Chapter 1

1 Algebra I

Objectives

- To solve **linear equations**.
- To solve problems with linear equations and **simultaneous linear equations**.
- To use **substitution** and **transposition** with formulas.
- To add and multiply algebraic fractions.
- To solve **literal equations**.
- To solve **simultaneous literal equations**.

Algebra is the language of mathematics. Algebra helps us to state ideas more simply. It also enables us to make general statements about mathematics, and to solve problems that would be difficult to solve otherwise.

We know by basic arithmetic that $9 \times 7 + 2 \times 7 = 11 \times 7$. We could replace the number 7 in this statement by any other number we like, and so we could write down infinitely many such statements. These can all be captured by the algebraic statement $9x + 2x = 11x$, for any number x. Thus algebra enables us to write down general statements.

Formulas enable mathematical ideas to be stated clearly and concisely. An example is the well-known formula for compound interest. Suppose that an initial amount P is invested at an interest rate R, with interest compounded annually. Then the amount, A_n, that the investment is worth after n years is given by $A_n = P(1 + R)^n$.

In this chapter we review some of the techniques which you have met in previous years. Algebra plays a central role in Specialist Mathematics at Years 11 and 12. It is important that you become fluent with the techniques introduced in this chapter and in Chapter 5.

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1A Indices

This section revises algebra involving indices.

Review of index laws

For all non-zero real numbers a and b and all integers m and n:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
- $a^{-n} = \dfrac{1}{a^n}$
- $\dfrac{1}{a^{-n}} = a^n$
- $a^0 = 1$

Rational indices

If a is a positive real number and n is a natural number, then $a^{\frac{1}{n}}$ is defined to be the nth root of a. That is, $a^{\frac{1}{n}}$ is the positive number whose nth power is a. For example: $9^{\frac{1}{2}} = 3$.

If n is odd, then we can define $a^{\frac{1}{n}}$ when a is negative. If a is negative and n is odd, define $a^{\frac{1}{n}}$ to be the number whose nth power is a. For example: $(-8)^{\frac{1}{3}} = -2$.

In both cases we can write:

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad \text{with} \quad \left(a^{\frac{1}{n}}\right)^n = a$$

In general, the expression a^x can be defined for rational indices, i.e. when $x = \dfrac{m}{n}$, where m and n are integers, by defining

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$$

To employ this definition, we will always first write the fractional power in simplest form.

Note: The index laws hold for rational indices m and n whenever both sides of the equation are defined (for example, if a and b are positive real numbers).

Example 1

Simplify each of the following:

a $x^2 \times x^3$ **b** $\dfrac{x^4}{x^2}$ **c** $x^{\frac{1}{2}} \div x^{\frac{4}{5}}$ **d** $(x^3)^{\frac{1}{2}}$

Solution	Explanation
a $x^2 \times x^3 = x^{2+3} = x^5$	$a^m \times a^n = a^{m+n}$
b $\dfrac{x^4}{x^2} = x^{4-2} = x^2$	$\dfrac{a^m}{a^n} = a^{m-n}$
c $x^{\frac{1}{2}} \div x^{\frac{4}{5}} = x^{\frac{1}{2}-\frac{4}{5}} = x^{-\frac{3}{10}}$	$\dfrac{a^m}{a^n} = a^{m-n}$
d $(x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$	$(a^m)^n = a^{mn}$

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Example 2

Evaluate:

a $125^{\frac{2}{3}}$ **b** $\left(\frac{1000}{27}\right)^{\frac{2}{3}}$

Solution

a $125^{\frac{2}{3}} = \left(125^{\frac{1}{3}}\right)^2 = 5^2 = 25$

b $\left(\frac{1000}{27}\right)^{\frac{2}{3}} = \left(\left(\frac{1000}{27}\right)^{\frac{1}{3}}\right)^2 = \left(\frac{10}{3}\right)^2 = \frac{100}{9}$

Explanation

$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

$\left(\frac{1000}{27}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{1000}{27}} = \frac{10}{3}$

Example 3

Simplify $\dfrac{\sqrt[4]{x^2y^3}}{x^{\frac{1}{2}}y^{\frac{2}{3}}}$.

Solution

$$\frac{\sqrt[4]{x^2y^3}}{x^{\frac{1}{2}}y^{\frac{2}{3}}} = \frac{(x^2y^3)^{\frac{1}{4}}}{x^{\frac{1}{2}}y^{\frac{2}{3}}} = \frac{x^{\frac{2}{4}}y^{\frac{3}{4}}}{x^{\frac{1}{2}}y^{\frac{2}{3}}}$$

$$= x^{\frac{2}{4}-\frac{1}{2}}y^{\frac{3}{4}-\frac{2}{3}}$$

$$= x^0y^{\frac{1}{12}}$$

$$= y^{\frac{1}{12}}$$

Explanation

$(ab)^n = a^nb^n$

$\frac{a^m}{a^n} = a^{m-n}$

$a^0 = 1$

Section summary

- **Index laws**
  - $a^m \times a^n = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $(ab)^n = a^nb^n$
  - $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
  - $a^{-n} = \frac{1}{a^n}$
  - $\frac{1}{a^{-n}} = a^n$
  - $a^0 = 1$
- **Rational indices**
  - $a^{\frac{1}{n}} = \sqrt[n]{a}$
  - $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$

Exercise 1A

Example 1

1 Simplify each of the following using the appropriate index laws:

a $x^3 \times x^4$ **b** $a^5 \times a^{-3}$ **c** $x^2 \times x^{-1} \times x^2$ **d** $\frac{y^3}{y^7}$

e $\frac{x^8}{x^{-4}}$ **f** $\frac{p^{-5}}{p^2}$ **g** $a^{\frac{1}{2}} \div a^{\frac{2}{3}}$ **h** $(a^{-2})^4$

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i $(y^{-2})^{-7}$ **j** $(x^5)^3$ **k** $(a^{-20})^{\frac{3}{5}}$ **l** $\left(x^{-\frac{1}{2}}\right)^{-4}$

m $(n^{10})^{\frac{1}{5}}$ **n** $2x^{\frac{1}{2}} \times 4x^3$ **o** $(a^2)^{\frac{5}{2}} \times a^{-4}$ **p** $\dfrac{1}{x^{-4}}$

q $\left(2n^{-\frac{2}{5}}\right)^5 \div (4^3n^4)$ **r** $x^3 \times 2x^{\frac{1}{2}} \times -4x^{-\frac{3}{2}}$

s $(ab^3)^2 \times a^{-2}b^{-4} \times \dfrac{1}{a^2b^{-3}}$ **t** $(2^2p^{-3} \times 4^3p^5 \div (6p^{-3}))^0$

Example 2

2 Evaluate each of the following:

a $25^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $\left(\dfrac{16}{9}\right)^{\frac{1}{2}}$ **d** $16^{-\frac{1}{2}}$

e $\left(\dfrac{49}{36}\right)^{-\frac{1}{2}}$ **f** $27^{\frac{1}{3}}$ **g** $144^{\frac{1}{2}}$ **h** $64^{\frac{2}{3}}$

i $9^{\frac{3}{2}}$ **j** $\left(\dfrac{81}{16}\right)^{\frac{1}{4}}$ **k** $\left(\dfrac{23}{5}\right)^0$ **l** $128^{\frac{3}{7}}$

3 Use your calculator to evaluate each of the following, correct to two decimal places:

a 4.35^2 **b** 2.4^5 **c** $\sqrt{34.6921}$ **d** $(0.02)^{-3}$ **e** $\sqrt[3]{0.729}$

f $\sqrt[4]{2.3045}$ **g** $(345.64)^{-\frac{1}{3}}$ **h** $(4.568)^{\frac{2}{5}}$ **i** $\dfrac{1}{(0.064)^{-\frac{1}{3}}}$

4 Simplify each of the following, giving your answer with positive index:

a $\dfrac{a^2b^3}{a^{-2}b^{-4}}$ **b** $\dfrac{2a^2(2b)^3}{(2a)^{-2}b^{-4}}$ **c** $\dfrac{a^{-2}b^{-3}}{a^{-2}b^{-4}}$

d $\dfrac{a^2b^3}{a^{-2}b^{-4}} \times \dfrac{ab}{a^{-1}b^{-1}}$ **e** $\dfrac{(2a)^2 \times 8b^3}{16a^{-2}b^{-4}}$ **f** $\dfrac{2a^2b^3}{8a^{-2}b^{-4}} \div \dfrac{16ab}{(2a)^{-1}b^{-1}}$

5 Write $\dfrac{2^n \times 8^n}{2^{2n} \times 16}$ in the form 2^{an+b}.

6 Write $2^{-x} \times 3^{-x} \times 6^{2x} \times 3^{2x} \times 2^{2x}$ as a power of 6.

7 Simplify each of the following:

a $2^{\frac{1}{3}} \times 2^{\frac{1}{6}} \times 2^{-\frac{2}{3}}$ **b** $a^{\frac{1}{4}} \times a^{\frac{2}{5}} \times a^{-\frac{1}{10}}$ **c** $2^{\frac{2}{3}} \times 2^{\frac{5}{6}} \times 2^{-\frac{2}{3}}$

d $\left(2^{\frac{1}{3}}\right)^2 \times \left(2^{\frac{1}{2}}\right)^5$ **e** $\left(2^{\frac{1}{3}}\right)^2 \times 2^{\frac{1}{3}} \times 2^{-\frac{2}{5}}$

Example 3

8 Simplify each of the following:

a $\sqrt[3]{a^3b^2} \div \sqrt[3]{a^2b^{-1}}$ **b** $\sqrt{a^3b^2} \times \sqrt{a^2b^{-1}}$ **c** $\sqrt[5]{a^3b^2} \times \sqrt[5]{a^2b^{-1}}$

d $\sqrt{a^{-4}b^2} \times \sqrt{a^3b^{-1}}$ **e** $\sqrt{a^3b^2c^{-3}} \times \sqrt{a^2b^{-1}c^{-5}}$ **f** $\sqrt[5]{a^3b^2} \div \sqrt[5]{a^2b^{-1}}$

g $\dfrac{\sqrt{a^3b^2}}{a^2b^{-1}c^{-5}} \times \dfrac{\sqrt{a^{-4}b^2}}{a^3b^{-1}} \times \sqrt{a^3b^{-1}}$

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1B Standard form

Often when dealing with real-world problems, the numbers involved may be very small or very large. For example:

- The distance from Earth to the Sun is approximately 150 000 000 kilometres.
- The mass of an oxygen atom is approximately 0.000 000 000 000 000 000 000 026 grams.

To help deal with such numbers, we can use a more convenient way to express them. This involves expressing the number as a product of a number between 1 and 10 and a power of 10 and is called **standard form** or **scientific notation**.

These examples written in standard form are:

- 1.5×10^8 kilometres
- 2.6×10^{-23} grams

Multiplication and division with very small or very large numbers can often be simplified by first converting the numbers into standard form. When simplifying algebraic expressions or manipulating numbers in standard form, a sound knowledge of the index laws is essential.

Example 4

Write each of the following in standard form:

a 3 453 000 **b** 0.00675

Solution

a $3\,453\,000 = 3.453 \times 10^6$ **b** $0.00675 = 6.75 \times 10^{-3}$

Example 5

Find the value of $\dfrac{32\,000\,000 \times 0.000\,004}{16\,000}$.

Solution

$$\begin{aligned}\frac{32\,000\,000 \times 0.000\,004}{16\,000} &= \frac{3.2 \times 10^7 \times 4 \times 10^{-6}}{1.6 \times 10^4}\\ &= \frac{12.8 \times 10^1}{1.6 \times 10^4}\\ &= 8 \times 10^{-3}\\ &= 0.008\end{aligned}$$

▶ Significant figures

When measurements are made, the result is recorded to a certain number of significant figures. For example, if we say that the length of a piece of ribbon is 156 cm to the nearest centimetre, this means that the length is between 155.5 cm and 156.5 cm. The number 156 is said to be correct to three significant figures. Similarly, we may record π as being 3.1416, correct to five significant figures.

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When rounding off to a given number of significant figures, first identify the last significant digit and then:

- if the next digit is 0, 1, 2, 3 or 4, round down
- if the next digit is 5, 6, 7, 8 or 9, round up.

It can help with rounding off if the original number is first written in scientific notation.

So $\pi = 3.141\,592\,653\ldots$ is rounded off to 3, 3.1, 3.14, 3.142, 3.1416, 3.14159, etc. depending on the number of significant figures required.

Writing a number in scientific notation makes it clear how many significant figures have been recorded. For example, it is unclear whether 600 is recorded to one, two or three significant figures. However, when written in scientific notation as 6.00×10^2, 6.0×10^2 or 6×10^2, it is clear how many significant figures are recorded.

Example 6

Evaluate $\dfrac{\sqrt[5]{a}}{b^2}$ if $a = 1.34 \times 10^{-10}$ and $b = 2.7 \times 10^{-8}$.

Solution

$$\begin{aligned}
\frac{\sqrt[5]{a}}{b^2} &= \frac{\sqrt[5]{1.34 \times 10^{-10}}}{(2.7 \times 10^{-8})^2} \\
&= \frac{(1.34 \times 10^{-10})^{\frac{1}{5}}}{2.7^2 \times (10^{-8})^2} \\
&= 1.45443\ldots \times 10^{13} \\
&= 1.45 \times 10^{13} \quad \text{to three significant figures}
\end{aligned}$$

Many calculators can display numbers in scientific notation. The format will vary from calculator to calculator. For example, the number $3\,245\,000 = 3.245 \times 10^6$ may appear as 3.245E6 or 3.245^{06}.

Using the TI-Nspire

Insert a **Calculator** page, then use on > **Settings** > **Document Settings** and change the **Exponential Format** field to **Scientific**. If you want this change to apply only to the current page, select OK to accept the change. Select **Current** to return to the current page.

Using the Casio ClassPad

The ClassPad calculator can be set to express decimal answers in various forms. To select a fixed number of decimal places, including specifying scientific notation with a fixed decimal accuracy, go to **Settings** and in **Basic format** tap the arrow to select from the various Number formats available.

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Section summary

- A number is said to be in **scientific notation** (or **standard form**) when it is written as a product of a number between 1 and 10 and an integer power of 10.
  For example: $6547 = 6.457 \times 10^3$ and $0.789 = 7.89 \times 10^{-1}$
- Writing a number in scientific notation makes it clear how many **significant figures** have been recorded.
- When rounding off to a given number of significant figures, first identify the last significant digit and then:
  - if the next digit is 0, 1, 2, 3 or 4, round down
  - if the next digit is 5, 6, 7, 8 or 9, round up.

Exercise 1B

Example 4

1 Express each of the following numbers in standard form:

a 47.8 **b** 6728 **c** 79.23 **d** 43 580
e 0.0023 **f** 0.000 000 56 **g** 12.000 34 **h** 50 million
i 23 000 000 000 **j** 0.000 000 0013 **k** 165 thousand **l** 0.000 014 567

2 Express each of the following in scientific notation:

a X-rays have a wavelength of 0.000 000 01 cm.
b The mass of a hydrogen atom is 0.000 000 000 000 000 000 000 001 67 g.
c Visible light has wavelength 0.000 05 cm.
d One nautical mile is 1853.18 m.
e A light year is 9 461 000 000 000 km.
f The speed of light is 29 980 000 000 cm/s.

3 Express each of the following as an ordinary number:

a The star Sirius is approximately 8.128×10^{13} km from Earth.
b A single red blood cell contains 2.7×10^8 molecules of haemoglobin.
c The radius of an electron is 2.8×10^{-13} cm.

4 Write each of the following in scientific notation, correct to the number of significant figures indicated in the brackets:

a 456.89 (4) **b** 34567.23 (2) **c** 5679.087 (5)
d 0.04536 (2) **e** 0.09045 (2) **f** 4568.234 (5)

Example 5

5 Find the value of:

a $\dfrac{324\,000 \times 0.000\,0007}{4000}$ **b** $\dfrac{5\,240\,000 \times 0.8}{42\,000\,000}$

Example 6

6 Evaluate the following correct to three significant figures:

a $\dfrac{\sqrt[3]{a}}{b^4}$ if $a = 2 \times 10^9$ and $b = 3.215$ **b** $\dfrac{\sqrt[4]{a}}{4b^4}$ if $a = 2 \times 10^{12}$ and $b = 0.05$

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1C Solving linear equations and simultaneous linear equations

Many problems may be solved by first translating them into mathematical equations and then solving the equations using algebraic techniques. An equation is solved by finding the value or values of the variables that would make the statement true.

Linear equations are simple equations that can be written in the form $ax + b = 0$. There are a number of standard techniques that can be used for solving linear equations.

Example 7

a Solve $\frac{x}{5} - 2 = \frac{x}{3}$.

b Solve $\frac{x-3}{2} - \frac{2x-4}{3} = 5$.

Solution

a Multiply both sides of the equation by the lowest common multiple of 3 and 5:

$$\frac{x}{5} - 2 = \frac{x}{3}$$

$$\frac{x}{5} \times 15 - 2 \times 15 = \frac{x}{3} \times 15$$

$$3x - 30 = 5x$$

$$3x - 5x = 30$$

$$-2x = 30$$

$$x = \frac{30}{-2}$$

$$\therefore \quad x = -15$$

b Multiply both sides of the equation by the lowest common multiple of 2 and 3:

$$\frac{x-3}{2} - \frac{2x-4}{3} = 5$$

$$\frac{x-3}{2} \times 6 - \frac{2x-4}{3} \times 6 = 5 \times 6$$

$$3(x-3) - 2(2x-4) = 30$$

$$3x - 9 - 4x + 8 = 30$$

$$3x - 4x = 30 + 9 - 8$$

$$-x = 31$$

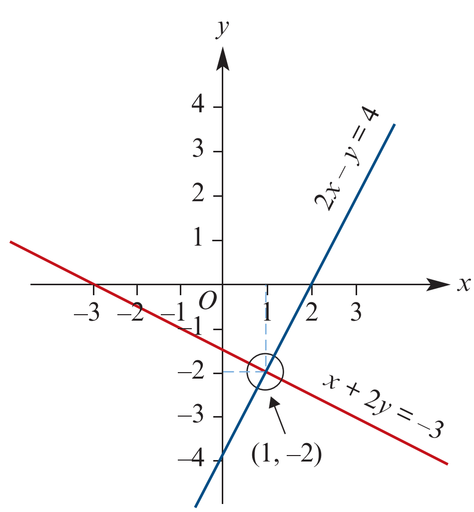
$$x = \frac{31}{-1}$$

$$\therefore \quad x = -31$$

▶ Simultaneous linear equations

The intersection point of two straight lines can be found graphically; however, the accuracy of the solution will depend on the accuracy of the graphs.

Alternatively, the intersection point may be found algebraically by solving the pair of simultaneous equations. We shall consider two techniques for solving simultaneous equations.



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Example 8

Solve the equations $2x - y = 4$ and $x + 2y = -3$.

Solution

Method 1: Substitution

$$2x - y = 4 \quad (1)$$
$$x + 2y = -3 \quad (2)$$

From equation (2), we get $x = -3 - 2y$.

Substitute in equation (1):

$$2(-3 - 2y) - y = 4$$
$$-6 - 4y - y = 4$$
$$-5y = 10$$
$$y = -2$$

Substitute the value of y into (2):

$$x + 2(-2) = -3$$
$$x = 1$$

Check in (1): LHS $= 2(1) - (-2) = 4$
RHS $= 4$

Method 2: Elimination

$$2x - y = 4 \quad (1)$$
$$x + 2y = -3 \quad (2)$$

To eliminate x, multiply equation (2) by 2 and subtract the result from equation (1).

When we multiply equation (2) by 2, the pair of equations becomes:

$$2x - y = 4 \quad (1)$$
$$2x + 4y = -6 \quad (2')$$

Subtract (2′) from (1):

$$-5y = 10$$
$$y = -2$$

Now substitute for y in equation (2) to find x, and check as in the substitution method.

Explanation

Using one of the two equations, express one variable in terms of the other variable.

Then substitute this expression into the other equation (reducing it to an equation in one variable, y). Solve the equation for y.

Substitute this value for y in one of the equations to find the other variable, x.

A check can be carried out with the other equation.

If one of the variables has the same coefficient in the two equations, we can eliminate that variable by subtracting one equation from the other.

It may be necessary to multiply one of the equations by a constant to make the coefficients of x or y the same in the two equations.

Note: This example shows that the point $(1, -2)$ is the point of intersection of the graphs of the two linear relations.

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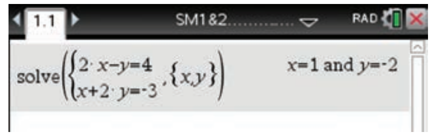
Using the TI-Nspire

Calculator application

Simultaneous equations can be solved in a **Calculator** application.

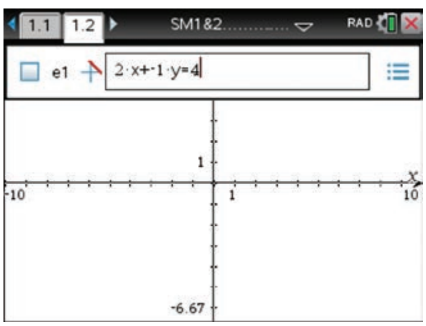
- Use (menu) > **Algebra** > **Solve System of Equations** > **Solve System of Equations**.
- Complete the pop-up screen.
- Enter the equations as shown to give the solution to the simultaneous equations $2x - y = 4$ and $x + 2y = -3$.

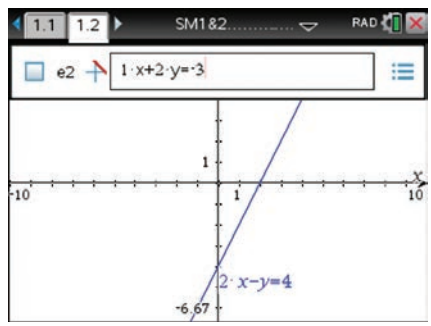


Graphs application

Simultaneous equations can also be solved graphically in a **Graphs** application.

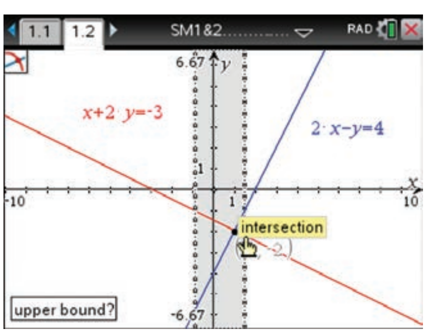
- Equations of the form $ax + by = c$ can be entered directly using (menu) > **Graph Entry/Edit** > **Equation** > **Line** > **Line Standard**.
- Alternatively, rearrange each equation to make y the subject, and enter as a standard function (e.g. $f1(x) = 2x - 4$).

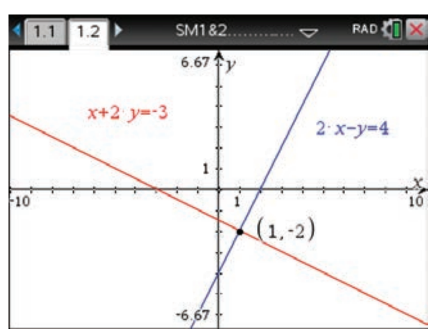




Note: If the Entry Line is not visible, press (tab). Pressing (enter) will hide the Entry Line. If you want to add more equations, use ▼ to add the next equation.

- The intersection point is found using (menu) > **Analyze Graph** > **Intersection**.
- Move the cursor to the left of the intersection point (lower bound), click, and move to the right of the intersection point (upper bound).
- Click to paste the coordinates to the screen.





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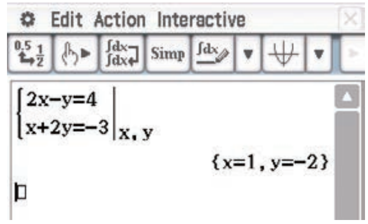
Using the Casio ClassPad

To solve the simultaneous equations algebraically:

- Open the Main $\sqrt{\alpha}$ application and turn on the keyboard.
- In Math1, tap the simultaneous equations icon.
- Enter the two equations as shown.
- Type x, y in the bottom-right square to indicate the variables.
- Tap EXE.

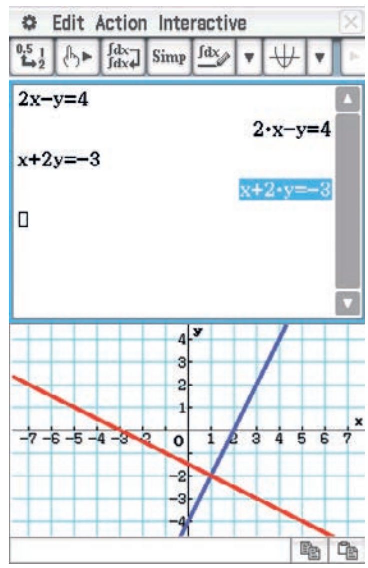


There are two methods for solving simultaneous equations graphically.

Method 1

In the Main $\sqrt{\alpha}$ application:

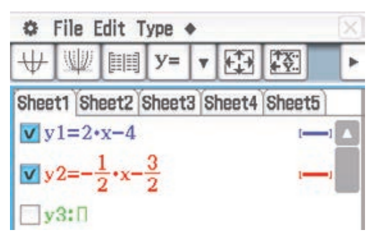
- Enter the equation $2x - y = 4$ and tap EXE.
- Enter the equation $x + 2y = -3$ and tap EXE.
- Select the graph icon from the toolbar to insert a graph window. An appropriate window can be set by selecting **Zoom** > **Quick** > **Quick Standard**.
- Highlight each equation and drag it into the graph window.
- To find the point of intersection, go to **Analysis** > **G-Solve** > **Intersection**.

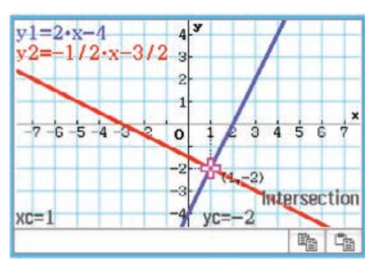


Method 2

For this method, the equations need to be rearranged to make y the subject. In this form, the equations are $y = 2x - 4$ and $y = -\frac{1}{2}x - \frac{3}{2}$.

- Open the menu; select **Graph & Table**.
- Tap in the working line of $y1$ and enter $2x - 4$.
- Tap in the working line of $y2$ and enter $-\frac{1}{2}x - \frac{3}{2}$.
- Tick the boxes for $y1$ and $y2$.
- Select the graph icon from the toolbar.
- Go to **Analysis** > **G-Solve** > **Intersection**.
- If necessary, the view window settings can be adjusted by tapping the view window icon or by selecting **Zoom** > **Quick** > **Quick Standard**.





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Section summary

- An equation is solved by finding the value or values of the variables that would make the statement true.
- A linear equation is one in which the 'unknown' is to the first power.
- There are often several different ways to solve a linear equation. The following steps provide some suggestions:
  1. Expand brackets and, if the equation involves fractions, multiply through by the lowest common denominator of the terms.
  2. Group all of the terms containing a variable on one side of the equation and the terms without the variable on the other side.
- Methods for solving simultaneous linear equations in two variables by hand:

  Substitution
  - Make one of the variables the subject in one of the equations.
  - Substitute for that variable in the other equation.

  Elimination
  - Choose one of the two variables to eliminate.
  - Obtain the same or opposite coefficients for this variable in the two equations. To do this, multiply both sides of one or both equations by a number.
  - Add or subtract the two equations to eliminate the chosen variable.

Exercise 1C

Example 7a

1 Solve the following linear equations:

a $3x + 7 = 15$ **b** $8 - \frac{x}{2} = -16$ **c** $42 + 3x = 22$

d $\frac{2x}{3} - 15 = 27$ **e** $5(2x + 4) = 13$ **f** $-3(4 - 5x) = 24$

g $3x + 5 = 8 - 7x$ **h** $2 + 3(x - 4) = 4(2x + 5)$ **i** $\frac{2x}{5} - \frac{3}{4} = 5x$

j $6x + 4 = \frac{x}{3} - 3$

Example 7b

2 Solve the following linear equations:

a $\frac{x}{2} + \frac{2x}{5} = 16$ **b** $\frac{3x}{4} - \frac{x}{3} = 8$

c $\frac{3x - 2}{2} + \frac{x}{4} = -8$ **d** $\frac{5x}{4} - \frac{4}{3} = \frac{2x}{5}$

e $\frac{x - 4}{2} + \frac{2x + 5}{4} = 6$ **f** $\frac{3 - 3x}{10} - \frac{2(x + 5)}{6} = \frac{1}{20}$

g $\frac{3 - x}{4} - \frac{2(x + 1)}{5} = -24$ **h** $\frac{-2(5 - x)}{8} + \frac{6}{7} = \frac{4(x - 2)}{3}$

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Example 8

3 Solve each of the following pairs of simultaneous equations:

a $3x + 2y = 2$
$2x - 3y = 6$

b $5x + 2y = 4$
$3x - y = 6$

c $2x - y = 7$
$3x - 2y = 2$

d $x + 2y = 12$
$x - 3y = 2$

e $7x - 3y = -6$
$x + 5y = 10$

f $15x + 2y = 27$
$3x + 7y = 45$

1D Solving problems with linear equations

Many problems can be solved by translating them into mathematical language and using an appropriate mathematical technique to find the solution. By representing the unknown quantity in a problem with a symbol (called a pronumeral or a variable) and constructing an equation from the information, the value of the unknown can be found by solving the equation.

Before constructing the equation, each variable and what it stands for (including the units) should be stated. All the elements of the equation must be in units of the same system.

Example 9

For each of the following, form the relevant linear equation and solve it for x:

a The length of the side of a square is $(x - 6)$ cm. Its perimeter is 52 cm.

b The perimeter of a square is $(2x + 8)$ cm. Its area is 100 cm^2.

Solution

a Perimeter = 4 × side length

Therefore

$$4(x - 6) = 52$$
$$x - 6 = 13$$
and so $x = 19$

b The perimeter of the square is $2x + 8$.

The length of one side is $\frac{2x + 8}{4} = \frac{x + 4}{2}$.

Thus the area is

$$\left(\frac{x + 4}{2}\right)^2 = 100$$

As the side length must be positive, this gives the linear equation

$$\frac{x + 4}{2} = 10$$

Therefore $x = 16$.

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Example 10

An athlete trains for an event by gradually increasing the distance she runs each week over a five-week period. If she runs an extra 5 km each successive week and over the five weeks runs a total of 175 km, how far did she run in the first week?

Solution

Let the distance run in the first week be x km.

Then the distance run in the second week is $x + 5$ km, and the distance run in the third week is $x + 10$ km, and so on.

The total distance run is $x + (x + 5) + (x + 10) + (x + 15) + (x + 20)$ km.

$$\begin{aligned} \therefore \quad 5x + 50 &= 175 \\ 5x &= 125 \\ x &= 25 \end{aligned}$$

The distance she ran in the first week was 25 km.

Example 11

A man bought 14 CDs at a sale. Some cost \$15 each and the remainder cost \$12.50 each. In total he spent \$190. How many \$15 CDs and how many \$12.50 CDs did he buy?

Solution

Let n be the number of CDs costing \$15.
Then $14 - n$ is the number of CDs costing \$12.50.

$$\begin{aligned} \therefore \quad 15n + 12.5(14 - n) &= 190 \\ 15n + 175 - 12.5n &= 190 \\ 2.5n + 175 &= 190 \\ 2.5n &= 15 \\ n &= 6 \end{aligned}$$

He bought 6 CDs costing \$15 and 8 CDs costing \$12.50.

Section summary

Steps for solving a word problem with a linear equation

- Read the question carefully and write down the known information clearly.
- Identify the unknown quantity that is to be found.
- Assign a variable to this quantity.
- Form an expression in terms of x (or the variable being used) and use the other relevant information to form the equation.
- Solve the equation.
- Write a sentence answering the initial question.

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Exercise 1D

Skillsheet

1 For each of the cases below, write down a relevant equation involving the variables defined, and solve the equation for parts **a**, **b** and **c**.

Example 9

- **a** The length of the side of a square is $(x - 2)$ cm. Its perimeter is 60 cm.
- **b** The perimeter of a square is $(2x + 7)$ cm. Its area is 49 cm^2.
- **c** The length of a rectangle is $(x - 5)$ cm. Its width is $(12 - x)$ cm. The rectangle is twice as long as it is wide.
- **d** The length of a rectangle is $(2x + 1)$ cm. Its width is $(x - 3)$ cm. The perimeter of the rectangle is y cm.
- **e** n people each have a meal costing $\$p$. The total cost of the meal is $\$Q$.
- **f** S people each have a meal costing $\$p$. A 10% service charge is added to the cost. The total cost of the meal is $\$R$.
- **g** A machine working at a constant rate produces n bolts in 5 minutes. It produces 2400 bolts in 1 hour.
- **h** The radius of a circle is $(x + 3)$ cm. A sector subtending an angle of 60° at the centre is cut off. The arc length of the minor sector is a cm.

Example 10

2 Bronwyn and Noel have a women's clothing shop in Summerland. Bronwyn manages the shop and her sales are going up steadily over a particular period of time. They are going up by \$500 per week. If over a five-week period her sales total \$17 500, how much did she earn in the first week?

Example 11

3 Bronwyn and Noel have a women's clothing shop in Summerland. Sally, Adam and baby Lana came into the shop and Sally bought dresses and handbags. The dresses cost \$65 each and the handbags cost \$26 each. Sally bought 11 items and in total she spent \$598. How many dresses and how many handbags did she buy?

4 A rectangular courtyard is three times as long as it is wide. If the perimeter of the courtyard is 67 m, find the dimensions of the courtyard.

5 A wine merchant buys 50 cases of wine. He pays full price for half of them, but gets a 40% discount on the remainder. If he paid a total of \$2260, how much was the full price of a single case?

6 A real-estate agent sells 22 houses in six months. He makes a commission of \$11 500 per house on some and \$13 000 per house on the remainder. If his total commission over the six months was \$272 500, on how many houses did he make a commission of \$11 500?

7 Three boys compare their marble collections. The first boy has 14 fewer than the second boy, who has twice as many as the third. If between them they have 71 marbles, how many does each boy have?

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8 Three girls are playing Scrabble. At the end of the game, their three scores add up to 504. Annie scored 10% more than Belinda, while Cassie scored 60% of the combined scores of the other two. What did each player score?

9 A biathlon event involves running and cycling. Kim can cycle 30 km/h faster than she can run. If Kim spends 48 minutes running and a third as much time again cycling in an event that covers a total distance of 60 km, how fast can she run?

10 The mass of a molecule of a certain chemical compound is 2.45×10^{-22} g. If each molecule is made up of two carbon atoms and six oxygen atoms and the mass of an oxygen atom is one-third that of a carbon atom, find the mass of an oxygen atom.

11 Mother's pearl necklace fell to the floor. One-sixth of the pearls rolled under the fridge, one-third rolled under the couch, one-fifth of them behind the book shelf, and nine were found at her feet. How many pearls are there?

12 Parents say they don't have favourites, but everyone knows that's a lie. A father distributes \$96 to his three children according to the following instructions: The middle child receives \$12 less than the oldest, and the youngest receives one-third as much as the middle child. How much does each receive?

13 Kavindi has achieved an average mark of 88% on her first four maths tests. What mark would she need on her fifth test to increase her average to 90%?

14 In a particular class, 72% of the students have black hair. Five black-haired students leave the class, so that now 65% of the students have black hair. How many students were originally in the class?

15 Two tanks are being emptied. Tank A contains 100 litres of water and tank B contains 120 litres of water. Water runs from Tank A at 2 litres per minute, and water runs from tank B at 3 litres per minute. After how many minutes will the amount of water in the two tanks be the same?

16 Suppose that candle A is initially 10 cm tall and burns out after 2 hours. Candle B is initially 8 cm tall and burns out after 4 hours. Both candles are lit at the same time. Assuming 'constant burning rates':

a When is the height of candle A the same as the height of candle B?

b When is the height of candle A half the height of candle B?

c When is candle A 1 cm taller than candle B?

17 A motorist drove 320 km in $\frac{10}{3}$ hours. He drove part of the way at an average speed of 100 km/h and the rest of the way at an average speed of 90 km/h. What is the distance he travelled at 100 km/h?

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18 Jarmila travels regularly between two cities. She takes $\frac{14}{3}$ hours if she travels at her usual speed. If she increase her speed by 3 km/h, she can reduce her time taken by 20 minutes. What is her usual speed?

1E Solving problems with simultaneous linear equations

When the relationship between two quantities is linear, we can find the constants which determine this linear relationship if we are given two sets of information satisfying the relationship. Simultaneous linear equations enable this to be done.

Another situation in which simultaneous linear equations may be used is where it is required to find the point of the Cartesian plane which satisfies two linear relations.

Example 12

There are two possible methods for paying gas bills:

Method A A fixed charge of \$25 per quarter + 50c per unit of gas used

Method B A fixed charge of \$50 per quarter + 25c per unit of gas used

Determine the number of units which must be used before method B becomes cheaper than method A.

Solution

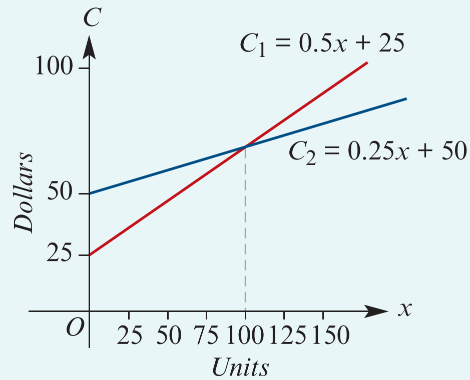
Let C_1 = charge (\$) using method A

C_2 = charge (\$) using method B

x = number of units of gas used

Then $C_1 = 25 + 0.5x$

$C_2 = 50 + 0.25x$



From the graph we see that method B is cheaper if the number of units exceeds 100.

The solution can be obtained by solving simultaneous linear equations:

$$\begin{aligned} C_1 &= C_2 \\ 25 + 0.5x &= 50 + 0.25x \\ 0.25x &= 25 \\ x &= 100 \end{aligned}$$

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Example 13

If 3 kg of jam and 2 kg of butter cost \$29, and 6 kg of jam and 3 kg of butter cost \$54, find the cost per kilogram of jam and butter.

Solution

Let the cost of 1 kg of jam be x dollars and the cost of 1 kg of butter be y dollars.

Then $3x + 2y = 29$ (1)

and $6x + 3y = 54$ (2)

Multiply (1) by 2: $6x + 4y = 58$ (1′)

Subtract (1′) from (2): $-y = -4$

$y = 4$

Substitute in (2): $6x + 3(4) = 54$

$6x = 42$

$x = 7$

Jam costs \$7 per kilogram and butter costs \$4 per kilogram.

Section summary

Steps for solving a word problem with simultaneous linear equations

- Read the question carefully and write down the known information clearly.
- Identify the two unknown quantities that are to be found.
- Assign variables to these two quantities.
- Form expressions in terms of x and y (or other suitable variables) and use the other relevant information to form the two equations.
- Solve the system of equations.
- Write a sentence answering the initial question.

Exercise 1E

Example 12

1 A car hire firm offers the option of paying \$108 per day with unlimited kilometres, or \$63 per day plus 32 cents per kilometre travelled. How many kilometres would you have to travel in a given day to make the unlimited-kilometres option more attractive?

2 Company A will cater for your party at a cost of \$450 plus \$40 per guest. Company B offers the same service for \$300 plus \$43 per guest. How many guests are needed before Company A's charge is less than Company B's?

Example 13

3 A basketball final is held in a stadium which can seat 15 000 people. All the tickets have been sold, some to adults at \$45 and the rest for children at \$15. If the revenue from the tickets was \$525 000, find the number of adults who bought tickets.

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4 A contractor employed eight men and three boys for one day and paid them a total of \$2240. Another day he employed six men and eighteen boys for \$4200. What was the daily rate he paid each man and each boy?

5 The sum of two numbers is 212 and their difference is 42. Find the two numbers.

6 A chemical manufacturer wishes to obtain 700 litres of a 24% acid solution by mixing a 40% solution with a 15% solution. How many litres of each solution should be used?

7 Two children had 220 marbles between them. After one child had lost half her marbles and the other had lost 40 marbles, they had an equal number of marbles. How many did each child start with and how many did each child finish with?

8 An investor received \$31 000 interest per annum from a sum of money, with part of it invested at 10% and the remainder at 7% simple interest. She found that if she interchanged the amounts she had invested she could increase her return by \$1000 per annum. Calculate the total amount she had invested.

9 Each adult paid \$30 to attend a concert and each student paid \$20. A total of 1600 people attended. The total paid was \$37 000. How many adults and how many students attended the concert?

1F Substitution and transposition of formulas

An equation that states a relationship between two or more quantities is called a **formula**; e.g. the area of a circle is given by $A = \pi r^2$. The value of A, the subject of the formula, may be found by substituting a given value of r and the value of π.

Example 14

Using the formula $A = \pi r^2$, find the value of A correct to two decimal places if $r = 2.3$ and $\pi = 3.142$ (correct to three decimal places).

Solution

$$\begin{aligned} A &= \pi r^2 \\ &= 3.142(2.3)^2 \\ &= 16.62118 \end{aligned}$$

$\therefore \quad A = 16.62$ correct to two decimal places

The formula $A = \pi r^2$ can also be transposed to make r the subject.

When transposing a formula, follow a similar procedure to solving a linear equation. Whatever has been done to the variable required is 'undone'.

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Example 15

a Transpose the formula $A = \pi r^2$ to make r the subject.

b Hence find the value of r correct to three decimal places if $A = 24.58$ and $\pi = 3.142$ (correct to three decimal places).

Solution

a $A = \pi r^2$

$\frac{A}{\pi} = r^2$

$r = \sqrt{\frac{A}{\pi}}$

b $r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{24.58}{3.142}}$

$= 2.79697\ldots$

$r = 2.797$ correct to three decimal places

Section summary

- A formula relates different quantities: for example, the formula $A = \pi r^2$ relates the radius r with the area A of the circle.
- The variable on the left is called the subject of the formula: for example, in the formula $A = \pi r^2$, the subject is A.
- To calculate the value of a variable which is not the subject of a formula:
  Method 1 Substitute the values for the known variables, then solve the resulting equation for the unknown variable.
  Method 2 Rearrange to make the required variable the subject, then substitute values.

Exercise 1F

Example 14

1 Substitute the specified values to evaluate each of the following, giving the answers correct to two decimal places:

a v if $v = u + at$ and $u = 15, a = 2, t = 5$

b I if $I = \frac{PrT}{100}$ and $P = 600, r = 5.5, T = 10$

c V if $V = \pi r^2 h$ and $r = 4.25, h = 6$

d S if $S = 2\pi r(r + h)$ and $r = 10.2, h = 15.6$

e V if $V = \frac{4}{3}\pi r^2 h$ and $r = 3.58, h = 11.4$

f s if $s = ut + \frac{1}{2}at^2$ and $u = 25.6, t = 3.3, a = -1.2$

g T if $T = 2\pi\sqrt{\frac{\ell}{g}}$ and $\ell = 1.45, g = 9.8$

h f if $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ and $v = 3, u = 7$

i c if $c^2 = a^2 + b^2$ and $a = 8.8, b = 3.4$

j v if $v^2 = u^2 + 2as$ and $u = 4.8, a = 2.5, s = 13.6$

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Example 15

2 Transpose each of the following to make the symbol in brackets the subject:

a $v = u + at$ (a)

b $S = \frac{n}{2}(a + \ell)$ (ℓ)

c $A = \frac{1}{2}bh$ (b)

d $P = I^2R$ (I)

e $s = ut + \frac{1}{2}at^2$ (a)

f $E = \frac{1}{2}mv^2$ (v)

g $Q = \sqrt{2gh}$ (h)

h $-xy - z = xy + z$ (x)

i $\frac{ax + by}{c} = x - b$ (x)

j $\frac{mx + b}{x - b} = c$ (x)

3 The formula $F = \frac{9C}{5} + 32$ is used to convert temperatures given in degrees Celsius (C) to degrees Fahrenheit (F).

a Convert 28 degrees Celsius to degrees Fahrenheit.

b Transpose the formula to make C the subject and find C if $F = 135°$.

4 The sum, S, of the interior angles of a polygon with n sides is given by the formula $S = 180(n - 2)$.

a Find the sum of the interior angles of an octagon.

b Transpose the formula to make n the subject and hence determine the number of sides of a polygon whose interior angles add up to 1260°.

5 The volume, V, of a right cone is given by the formula $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height of the cone.

a Find the volume of a cone with radius 3.5 cm and height 9 cm.

b Transpose the formula to make h the subject and hence find the height of a cone with base radius 4 cm and volume 210 cm^3.

c Transpose the formula to make r the subject and hence find the radius of a cone with height 10 cm and volume 262 cm^3.

6 For a particular type of sequence of numbers, the sum (S) of the terms in the sequence is given by the formula

$$S = \frac{n}{2}(a + \ell)$$

where n is the number of terms in the sequence, a is the first term and ℓ is the last term.

a Find the sum of such a sequence of seven numbers whose first term is -3 and whose last term is 22.

b What is the first term of such a sequence of 13 numbers whose last term is 156 and whose sum is 1040?

c How many terms are there in the sequence $25 + 22 + 19 + \cdots + (-5) = 110$?

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1G Algebraic fractions

The principles involved in addition, subtraction, multiplication and division of algebraic fractions are the same as for simple numerical fractions.

► Addition and subtraction

To add or subtract, all fractions must be written with a common denominator.

Example 16

Simplify:

a $\frac{x}{3}+\frac{x}{4}$

b $\frac{2}{x}+\frac{3a}{4}$

c $\frac{5}{x+2}-\frac{4}{x-1}$

d $\frac{4}{x+2}-\frac{7}{(x+2)^2}$

Solution

a $\frac{x}{3}+\frac{x}{4}=\frac{4x+3x}{12}$

$=\frac{7x}{12}$

b $\frac{2}{x}+\frac{3a}{4}=\frac{8+3ax}{4x}$

c $\frac{5}{x+2}-\frac{4}{x-1}=\frac{5(x-1)-4(x+2)}{(x+2)(x-1)}$

$=\frac{5x-5-4x-8}{(x+2)(x-1)}$

$=\frac{x-13}{(x+2)(x-1)}$

d $\frac{4}{x+2}-\frac{7}{(x+2)^2}=\frac{4(x+2)-7}{(x+2)^2}$

$=\frac{4x+1}{(x+2)^2}$

► Multiplication and division

Before multiplying and dividing algebraic fractions, it is best to factorise numerators and denominators where possible so that common factors can be readily identified.

Example 17

Simplify:

a $\frac{3x^2}{10y^2}\times\frac{5y}{12x}$

b $\frac{2x-4}{x-1}\times\frac{x^2-1}{x-2}$

c $\frac{x^2-1}{2x-2}\times\frac{4x}{x^2+4x+3}$

d $\frac{x^2+3x-10}{x^2-x-2}\div\frac{x^2+6x+5}{3x+3}$

Solution

a $\frac{3x^2}{10y^2}\times\frac{5y}{12x}=\frac{x}{8y}$

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b $\dfrac{2x-4}{x-1} \times \dfrac{x^2-1}{x-2} = \dfrac{2(x-2)}{x-1} \times \dfrac{(x-1)(x+1)}{x-2}$

$= 2(x+1)$

c $\dfrac{x^2-1}{2x-2} \times \dfrac{4x}{x^2+4x+3} = \dfrac{(x-1)(x+1)}{2(x-1)} \times \dfrac{4x}{(x+1)(x+3)}$

$= \dfrac{2x}{x+3}$

d $\dfrac{x^2+3x-10}{x^2-x-2} \div \dfrac{x^2+6x+5}{3x+3} = \dfrac{(x+5)(x-2)}{(x-2)(x+1)} \times \dfrac{3(x+1)}{(x+1)(x+5)}$

$= \dfrac{3}{x+1}$

More examples

The following two examples involve algebraic fractions and rational indices.

Example 18

Express $\dfrac{3x^3}{\sqrt{4-x}} + 3x^2\sqrt{4-x}$ as a single fraction.

Solution

$$\begin{aligned}
\frac{3x^3}{\sqrt{4-x}} + 3x^2\sqrt{4-x} &= \frac{3x^3 + 3x^2\sqrt{4-x}\,\sqrt{4-x}}{\sqrt{4-x}} \\
&= \frac{3x^3 + 3x^2(4-x)}{\sqrt{4-x}} \\
&= \frac{12x^2}{\sqrt{4-x}}
\end{aligned}$$

Example 19

Express $(x-4)^{\frac{1}{5}} - (x-4)^{-\frac{4}{5}}$ as a single fraction.

Solution

$$\begin{aligned}
(x-4)^{\frac{1}{5}} - (x-4)^{-\frac{4}{5}} &= (x-4)^{\frac{1}{5}} - \frac{1}{(x-4)^{\frac{4}{5}}} \\
&= \frac{(x-4)^{\frac{1}{5}}(x-4)^{\frac{4}{5}} - 1}{(x-4)^{\frac{4}{5}}} \\
&= \frac{x-5}{(x-4)^{\frac{4}{5}}}
\end{aligned}$$

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Section summary

- Simplifying algebraic fractions
  - First factorise the numerator and denominator.
  - Then cancel any factors common to the numerator and denominator.
- Adding and subtracting algebraic fractions
  - First obtain a common denominator and then add or subtract.
- Multiplying and dividing algebraic fractions
  - First factorise each numerator and denominator completely.
  - Then complete the calculation by cancelling common factors.

Exercise 1G

Skillsheet

Example 16

1 Simplify each of the following:

a $\dfrac{2x}{3}+\dfrac{3x}{2}$ **b** $\dfrac{3a}{2}-\dfrac{a}{4}$ **c** $\dfrac{3h}{4}+\dfrac{5h}{8}-\dfrac{3h}{2}$

d $\dfrac{3x}{4}-\dfrac{y}{6}-\dfrac{x}{3}$ **e** $\dfrac{3}{x}+\dfrac{2}{y}$ **f** $\dfrac{5}{x-1}+\dfrac{2}{x}$

g $\dfrac{3}{x-2}+\dfrac{2}{x+1}$ **h** $\dfrac{2x}{x+3}-\dfrac{4x}{x-3}-\dfrac{3}{2}$ **i** $\dfrac{4}{x+1}+\dfrac{3}{(x+1)^2}$

j $\dfrac{a-2}{a}+\dfrac{a}{4}+\dfrac{3a}{8}$ **k** $2x-\dfrac{6x^2-4}{5x}$ **l** $\dfrac{2}{x+4}-\dfrac{3}{x^2+8x+16}$

m $\dfrac{3}{x-1}+\dfrac{2}{(x-1)(x+4)}$ **n** $\dfrac{3}{x-2}-\dfrac{2}{x+2}+\dfrac{4}{x^2-4}$

o $\dfrac{5}{x-2}+\dfrac{3}{x^2+5x+6}+\dfrac{2}{x+3}$ **p** $x-y-\dfrac{1}{x-y}$

q $\dfrac{3}{x-1}-\dfrac{4x}{1-x}$ **r** $\dfrac{3}{x-2}+\dfrac{2x}{2-x}$

Example 17

2 Simplify each of the following:

a $\dfrac{x^2}{2y}\times\dfrac{4y^3}{x}$ **b** $\dfrac{3x^2}{4y}\times\dfrac{y^2}{6x}$ **c** $\dfrac{4x^3}{3}\times\dfrac{12}{8x^4}$

d $\dfrac{x^2}{2y}\div\dfrac{3xy}{6}$ **e** $\dfrac{4-x}{3a}\times\dfrac{a^2}{4-x}$ **f** $\dfrac{2x+5}{4x^2+10x}$

g $\dfrac{(x-1)^2}{x^2+3x-4}$ **h** $\dfrac{x^2-x-6}{x-3}$ **i** $\dfrac{x^2-5x+4}{x^2-4x}$

j $\dfrac{5a^2}{12b^2}\div\dfrac{10a}{6b}$ **k** $\dfrac{x-2}{x}\div\dfrac{x^2-4}{2x^2}$ **l** $\dfrac{x+2}{x(x-3)}\div\dfrac{4x+8}{x^2-4x+3}$

m $\dfrac{2x}{x-1}\div\dfrac{4x^2}{x^2-1}$ **n** $\dfrac{x^2-9}{x+2}\times\dfrac{3x+6}{x-3}\div\dfrac{9}{x}$ **o** $\dfrac{3x}{9x-6}\div\dfrac{6x^2}{x-2}\times\dfrac{2}{x+5}$

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3 Express each of the following as a single fraction:

a $\dfrac{1}{x-3}+\dfrac{2}{x-3}$ **b** $\dfrac{2}{x-4}+\dfrac{2}{x-3}$ **c** $\dfrac{3}{x+4}+\dfrac{2}{x-3}$

d $\dfrac{2x}{x-3}+\dfrac{2}{x+4}$ **e** $\dfrac{1}{(x-5)^2}+\dfrac{2}{x-5}$ **f** $\dfrac{3x}{(x-4)^2}+\dfrac{2}{x-4}$

g $\dfrac{1}{x-3}-\dfrac{2}{x-3}$ **h** $\dfrac{2}{x-3}-\dfrac{5}{x+4}$ **i** $\dfrac{2x}{x-3}+\dfrac{3x}{x+3}$

j $\dfrac{1}{(x-5)^2}-\dfrac{2}{x-5}$ **k** $\dfrac{2x}{(x-6)^3}-\dfrac{2}{(x-6)^2}$ **l** $\dfrac{2x+3}{x-4}-\dfrac{2x-4}{x-3}$

Example 18

4 Express each of the following as a single fraction:

a $\sqrt{1-x}+\dfrac{2}{\sqrt{1-x}}$ **b** $\dfrac{2}{\sqrt{x-4}}+\dfrac{2}{3}$ **c** $\dfrac{3}{\sqrt{x+4}}+\dfrac{2}{\sqrt{x+4}}$

d $\dfrac{3}{\sqrt{x+4}}+\sqrt{x+4}$ **e** $\dfrac{3x^3}{\sqrt{x+4}}-3x^2\sqrt{x+4}$ **f** $\dfrac{3x^3}{2\sqrt{x+3}}+3x^2\sqrt{x+3}$

Example 19

5 Simplify each of the following:

a $(6x-3)^{\frac{1}{3}}-(6x-3)^{-\frac{2}{3}}$ **b** $(2x+3)^{\frac{1}{3}}-2x(2x+3)^{-\frac{2}{3}}$ **c** $(3-x)^{\frac{1}{3}}-2x(3-x)^{-\frac{2}{3}}$

1H Literal equations

A literal equation in x is an equation whose solution will be expressed in terms of pronumerals rather than numbers.

For the equation $2x+5=7$, the solution is the number 1.

For the literal equation $ax+b=c$, the solution is $x=\dfrac{c-b}{a}$.

Literal equations are solved in the same way as numerical equations. Essentially, the literal equation is transposed to make x the subject.

Example 20

Solve the following for x:

a $px-q=r$ **b** $ax+b=cx+d$ **c** $\dfrac{a}{x}=\dfrac{b}{2x}+c$

Solution

a
$$\begin{aligned} px-q&=r\\ px&=r+q\\ \therefore\quad x&=\frac{r+q}{p}\end{aligned}$$

b
$$\begin{aligned} ax+b&=cx+d\\ ax-cx&=d-b\\ x(a-c)&=d-b\\ \therefore\quad x&=\frac{d-b}{a-c}\end{aligned}$$

c Multiply both sides by $2x$:
$$\begin{aligned} \frac{a}{x}&=\frac{b}{2x}+c\\ 2a&=b+2xc\\ 2a-b&=2xc\\ \therefore\quad x&=\frac{2a-b}{2c}\end{aligned}$$

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Simultaneous literal equations

Simultaneous literal equations are solved by the same methods that are used for solving simultaneous equations, i.e. substitution and elimination.

Example 21

Solve each of the following pairs of simultaneous equations for x and y:

a $y = ax + c$
$y = bx + d$

b $ax - y = c$
$x + by = d$

Solution

a Equate the two expressions for y:

$$ax + c = bx + d$$
$$ax - bx = d - c$$
$$x(a - b) = d - c$$

Thus $$x = \frac{d - c}{a - b}$$

and $$y = a\left(\frac{d - c}{a - b}\right) + c$$
$$= \frac{ad - ac + ac - bc}{a - b}$$
$$= \frac{ad - bc}{a - b}$$

b We will use the method of elimination, and eliminate y.

First number the two equations:

$$ax - y = c \quad (1)$$
$$x + by = d \quad (2)$$

Multiply (1) by b:

$$abx - by = bc \quad (1')$$

Add (1′) and (2):

$$abx + x = bc + d$$
$$x(ab + 1) = bc + d$$
$$\therefore \quad x = \frac{bc + d}{ab + 1}$$

Substitute in (1):

$$y = ax - c$$
$$= a\left(\frac{bc + d}{ab + 1}\right) - c$$
$$= \frac{ad - c}{ab + 1}$$

Section summary

- An equation for the variable x in which all the coefficients of x, including the constants, are pronumerals is known as a **literal equation**.
- The methods for solving linear literal equations or simultaneous linear literal equations are exactly the same as when the coefficients are given numbers.

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Exercise 1H

Example 20

1 Solve each of the following for x:

a $ax + n = m$

b $ax + b = bx$

c $\dfrac{ax}{b} + c = 0$

d $px = qx + 5$

e $mx + n = nx - m$

f $\dfrac{1}{x + a} = \dfrac{b}{x}$

g $\dfrac{b}{x - a} = \dfrac{2b}{x + a}$

h $\dfrac{x}{m} + n = \dfrac{x}{n} + m$

i $-b(ax + b) = a(bx - a)$

j $p^2(1 - x) - 2pqx = q^2(1 + x)$

k $\dfrac{x}{a} - 1 = \dfrac{x}{b} + 2$

l $\dfrac{x}{a - b} + \dfrac{2x}{a + b} = \dfrac{1}{a^2 - b^2}$

m $\dfrac{p - qx}{t} + p = \dfrac{qx - t}{p}$

n $\dfrac{1}{x + a} + \dfrac{1}{x + 2a} = \dfrac{2}{x + 3a}$

2 For the simultaneous equations $ax + by = p$ and $bx - ay = q$, show that $x = \dfrac{ap + bq}{a^2 + b^2}$ and $y = \dfrac{bp - aq}{a^2 + b^2}$.

3 For the simultaneous equations $\dfrac{x}{a} + \dfrac{y}{b} = 1$ and $\dfrac{x}{b} + \dfrac{y}{a} = 1$, show that $x = y = \dfrac{ab}{a + b}$.

Example 21

4 Solve each of the following pairs of simultaneous equations for x and y:

a $ax + y = c$
$x + by = d$

b $ax - by = a^2$
$bx - ay = b^2$

c $ax + by = t$
$ax - by = s$

d $ax + by = a^2 + 2ab - b^2$
$bx + ay = a^2 + b^2$

e $(a + b)x + cy = bc$
$(b + c)y + ax = -ab$

f $3(x - a) - 2(y + a) = 5 - 4a$
$2(x + a) + 3(y - a) = 4a - 1$

5 Write s in terms of a only in the following pairs of equations:

a $s = ah$
$h = 2a + 1$

b $s = ah$
$h = a(2 + h)$

c $as = a + h$
$h + ah = 1$

d $as = s + h$
$ah = a + h$

e $s = h^2 + ah$
$h = 3a^2$

f $as = a + 2h$
$h = a - s$

g $s = 2 + ah + h^2$
$h = a - \dfrac{1}{a}$

h $3s - ah = a^2$
$as + 2h = 3a$

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1I Using a CAS calculator for algebra

Using the TI-Nspire

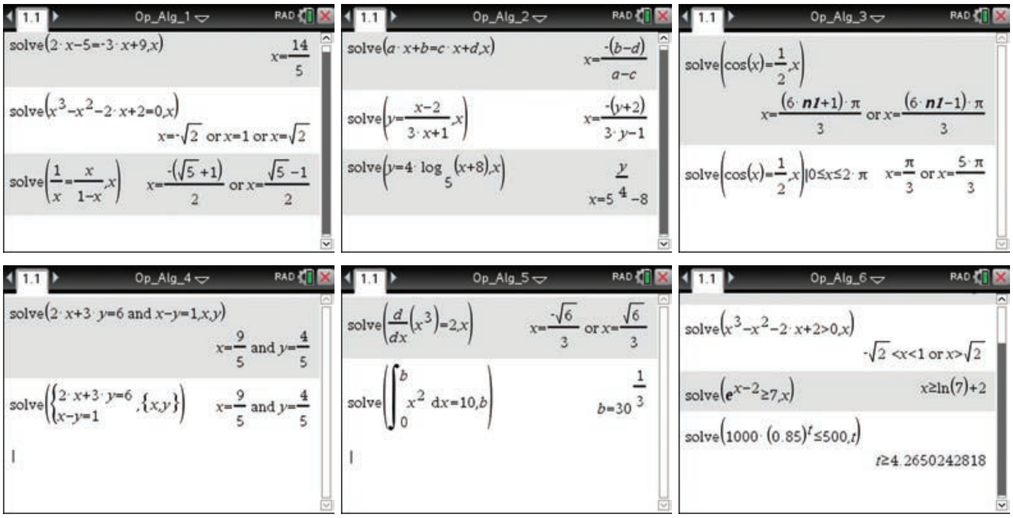
This section demonstrates the basic algebra commands of the TI-Nspire. To access them, open a **Calculator** application (on > **New Document** > **Add Calculator**) and select menu > **Algebra**. The three main commands are solve, factor and expand.

1: Solve

This command is used to solve equations, simultaneous equations and some inequalities.

An approximate (decimal) answer can be obtained by pressing ctrl enter or by including a decimal number in the expression.

The following screens illustrate its use.

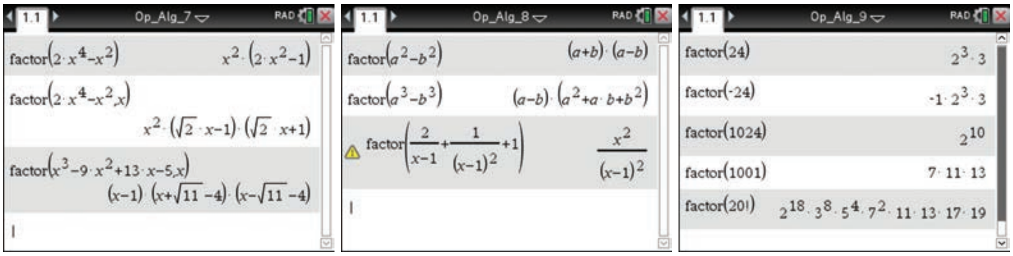


2: Factor

This command is used for factorisation.

Factorisation over the rational numbers is obtained by not specifying the variable, whereas factorisation over the real numbers is obtained by specifying the variable.

The following screens illustrate its use.



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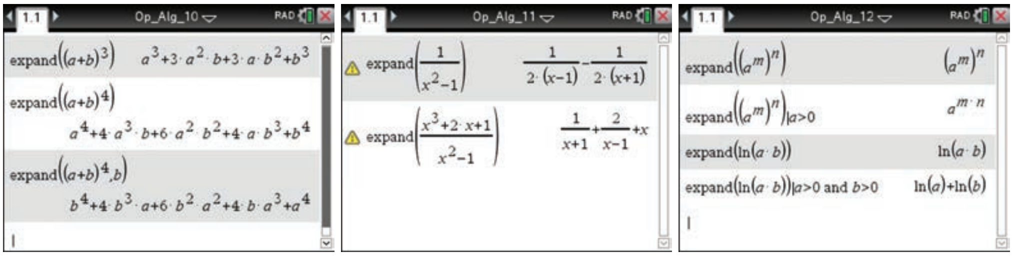
3: Expand

This command is used for expanding out expressions.

By specifying the variable, the expanded expression will be ordered in decreasing powers of that variable. Symbolic expressions can only be expanded for an appropriate domain.



Using the Casio ClassPad

This section explores the $\sqrt[\text{Main}]{\alpha}$ application.

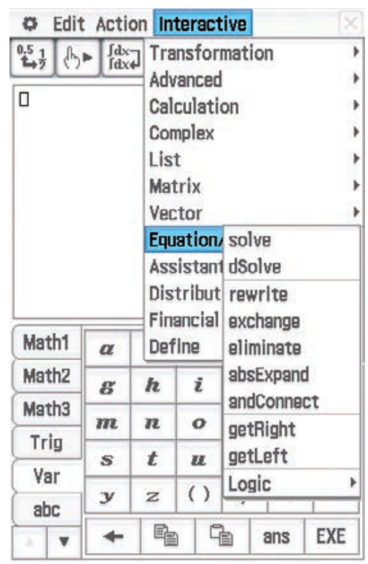
The **Interactive** menu is easiest to use with the stylus and the soft keyboards Math1, Math2 and Math3.

Solve

This is used to solve equations and inequalities.

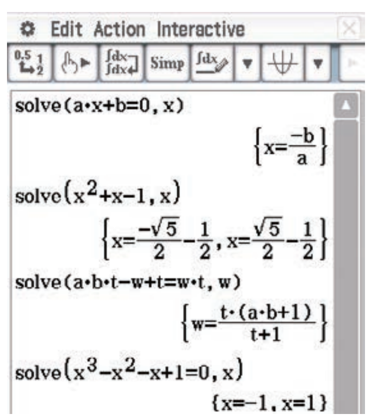
The variables x, y and z are found on the hard keyboard. Other variables may be entered using the Var keyboard. Variables are shown in bold italics.

Note: The abc keyboard allows you to type text; however, the letters are not always recognised as variables. If you use the abc keyboard for variables, then you must type $a \times x$, for example, because ax will be treated as text.



Examples:

- Enter $ax + b = 0$ and highlight it with the stylus. Go to **Interactive** > **Equation/Inequality** > **solve** and ensure the variable selected is x.
- Enter $x^2 + x - 1$ and follow the same instructions as above. Note that '= 0' has been omitted in this example. It is not necessary to enter the right-hand side of an equation if it is zero.
- To solve $abt - w + t = wt$ for w, select w as the variable.
- Solve $x^3 - x^2 - x + 1 = 0$ for x.



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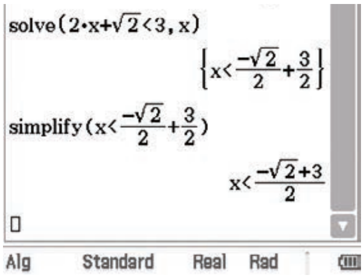
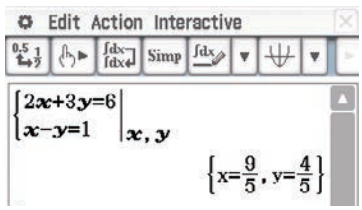
More examples:

- Solve $2x + \sqrt{2} < 3$ for x.



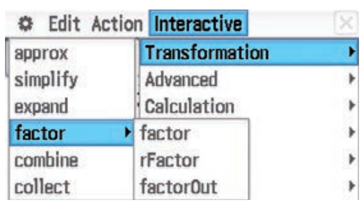
Note: For the square root, use [√■] from [Math1].
The inequality signs (<, >, ≤, ≥) are in [Math3].

- If the answer is not in the form required, it is often possible to cut and paste it into the next entry line and use **Interactive** > **Transformation** > **simplify** as shown on the right.
- To solve a pair of simultaneous equations, tap [{■] from the [Math1] keyboard and enter the equations and variables as shown.
- For more than two equations, tap [{■] until the required number of equations is displayed.



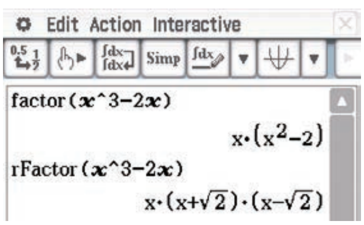
Factor

To factorise is to write an expression as a product of simpler expressions. This command is found in **Interactive** > **Transformation** > **factor**.



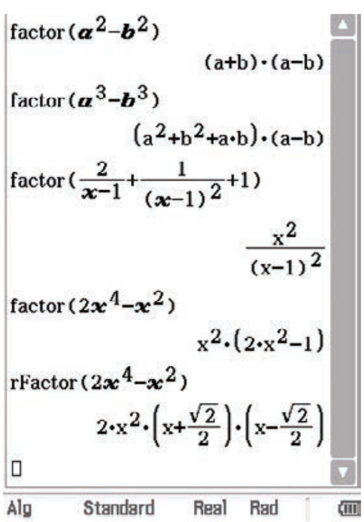
Examples:

- To factorise $x^3 - 2x$ over the rational numbers, use **factor**.
- To factorise over the real numbers, use **rFactor**.



More examples:

- Factorise $a^2 - b^2$.
- Factorise $a^3 - b^3$.
- Factorise $\dfrac{2}{x-1} + \dfrac{1}{(x-1)^2} + 1$.
- Factorise $2x^4 - x^2$ over the rationals.
- Factorise $2x^4 - x^2$ over the reals.



This command can also be used to give the prime decomposition (factors) of integers.

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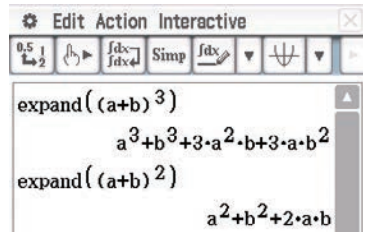
Expand

An expression can be expanded out by using **Interactive** > **Transformation** > **expand**.

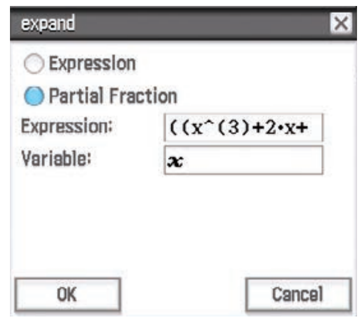


Examples:

- Expand $(a + b)^3$.
- Expand $(a + b)^2$.

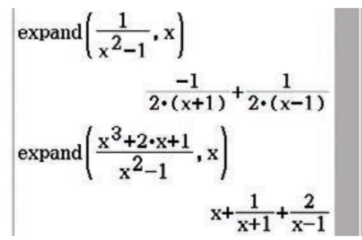
This command can also be used to form partial fractions.

In this case, enter and highlight the expression, go to **Interactive** > **Transformation** > **expand**, select the **Partial Fraction** option as shown on the right, and set the variable as x.



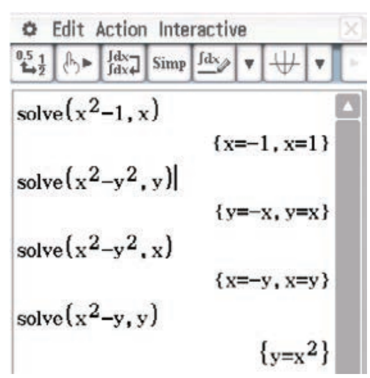
Examples:

- Expand $\dfrac{1}{x^2 - 1}$.
- Expand $\dfrac{x^3 + 2x + 1}{x^2 - 1}$.



Zeroes

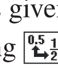
To find the zeroes of an expression in Main $\sqrt{\alpha}$, select **Interactive** > **Equation/Inequality** > **solve** and ensure that you set the variable. The calculator assumes that you are solving an equation for which one side is zero.



Examples:

- Zeroes of $x^2 - 1$ for x.
- Zeroes of $x^2 - y^2$ for y.
- Zeroes of $x^2 - y^2$ for x.
- Zeroes of $x^2 - y$ for y.
- Zeroes of $x^2 - 4x + 8$ for x. No solutions.
- Zeroes of $x^2 - 4x + 1$ for x. Two solutions.
- Zeroes of $x^2 - 4x + 4$ for x. One solution.

Approximate

Switch mode in the status bar to Decimal. If an answer is given in Standard (exact) mode, it can be converted by highlighting the answer and tapping  in the toolbar.

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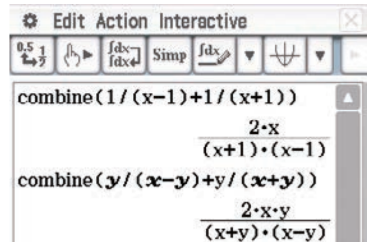
Combining fractions

This command returns the answer as a single fraction with the denominator in factored form.

Examples:

- Enter and highlight $1/(x-1)+1/(x+1)$. Then select **Interactive** > **Transformation** > **combine**.
- Enter and highlight $y/(x-y)+y/(x+y)$. Then select **Interactive** > **Transformation** > **combine**.



Exercise 1I

This exercise provides practice in some of the skills associated with a CAS calculator. Other exercises in this chapter can be attempted with CAS, but it is recommended that you also use this chapter to develop your 'by hand' skills.

1 Solve each of the following equations for x:

a $\dfrac{a(a-x)}{b}-\dfrac{b(b+x)}{a}=x$

b $2(x-3)+(x-2)(x-4)=x(x+1)-33$

c $\dfrac{x+a}{x+b}=1-\dfrac{x}{x-b}$

d $\dfrac{x+a}{x-c}+\dfrac{x+c}{x-a}=2$

2 Factorise each of the following:

a $x^2y^2-x^2-y^2+1$

b $x^3-2-x+2x^2$

c $a^4-8a^2b-48b^2$

d $a^2+2bc-(c^2+2ab)$

3 Solve each of the following pairs of simultaneous equations for x and y:

a $axy+b=(a+c)y$
$bxy+a=(b+c)y$

b $x(b-c)+by-c=0$
$y(c-a)-ax+c=0$

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Chapter summary

AS

Nrich

- **Indices**
  - $a^m \times a^n = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $(ab)^n = a^n b^n$
  - $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
  - $a^{-n} = \frac{1}{a^n}$
  - $a^0 = 1$
  - $a^{\frac{1}{n}} = \sqrt[n]{a}$
- A number is expressed in **standard form** or **scientific notation** when written as a product of a number between 1 and 10 and an integer power of 10; e.g. 1.5×10^8
- **Linear equations**

  First identify the steps done to construct an equation; the equation is then solved by 'undoing' these steps. This is achieved by doing 'the opposite' in 'reverse order'.

  e.g.: Solve $3x + 4 = 16$ for x.

  Note that x has been multiplied by 3 and then 4 has been added.

  Subtract 4 from both sides: $3x = 12$

  Divide both sides by 3: $x = 4$
- An equation that states a relationship between two or more quantities is called a **formula**; e.g. the area of a circle is given by $A = \pi r^2$. The value of A, the subject of the formula, may be found by substituting a given value of r and the value of π.

  A formula can be transposed to make a different variable the subject by using a similar procedure to solving linear equations, i.e. whatever has been done to the variable required is 'undone'.
- A **literal equation** is solved using the same techniques as for a numerical equation: transpose the literal equation to make the required variable the subject.

Technology-free questions

1 Simplify the following:

a $(x^3)^4$ **b** $(y^{-12})^{\frac{3}{4}}$ **c** $3x^{\frac{3}{2}} \times -5x^4$ **d** $(x^3)^{\frac{4}{3}} \times x^{-5}$

2 Express the product $32 \times 10^{11} \times 12 \times 10^{-5}$ in standard form.

3 Simplify the following:

a $\frac{3x}{5} + \frac{y}{10} - \frac{2x}{5}$ **b** $\frac{4}{x} - \frac{7}{y}$ **c** $\frac{5}{x+2} + \frac{2}{x-1}$

d $\frac{3}{x+2} + \frac{4}{x+4}$ **e** $\frac{5x}{x+4} + \frac{4x}{x-2} - \frac{5}{2}$ **f** $\frac{3}{x-2} - \frac{6}{(x-2)^2}$

4 Simplify the following:

a $\frac{x+5}{2x-6} \div \frac{x^2+5x}{4x-12}$ **b** $\frac{3x}{x+4} \div \frac{12x^2}{x^2-16}$

c $\frac{x^2-4}{x-3} \times \frac{3x-9}{x+2} \div \frac{9}{x+2}$ **d** $\frac{4x+20}{9x-6} \times \frac{6x^2}{x+5} \div \frac{2}{3x-2}$

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5 The human body can produce 2.5 million red blood cells per second. If a person donates 500 mL of blood, how long will it take to replace the red blood cells if a litre of blood contains 5×10^{12} red blood cells?

6 The Sun is approximately 1.5×10^8 km from Earth and a comet is approximately 3×10^6 km from Earth. How many times further from Earth than the comet is the Sun?

7 Swifts Creek Soccer Team has played 54 matches over the past three seasons. They have drawn one-third of their games and won twice as many games as they have lost. How many games have they lost?

8 A music store specialises in three types of CDs: classical, blues and heavy metal. In one week they sold a total of 420 CDs. They sold 10% more classical than blues, while sales of heavy metal CDs constituted 50% more than the combined sales of classical and blues CDs. How many of each type of CD did they sell?

9 The volume, V, of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the base and h is the height of the cylinder.

a Find the volume of a cylinder with base radius 5 cm and height 12 cm.

b Transpose the formula to make h the subject and hence find the height of a cylinder with a base radius of 5 cm and a volume of 585 cm^3.

c Transpose the formula to make r the subject and hence find the radius of a cylinder with a height of 6 cm and a volume of 768 cm^3.

10 Solve for x:

a $xy + ax = b$

b $\dfrac{a}{x} + \dfrac{b}{x} = c$

c $\dfrac{x}{a} = \dfrac{x}{b} + 2$

d $\dfrac{a - dx}{d} + b = \dfrac{ax + d}{b}$

11 Simplify:

a $\dfrac{p}{p+q} + \dfrac{q}{p-q}$

b $\dfrac{1}{x} - \dfrac{2y}{xy - y^2}$

c $\dfrac{x^2 + x - 6}{x + 1} \times \dfrac{2x^2 + x - 1}{x + 3}$

d $\dfrac{2a}{2a + b} \times \dfrac{2ab + b^2}{ba^2}$

12 A is three times as old as B. In three years' time, B will be three times as old as C. In fifteen years' time, A will be three times as old as C. What are their present ages?

13 **a** Solve the following simultaneous equations for a and b:

$$a - 5 = \frac{1}{7}(b + 3) \qquad b - 12 = \frac{1}{5}(4a - 2)$$

b Solve the following simultaneous equations for x and y:

$$(p - q)x + (p + q)y = (p + q)^2$$
$$qx - py = q^2 - pq$$

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14 A man has to travel 50 km in 4 hours. He does it by walking the first 7 km at x km/h, cycling the next 7 km at $4x$ km/h and motoring the remainder at $(6x + 3)$ km/h. Find x.

15 Simplify each of the following:

a $2n^2 \times 6nk^2 \div (3n)$ **b** $\dfrac{8c^2x^3y}{6a^2b^3c^3} \div \dfrac{\frac{1}{2}xy}{15abc^2}$

16 Solve the equation $\dfrac{x+5}{15} - \dfrac{x-5}{10} = 1 + \dfrac{2x}{15}$.

Multiple-choice questions

1 For non-zero values of x and y, if $5x + 2y = 0$, then the ratio $\dfrac{y}{x}$ is equal to

A $-\dfrac{5}{2}$ **B** $-\dfrac{2}{5}$ **C** $\dfrac{2}{5}$ **D** 1 **E** $\dfrac{5}{4}$

2 The solution of the simultaneous equations $3x + 2y = 36$ and $3x - y = 12$ is

A $x = \dfrac{20}{3},\ y = 8$ **B** $x = 2,\ y = 0$ **C** $x = 1,\ y = -3$

D $x = \dfrac{20}{3},\ y = 6$ **E** $x = \dfrac{3}{2},\ y = -\dfrac{3}{2}$

3 The solution of the equation $t - 9 = 3t - 17$ is

A $t = -4$ **B** $t = \dfrac{11}{2}$ **C** $t = 4$ **D** $t = 2$ **E** $t = -2$

4 If $m = \dfrac{n-p}{n+p}$, then $p =$

A $\dfrac{n(1-m)}{1+m}$ **B** $\dfrac{n(m-1)}{1+m}$ **C** $\dfrac{n(1+m)}{1-m}$ **D** $\dfrac{n(1+m)}{m-1}$ **E** $\dfrac{m(n-1)}{m+1}$

5 $\dfrac{3}{x-3} - \dfrac{2}{x+3} =$

A 1 **B** $\dfrac{x+15}{x^2-9}$ **C** $\dfrac{15}{x-9}$ **D** $\dfrac{x+3}{x^2-9}$ **E** $-\dfrac{1}{9}$

6 $9x^2y^3 \div (15(xy)^3)$ is equal to

A $\dfrac{9x}{15}$ **B** $\dfrac{18xy}{5}$ **C** $\dfrac{3y}{5x}$ **D** $\dfrac{3x}{5}$ **E** $\dfrac{3}{5x}$

7 Transposing the formula $V = \dfrac{1}{3}h(\ell + w)$ gives $\ell =$

A $\dfrac{hw}{3V}$ **B** $\dfrac{3V}{h} - w$ **C** $\dfrac{3V-2w}{h}$ **D** $\dfrac{3Vh}{2} - w$ **E** $\dfrac{1}{3}h(V+w)$

8 $\dfrac{(3x^2y^3)^2}{2x^2y} =$

A $\dfrac{9}{2}x^2y^7$ **B** $\dfrac{9}{2}x^2y^5$ **C** $\dfrac{9}{2}x^6y^7$ **D** $\dfrac{9}{2}x^6y^6$ **E** $\dfrac{9}{2}x^2y^4$

Review

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9 If X is 50% greater than Y and Y is 20% less than Z, then

A X is 30% greater than Z **B** X is 20% greater than Z **C** X is 20% less than Z
D X is 10% less than Z **E** X is 10% greater than Z

10 The average of two numbers is $5x + 4$. One of the numbers is x. The other number is

A $4x + 4$ **B** $9x + 8$ **C** $9x + 4$ **D** $10x + 8$ **E** $3x + 1$

Extended-response questions

1 Jack cycles home from work, a distance of $10x$ km. Benny leaves at the same time and drives the $40x$ km to his home.

a Write an expression in terms of x for the time taken for Jack to reach home if he cycles at an average speed of 8 km/h.

b Write an expression in terms of x for the time taken for Benny to reach home if he drives at an average speed of 70 km/h.

c In terms of x, find the difference in times of the two journeys.

d If Jack and Benny arrive at their homes 30 minutes apart:

i find x, correct to three decimal places

ii find the distance from work of each home, correct to the nearest kilometre.

2 Sam's plastic dinghy has sprung a leak and water is pouring in the hole at a rate of 27 000 cm^3 per minute. He grabs a cup and frantically starts bailing the water out at a rate of 9000 cm^3 per minute. The dinghy is shaped like a circular prism (cylinder) with a base radius of 40 cm and a height of 30 cm.

a How fast is the dinghy filling with water?

b Write an equation showing the volume of water, V cm^3, in the dinghy after t minutes.

c Find an expression for the depth of water, h cm, in the dinghy after t minutes.

d If Sam is rescued after 9 minutes, is this before or after the dinghy has completely filled with water?

3 Henry and Thomas Wong collect basketball cards. Henry has five-sixths the number of cards that Thomas has. The Wright family also collect cards. George Wright has half as many cards again as Thomas, Sally Wright has 18 fewer than Thomas, and Zeb Wright has one-third the number Thomas has.

a Write an expression for each child's number of cards in terms of the number Thomas has.

b The Wright family owns six more cards than the Wong family. Write an equation representing this information.

c Solve the equation from part **b** and use the result to find the number of cards each child has collected.

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4 The gravitational force between two objects, F N, is given by the formula

$$F = \frac{6.67 \times 10^{-11} m_1 m_2}{r^2}$$

where m_1 and m_2 are the masses (in kilograms) of the two objects and r is the distance (in metres) between them.

a What is the gravitational force between two objects each weighing 200 kg if they are 12 m apart? Express the answer in standard form (to two significant figures).

b Transpose the above formula to make m_1 the subject.

c The gravitational force between a planet and an object 6.4×10^6 m away from the centre of the planet is found to be 2.4×10^4 N. If the object has a mass of 1500 kg, calculate the approximate mass of the planet, giving the answer in standard form (to two significant figures).

5 A water storage reservoir is 3 km wide, 6 km long and 30 m deep. (The water storage reservoir is assumed to be a cuboid.)

a Write an equation to show the volume of water, V m^3, in the reservoir when it is d metres full.

b Calculate the volume of water, V_F m^3, in the reservoir when it is completely filled.

The water flows from the reservoir down a long pipe to a hydro-electric power station in a valley below. The amount of energy, E J, that can be obtained from a full reservoir is given by the formula

$$E = kV_F h$$

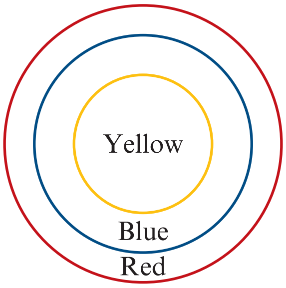
where k is a constant and h m is the length of the pipe.

c Find k, given that $E = 1.06 \times 10^{15}$ when $h = 200$, expressing the answer in standard form correct to three significant figures.

d How much energy could be obtained from a full reservoir if the pipe was 250 m long?

e If the rate of water falling through the pipe is 5.2 m^3/s, how many days without rain could the station operate before emptying an initially full reservoir?

6 A new advertising symbol is to consist of three concentric circles as shown, with the outer circle having a radius of 10 cm. It is desired that the three coloured regions cover the same area. Find the radius of the innermost circle in the figure shown.



7 Temperatures in Fahrenheit (F) can be converted to Celsius (C) by the formula

$$C = \frac{5}{9}(F - 32)$$

Find the temperature which has the same numerical value in both scales.

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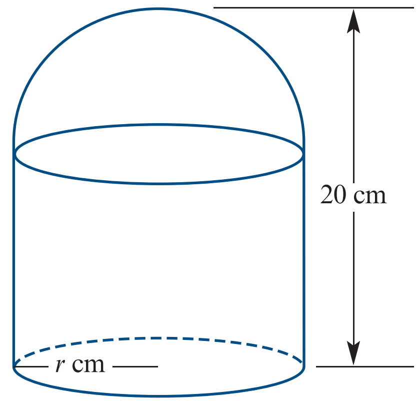
8 A cyclist goes up a long slope at a constant speed of 15 km/h. He turns around and comes down the slope at a constant speed of 40 km/h. Find his average speed over a full circuit.

9 A container has a cylindrical base and a hemispherical top, as shown in the figure. The height of the container is 20 cm and its capacity is to be exactly 2 litres. Let r cm be the radius of the base.



a Express the height of the cylinder, h cm, in terms of r.

b **i** Express the volume of the container in terms of r.

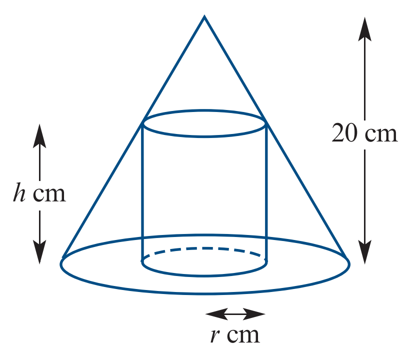
ii Find r and h if the volume is 2 litres.

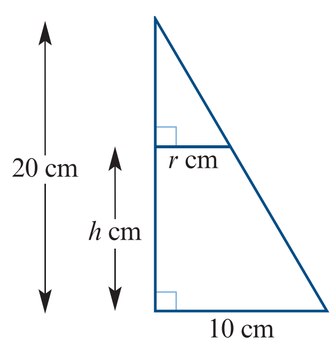
10 **a** Two bottles contain mixtures of wine and water. In bottle A there is two times as much wine as water. In bottle B there is five times as much water as wine. Bottle A and bottle B are used to fill a third bottle, which has a capacity of 1 litre. How much liquid must be taken from each of bottle A and bottle B if the third bottle is to contain equal amounts of wine and water?

b Repeat for the situation where the ratio of wine to water in bottle A is $1 : 2$ and the ratio of wine to water in bottle B is $3 : 1$.

c Generalise the result for the ratio $m : n$ in bottle A and $p : q$ in bottle B.

11 A cylinder is placed so as to fit into a cone as shown in the diagram.
The cone has a height of 20 cm and a base radius of 10 cm. The cylinder has a height of h cm and a base radius of r cm.





a Use similar triangles to find h in terms of r.

b The volume of the cylinder is given by the formula $V = \pi r^2 h$. Find the volume of the cylinder in terms of r.

c Use a CAS calculator to find the values of r and h for which the volume of the cylinder is 500 cm^3.

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