.004)
Initial probability of the state m, μ_0	0.614 (0.000)	0.607 (0.002)
Persistence of the state m, μ	0.654 (0.001)	0.618 (0.001)
Persistence of the state b, β	0.501 (0.001)	0.508 (0.002)
Amendment cost, η_a	0.009 (0.000)	0.007 (0.000)
Waiver friction, η_w	0.700 (0.001)	0.793 (0.034)

Standard errors are in parentheses. In estimation, the parameters are set $\bar{c} = 1$, $\rho = 0.9$.

Table 17: Parameter estimates for a sample without covenant amendments that also change contract maturity date

Parameter	Main specification	Sample without amendments to maturity
Degree of contractual completeness, q	0.687 (0.002)	0.676 (0.004)
Initial probability of the state m, μ_0	0.614 (0.000)	0.606 (0.001)
Persistence of the state m, μ	0.654 (0.001)	0.655 (0.001)
Persistence of the state b, β	0.501 (0.001)	0.511 (0.002)
Amendment cost, η_a	0.009 (0.000)	0.012 (0.000)
Waiver friction, η_w	0.700 (0.001)	0.853 (0.066)

Standard errors are in parentheses. In estimation, the parameters are set $\bar{c} = 1$, $\rho = 0.9$.

9.9 Moments for the estimation of the monetary value of costs

I add two moments to the moments used in the baseline estimation.

Let us consider the case when, at time $t = 1$, investors observe an amendment and no violation of covenants. First, investors update their beliefs about the probabilities of states m and b:

$$Pr\left[r_1 = m|\text{amendment \& no viol}\right] = \frac{\mu_0\left(1-\pi^0_{m_1}\right)(1-H_m)}{\mu_0\left(1-\pi^0_{m_1}\right)(1-H_m)+(1-\mu_0)\pi^0_{b_1}(1-H_b)}$$
$$Pr\left[r_1 = b|\text{amendment \& no viol}\right] = \frac{(1-\mu_0)\pi^0_{b_1}(1-H_b)}{\mu_0\left(1-\pi^0_{m_1}\right)(1-H_m)+(1-\mu_0)\pi^0_{b_1}(1-H_b)}$$

Second, investors update their beliefs about the losses to the enterprise value given the new contract. If $r_1 = m$, this loss is

$$-\left(\eta_a + E\left[(1-q)\left(\mu f^m_m + (1-\mu)e^b\right) - (1-q)\sqrt{\left(\mu f^m_m\right)^2 + \left((1-\mu)e^b\right)^2}|\text{amendment}\right]\right).$$

If $r_1 = b$, this loss is

$$-\left(e^b + \eta_a + E\left[(1-q)\left(\beta f^b_b + (1-\beta)e^m\right) - (1-q)\sqrt{\left(\beta f^b_b\right)^2 + \left((1-\beta)e^m\right)^2}|\text{amendment}\right]\right).$$

The change in the market value after observing amendment and no violation is

$$\begin{aligned}
&-\frac{\mu_0\left(1-\pi^0_{m_1}\right)(1-H_m)}{\mu_0\left(1-\pi^0_{m_1}\right)(1-H_m)+(1-\mu_0)\pi^0_{b_1}(1-H_b)}\\
&\times\left(\eta_a + E\left[(1-q)\left(\mu f^m_m + (1-\mu)e^b\right) - (1-q)\sqrt{\left(\mu f^m_m\right)^2 + \left((1-\mu)e^b\right)^2}|\text{amendment}\right]\right)\\
&-\frac{(1-\mu_0)\pi^0_{b_1}(1-H_b)}{\mu_0\left(1-\pi^0_{m_1}\right)(1-H_m)+(1-\mu_0)\pi^0_{b_1}(1-H_b)}\\
&\times\left(e^b + \eta_a + E\left[(1-q)\left(\beta f^b_b + (1-\beta)e^m\right) - (1-q)\sqrt{\left(\beta f^b_b\right)^2 + \left((1-\beta)e^m\right)^2}|\text{amendment}\right]\right)\\
&-V,
\end{aligned}$$

where V is the value of the objective function at the solution to the problem at time $t = 0$ outlined in section 1.3.

The derivation of the change in market value after an amendment and a violation is derived analogously:

$$
\begin{aligned}
&-\frac{\mu_0\pi^0_{m_1}(1-H_m)}{\mu_0\pi^0_{m_1}(1-H_m)+(1-\mu_0)(1-\pi^0_{b_1})(1-H_b)}\\
&\times\left(e^m+\eta_a+E\left[(1-q)\left(\mu f^m_m+(1-\mu)e^b\right)-(1-q)\sqrt{(\mu f^m_m)^2+((1-\mu)e^b)^2}|\text{amendment}\right]\right)\\
&-\frac{(1-\mu_0)(1-\pi^0_{b_1})(1-H_b)}{\mu_0\pi^0_{m_1}(1-H_m)+(1-\mu_0)(1-\pi^0_{b_1})(1-H_b)}\\
&\times\left(\eta_a+E\left[(1-q)\left(\beta f^b_b+(1-\beta)e^m\right)-(1-q)\sqrt{\left(\beta f^b_b\right)^2+((1-\beta)e^m)^2}|\text{amendment}\right]\right)\\
&-V
\end{aligned}
$$

9.10 An extension with different amendment costs

In this section, I present a modification to the main model, where I allow the cost of amendments to be different depending on whether the borrower violated a financial covenant. I discuss how the model's solution and theoretical moments change, and also which additional moments are used to estimate the extended model.

In this extension of the model, instead of having a common cost of amendments, η_a, I introduce two costs of amendments: η_a^v and η_a^{nv} – costs of amendments upon a violation and no violation, respectively.

In the solution, upon an amendment, the levels of covenants chosen at time $t = 1$ for time $t = 2$ do not change. The conditions for the amendment, however, are different in violation and no-violation cases. Specifically, the condition (2) has η_a^v or η_a^{nv} instead of η_a. Therefore, x_m^l, x_m^u, x_b^l, and x_b^u are also different for violation and no-violation cases as they are derived from different versions of conditions like in (2). As a result, the probabilities of an amendment, H_m and H_b, as well as expected amended covenant levels in case of an amendment are different depending on whether a violation occurred or not. In what follows, a superscript v (nv) denotes the value in the case of covenant violation (no covenant violation).

The problem at time $t = 0$ also changes. First, the optimal covenants for time $t = 2$, π_{m2}^{0*} and π_{b2}^{0*}, change.[18] Second, the problem of finding π_{m1}^{0*} and π_{b1}^{0*} is no longer separable from the problem of finding π_{m2}^{0*} and π_{b2}^{0*}. The covenants for time $t = 1$ now not only determine whether there is a violation at time $t = 1$ but also whether an amendment will occur and the expected costs of this amendment. Because finding closed-form solutions to $\{\pi_{m1}^{0*}, \pi_{b1}^{0*}, \pi_{m2}^{0*}, \pi_{b2}^{0*}\}$ is impossible, I solve for the optimal

[18] I could have allowed the parties to choose different π_{m2}^{0*} and π_{b2}^{0*} for violation and non-violation cases, or, in other words, condition the level of time-2 covenants on the instance of a covenant violation at time $t = 1$. Empirically, however, such clauses are uncommon.

covenants chosen at time $t = 0$ numerically.

Below I provide updated theoretical moments for the extended model.

1. Ex ante probability of violating a covenant at time $t = 1$, as set at time $t = 0$. The optimal $\{\pi_{m1}^{0*}, \pi_{b1}^{0*}\}$ are different from the main specification.

2. Ex ante probability of violating a covenant at time $t = 2$, as set at time $t = 0$. The optimal $\{\pi_{m2}^{0*}, \pi_{b2}^{0*}\}$ are different from the main specification.

3. Ex ante unconditional probability of amending the covenant schedule:

$$\begin{aligned} \mathbf{E}_0[\mathbf{1}(\text{amendment}_{t=1})] = \\ \mu_0\pi_{m1}^{0*}\left(1 - H_m^v\right) + \mu_0(1 - \pi_{m1}^{0*})\left(1 - H_m^{nv}\right) + \\ (1 - \mu_0)\pi_{b1}^{0*}\left(1 - H_b^{nv}\right) + (1 - \mu_0)(1 - \pi_{b1}^{0*})\left(1 - H_b^v\right) \end{aligned} \tag{12}$$

4. Ex ante probability of amending the contract at time $t = 1$, conditional on no covenant violation:

$$\begin{gathered} \mathbf{E}_0[\mathbf{1}(\text{amendment}_{t=1})|\mathbf{1}(\text{violation}_{t=1}) = 0] = \\ \frac{\mu_0(1 - \pi_{m1}^{0*})\left(1 - H_m^{nv}\right) + (1 - \mu_0)\pi_{b1}^{0*}\left(1 - H_b^{nv}\right)}{\mu_0 \times (1 - \pi_{m1}^{0*}) + (1 - \mu_0) \times \pi_{b1}^{0*}} \end{gathered} \tag{13}$$

5. The amended probability of violating a covenant at time $t = 2$, conditional on amendment at time $t = 1$:

$$\begin{aligned} \mathbf{E}_0[\mathbf{1}(\text{violation}_{t=2})|\mathbf{1}(\text{amendment}_{t=1})] = \\ \frac{1}{\mu_0\pi_{m1}^{0*}\left(1 - H_m^v\right) + \mu_0(1 - \pi_{m1}^{0*})\left(1 - H_m^{nv}\right) + (1 - \mu_0)\pi_{b1}^{0*}\left(1 - H_b^{nv}\right) + (1 - \mu_0)(1 - \pi_{b1}^{0*})\left(1 - H_b^v\right)} \\ \times \left(\mu_0\pi_{m1}^{0*}(1 - H_m^v)\left(\mu E\left[\pi_{m2}^{1m*}|f_m^m \notin \left(x_m^{lv}, x_m^{uv}\right)\right] + (1 - \mu)\left(1 - E\left[\pi_{b2}^{1m*}|f_m^m \notin \left(x_m^{lv}, x_m^{uv}\right)\right]\right)\right)\right. \\ +\mu_0(1 - \pi_{m1}^{0*})(1 - H_m^{nv})\left(\mu E\left[\pi_{m2}^{1m*}|f_m^m \notin \left(x_m^{lnv}, x_m^{unv}\right)\right] + (1 - \mu)\left(1 - E\left[\pi_{b2}^{1m*}|f_m^m \notin \left(x_m^{lnv}, x_m^{unv}\right)\right]\right)\right) \\ +(1 - \mu_0)\pi_{b1}^{0*}(1 - H_b^{nv})\left(\beta\left(1 - E\left[\pi_{b2}^{1b*}|f_b^b \notin \left(x_b^{lnv}, x_b^{unv}\right)\right]\right) + (1 - \beta)E\left[\pi_{m2}^{1b*}|f_b^b \notin \left(x_b^{lnv}, x_b^{nv}\right)\right]\right) \\ \left.+(1 - \mu_0)(1 - \pi_{b1}^{0*})(1 - H_b^v)\left(\beta\left(1 - E\left[\pi_{b2}^{1b*}|f_b^b \notin \left(x_b^{lv}, x_b^{uv}\right)\right]\right) + (1 - \beta)E\left[\pi_{m2}^{1b*}|f_b^b \notin \left(x_b^{lv}, x_b^{uv}\right)\right]\right)\right) \end{aligned} \tag{14}$$

6. The amended probability of violating a covenant at time $t = 2$, conditional on no covenant viola-

tion and amendment at time $t = 1$:

$$\begin{aligned}
&\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=2})|\mathbf{1}(\text{amendment}_{t=1}) \times (1 - \mathbf{1}(\text{violation}_{t=1}))] = \\
&\frac{1}{\mu_0(1 - \pi_{m1}^{0*})\,(1 - H_m^{nv}) + (1 - \mu_0)\pi_{b1}^{0*}\,(1 - H_b^{nv})} \\
&\times \left(\mu_0(1 - H_m^{nv})(1 - \pi_{m1}^{0*})\left(\mu E\left[\pi_{m2}^{1m*}|f_m^m \notin \left(x_m^{lnv}, x_m^{unv}\right)\right] + (1-\mu)\left(1 - E\left[\pi_{b2}^{1m*}|f_m^m \notin \left(x_m^{lnv}, x_m^{unv}\right)\right]\right)\right)\right. \\
&\left. +(1 - \mu_0)(1 - H_b^{nv})\pi_{b1}^{0*}\left(\beta\left(1 - E\left[\pi_{b2}^{1b*}|f_b^b \notin \left(x_b^{lnv}, x_b^{unv}\right)\right]\right) + (1-\beta)E\left[\pi_{m2}^{1b*}|f_b^b \notin \left(x_b^{lnv}, x_b^{unv}\right)\right]\right)\right)
\end{aligned} \tag{15}$$

In addition, because in the extended model there is one more parameter to estimate, I introduce two new moments:

7. Ex ante probability of amending the contract at time $t = 1$, conditional on a covenant violation:

$$\begin{aligned}
&\mathbf{E}_0[\mathbf{1}(\text{amendment}_{t=1})|\mathbf{1}(\text{violation}_{t=1}) = 1] = \\
&\frac{\mu_0\pi_{m1}^{0*}\,(1 - H_m^v) + (1 - \mu_0)(1 - \pi_{b1}^{0*})\,(1 - H_b^v)}{\mu_0 \times \pi_{m1}^{0*} + (1 - \mu_0) \times (1 - \pi_{b1}^{0*})}
\end{aligned} \tag{16}$$

8. The amended probability of violating a covenant at time $t = 2$, conditional on a covenant violation and an amendment at time $t = 1$:

$$\begin{aligned}
&\mathbf{E}_0[\mathbf{1}(\text{violation}_{t=2})|\mathbf{1}(\text{amendment}_{t=1}) \times \mathbf{1}(\text{violation}_{t=1})] = \\
&\frac{1}{\mu_0\pi_{m1}^{0*}\,(1 - H_m^v) + (1 - \mu_0)(1 - \pi_{b1}^{0*})\,(1 - H_b^v)} \\
&\times \left(\mu_0(1 - H_m^v)\pi_{m1}^{0*}\left(\mu E\left[\pi_{m2}^{1m*}|f_m^m \notin \left(x_m^{lv}, x_m^{uv}\right)\right] + (1-\mu)\left(1 - E\left[\pi_{b2}^{1m*}|f_m^m \notin \left(x_m^{lv}, x_m^{uv}\right)\right]\right)\right)\right. \\
&\left. +(1 - \mu_0)(1 - H_b^v)(1 - \pi_{b1}^{0*})\left(\beta\left(1 - E\left[\pi_{b2}^{1b*}|f_b^b \notin \left(x_b^{lv}, x_b^{uv}\right)\right]\right) + (1-\beta)E\left[\pi_{m2}^{1b*}|f_b^b \notin \left(x_b^{lv}, x_b^{uv}\right)\right]\right)\right)
\end{aligned} \tag{17}$$