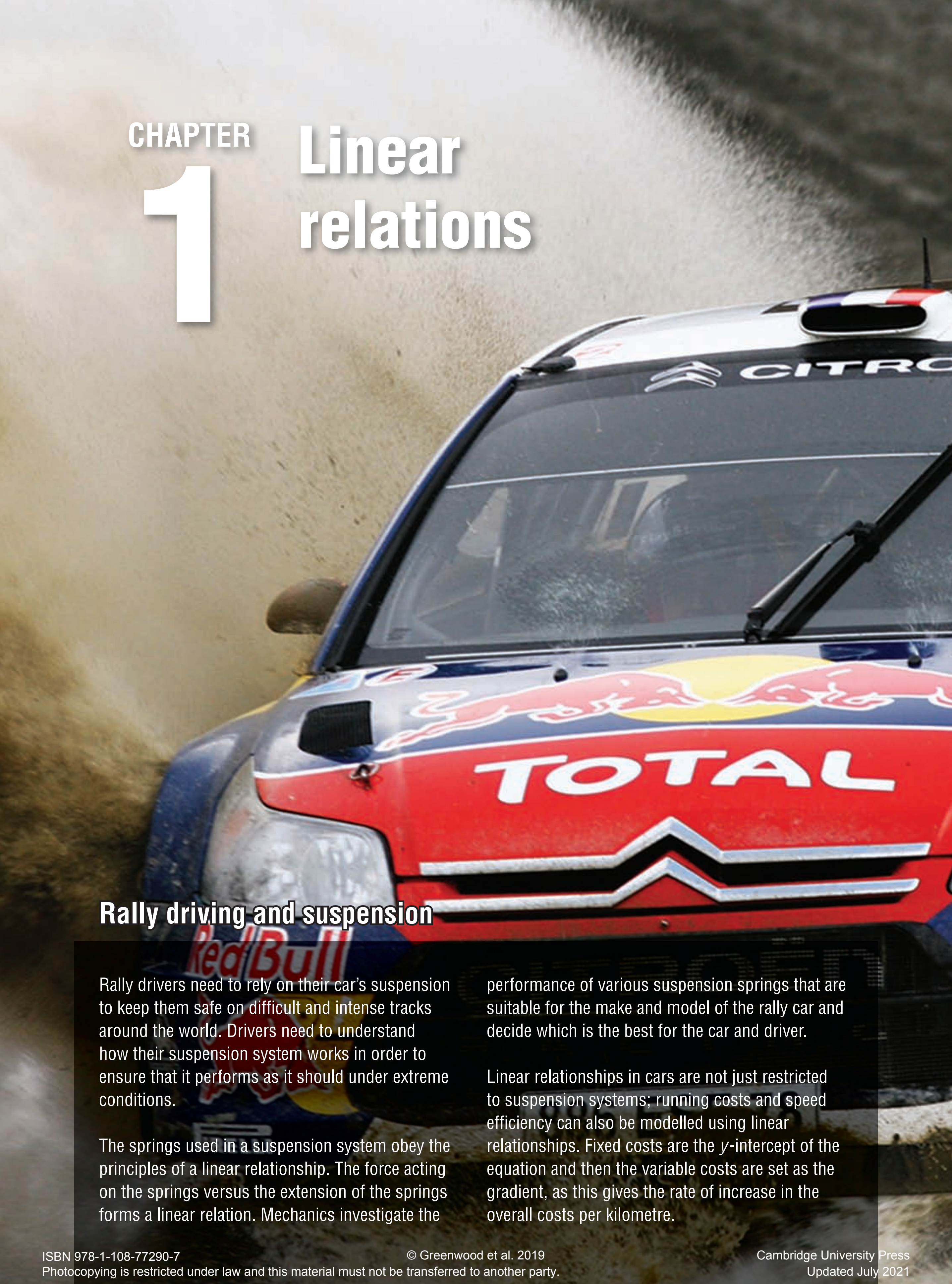
CHAPTER 1

Linear relations



Rally driving and suspension

Rally drivers need to rely on their car's suspension to keep them safe on difficult and intense tracks around the world. Drivers need to understand how their suspension system works in order to ensure that it performs as it should under extreme conditions.

The springs used in a suspension system obey the principles of a linear relationship. The force acting on the springs versus the extension of the springs forms a linear relation. Mechanics investigate the performance of various suspension springs that are suitable for the make and model of the rally car and decide which is the best for the car and driver.

Linear relationships in cars are not just restricted to suspension systems; running costs and speed efficiency can also be modelled using linear relationships. Fixed costs are the y-intercept of the equation and then the variable costs are set as the gradient, as this gives the rate of increase in the overall costs per kilometre.

ISBN 978-1-108-77290-7

Cambridge University Press

Updated July 2021

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In this chapter

Victorian Curriculum

NUMBER AND ALGEBRA
Patterns and algebra

Factorise algebraic expressions by taking out a common algebraic factor (VCMNA329)

Apply the four operations to simple algebraic fractions with numerical denominators (VCMNA331)

Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term (VCMNA333)

Linear and non-linear relationships

Solve problems involving linear equations, including those derived from formulas (VCMNA335)

Solve linear inequalities and graph their solutions on a number line (VCMNA336)

Solve simultaneous linear equations, using algebraic and graphical techniques including using digital technology (VCMNA337)

Solve problems involving gradients of parallel and perpendicular lines (VCMNA338)

Solve linear equations involving simple algebraic fractions (VCMNA340)

1A Review of algebra CONSOLIDATING

Learning intentions

- To review the key words of algebra: term, coefficient, expression, equation
- To review how to combine like terms under addition and subtraction
- To review how to multiply and divide algebraic terms and apply the distributive law to expand brackets
- To review how to factorise an expression using the highest common factor
- To be able to substitute values for pronumerals and evaluate expressions

Algebra involves the use of pronumerals (or variables), which are letters representing numbers. Combinations of numbers and pronumerals form terms (numbers and pronumerals connected by multiplication and division), expressions (a term or terms connected by addition and subtraction) and equations (mathematical statements that include an equals sign). Skills in algebra are important when dealing with the precise and concise nature of mathematics. The complex nature of many problems in finance and engineering usually result in algebraic expressions and equations that need to be simplified and solved.

Stockmarket traders rely on financial modelling based on complex algebraic expressions. Financial market analysts and computer systems analysts require advanced algebraic skills.

LESSON STARTER Mystery problem

Between one school day and the next, the following problem appeared on a student noticeboard.

Prove that $8 - x^2 + \frac{3x - 9}{3} + 5(x - 1) - x(6 - x) = 0$.

- By working with the left-hand side of the equation, show that this equation is true for any value of x.
- At each step of your working, discuss what algebraic processes you have used.

KEY IDEAS

- Key words in algebra:
  - **term:** $5x$, $7x^2y$, $\frac{2a}{3}$, 7 (a constant term)
  - **coefficient:** -3 is the coefficient of x^2 in $7 - 3x^2$; 1 is the coefficient of y in $y + 7x$.
  - **expression:** $7x$, $3x + 2xy$, $\frac{x + 3}{2}$, $\sqrt{2a^2 - b}$
  - **equation:** $x = 5$, $7x - 1 = 2$, $x^2 + 2x = -4$
- Expressions can be evaluated by substituting a value for each pronumeral (variable).
  - Order of operations are followed: First brackets, then indices, then multiplication and division, then addition and subtraction, working then from left to right.

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

- **Like terms** have the same pronumeral part and, using addition and subtraction, can be collected to form a single term.
  For example, $3x - 7x + x = -3x$
  $6a^2b - ba^2 = 5a^2b$
  Note that $a^2b = ba^2$

- The symbols for multiplication (×) and division (÷) are usually not shown.
  $7 \times x \div y = \frac{7x}{y}$
  $-6a^2b \div (ab) = \frac{-6a^2b}{ab}$
  $= -6a$

- The **distributive law** is used to expand brackets.
  - $a(b + c) = ab + ac$ $\quad 2(x + 7) = 2x + 14$
  - $a(b - c) = ab - ac$ $\quad -x(3 - x) = -3x + x^2$

- **Factorisation** involves writing expressions as a product of factors.
  - Many expressions can be factorised by taking out the highest common factor (HCF).
    $15 = 3 \times 5$
    $3x - 12 = 3(x - 4)$
    $9x^2y - 6xy + 3x = 3x(3xy - 2y + 1)$

- Other general properties are:
  - **associative** $a \times (b \times c) = (a \times b) \times c$ or $a + (b + c) = (a + b) + c$
  - **commutative** $ab = ba$ or $a + b = b + a$ $\left(\text{Note: } \frac{a}{b} \neq \frac{b}{a} \text{ and } a - b \neq b - a.\right)$
  - **identity** $a \times 1 = a$ or $a + 0 = a$
  - **inverse** $a \times \frac{1}{a} = 1$ or $a + (-a) = 0$

BUILDING UNDERSTANDING

1. Which of the following is an equation?
   A $3x - 1$ **B** $\frac{x + 1}{4}$ **C** $7x + 2 = 5$ **D** $3x^2y$
2. Which expression contains a term with a coefficient of −9?
   A $8 + 9x$ **B** $2x + 9x^2y$ **C** $9a - 2ab$ **D** $b - 9a^2$
3. State the coefficient of a^2 in these expressions.
   a $a + a^2$ **b** $\frac{3}{2} - 4a^2$ **c** $1 - \frac{a^2}{5}$ **d** $-\frac{7a^2}{3} - 1$
4. Decide whether the following pairs of terms are like terms.
   a xy and $2yx$ **b** $7a^2b$ and $-7ba^2$ **c** $-4abc^2$ and $8ab^2c$
5. Evaluate:
   a $(-3)^2$ **b** $(-2)^3$ **c** -2^3 **d** -3^2

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Example 1 Collecting like terms

Simplify by collecting like terms.

a $7a + 3a$ **b** $3a^2b - 2a^2b$ **c** $5xy + 2xy^2 - 2xy + 3y^2x$

SOLUTION

a $7a + 3a = 10a$

EXPLANATION

Keep the pronumeral and add the coefficients.

b $3a^2b - 2a^2b = a^2b$

$3a^2b$ and $2a^2b$ have the same pronumeral part, so they are like terms. Subtract coefficients and recall that $1a^2b = a^2b$.

c $5xy + 2xy^2 - 2xy + 3y^2x = 3xy + 5xy^2$

Collect like terms, noting that $3y^2x = 3xy^2$. The + or − sign belongs to the term that directly follows it.

Now you try

Simplify by collecting like terms.

a $4a + 13a$ **b** $5ab^2 - 2ab^2$ **c** $3xy + 4x^2y - xy + 2yx^2$

Example 2 Multiplying and dividing expressions

Simplify the following.

a $2h \times 7l$ **b** $-3p^2r \times 2pr$ **c** $-\frac{7xy}{14y}$

SOLUTION

a $2h \times 7l = 14hl$

EXPLANATION

Multiply the numbers and remove the × sign.

b $-3p^2r \times 2pr = -6p^3r^2$

Remember the basic index law: When you multiply terms with the same base you add the powers.

c $-\frac{7xy}{14y} = -\frac{x}{2}$

Cancel the highest common factor of 7 and 14 and cancel the y.

Now you try

Simplify the following.

a $3a \times 6b$ **b** $-2x^2y \times 5xy$ **c** $-\frac{4ab}{8a}$

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Example 3 Expanding the brackets

Expand the following using the distributive law. Simplify where possible.

a $2(x + 4)$ **b** $-3x(x - y)$ **c** $3(x + 2) - 4(2x - 4)$

SOLUTION

a $2(x + 4) = 2x + 8$

b $-3x(x - y) = -3x^2 + 3xy$

c $3(x + 2) - 4(2x - 4) = 3x + 6 - 8x + 16$
$= -5x + 22$

EXPLANATION

$2(x + 4) = 2 \times x + 2 \times 4$

Note that $x \times x = x^2$ and $-3 \times (-1) = 3$.

Expand each pair of brackets and simplify by collecting like terms.

Now you try

Expand the following using the distributive law. Simplify where possible.

a $3(x + 2)$ **b** $-2x(x - y)$ **c** $2(x + 3) - 3(2x - 1)$

Example 4 Factorising simple algebraic expressions

Factorise:

a $3x - 9$ **b** $2x^2 + 4x$

SOLUTION

a $3x - 9 = 3(x - 3)$

b $2x^2 + 4x = 2x(x + 2)$

EXPLANATION

HCF of $3x$ and 9 is 3.

Check that $3(x - 3) = 3x - 9$.

HCF of $2x^2$ and $4x$ is $2x$.

Check that $2x(x + 2) = 2x^2 + 4x$.

Now you try

Factorise:

a $2x - 10$ **b** $3x^2 + 9x$

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Example 5 Evaluating expressions

Evaluate $a^2 - 2bc$ if $a = -3$, $b = 5$ and $c = -1$.

SOLUTION	EXPLANATION
$a^2 - 2bc = (-3)^2 - 2(5)(-1)$	Substitute for each pronumeral:
$= 9 - (-10)$	$(-3)^2 = -3 \times (-3)$ and $2 \times 5 \times (-1) = -10$
$= 19$	To subtract a negative number, add its opposite.

Now you try

Evaluate $b^2 - 3ac$ if $a = 1$, $b = -2$ and $c = -3$.

Exercise 1A

FLUENCY	1, 2–7(1/2)	2–7(1/2)	2–7(1/3)

1 Simplify by collecting like terms.

Example 1a **a** i $5a + 9a$ ii $7a - 2a$

Example 1b **b** i $4a^2b - 2a^2b$ ii $5x^2y - 4x^2y$

Example 1c **c** i $4xy + 3xy^2 - 3xy + 2y^2x$ ii $6ab + 2ab^2 - 2ab + 4b^2a$

Example 1

2 Simplify by collecting like terms.

a $6a + 4a$ **b** $8d + 7d$ **c** $5y - 5y$
d $2xy + 3xy$ **e** $9ab - 5ab$ **f** $4t + 3t + 2t$
g $7b - b + 3b$ **h** $3st^2 - 4st^2$ **i** $4m^2n - 7nm^2$
j $0.3a^2b - ba^2$ **k** $4gh + 5 - 2gh$ **l** $7xy + 5xy - 3y$
m $4a + 5b - a + 2b$ **n** $3jk - 4j + 5jk - 3j$ **o** $2ab^2 + 5a^2b - ab^2 + 5ba^2$
p $3mn - 7m^2n + 6nm^2 - mn$ **q** $4st + 3ts^2 + st - 4s^2t$ **r** $7x^3y^4 - 3xy^2 - 4y^4x^3 + 5y^2x$

Example 2

3 Simplify the following.

a $4a \times 3b$ **b** $5a \times 5b$ **c** $-2a \times 3d$ **d** $5h \times (-2m)$
e $-6h \times (-5t)$ **f** $-5b \times (-6l)$ **g** $2s^2 \times 6t$ **h** $-3b^2 \times 7d^5$
i $4ab \times 2ab^3$ **j** $-6p^2 \times (-4pq)$ **k** $6hi^4 \times (-3h^4i)$ **l** $7mp \times 9mr$
m $\frac{7x}{7}$ **n** $\frac{6ab}{2}$ **o** $-\frac{3a}{9}$ **p** $-\frac{2ab}{8}$
q $\frac{4ab}{2a}$ **r** $-\frac{15xy}{5y}$ **s** $-\frac{4xy}{8x}$ **t** $-\frac{28ab}{56b}$

Example 3a,b

4 Expand the following, using the distributive law.

a $5(x + 1)$ **b** $2(x + 4)$ **c** $3(x - 5)$
d $-5(4 + b)$ **e** $-2(y - 3)$ **f** $-7(a + c)$
g $-6(-m - 3)$ **h** $4(m - 3n + 5)$ **i** $-2(p - 3q - 2)$
j $2x(x + 5)$ **k** $6a(a - 4)$ **l** $-4x(3x - 4y)$
m $3y(5y + z - 8)$ **n** $9g(4 - 2g - 5h)$ **o** $-2a(4b - 7a + 10)$
p $7y(2y - 2y^2 - 4)$ **q** $-3a(2a^2 - a - 1)$ **r** $-t(5t^3 + 6t^2 + 2)$
s $2m(3m^3 - m^2 + 5m)$ **t** $-x(1 - x^3)$ **u** $-3s(2t - s^3)$

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Example 3c

5 Expand and simplify the following, using the distributive law.

a $2(x + 4) + 3(x + 5)$
b $4(a + 2) + 6(a + 3)$
c $6(3y + 2) + 3(y - 3)$
d $3(2m + 3) + 3(3m - 1)$
e $2(2 + 6b) - 3(4b - 2)$
f $3(2t + 3) - 5(2 - t)$
g $2x(x + 4) + x(x + 7)$
h $4(6z - 4) - 3(3z - 3)$
i $3d^2(2d^3 - d) - 2d(3d^4 + 4d^2)$
j $q^3(2q - 5) + q^2(7q^2 - 4q)$

Example 4

6 Factorise:

a $3x - 9$
b $4x - 8$
c $10y + 20$
d $6y + 30$
e $x^2 + 7x$
f $2a^2 + 8a$
g $5x^2 - 5x$
h $9y^2 - 63y$
i $xy - xy^2$
j $x^2y - 4x^2y^2$
k $8a^2b + 40a^2$
l $7a^2b + ab$
m $-5t^2 - 5t$
n $-6mn - 18mn^2$
o $-y^2 - 8yz$

Example 5

7 Evaluate these expressions if $a = -4$, $b = 3$ and $c = -5$.

a $-2a^2$
b $b - a$
c $abc + 1$
d $-ab$
e $\frac{a + b}{2}$
f $\frac{3b - a}{5}$
g $\frac{a^2 - b^2}{c}$
h $\frac{\sqrt{a^2 + b^2}}{\sqrt{c^2}}$

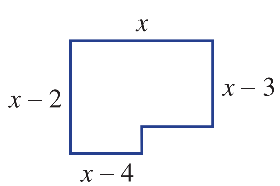
PROBLEM-SOLVING	8	8, 9	9

8 Find an expression for the area of a floor of a rectangular room with the following side lengths. Expand and simplify your answer.

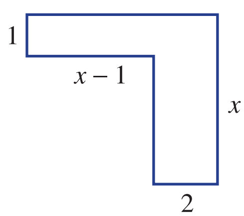
a $x + 3$ and $2x$
b x and $x - 5$

9 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes. (Note: All angles are right angles.)

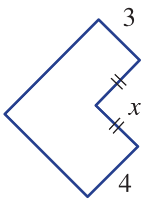
a



b



c



REASONING	10	10, 11	11, 12

10 When $a = -2$ give reasons why:

a $a^2 > 0$
b $-a^2 < 0$
c $a^3 < 0$

11 Decide whether the following are true or false for all values of a and b. If false, give an example to show that it is false.

a $a + b = b + a$
b $a - b = b - a$
c $ab = ba$
d $\frac{a}{b} = \frac{b}{a}$
e $a + (b + c) = (a + b) + c$
f $a - (b - c) = (a - b) - c$
g $a \times (b \times c) = (a \times b) \times c$
h $a \div (b \div c) = (a \div b) \div c$

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

12 **a** Write an expression for the statement ‘the sum of x and y divided by 2’.

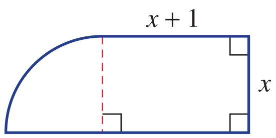
b Explain why the statement above is ambiguous.

c Write an unambiguous statement describing $\frac{a + b}{2}$.

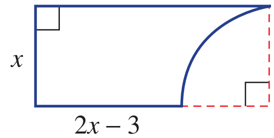
ENRICHMENT: Algebraic circular spaces	–	–	13

13 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes. Your answers may contain π, for example 4π. Do not use decimals.

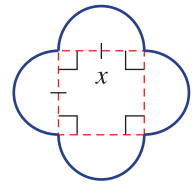
a



b



c



Architects, builders, carpenters and landscapers are among the many occupations that use algebraic formulas to calculate areas and perimeters in daily work.

ISBN 978-1-108-77290-7

1B Multiplying and dividing algebraic fractions

Learning intentions

- To understand that expressions need to be in factorised form in order to cancel common factors
- To know that it is helpful to cancel common factors in fractions before multiplying or dividing
- To be able to multiply and divide fractions involving algebraic expressions

Since pronumerals represent numbers, the rules for algebraic fractions are the same as those for simple numerical fractions. This includes processes such as cancelling common factors, adding or subtracting with a lowest common denominator (LCD) and dividing, by multiplying by the reciprocal of the fraction that follows the division sign. In this section we focus on multiplying and dividing algebraic fractions.

The study of air-conditioning uses algebraic fractions to model airflow, air temperatures and humidity. The mechanical engineers who design ventilation systems, and the electricians who install and repair them, all require algebraic skills.

LESSON STARTER Describe the error

Here are three problems involving algebraic fractions. Each simplification contains one critical error. Find and describe the errors, then give the correct answer.

a $\dfrac{6x - \cancel{8}^{2}}{\cancel{4}_{1}} = \dfrac{6x - 2}{1} = 6x - 2$

b $\dfrac{2a}{9} \div \dfrac{2}{3} = \dfrac{2a}{9} \times \dfrac{2}{3} = \dfrac{4a}{27}$

c $\dfrac{3b}{7} \div \dfrac{2b}{3} = \dfrac{3b}{7} \times \dfrac{3b}{2} = \dfrac{9b^2}{14}$

KEY IDEAS

- Simplify **algebraic fractions** by factorising expressions where possible and cancelling common factors.
- For multiplication, cancel common factors and then multiply the numerators together and the denominators together.
- For division, multiply by the **reciprocal** of the fraction that follows the division sign. The reciprocal of a is $\dfrac{1}{a}$ and the reciprocal of $\dfrac{a}{b}$ is $\dfrac{b}{a}$.

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BUILDING UNDERSTANDING

1 Simplify to find the answer in simplest form.

a $\frac{2}{3} \times \frac{6}{4}$ **b** $\frac{3}{4} \times \frac{10}{9}$ **c** $\frac{4}{7} \div \frac{2}{7}$ **d** $\frac{3}{6} \div \frac{6}{9}$

2 What is the reciprocal of each fraction?

a $\frac{3}{2}$ **b** $\frac{7a}{3}$ **c** $\frac{-4xy}{7t}$ **d** $\frac{-8x^2a}{b^2c}$

3 Simplify by cancelling common factors.

a $\frac{10x}{2}$ **b** $\frac{24x}{6}$ **c** $\frac{5a}{20}$ **d** $\frac{7}{21a}$

Example 6 Cancelling common factors

Simplify by cancelling common factors.

a $\frac{8a^2b}{2a}$ **b** $\frac{3-9x}{3}$

SOLUTION

a $\frac{8a^2b}{2a} = \frac{8^4 \times a^1 \times a \times b}{2_1 \times a_1}$

$= 4ab$

b $\frac{3-9x}{3} = \frac{3^1(1-3x)}{3^1}$

$= 1 - 3x$

EXPLANATION

Cancel the common factors 2 and a.

Factorise the numerator, then cancel the common factor of 3.

Now you try

Simplify by cancelling common factors.

a $\frac{9ab^2}{3b}$ **b** $\frac{5-10x}{5}$

Example 7 Multiplying and dividing algebraic fractions

Simplify the following.

a $\frac{2}{a} \times \frac{a+2}{4}$ **b** $\frac{2x-4}{3} \div \frac{x-2}{6}$

SOLUTION

a $\frac{2^1}{a} \times \frac{a+2}{4_2} = \frac{a+2}{2a}$

EXPLANATION

Cancel the common factor of 2 and then multiply the numerators and the denominators. a cannot be cancelled as it is not a common factor in $a + 2$.

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

b $\frac{2x-4}{3} \div \frac{x-2}{6} = \frac{2x-4}{3} \times \frac{6}{x-2}$

$= \frac{2\cancel{(x-2)}^{1}}{\cancel{3}^{1}} \times \frac{\cancel{6}^{2}}{\cancel{(x-2)}^{1}}$

$= 4$

Multiply by the reciprocal of the second fraction.
Factorise $2x - 4$ and cancel the common factors.

Now you try

Simplify the following.

a $\frac{6}{a} \times \frac{a+1}{12}$

b $\frac{3x-12}{2} \div \frac{x-4}{4}$

Exercise 1B

FLUENCY	1, 2–5(1/2)	2–5(1/2)	2–5(1/3)

1 Simplify by cancelling common factors.

Example 6a

a i $\frac{6a^2b}{2a}$ ii $\frac{10xy^2}{5y}$

Example 6b

b i $\frac{4-8x}{4}$ ii $\frac{5-5x}{5}$

Example 6a

2 Simplify by cancelling common factors.

a $\frac{35x^2}{7x}$
b $\frac{-14x^2y}{7xy}$
c $\frac{-36ab^2}{4ab}$
d $\frac{8xy^3}{-4xy^2}$
e $\frac{-15pq^2}{30p^2q^2}$
f $\frac{-20s}{45s^2t}$
g $\frac{-48x^2}{16xy}$
h $\frac{120ab^2}{140ab}$

Example 6b

3 Simplify by cancelling common factors.

a $\frac{4x+8}{4}$
b $\frac{6a-30}{6}$
c $\frac{6x-18}{2}$
d $\frac{5-15y}{5}$
e $\frac{-2-12b}{-2}$
f $\frac{21x-7}{-7}$
g $\frac{9t-27}{-9}$
h $\frac{44-11x}{-11}$
i $\frac{x^2+2x}{x}$
j $\frac{6x-4x^2}{2x}$
k $\frac{a^2-a}{a}$
l $\frac{7a+14a^2}{21a}$

Example 7a

4 Simplify the following.

a $\frac{3}{x} \times \frac{x-1}{6}$
b $\frac{x+4}{10} \times \frac{2}{x}$
c $\frac{-8a}{7} \times \frac{7}{2a}$
d $\frac{x+3}{9} \times \frac{4}{x+3}$
e $\frac{y-7}{y} \times \frac{5y}{y-7}$
f $\frac{10a^2}{a+6} \times \frac{a+6}{4a}$
g $\frac{2m+4}{m} \times \frac{m}{m+2}$
h $\frac{6-18x}{2} \times \frac{5}{1-3x}$
i $\frac{b-1}{10} \times \frac{-5}{b-1}$

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Example 7b

5 Simplify the following.

a $\frac{x}{5} \div \frac{x}{15}$ **b** $\frac{x+4}{2} \div \frac{x+4}{6}$ **c** $\frac{6x-12}{5} \div \frac{x-2}{3}$

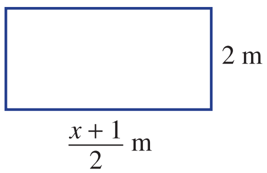
d $\frac{3-6y}{8} \div \frac{1-2y}{2}$ **e** $\frac{2}{a-1} \div \frac{3}{2a-2}$ **f** $\frac{2}{10x-5} \div \frac{10}{2x-1}$

g $\frac{5}{3a+4} \div \frac{15}{-15a-20}$ **h** $\frac{2x-6}{5x-20} \div \frac{x-3}{x-4}$ **i** $\frac{t+1}{9} \div \frac{-t-1}{3}$

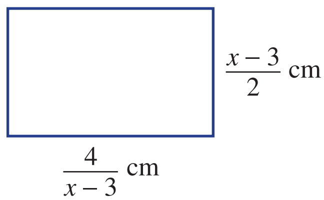
PROBLEM-SOLVING	6	6, 7(1/2)	7–8(1/2)

6 Find a simplified expression for the area of these rectangles.

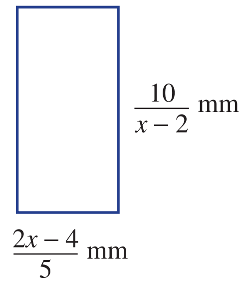
a



b



c



7 Simplify these expressions.

a $\frac{x}{3} \times \frac{9x}{5} \times \frac{15}{3x}$ **b** $\frac{2}{a} \times \frac{a}{5} \times \frac{10}{3a}$

c $\frac{x-1}{2} \times \frac{4x}{2x-2} \times \frac{x+3}{5x}$ **d** $\frac{2x-1}{x} \div \frac{2x-1}{2} \div \frac{1}{2}$

e $\frac{2x-3}{5} \div \frac{14x-21}{10} \div \frac{x}{2}$ **f** $\frac{b^2-b}{b} \div \frac{b-1}{b^2} \times \frac{2}{b-1}$

8 Write the missing algebraic fraction.

a $\frac{x+3}{5} \times \square = 2$ **b** $\frac{1-x}{x} \times \square = 3$

c $\square \div \frac{x}{2} = \frac{3(x+2)}{x}$ **d** $\square \div \frac{2x-2}{3} = \frac{5x}{x-1}$

e $\frac{1}{x} \div \square \times \frac{x-1}{2} = 1$ **f** $\frac{2-x}{7} \times \square \div \frac{5x}{x-1} = x$

REASONING	9(1/2)	9(1/2), 10	9(1/2), 10, 11

9 Recall that $(x-1)^2 = (x-1)(x-1)$. Use this idea to simplify the following.

a $\frac{(x-1)^2}{x-1}$ **b** $\frac{3(x+2)^2}{x+2}$ **c** $\frac{4(x-3)^2}{2(x-3)}$

d $\frac{4(x+2)}{(x+2)^2}$ **e** $\frac{-5(1-x)}{(1-x)^2}$ **f** $\frac{(2x-2)^2}{x-1}$

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10 Prove that the following all simplify to 1.

a $\frac{5x+5}{15} \times \frac{3}{x+1}$ **b** $\frac{3x-21}{2-x} \times \frac{4-2x}{6x-42}$ **c** $\frac{10-5x}{2x+6} \div \frac{20-10x}{4x+12}$

11 **a** Explain why $\frac{x-1}{2} \times \frac{4}{1-x} = \frac{x-1}{2} \times \frac{-4}{x-1}$.

b Use this idea to simplify these expressions.

i $\frac{2-a}{3} \times \frac{7}{a-2}$ **ii** $\frac{6x-3}{x} \div \frac{1-2x}{4}$ **iii** $\frac{18-x}{3x-1} \div \frac{2x-36}{7-21x}$

ENRICHMENT: Simplifying with quadratics	–	–	12(½)

12 You may recall that to factorise a monic quadratic of the form $x^2 + bx + c$ we look for factors of c which add to b. So for example: $x^2 - x - 6 = (x-3)(x+2)$.

So:

$$\frac{x^2-x-6}{6} \times \frac{3}{x-3} = \frac{{}^{1}\cancel{(x-3)}(x+2)}{\cancel{6}^{2}} \times \frac{\cancel{3}^{1}}{\cancel{(x-3)}^{1}}$$
$$= \frac{x+2}{2}$$

Now simplify these algebraic fractions which involve quadratics.

a $\frac{x^2-2x-8}{4} \times \frac{2}{x-4}$ **b** $\frac{x^2+5x+6}{x+2} \times \frac{x}{x+3}$

c $\frac{x+1}{x^2-4x-5} \times \frac{x-5}{3}$ **d** $\frac{3x-27}{4x} \times \frac{2x}{x^2-7x-18}$

e $\frac{4ab}{a^2+a} \div \frac{b}{a^2+2a+1}$ **f** $\frac{a+8}{a^2-5a-6} \div \frac{a^2+5a-24}{a-6}$

g $\frac{(x-y)^2}{xy} \div \frac{x^2-y^2}{x+y}$ **h** $\frac{y^2+4y+4}{x^2y} \div \frac{(y+2)^2}{xy^2+2xy}$

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

1C Adding and subtracting algebraic fractions

Learning intentions

- To know how to find the lowest common denominator of algebraic fractions
- To be able to combine numerators using expansion and addition of like terms
- To be able to add and subtract algebraic fractions

The sum or difference of two or more algebraic fractions can be simplified in a similar way to numerical fractions with the use of a common denominator.

Electricians, electrical and electronic engineers work with algebraic fractions when modelling the flow of electric energy in circuits. The application of algebra when using electrical formulas is essential in these professions.

LESSON STARTER Spot the difference

Here are two sets of simplification steps. One set has one critical error. Can you find and correct it?

$$\frac{2}{3} - \frac{5}{2} = \frac{4}{6} - \frac{15}{6} = \frac{-11}{6}$$

$$\frac{x}{3} - \frac{x+1}{2} = \frac{2x}{6} - \frac{3(x+1)}{6} = \frac{2x - 3x + 3}{6} = \frac{-x+3}{6}$$

KEY IDEAS

- Add and subtract algebraic fractions by firstly finding the lowest common denominator (LCD) and then combine the numerators.
- Expand numerators correctly by taking into account addition and subtraction signs. E.g. $-2(x + 1) = -2x - 2$ and $-5(2x - 3) = -10x + 15$.

BUILDING UNDERSTANDING

1. Expand the following.
   a $2(x - 2)$ b $-(x + 6)$ c $-6(x - 2)$
2. Simplify these by firstly finding the lowest common denominator (LCD).
   a $\frac{1}{2} + \frac{1}{3}$ b $\frac{4}{3} - \frac{1}{5}$ c $\frac{3}{7} - \frac{1}{14}$ d $\frac{5}{3} + \frac{7}{6}$
3. State the lowest common denominator for these pairs of fractions.
   a $\frac{a}{3}, \frac{7a}{4}$ b $\frac{x}{2}, \frac{4xy}{6}$ c $\frac{3xy}{7}, \frac{-3x}{14}$ d $\frac{2}{x}, \frac{3}{2x}$

Example 8 Adding and subtracting simple algebraic fractions

Simplify the following.

a $\frac{3}{4} - \frac{a}{2}$

b $\frac{2}{5} + \frac{3}{a}$

SOLUTION

a $\frac{3}{4} - \frac{a}{2} = \frac{3}{4} - \frac{2a}{4}$

$= \frac{3 - 2a}{4}$

b $\frac{2}{5} + \frac{3}{a} = \frac{2a}{5a} + \frac{15}{5a}$

$= \frac{2a + 15}{5a}$

EXPLANATION

The LCD of 2 and 4 is 4. Express each fraction as an equivalent fraction with a denominator of 4. Subtract the numerators.

The LCD of 5 and a is $5a$.

Add the numerators.

Now you try

Simplify the following.

a $\frac{5}{6} - \frac{a}{3}$

b $\frac{3}{4} + \frac{2}{a}$

Example 9 Adding and subtracting more complex algebraic fractions

Simplify the following algebraic expressions.

a $\frac{x + 3}{2} + \frac{x - 2}{5}$

b $\frac{2x - 1}{3} - \frac{x - 1}{4}$

SOLUTION

a $\frac{x + 3}{2} + \frac{x - 2}{5} = \frac{5(x + 3)}{10} + \frac{2(x - 2)}{10}$

$= \frac{5(x + 3) + 2(x - 2)}{10}$

$= \frac{5x + 15 + 2x - 4}{10}$

$= \frac{7x + 11}{10}$

EXPLANATION

LCD is 10.

Use brackets to ensure you retain equivalent fractions.

Combine the numerators, then expand the brackets and simplify.

Continued on next page

b $\dfrac{2x-1}{3}-\dfrac{x-1}{4}=\dfrac{4(2x-1)}{12}-\dfrac{3(x-1)}{12}$ Express each fraction with the LCD of 12.

$=\dfrac{4(2x-1)-3(x-1)}{12}$ Combine the numberators.

$=\dfrac{8x-4-3x+3}{12}$ Expand the brackets: $4(2x-1)=8x-4$ and $-3(x-1)=-3x+3$.

$=\dfrac{5x-1}{12}$ Simplify by collecting like terms.

Now you try

Simplify the following algebraic expressions.

a $\dfrac{x+1}{3}+\dfrac{x-2}{2}$

b $\dfrac{3x-2}{2}-\dfrac{x-2}{5}$

Example 10 Adding and subtracting with algebraic denominators

Simplify the algebraic expression $\dfrac{3}{x-6}-\dfrac{2}{x+2}$.

SOLUTION

$$\frac{3}{x-6}-\frac{2}{x+2}=\frac{3(x+2)}{(x-6)(x+2)}-\frac{2(x-6)}{(x-6)(x+2)}$$

$$=\frac{3(x+2)-2(x-6)}{(x-6)(x+2)}$$

$$=\frac{3x+6-2x+12}{(x-6)(x+2)}$$

$$=\frac{x+18}{(x-6)(x+2)}$$

EXPLANATION

$(x-6)(x+2)$ is the lowest common multiple of $(x-6)$ and $(x+2)$.
Combine the numberators and then expand the brackets.

Recall that $-2\times(-6)=12$.

Collect like terms to simplify.

Now you try

Simplify the expression $\dfrac{4}{x-5}-\dfrac{3}{x+1}$.

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Updated July 2021

Exercise 1C

FLUENCY	1, 2–4(1/2)	2–5(1/2)	2–5(1/3)

1 Simplify the following.

Example 8a
a **i** $\frac{1}{4} - \frac{a}{2}$ **ii** $\frac{3}{10} - \frac{a}{5}$

Example 8b
b **i** $\frac{1}{3} + \frac{2}{a}$ **ii** $\frac{3}{7} + \frac{5}{a}$

Example 8a
2 Simplify the following.

a $\frac{2}{3} + \frac{a}{7}$ **b** $\frac{3}{8} + \frac{a}{2}$ **c** $\frac{3}{10} - \frac{3b}{2}$ **d** $\frac{2}{5} + \frac{4x}{15}$

e $\frac{1}{9} - \frac{2a}{3}$ **f** $\frac{a}{3} - \frac{a}{5}$ **g** $\frac{2x}{5} - \frac{x}{4}$ **h** $\frac{6b}{7} - \frac{b}{14}$

Example 8b
3 Simplify the following.

a $\frac{2}{3} + \frac{5}{a}$ **b** $\frac{3}{4} + \frac{2}{a}$ **c** $\frac{7}{9} - \frac{3}{a}$ **d** $\frac{4}{b} - \frac{3}{4}$

e $\frac{2}{7} - \frac{3}{2b}$ **f** $\frac{3}{2y} - \frac{7}{9}$ **g** $\frac{-4}{x} - \frac{2}{3}$ **h** $\frac{-9}{2x} - \frac{1}{3}$

Example 9a
4 Simplify the following algebraic expressions.

a $\frac{x+3}{4} + \frac{x+2}{5}$ **b** $\frac{x+2}{3} + \frac{x+1}{4}$ **c** $\frac{x-3}{4} + \frac{x+2}{2}$

d $\frac{x+4}{3} + \frac{x-3}{9}$ **e** $\frac{2x+1}{2} + \frac{x-2}{3}$ **f** $\frac{3x+1}{5} + \frac{2x+1}{10}$

g $\frac{x-2}{8} + \frac{2x+4}{12}$ **h** $\frac{5x+3}{10} + \frac{2x-2}{4}$ **i** $\frac{3-x}{14} + \frac{x-1}{7}$

Example 9b
5 Simplify these algebraic fractions.

a $\frac{2x+1}{3} - \frac{x-1}{2}$ **b** $\frac{3x-1}{3} - \frac{2x-3}{4}$ **c** $\frac{x+6}{5} - \frac{x-4}{3}$

d $\frac{x-3}{2} - \frac{2x+1}{7}$ **e** $\frac{7x+2}{7} - \frac{x+2}{3}$ **f** $\frac{10x-4}{3} - \frac{2x+1}{6}$

g $\frac{4-x}{6} - \frac{1-x}{5}$ **h** $\frac{1-3x}{5} - \frac{x+2}{3}$ **i** $\frac{6-5x}{2} - \frac{2-7x}{4}$

PROBLEM-SOLVING	6(1/2)	6(1/2)	6(1/2), 7

Example 10
6 Simplify the following algebraic expressions.

a $\frac{5}{x+1} + \frac{2}{x+4}$ **b** $\frac{4}{x-7} + \frac{3}{x+2}$ **c** $\frac{1}{x-3} + \frac{2}{x+5}$

d $\frac{3}{x+3} - \frac{2}{x-4}$ **e** $\frac{6}{2x-1} - \frac{3}{x-4}$ **f** $\frac{4}{x-5} + \frac{2}{3x-4}$

g $\frac{5}{2x-1} - \frac{6}{x+7}$ **h** $\frac{2}{x-3} - \frac{3}{3x+4}$ **i** $\frac{8}{3x-2} - \frac{3}{1-x}$

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7 **a** Write the LCD for these pairs of fractions.

i $\frac{3}{a}, \frac{2}{a^2}$ **ii** $\frac{7}{x^2}, \frac{3+x}{x}$

b Now simplify these expressions.

i $\frac{2}{a} - \frac{3}{a^2}$ **ii** $\frac{a+1}{a} - \frac{4}{a^2}$ **iii** $\frac{7}{2x^2} + \frac{3}{4x}$

REASONING	8	8, 9	8, 9

8 Describe the error in this working, then fix the solution.

$$\begin{aligned}\frac{x}{2} - \frac{x+1}{3} &= \frac{3x}{6} - \frac{2(x+1)}{6}\\ &= \frac{3x}{6} - \frac{2x+2}{6}\\ &= \frac{x+2}{6}\end{aligned}$$

9 **a** Explain why $2x - 3 = -(3 - 2x)$.

b Use this idea to help simplify these expressions.

i $\frac{1}{x-1} - \frac{1}{1-x}$ **ii** $\frac{3x}{3-x} + \frac{x}{x-3}$ **iii** $\frac{x+1}{7-x} - \frac{2}{x-7}$

ENRICHMENT: Fraction challenges	–	–	10, 11

10 Simplify these expressions.

a $\frac{a-b}{b-a}$

b $\frac{5}{a} + \frac{2}{a^2}$

c $\frac{3}{x+1} + \frac{2}{(x+1)^2}$

d $\frac{x}{(x-2)^2} - \frac{x}{x-2}$

e $\frac{x}{2(3-x)} - \frac{x^2}{7(x-3)^2}$

f $\frac{1}{x} - \frac{1}{y} - \frac{1}{z}$

11 By first simplifying the left-hand side of these equations, find the value of a.

a $\frac{a}{x-1} - \frac{2}{x+1} = \frac{4}{(x-1)(x+1)}$

b $\frac{3}{2x-1} + \frac{a}{x+1} = \frac{5x+2}{(2x-1)(x+1)}$

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

1D Solving linear equations

Learning intentions

- To know the form of a linear equation
- To understand that an equivalent equation can be generated by applying the same operation to each side of the equation
- To be able to solve a linear equation involving two or more steps, including brackets and variables on both sides
- To be able to solve linear equations involving algebraic fractions
- To understand that solutions can be checked by substituting into both sides of an equation

A linear equation is a statement that contains an equals sign and includes constants and pronumerals with a power of 1 only. Here are some examples:

$$2x - 5 = 7$$

$$\frac{x+1}{3} = x + 4$$

$$-3(x + 2) = \frac{1}{2}$$

We solve linear equations by operating on both sides of the equation until a solution is found.

A small business, such as a garden nursery, generates revenue from its sales. To calculate the number of employees (x) a business can afford, a linear revenue equation is solved for x:
Revenue (y) = pay (m) × employees (x) + costs (c)

LESSON STARTER What's the best method?

Here are four linear equations.

- Discuss what you think is the best method to solve them using 'by hand' techniques.
- Discuss how it might be possible to check that a solution is correct.

a $\frac{7x - 2}{3} = 4$ **b** $3(x - 1) = 6$ **c** $4x + 1 = x - 2$ **d** $\frac{2x + 1}{5} = \frac{x - 1}{3}$

KEY IDEAS

- An equation is true for the given values of the pronumerals when the left-hand side equals the right-hand side.
  $2x - 4 = 6$ is true when $x = 5$ but false when $x \neq 5$.
- A **linear equation** contains pronumerals with a highest power of 1.
- Useful steps in solving linear equations are:
  - using inverse operations (backtracking)
  - collecting like terms
  - expanding brackets
  - multiplying by the lowest common denominator.

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

BUILDING UNDERSTANDING

1 Decide whether the following are linear equations.

a $x^2 - 1 = 0$ b $\sqrt{x} + x = 3$ c $\frac{x-1}{2} = 5$ d $\frac{3x}{4} = 2x - 1$

2 Decide whether these equations are true when $x = 2$.

a $3x - 1 = 5$ b $4 - x = 1$ c $\frac{2x+1}{5} = x + 4$

3 Decide whether these equations are true when $x = -6$.

a $-3x + 17 = x$ b $2(4 - x) = 20$ c $\frac{2-3x}{10} = \frac{-12}{x}$

4 Solve the following equations and check your solution using substitution.

a $x + 8 = 13$ b $x - 5 = 3$ c $-x + 4 = 7$ d $-x - 5 = -9$

Example 11 Solving linear equations

Solve the following equations and check your solution using substitution.

a $4x + 5 = 17$ b $3(2x + 5) = 4x$

SOLUTION	EXPLANATION
a $4x + 5 = 17$ $4x = 12$ $x = 3$	Subtract 5 from both sides and then divide both sides by 4.
Check: LHS $= 4 \times 3 + 5 = 17$, RHS $= 17$.	Check by seeing if $x = 3$ makes the equation true.
b $3(2x + 5) = 4x$ $6x + 15 = 4x$ $2x + 15 = 0$ $2x = -15$ $x = -\frac{15}{2}$	Expand the brackets. Gather like terms by subtracting $4x$ from both sides. Subtract 15 from both sides and then divide both sides by 2.
Check: LHS $= 3\left(2 \times \left(-\frac{15}{2}\right) + 5\right)$ $= -30$ RHS $= 4 \times \left(-\frac{15}{2}\right)$ $= -30$	Check by seeing if $x = -\frac{15}{2}$ makes the equation true by substituting into the equation's left-hand side (LHS) and right-hand side (RHS) and confirming they are equal.

Now you try

Solve the following equations and check your solution using substitution.

a $2x + 7 = 13$ b $4(2x + 1) = 2x$

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

Example 12 Solving equations involving algebraic fractions

Solve the following equations and check your solution using substitution.

a $\frac{x+3}{4} = 2$ **b** $\frac{4x-2}{3} = \frac{3x-1}{2}$ **c** $\frac{x+2}{3} - \frac{2x-1}{2} = 4$

SOLUTION	EXPLANATION
a $\frac{x+3}{4} = 2$	Multiply both sides by 4.
$x + 3 = 8$	Subtract 3 from both sides.
$x = 5$	
Check: LHS $= \frac{5+3}{4} = 2$, RHS $= 2$.	Check by seeing if $x = 5$ makes the equation true.
b $\frac{4x-2}{3} = \frac{3x-1}{2}$	Multiply both sides by the LCD of 2 and 3, which is 6.
$\frac{\cancel{6}^{2}(4x-2)}{\cancel{3}_{1}} = \frac{\cancel{6}^{3}(3x-1)}{\cancel{2}_{1}}$	Cancel common factors.
$2(4x-2) = 3(3x-1)$	
$8x - 4 = 9x - 3$	Expand the brackets and gather terms containing x by subtracting $8x$ from both sides.
$-4 = x - 3$	
$-1 = x$	Rewrite with x as the subject. Check by seeing if $x = -1$ makes the equation true.
$\therefore x = -1$	
c $\frac{x+2}{3} - \frac{2x-1}{2} = 4$	Multiply both sides by the LCD of 2 and 3, which is 6.
$\frac{6(x+2)}{3} - \frac{6(2x-1)}{2} = 4 \times 6$	
$2(x+2) - 3(2x-1) = 24$	Cancel common factors.
$2x + 4 - 6x + 3 = 24$	Expand, nothing that $-3 \times (-1) = 3$.
$-4x + 7 = 24$	Simplify and solve for x.
$-4x = 17$	
$x = -\frac{17}{4}$	Check your solution using substitution.

Now you try

Solve the following equations and check your solution using substitution.

a $\frac{x+1}{3} = 4$ **b** $\frac{2x-1}{3} = \frac{x-3}{4}$ **c** $\frac{x+1}{2} - \frac{2x-1}{3} = 2$

ISBN 978-1-108-77290-7

Exercise 1D

FLUENCY	1, 2–5(1/2)	2–5(1/3)	2–5(1/4)

1 Solve the following equations and check your solution using substitution.

Example 11a
a i $3x + 4 = 13$ ii $5x + 2 = 27$

Example 11b
b i $2(2x + 3) = 2x$ ii $5(x + 2) = 3x$

Example 11a
2 Solve the following equations and check your solution using substitution.

a $2x + 9 = 14$ b $4x + 3 = 14$ c $3x - 3 = -4$ d $6x + 5 = -6$
e $-3x + 5 = 17$ f $-2x + 7 = 4$ g $-4x - 9 = 9$ h $-3x - 7 = -3$
i $8 - x = 10$ j $5 - x = -2$ k $6 - 5x = 16$ l $4 - 9x = -7$

Example 11b
3 Solve the following equations and check your solution using substitution.

a $4(x + 3) = 16$ b $2(x - 3) = 12$ c $2(x - 4) = 15$
d $3(1 - 2x) = 8$ e $3(2x + 3) = -5x$ f $2(4x - 5) = -7x$
g $3(2x + 3) + 2(x + 4) = 25$ h $2(2x - 3) + 3(4x - 1) = 23$ i $2(3x - 2) - 3(x + 1) = 5$
j $5(2x + 1) - 3(x - 3) = 63$ k $5(x - 3) = 4(x - 6)$ l $4(2x + 5) = 3(x + 15)$
m $5(x + 2) = 3(2x - 3)$ n $3(4x - 1) = 7(2x - 7)$ o $7(2 - x) = 8 - x$

Example 12a
4 Solve the following equations and check your solution using substitution.

a $\frac{x - 4}{2} = 3$ b $\frac{x + 2}{3} = 5$ c $\frac{x + 4}{3} = -6$ d $\frac{2x + 7}{3} = 5$
e $\frac{2x + 1}{3} = -3$ f $\frac{3x - 2}{4} = 4$ g $\frac{x}{2} - 5 = 3$ h $\frac{3x}{2} + 2 = 8$
i $\frac{2x}{3} - 2 = -8$ j $5 - \frac{x}{2} = 1$ k $4 - \frac{2x}{3} = 0$ l $5 - \frac{4x}{7} = 9$
m $\frac{x + 1}{3} + 2 = 9$ n $\frac{x - 3}{2} - 4 = 2$ o $4 + \frac{x - 5}{2} = -3$ p $1 - \frac{2 - x}{3} = 2$

5 For each of the following statements, write an equation and solve it to find x.

a When 3 is added to x, the result is 7.
b When x is added to 8, the result is 5.
c When 4 is subtracted from x, the result is 5.
d When x is subtracted from 15, the result is 22.
e Twice the value of x is added to 5 and the result is 13.
f 5 less than x when doubled is -15.
g When 8 is added to 3 times x, the result is 23.
h 5 less than twice x is 3 less than x.

PROBLEM-SOLVING	7, 8, 11	6(1/2), 7, 9, 11, 13(1/2)	6–7(1/2), 9–12, 13(1/2)

Example 12b
6 Solve the following equations, which involve algebraic fractions.

a $\frac{2x + 12}{7} = \frac{3x + 5}{4}$ b $\frac{5x - 4}{4} = \frac{x - 5}{5}$ c $\frac{3x - 5}{4} = \frac{2x - 8}{3}$
d $\frac{1 - x}{5} = \frac{2 - x}{3}$ e $\frac{6 - 2x}{3} = \frac{5x - 1}{4}$ f $\frac{10 - x}{2} = \frac{x + 1}{3}$
g $\frac{2(x + 1)}{3} = \frac{3(2x - 1)}{2}$ h $\frac{-2(x - 1)}{3} = \frac{2 - x}{4}$ i $\frac{3(6 - x)}{2} = \frac{2(x + 1)}{5}$

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7 Substitute the given values and then solve for the unknown in each of the following common formulas.

a $v = u + at$ Solve for a given $v = 6$, $u = 2$ and $t = 4$.

b $s = ut + \frac{1}{2}at^2$ Solve for u given $s = 20$, $t = 2$ and $a = 4$.

c $A = h\left(\frac{a + b}{2}\right)$ Solve for b given $A = 10$, $h = 4$ and $a = 3$.

d $A = P\left(1 + \frac{r}{100}\right)$ Solve for r given $A = 1000$ and $P = 800$.

8 The perimeter of a square is 68 cm. Determine its side length.

9 The sum of two consecutive numbers is 35. What are the numbers?

10 I ride four times faster than I jog. If a trip took me 45 minutes and I spent 15 of these minutes jogging 3 km, how far did I ride?

11 A service technician charges \$30 up front and \$46 for each hour that he works.

a What will a 4-hour job cost?

b If the technician works on a job for 2 days and averages 6 hours per day, what will be the overall cost?

c Find how many hours the technician worked if the cost is:

i \$76

ii \$513

iii \$1000 (round to the nearest half hour).

12 The capacity of a petrol tank is 80 litres. If it initially contains 5 litres and a petrol pump fills it at 3 litres per 10 seconds, find:

a the amount of fuel in the tank after 2 minutes

b how long it will take to fill the tank to 32 litres

c how long it will take to fill the tank.

Example 12c

13 Solve the following equations by multiplying both sides by the LCD.

a $\frac{x}{2} + \frac{2x}{3} = 7$

b $\frac{x}{4} + \frac{3x}{3} = 5$

c $\frac{3x}{5} - \frac{2x}{3} = 1$

d $\frac{2x}{5} - \frac{x}{4} = 3$

e $\frac{x - 1}{2} + \frac{x + 2}{5} = 2$

f $\frac{x + 3}{3} + \frac{x - 4}{2} = 4$

g $\frac{x + 2}{3} - \frac{x - 1}{2} = 1$

h $\frac{x - 4}{5} - \frac{x + 2}{3} = 2$

i $\frac{7 - 2x}{3} - \frac{6 - x}{2} = 1$

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REASONING	14	14	15

14 Solve $2(x - 5) = 8$ using the following two methods and then decide which method you prefer. Give a reason.

a Method 1: First expand the brackets.
b Method 2: First divide both sides by 2.

15 A family of equations can be represented using other pronumerals (sometimes called parameters). For example, the solution to the family of equations $2x - a = 4$ is $x = \frac{4 + a}{2}$.

Find the solution for x in these equation families.

a $x + a = 5$
b $6x + 2a = 3a$
c $ax + 2 = 7$
d $ax - 1 = 2a$
e $\frac{ax - 1}{3} = a$
f $ax + b = c$

ENRICHMENT: More complex equations	–	–	16, 17

16 Make a the subject in these equations.

a $a(b + 1) = c$
b $ab + a = b$
c $\frac{1}{a} + b = c$
d $a - \frac{a}{b} = 1$
e $\frac{1}{a} + \frac{1}{b} = 0$
f $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$

17 Solve for x in terms of the other pronumerals.

a $\frac{x}{2} - \frac{x}{3} = a$
b $\frac{x}{a} + \frac{x}{b} = 1$
c $\frac{x}{a} - \frac{x}{b} = c$

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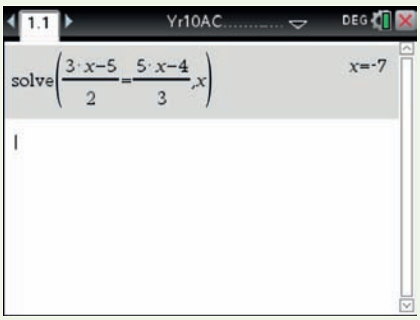
Using calculators to solve equations and inequalities

1 Solve the equation $\frac{3x-5}{2} = \frac{5x-4}{3}$.

2 Solve the inequality $5 < \frac{2x+3}{5}$.

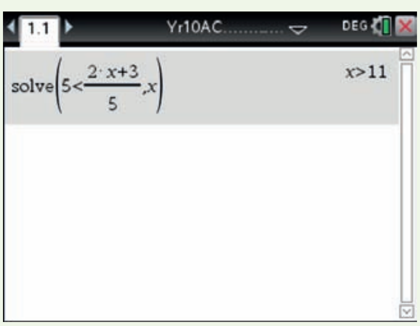
Using the TI-Nspire:

1 In a **Calculator** page use menu **>Algebra>Solve** and type the equation as shown.



Hint: use the fraction template (ctrl ÷)

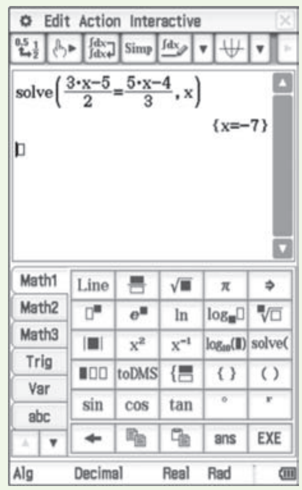
2 In a **Calculator** page use menu **>Algebra>Solve** and type the equation as shown.



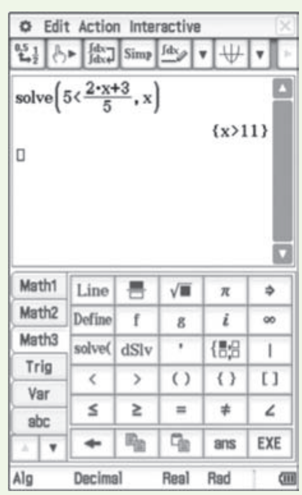
Hint: the inequality symbols (e.g. <) are accessed using ctrl =
Hint: use the fraction template (ctrl ÷)

Using the ClassPad:

1 Tap **solv(**, then and type the equation as shown.



2 Tap **solv(** and type the equation as shown.



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1E Linear inequalities

Learning intentions

- To know the meaning of the term inequality
- To be able to use and interpret the signs $>, \geqslant, \leqslant, <$
- To know how to interpret and represent an inequality on a number line
- To understand when to reverse the sign in an inequality
- To be able to solve a linear inequality

There are many situations in which a solution to the problem is best described using one of the symbols $<, \leqslant, >$ or $\geqslant$. For example, a pharmaceutical company might need to determine the possible number of packets of a particular drug that need to be sold so that the product is financially viable. This range of values may be expressed using inequality symbols.

An inequality is a mathematical statement that uses an *is less than* ($<$), an *is less than or equal to* ($\leqslant$), an *is greater than* ($>$) or an *is greater than or equal to* ($\geqslant$) symbol. Inequalities may result in an infinite number of solutions and these can be illustrated using a number line.

Doctors, nurses and pharmacists can use an inequality to express the dosage range of a medication from the lowest effective level to the highest safe level.

LESSON STARTER What does it mean for x?

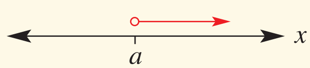
The following inequalities provide some information about the value of x.

a $2 \geqslant x$ **b** $-2x < 4$ **c** $3 - x \leqslant -1$

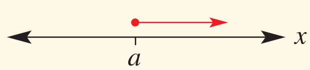
- Can you describe the possible values for x that satisfy each inequality?
- Test some values to check.
- How would you write the solution for x? Illustrate this on a number line.

KEY IDEAS

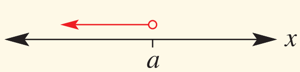
- The four **inequality signs** are $<, \leqslant, >$ and $\geqslant$.
  - $x > a$ means x is greater than a.



  - $x \geqslant a$ means x is greater than or equal to a.



  - $x < a$ means x is less than a.



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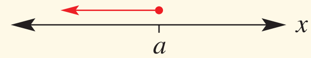
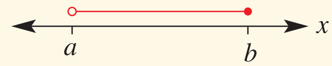
- $x \leqslant a$ means x is less than or equal to a.



- Also $a < x \leqslant b$ could be illustrated as shown.



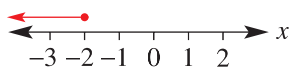
- An open circle is used for $<$ or $>$ where the end point is not included.
- A closed circle is used for $\leqslant$ or $\geqslant$ where the end point is included.

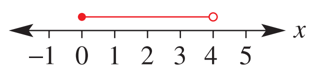
■ Solving **linear inequalities** follows the same rules as solving linear equations, except:

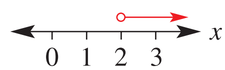
- We reverse the inequality sign if we multiply or divide by a negative number.
  For example, $-5 < -3$ is equivalent to $5 > 3$ and if $-2x < 4$, then $x > -2$.
- We reverse the inequality sign if the sides are switched.
  For example, if $2 \geqslant x$, then $x \leqslant 2$.

BUILDING UNDERSTANDING

1. State three numbers that satisfy each of these inequalities.
   a $x \geqslant 3$ **b** $x < -1.5$ **c** $0 < x \leqslant 7$ **d** $-8.7 \leqslant x < -8.1$
2. Match the graph **a**–**c** with the inequality **A**–**C**.
   A $x > 2$ **B** $x \leqslant -2$ **C** $0 \leqslant x < 4$

   a
   

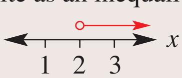
   b
   

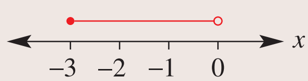
   c
   

3. Phil houses x rabbits. If $10 < x \leqslant 13$, how many rabbits could Phil have?

Example 13 Writing inequalities from number lines

Write as an inequality.

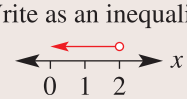
a


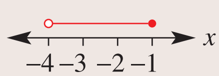
b


SOLUTION	EXPLANATION
a $x > 2$	An open circle means 2 is not included.
b $-3 \leqslant x < 0$	-3 is included but 0 is not.

Now you try

Write as an inequality.

a


b


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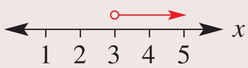
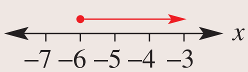
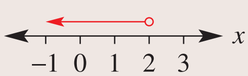
Example 14 Solving linear inequalities

Solve the following inequalities and graph their solutions on a number line.

a $3x + 4 > 13$ **b** $4 - \frac{x}{3} \leqslant 6$ **c** $3x + 2 > 6x - 4$

SOLUTION	EXPLANATION
a $3x + 4 > 13$	Subtract 4 from both sides and then divide both sides by 3.
$3x > 9$	
$\therefore\ x > 3$	
	Use an open circle since x does not include 3.
b $4 - \frac{x}{3} \leqslant 6$	Subtract 4 from both sides.
$-\frac{x}{3} \leqslant 2$	Multiply both sides by -3 and reverse the inequality sign.
$\therefore x \geqslant -6$	
	Use a closed circle since x includes the number -6.
c $3x + 2 > 6x - 4$	Subtract $3x$ from both sides to gather the terms containing x.
$2 > 3x - 4$	Add 4 to both sides and then divide both sides by 3.
$6 > 3x$	
$2 > x$	
$\therefore\ x < 2$	Make x the subject. Switching sides means the inequality sign must be reversed.
	Use an open circle since x does not include 2.

Now you try

Solve the following inequalities and graph their solutions on a number line.

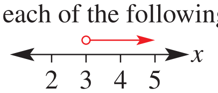
a $2x + 5 > 11$ **b** $2 - \frac{x}{3} \leqslant 4$ **c** $4x + 1 > 7x - 2$

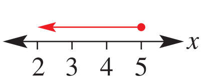
Exercise 1E

FLUENCY	1, 2–4(½)	2–5(½)	2–5(½)

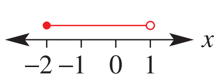
1 Write each of the following as an inequality.

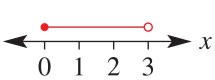
Example 13a

a **i** 

ii 

Example 13b

b **i** 

ii 

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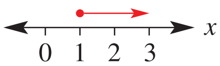
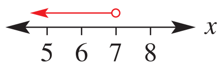
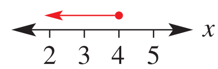
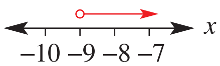
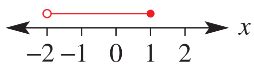
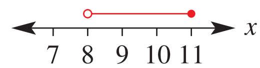
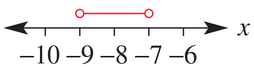
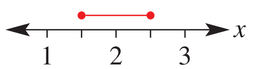
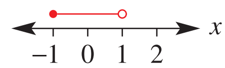
Example 13

2 Write each of the following as an inequality.

a 
b 
c 
d 
e 
f 
g 
h 
i 

Example 14a

3 Solve the following inequalities and graph their solutions on a number line.

a $2x + 6 < 14$
b $3x + 5 \geqslant 20$
c $4x - 7 \geqslant 9$
d $\frac{x}{5} \leqslant 2$
e $\frac{x+4}{3} \leqslant 2$
f $\frac{5x-3}{2} > 6$
g $\frac{2x+3}{5} > 3$
h $\frac{x}{3} + 4 \leqslant 6$
i $\frac{x}{9} + 6 < 4$
j $-3 + \frac{x}{4} > 5$
k $3(3x - 1) \leqslant 7$
l $2(4x + 4) < 5$

Example 14b

4 Solve the following inequalities. Remember: if you multiply or divide by a negative number, you must reverse the inequality sign.

a $-5x + 7 \leqslant 9$
b $4 - 3x > -2$
c $-5x - 7 \geqslant 18$
d $\frac{3-x}{2} \geqslant 5$
e $\frac{5-2x}{3} > 7$
f $\frac{4-6x}{5} \leqslant -4$
g $3 - \frac{x}{2} \leqslant 8$
h $-\frac{x}{3} - 5 > 2$

Example 14c

5 Solve the following inequalities.

a $x + 1 < 2x - 5$
b $5x + 2 \geqslant 8x - 4$
c $7 - x > 2 + x$
d $3(x + 2) \leqslant 4(x - 1)$
e $7(1 - x) \geqslant 3(2 + 3x)$
f $-(2 - 3x) < 5(4 - x)$

PROBLEM-SOLVING	6, 7	6, 7, 8($^1/_2$)	7, 8

6 For the following situations, write an inequality and solve it to find the possible values for x.

a 7 more than twice a number x is less than 12.
b Half of a number x subtracted from 4 is greater than or equal to -2.
c The product of 3 and one more than a number x is at least 2.
d The sum of two consecutive even integers, of which the smaller is x, is no more than 24.
e The sum of four consecutive even integers, of which x is the largest, is at most 148.

7 The cost of a satellite phone call is 30 cents plus 20 cents per minute.

a Find the possible cost of a call if it is:
  i shorter than 5 minutes
  ii longer than 10 minutes.

b For how many minutes can the phone be used if the cost per call is:
  i less than \$2.10?
  ii greater than or equal to \$3.50?

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8 Solve these inequalities by first multiplying by the LCD.

a $\frac{1+x}{2} < \frac{x-1}{3}$ **b** $\frac{2x-3}{2} \geqslant \frac{x+1}{3}$

c $\frac{3-2x}{5} \leqslant \frac{5x-1}{2}$ **d** $\frac{x}{2} \leqslant \frac{7-x}{3}$

e $\frac{5x}{3} \geqslant \frac{3(3-x)}{4}$ **f** $\frac{2(4-3x)}{5} > \frac{2(1+x)}{3}$

REASONING	9	9, 10	10, 11

9 How many whole numbers satisfy these inequalities? Give a reason.

a $x > 8$ **b** $2 < x \leqslant 3$

10 Solve these families of inequalities by writing x in terms of a. Consider cases where $a > 0$ and $a < 0$.

a $10x - 1 \geqslant a + 2$ **b** $\frac{2-x}{a} > 4$ **c** $a(1-x) > 7$

11 Describe the sets (in a form like $2 < x \leqslant 3$ or $-1 \leqslant x < 5$) that simultaneously satisfy these pairs of inequalities.

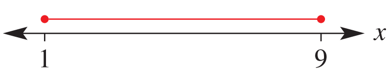
a $x < 5$, $x \geqslant -4$ **b** $x \leqslant -7$, $x > -9.5$ **c** $x \leqslant 10$, $x \geqslant 10$

ENRICHMENT: Mixed inequalities	–	–	12(½), 13

12 Solve the inequalities and graph their solutions on a number line. Consider this example first.

Solve $-2 \leqslant x - 3 \leqslant 6$.

$1 \leqslant x \leqslant 9$ (add 3 to both sides)



a $1 \leqslant x - 2 \leqslant 7$ **b** $-4 \leqslant x + 3 \leqslant 6$

c $-2 \leqslant x + 7 < 0$ **d** $0 \leqslant 2x + 3 \leqslant 7$

e $-5 \leqslant 3x + 4 \leqslant 11$ **f** $-16 \leqslant 3x - 4 \leqslant -10$

g $7 \leqslant 7x - 70 \leqslant 14$ **h** $-2.5 < 5 - 2x \leqslant 3$

13 Solve these inequalities for x.

a $\frac{3-x}{5} + \frac{1+x}{4} \geqslant 2$ **b** $\frac{2x-1}{3} - \frac{x+1}{4} < 1$ **c** $\frac{7x-1}{6} - \frac{2x-1}{2} \leqslant \frac{1}{2}$

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

1F Graphing straight lines CONSOLIDATING

Learning intentions
- To understand what it means for a point to lie on a line: graphically and algebraically
- To understand that straight lines have a constant gradient that can be positive, negative, zero or undefined
- To know how to determine the gradient of a line from its equation and use it and a point to sketch its graph
- To be able to find the axis intercepts of a linear graph and use them to sketch the graph
- To be able to sketch straight lines with only one intercept

In two dimensions, a straight-line graph can be described by a linear equation. Common forms of such equations are $y = mx + c$ and $ax + by = d$, where a, b, c, d and m are constants. From a linear equation a graph can be drawn by considering such features as x- and y-intercepts and the gradient.

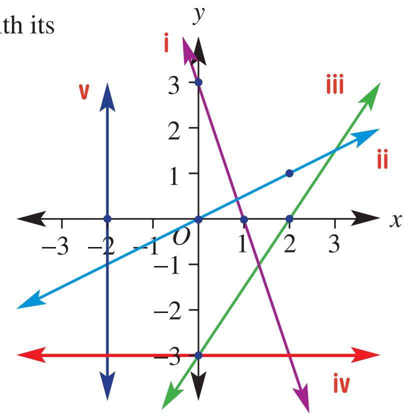
For any given straight-line graph the y-value changes by a fixed amount for each 1 unit change in the x-value. This change in y tells us the gradient of the line.

A financial analyst can use linear graphs to predict possible profit. The profit made by a lawn mower shop could be analysed with a straight-line graph of the equation:
Profit (y) = mower price (m) × sales (x) − costs (c)

LESSON STARTER Five graphs, five equations

Here are five equations and five graphs. Match each equation with its graph. Give reasons for each of your choices.

a $y = -3$
b $x = -2$
c $y = \frac{1}{2}x$
d $y = -3x + 3$
e $3x - 2y = 6$



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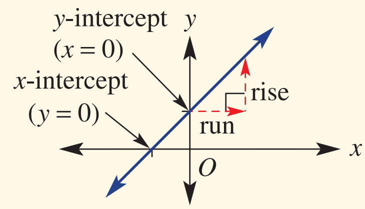
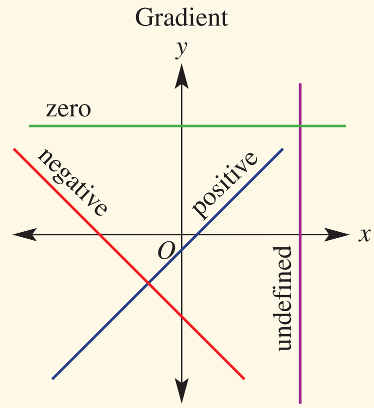
KEY IDEAS

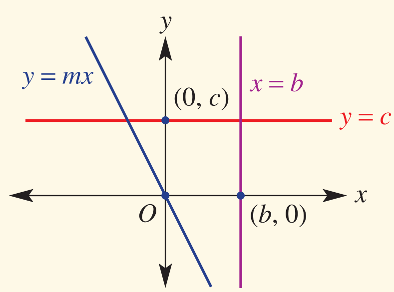
- The **gradient**, m, is a number that describes the slope of a line.
  - Gradient $= \frac{\text{rise}}{\text{run}}$
  - The gradient is the change in y per 1 unit change in x. Gradient is also referred to as the 'rate of change of y'.



- The **intercepts** are the points where the line crosses the x- and the y-axis.
  - The y-intercept is the y-value where $x = 0$.
  - The x-intercept is the x-value where $y = 0$.
- The gradient of a line can be positive, negative, zero (i.e. a horizontal line) or undefined (i.e. a vertical line).
- The gradient–intercept form of a straight line is $y = mx + c$, where m is the gradient and c is the y-intercept.



- Two points are needed to sketch most straight-line graphs.
- Special lines include those with only one axis intercept:
  - horizontal lines $y = c$
  - vertical lines $x = b$
  - lines passing through the origin $y = mx$.

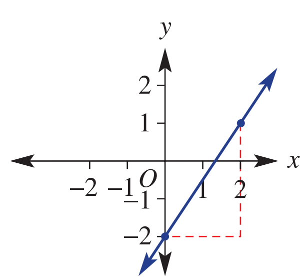


BUILDING UNDERSTANDING

1 Rearrange these equations into the form $y = mx + c$. Then state the gradient (m) and y-intercept (c).

a $y + 2x = 5$ **b** $2y = 4x - 6$ **c** $x - y = 7$ **d** $-2x - 5y = 3$

2 The graph of $y = \frac{3x}{2} - 2$ is shown.



a State the rise of the line if the run is:

i 2 **ii** 4 **iii** 7

b State the run in the line if the rise is:

i 3 **ii** 9 **iii** 4

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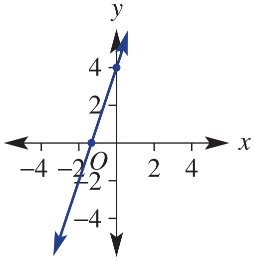
3 Match each of the following equations to one of the graphs shown.

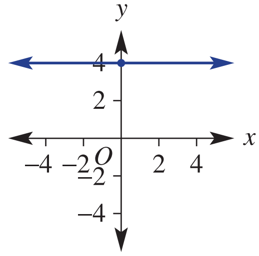
a $y = 3x + 4$ **b** $y = -2x - 4$ **c** $y = 4$

d $y = x$ **e** $y = -2x + 4$ **f** $x = -3$

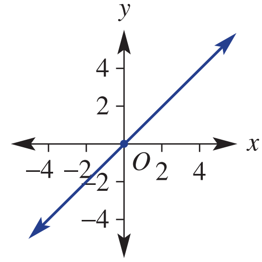
A



B



C



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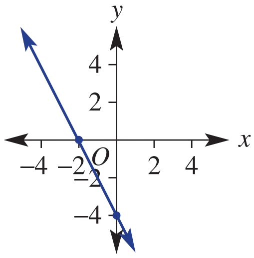
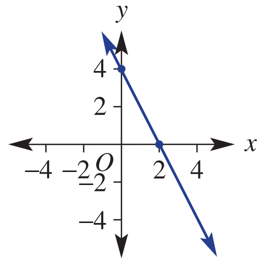
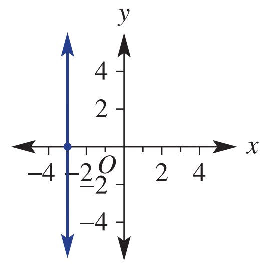


E



F



4 Find the value of the unknown in the following.

a $y = 2x - 4$ where $y = 0$ **b** $3x + 8y = 16$ where $x = 0$

Example 15 Deciding if a point is on a line

Decide if the point $(-2, 7)$ is on the line with the given equations.

a $y = -3x + 1$ **b** $2x + 2y = 1$

SOLUTION

a $y = -3x + 1$

$7 = -3(-2) + 1$

$7 = 7$ True

$\therefore$ $(-2, 7)$ is on the line.

b $2x + 2y = 1$

$2(-2) + 2(7) = 1$

$-4 + 14 = 1$

$10 = 1$ False

$\therefore$ $(-2, 7)$ is not on the line.

EXPLANATION

Substitute $x = -2$ and $y = 7$ into the equation of the line.

If the equation is true, then the point is on the line.

By substituting the point we find that the equation is false, so the point is not on the line.

Now you try

Decide if the point $(-1, 4)$ is on the line with the given equations.

a $y = -2x + 2$ **b** $3x + 3y = 2$

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Example 16 Sketching linear graphs using the gradient–intercept method

Find the gradient and y-intercept for these linear relations and sketch each graph.

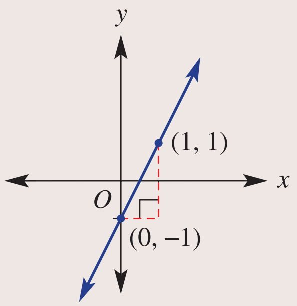
a $y = 2x - 1$ **b** $2x + 3y = 3$

SOLUTION

a $y = 2x - 1$

Gradient $= 2$

y-intercept $= -1$



EXPLANATION

In the form $y = mx + c$, the gradient is m (the coefficient of x) and c is the y-intercept.

Start by plotting the y-intercept at $(0, -1)$ on the graph.

Gradient $= 2 = \frac{2}{1}$, thus rise $= 2$ and run $= 1$.

From the y-intercept move 1 unit right (run) and 2 units up (rise) to the point $(1, 1)$. Join the two points with a line.

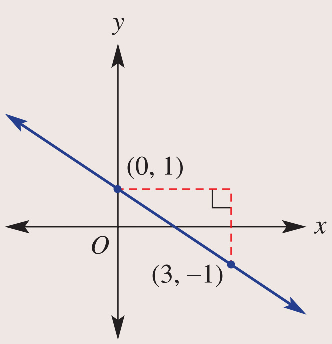
b $2x + 3y = 3$

$3y = -2x + 3$

$y = -\frac{2}{3}x + 1$

Gradient $= -\frac{2}{3}$

y-intercept $= 1$



Rewrite in the form $y = mx + c$ by subtracting $2x$ from both sides and then dividing both sides by 3.

Note: $\frac{-2x}{3}$ can also be written as $-\frac{2x}{3}$.

The gradient is the coefficient of x and the y-intercept is the constant term.

Start the graph by plotting the y-intercept at $(0, 1)$.

Gradient $= -\frac{2}{3}$ (run $= 3$ and fall $= 2$). From the point $(0, 1)$ move 3 units right (run) and 2 units down (fall) to $(3, -1)$.

Now you try

Find the gradient and y-intercept for these linear relations and sketch each graph.

a $y = 3x - 1$ **b** $3x + 4y = 4$

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Example 17 Sketching linear graphs using the *x*- and *y*-intercepts

Sketch the following by finding the x- and y-intercepts.

a $y = 2x - 8$ **b** $-3x - 2y = 6$

SOLUTION

a $y = 2x - 8$

y-intercept ($x = 0$):

$y = 2(0) - 8$

$y = -8$

The y-intercept is -8.

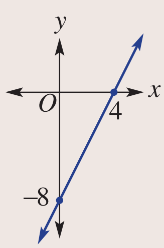
x-intercept ($y = 0$):

$0 = 2x - 8$

$8 = 2x$

$x = 4$

The x-intercept is 4.



b $-3x - 2y = 6$

y-intercept ($x = 0$):

$-3(0) - 2y = 6$

$-2y = 6$

$y = -3$

The y-intercept is -3

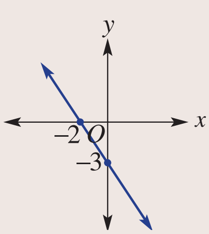
x-intercept ($y = 0$):

$-3x - 2(0) = 6$

$-3x = 6$

$x = -2$

The x-intercept is -2.



EXPLANATION

The y-intercept is at $x = 0$. For $y = mx + c$, c is the y-intercept.

The x-intercept is on the x-axis, so $y = 0$. Solve the equation for x.

Plot and label the intercepts and join with a straight line.

The y-intercept is on the y-axis so substitute $x = 0$. Simplify and solve for y.

The x-intercept is on the x-axis so substitute $y = 0$. Simplify and solve for x.

Sketch by drawing a line passing through the two axes intercepts. Label the intercepts.

Now you try

Sketch the following by finding the x- and y-intercepts.

a $y = 2x - 4$ **b** $-2x - 5y = 10$

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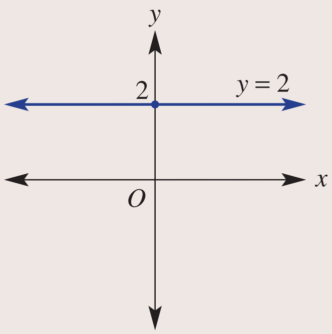
Example 18 Sketching lines with one intercept

Sketch these special lines.

a $y = 2$ **b** $x = -3$ **c** $y = -\frac{1}{2}x$

SOLUTION

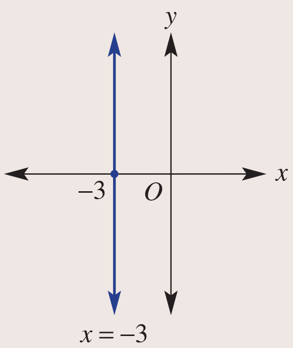
a



EXPLANATION

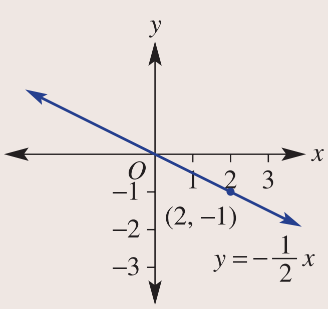
The y-coordinate of every point must be 2, hence $y = 2$ is a horizontal line passing through (0, 2).

b



The x-coordinate of every point must be -3, hence $x = -3$ is a vertical line passing through $(-3, 0)$.

c



Both the x- and y-intercepts are (0, 0), so the gradient can be used to find a second point.

The gradient $= -\frac{1}{2}$, hence use run = 2 and fall = 1.

Alternatively, substitute $x = 1$ to find a second point:

$$x = 1, y = -\frac{1}{2} \times (1)$$
$$= -\frac{1}{2}$$

Now you try

Sketch these special lines.

a $y = -1$ **b** $x = 2$ **c** $y = -\frac{1}{3}x$

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Exercise 1F

FLUENCY	1, 2–6(½)	2–6(½)	2–6(⅓)

Example 15a
Example 15b

1 Decide if the point $(-1, 5)$ is on the line with the given equations.

a i $y = -3x + 2$ ii $y = -2x - 1$

b i $2x + 2y = 5$ ii $3x + 3y = 12$

Example 15

2 Decide if the point $(3, -1)$ is on the line with the following equations.

a $y = x - 4$ b $y = -x + 2$ c $y = -3x + 9$

d $x + 2y = 6$ e $-2x - y = -5$ f $3y - 4x = -9$

Example 16a

3 Find the gradient and y-intercept for these linear relations and sketch a graph.

a $y = 5x - 3$ b $y = 2x + 3$ c $y = -2x - 1$

d $y = -x + 2$ e $y = x - 4$ f $y = -\frac{3}{2}x + 1$

g $y = \frac{4}{3}x - 2$ h $y = -\frac{7}{2}x + 6$ i $y = 0.5x - 0.5$

j $y = 1 - x$ k $y = 3 + \frac{2}{3}x$ l $y = 0.4 - 0.2x$

Example 16b

4 Find the gradient and y-intercept for these linear relations and sketch each graph.

a $3x + y = 12$ b $10x + 2y = 5$ c $x - y = 7$ d $3x - 3y = 6$

e $4x - 3y = 9$ f $-x - y = \frac{1}{3}$ g $-y - 4x = 8$ h $2y + x = \frac{1}{2}$

Example 17

5 Sketch the following by finding the x- and y-intercepts.

a $y = 3x - 6$ b $y = 2x + 4$ c $y = 4x + 10$

d $y = 3x - 4$ e $y = 7 - 2x$ f $y = 4 - \frac{x}{2}$

g $3x + 2y = 12$ h $2x + 5y = 10$ i $4y - 3x = 24$

j $x + 2y = 5$ k $3x + 4y = 7$ l $5y - 2x = 12$

Example 18

6 Sketch these special lines.

a $y = -4$ b $y = 1$ c $x = 2$ d $x = -\frac{5}{2}$

e $y = 0$ f $x = 0$ g $y = 4x$ h $y = -3x$

i $y = -\frac{1}{3}x$ j $y = \frac{5x}{2}$ k $x + y = 0$ l $4 - y = 0$

PROBLEM-SOLVING	7, 8	7, 8, 10	8–11

7 Sam is earning some money picking apples. She gets paid \$10 plus \$2 per kilogram of apples that she picks. If Sam earns \$$C$ for n kg of apples picked, complete the following.

a Write a rule for C in terms of n.

b Sketch a graph for $0 \leqslant n \leqslant 10$, labelling the endpoints.

c Use your rule to find:

i the amount Sam earned after picking 9 kg of apples

ii the number of kilograms of apples Sam picked if she earned \$57.

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8 A 90 L tank full of water begins to leak at a rate of 1.5 litres per hour. If V litres is the volume of water in the tank after t hours, complete the following.

a Write a rule for V in terms of t.
b Sketch a graph for $0 \leqslant t \leqslant 60$, labelling the endpoints.
c Use your rule to find:
  i the volume of water after 5 hours
  ii the time taken to completely empty the tank.

9 Alex earns \$84 for 12 hours of work.
a Write Alex's rate of pay per hour.
b Write the equation for Alex's total pay, \$$P$, after t hours of work.

10 It costs Jesse \$1600 to maintain and drive his car for 32 000 km.
a Find the cost in \$ per km.
b Write a formula for the cost, \$$C$, of driving Jesse's car for k kilometres.
c If Jesse also pays a total of \$1200 for registration and insurance, write the new formula for the cost to Jesse of owning and driving his car for k kilometres.

11 $D = 25t + 30$ is an equation for calculating the distance, D km, from home that a cyclist has travelled after t hours.
a What is the gradient of the graph of the given equation? What does it represent?
b What could the 30 represent?
c If a graph of D against t is drawn, what would be the intercept on the D-axis?

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REASONING	12	12, 13	13, 14

12 A student with a poor understanding of straight-line graphs writes down some incorrect information next to each equation. Decide how the error might have been made and then correct the information.

a $y = \dfrac{2x + 1}{2}$ (gradient = 2)

b $y = 0.5(x + 3)$ (y-intercept = 3)

c $3x + y = 7$ (gradient = 3)

d $x - 2y = 4$ (gradient = 1)

13 Write expressions for the gradient and y-intercept of these equations.

a $ay = 3x + 7$

b $ax - y = b$

c $by = 3 - ax$

14 A straight line is written in the form $ax + by = d$. In terms of a, b and d, find:

a the x-intercept

b the y-intercept

c the gradient

ENRICHMENT: Graphical areas	–	–	15

15 Find the area enclosed by these lines.

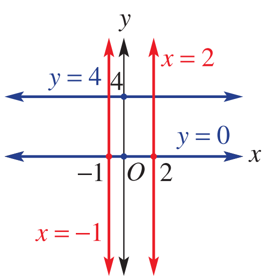
a $x = 2, x = -1, y = 0, y = 4$

b $x = 3, y = 2x, y = 0$

c $x = -3, y = -\dfrac{1}{2}x + 2, y = -2$

d $2x - 5y = -10, y = -2, x = 1$

e $y = 3x - 2, y = -3, y = 2 - x$



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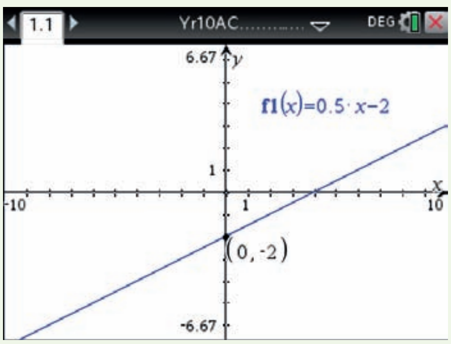
Using calculators to sketch straight lines

1. Sketch a graph of $y = 0.5x - 2$ and locate the x- and y-intercepts.
2. Construct a table of values for $y = 0.5x - 2$.

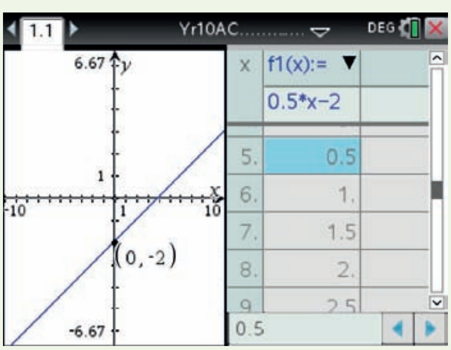
Using the TI-Nspire:

1. In a **Graphs** page, enter the rule $f1(x) = 0.5x - 2$. Use menu **>Trace>Graph Trace** and use the arrow keys to move left or right to observe intercepts. **Analyze Graph > Zero** can also be used for the x-intercept.



Hint: pressing enter will paste the intercept coordinates on the graph.

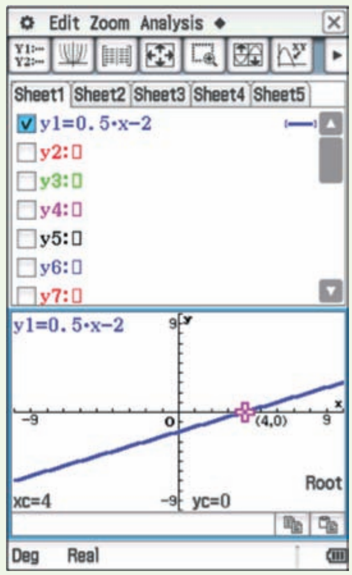
2. Press menu **>Table>Split-screen Table** to show the Table of Values.



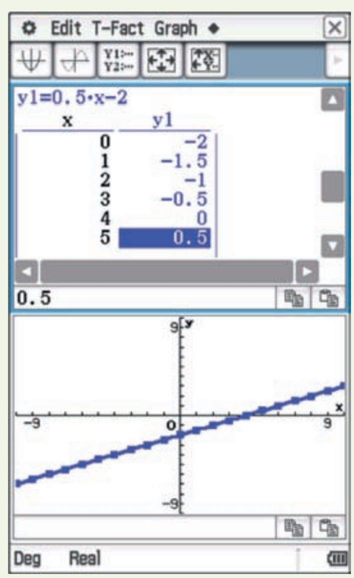
Hint: use ctrl + T as a shortcut to show the Table of Values.

Using the ClassPad:

1. In the **Graph&table** application enter the rule $y1 = 0.5x - 2$ followed by **EXE**. Tap [graph icon] to see the graph. Tap **Zoom**, **Quick Standard**. Tap **Analysis**, **G-Solve**, **Root** to locate the x-intercept. Tap **Analysis**, **G-Solve**, ***y*-intercept** to locate the y-intercept.



2. Tap [table settings icon] and set the table preferences to start at –10 and end at 10 with steps 1. Tap [table icon] to see the table.



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1G Finding an equation of a line

Learning intentions

- To understand that the gradient is the same between any two points on a straight line
- To know how to find the gradient of a line using two points
- To understand the gradient–intercept form, $y = mx + c$, of a straight line equation
- To be able to find the equation of a line given two points on the line
- To know the form of the equation of horizontal and vertical lines

It is a common procedure in mathematics to find the equation (or rule) of a straight line. Once the equation of a line is determined, it can be used to find the exact coordinates of other points on the line. Mathematics such as this can be used, for example, to predict a future company share price or the water level in a dam after a period of time.

Business equipment, such as a parcel courier's van, must eventually be replaced. For tax purposes, accountants calculate annual depreciation using the straight-line method. This reduces the equipment's value by an equal amount each year.

LESSON STARTER Fancy formula

Here is a proof of a rule for the equation of a straight line between any two given points.
Some of the steps are missing. See if you can fill them in.

$y = mx + c$

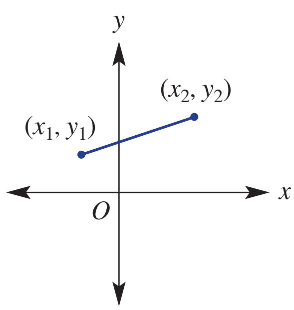
$y_1 = mx_1 + c$ (Substitute $(x, y) = (x_1, y_1)$.)

$\therefore c =$ ________

$\therefore y = mx +$ ________

$\therefore y - y_1 = m($________$)$

where $m = \dfrac{\quad}{x_2 - x_1}$



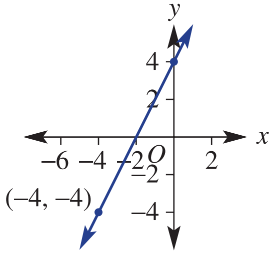
KEY IDEAS

- Horizontal lines have the equation $y = c$, where c is the y-intercept.
- Vertical lines have the equation $x = k$, where k is the x-intercept.
- Given the gradient (m) and the y-intercept (c), use $y = mx + c$ to state the equation of the line.
- To find the equation of a line when given any two points, find the gradient (m), then:
  - substitute a point to find c in $y = mx + c$, or
  - use $y - y_1 = m(x - x_1)$, where $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and (x_1, y_1), (x_2, y_2) are points on the line.

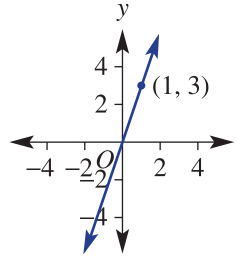
BUILDING UNDERSTANDING

1 State the gradient of the following lines.

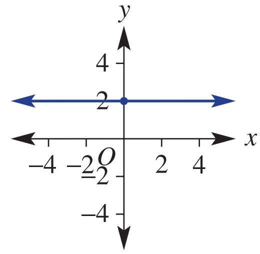
a



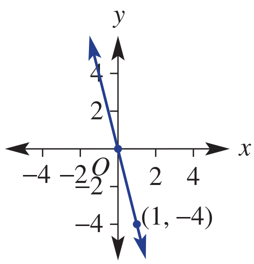
b



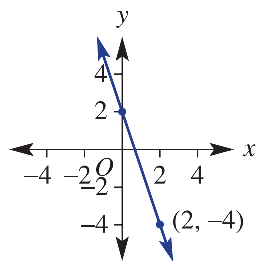
c



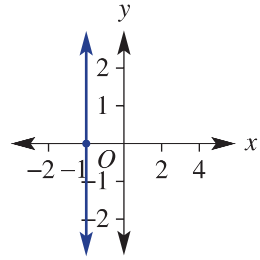
d



e



f



2 Find the value of c in $y = -2x + c$ when:

a $x = 3$ and $y = 2$

b $x = -1$ and $y = -4$

c $x = \frac{5}{2}$ and $y = 7$

Example 19 Finding the gradient of a line joining two points

Determine the gradient of the line joining the pair of points $(-3, 8)$ and $(5, -2)$.

SOLUTION

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

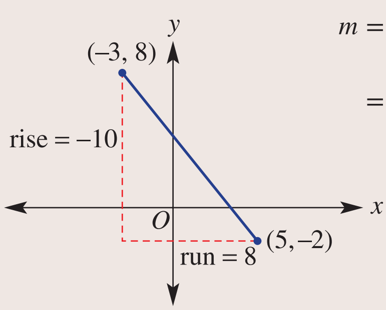
$$= \frac{-2 - 8}{5 - (-3)}$$

$$= -\frac{10}{8}$$

$$= -\frac{5}{4}$$

EXPLANATION

Use $(x_1, y_1) = (-3, 8)$
and $(x_2, y_2) = (5, -2)$.
Remember that $5 - (-3) = 5 + 3$.
Alternatively,



$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{-10}{8} \text{ (from diagram)}$$

Now you try

Determine the gradient of the line joining the pair of points $(-2, 6)$ and $(3, -1)$.

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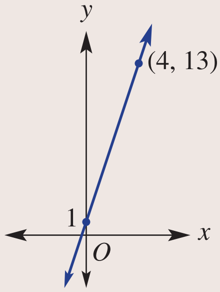
Example 20 Finding the equation of a line given the y-intercept and a point

Find the equation of the straight line shown.



SOLUTION

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{13 - 1}{4 - 0}$

$= \dfrac{12}{4}$

$= 3$

and $c = 1$

$\therefore y = 3x + 1$

EXPLANATION

The equation of a straight line is of the form $y = mx + c$.

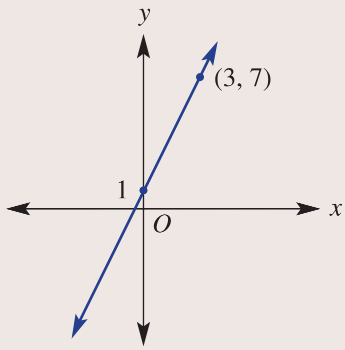
Find m using $(x_1, y_1) = (0, 1)$ and $(x_2, y_2) = (4, 13)$, or using $m = \dfrac{\text{rise}}{\text{run}}$ from the graph.

The y-intercept is 1.

Substitute $m = 3$ and $c = 1$.

Now you try

Find the equation of the line shown.



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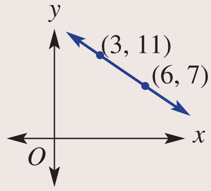
Example 21 Finding the equation of a line given two points

Find the equation of the straight line shown.



SOLUTION

Method 1

$y = mx + c$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{7 - 11}{6 - 3}$

$= -\dfrac{4}{3}$

$y = -\dfrac{4}{3}x + c$

$7 = -\dfrac{4}{3} \times (6) + c$

$7 = -8 + c$

$15 = c$

$\therefore y = -\dfrac{4}{3}x + 15$

Method 2

$y - y_1 = m(x - x_1)$

$y - 11 = -\dfrac{4}{3}(x - 3)$

$y = -\dfrac{4}{3}x + 4 + 11$

$= -\dfrac{4}{3}x + 15$

EXPLANATION

Use $(x_1, y_1) = (3, 11)$ and $(x_2, y_2) = (6, 7)$ in gradient formula, or $m = \dfrac{\text{rise}}{\text{run}}$ from the graph where rise $= -4$ (fall).

Substitute $m = -\dfrac{4}{3}$ into $y = mx + c$.

Substitute the point $(6, 7)$ or $(3, 11)$ to find the value of c.

Write the rule with both m and c.

Choose $(x_1, y_1) = (3, 11)$ or alternatively choose $(6, 7)$.

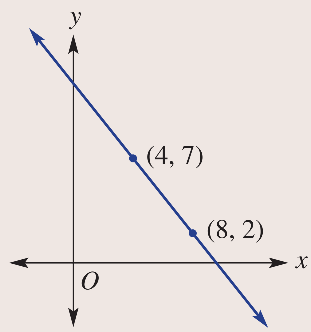
$m = -\dfrac{4}{3}$ was found using method 1.

Expand brackets and make y the subject.

$-\dfrac{4}{3} \times (-3) = 4$

Now you try

Find the equation of the straight line shown.



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Exercise 1G

FLUENCY	1, 2(1/2), 3, 4–5(1/2)	2–5(1/2)	2(1/3), 3–5(1/2)

Example 19

1 Determine the gradient of the line joining the following pairs of points.

a $(-2, 5)$ and $(2, -3)$ **b** $(-1, 4)$ and $(4, -2)$

Example 19

2 Determine the gradient of the line joining the following pairs of points.

a $(4, 2), (12, 4)$ **b** $(1, 4), (3, 8)$

c $(0, 2), (2, 7)$ **d** $(3, 4), (6, 13)$

e $(8, 4), (5, 4)$ **f** $(2, 7), (4, 7)$

g $(-1, 3), (2, 0)$ **h** $(-3, 2), (-1, 7)$

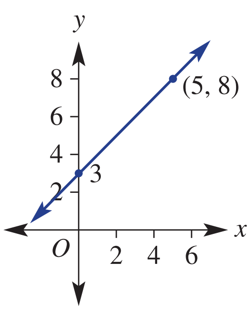
i $(-3, 4), (4, -1)$ **j** $(2, -3), (2, -5)$

k $(2, -3), (-4, -12)$ **l** $(-2, -5), (-4, -2)$

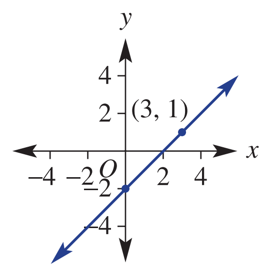
Example 20

3 Find the equation of the straight lines with the given y-intercepts.

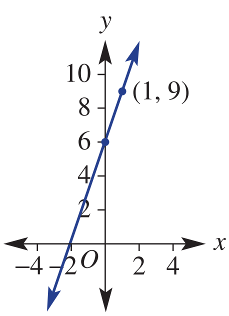
a



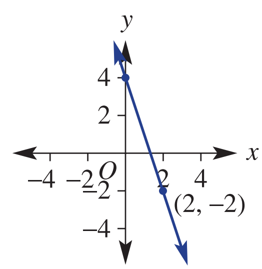
b



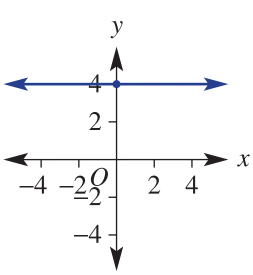
c



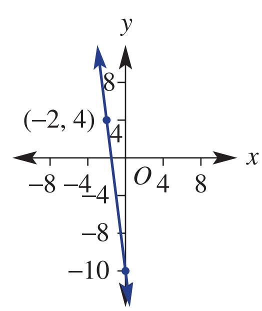
d



e



f



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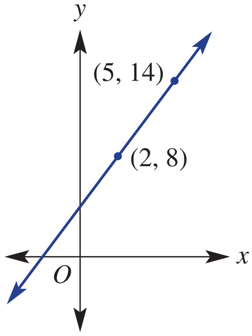
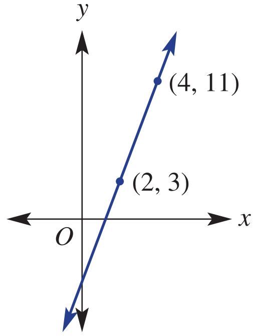
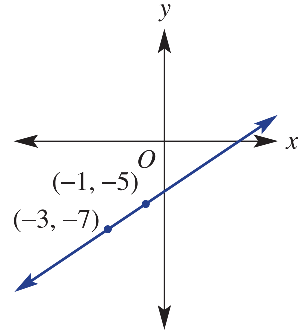
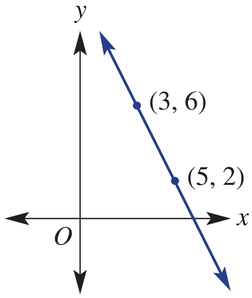
Example 21

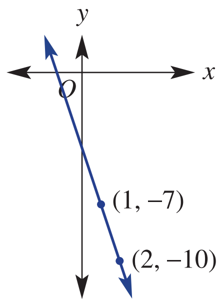
4 Find the equation of the straight lines with the given points.

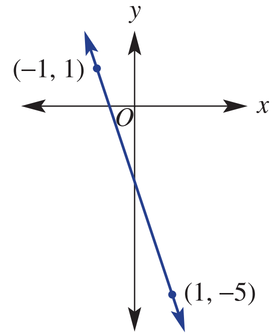
a


b


c


d


e


f


5 Given the following tables of values, determine the linear equation relating x and y in each case.

a

x	0	3
y	5	14

b

x	4	6
y	−4	−8

c

x	−1	3
y	−2	0

d

x	−2	1
y	2	−4

PROBLEM-SOLVING	6	6, 7	7, 8

6 Kyle invests some money in a simple savings fund and the amount increases at a constant rate over time. He hopes to buy a boat when the investment amount reaches \$20 000.
After 3 years the amount is \$16 500 and after 6 years the amount is \$18 000.

a Find a rule linking the investment amount (\$$A$) and time ($t$ years).

b How much did Kyle invest initially (i.e. when $t = 0$)?

c How long does Kyle have to wait before he can buy his boat?

d What would be the value of the investment after $12\frac{1}{2}$ years?

7 The cost of hiring a surfboard involves an up-front fee plus an hourly rate. Three hours of hire costs \$50 and 7 hours costs \$90.

a Sketch a graph of cost, \$$C$, for t hours of hire using the information given above.

b Find a rule linking \$$C$ in terms of t hours.

c i State the cost per hour.

ii State the up-front fee.

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8 **a** The following information applies to the filling of a flask with water, at a constant rate. In each case, find a rule for the volume, V litres, in terms of t minutes.

- **i** Initially (i.e. at $t = 0$) the flask is empty (i.e. $V = 0$) and after 1 minute it contains 4 litres of water.
- **ii** Initially (i.e. at $t = 0$) the flask is empty (i.e. $V = 0$) and after 3 minutes it contains 9 litres of water.
- **iii** After 1 and 2 minutes, the flask has 2 and 3 litres of water, respectively.
- **iv** After 1 and 2 minutes, the flask has 3.5 and 5 litres of water, respectively.

b For parts **iii** and **iv** above, find how much water was in the flask initially.

c Write your own information that would give the rule $V = -t + b$.

REASONING	9	9, 10	9, 10

9 A line joins the two points $(-1, 3)$ and $(4, -2)$.

a Calculate the gradient of the line using $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ where $(x_1, y_1) = (-1, 3)$ and $(x_2, y_2) = (4, -2)$.

b Calculate the gradient of the line using $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ where $(x_1, y_1) = (4, -2)$ and $(x_2, y_2) = (-1, 3)$.

c What conclusions can you draw from your results from parts **a** and **b** above? Give an explanation.

10 A line passes through the points $(1, 3)$ and $(4, -1)$.

a Calculate the gradient.

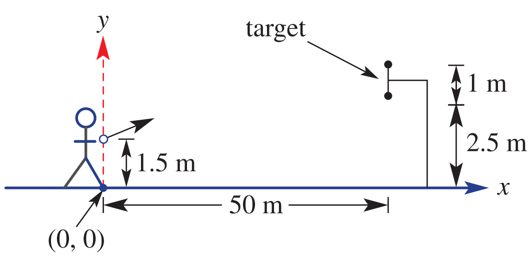
b Using $y - y_1 = m(x - x_1)$ and $(x_1, y_1) = (1, 3)$, find the rule for the line.

c Using $y - y_1 = m(x - x_1)$ and $(x_1, y_1) = (4, -1)$, find the rule for the line.

d What do you notice about your results from parts **b** and **c**? Can you explain why this is the case?

ENRICHMENT: Linear archery	–	–	11

11 An archer's target is 50 m away and is 2.5 m off the ground, as shown. Arrows are fired from a height of 1.5 m and the circular target has a diameter of 1 m.



a Find the gradient of the straight trajectory from the arrow (in firing position) to:

- **i** the bottom of the target
- **ii** the top of the target.

b If the position of the ground directly below the firing arrow is the point $(0, 0)$ on a Cartesian plane, find the equation of the straight trajectory to:

- **i** the bottom of the target
- **ii** the top of the target.

c If $y = mx + c$ is the equation of the arrow's trajectory, what are the possible values of m if the arrow is to hit the target?

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Progress quiz

1A **1** Simplify the following.

a $15a^2b + 2ab - 6ba^2 + 8b$ b $-3xy \times 4x^2$ c $4(m + 5) + 3(3m - 2)$

1B **2** Simplify by cancelling common factors.

a $\dfrac{36mk^2}{9mk}$ b $\dfrac{3a - 12}{3}$ c $\dfrac{21x - 3x^2}{6x}$

d $\dfrac{4}{m} \times \dfrac{m + 3}{12}$ e $\dfrac{a + 4}{4a} \times \dfrac{18a^2}{a + 4}$ f $\dfrac{6h - 15}{6} \div \dfrac{2h - 5}{5}$

1C **3** Simplify the following algebraic fractions.

a $\dfrac{3}{4} + \dfrac{m}{8}$ b $\dfrac{2}{3} - \dfrac{5}{2x}$ c $\dfrac{a + 4}{8} + \dfrac{1 - 3a}{12}$ d $\dfrac{5}{m - 1} - \dfrac{2}{m - 3}$

1D **4** Solve the following equations and check your solution by substitution.

a $2x + 8 = 18$ b $2(3k - 4) = -17$ c $\dfrac{m + 5}{5} = 7$ d $\dfrac{2a - 3}{3} = \dfrac{4a + 2}{4}$

1E **5** Solve the following inequalities and graph their solutions on a number line.

a $2a + 3 > 9$ b $8 - \dfrac{x}{2} \leqslant 10$

c $5m + 2 > 7m - 6$ d $-(a - 3) \leqslant 5(a + 3)$

1F **6** Decide if the point $(-3, 2)$ is on the line with the given equations.

a $y = x + 2$ b $-2x + y = 8$

1F **7** Sketch the following linear relations. For parts **a** and **b**, use the method suggested.

a $y = -\dfrac{3}{2}x + 1$; Use the gradient and y-intercept.

b $-2x - 3y = 6$; Use the x- and y-intercepts.

c $y = 3$

d $x = -2$

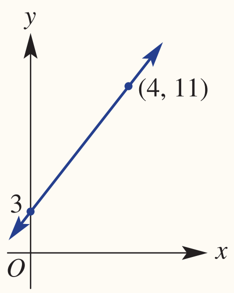
e $y = -\dfrac{3}{4}x$

1F **8** State the gradient and y-intercept of the following lines.

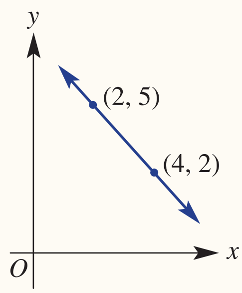
a $y = 3x - 2$ b $3x + 5y = 15$

1G **9** Find the equation of each straight line shown.

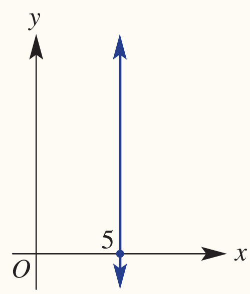
a



b



c



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1H Length and midpoint of a line segment

Learning intentions
- To know the meaning of the terms line segment and midpoint
- To understand that Pythagoras' theorem can be used to find the distance between two points
- To be able to find the length of a line segment (or distance between two points)
- To know how to find the midpoint of a line segment

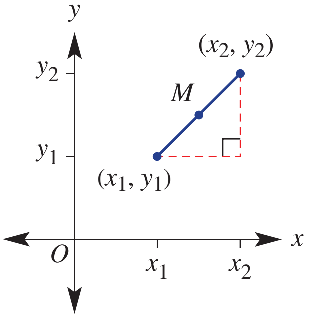
Two important features of a line segment (or line interval) are its length and midpoint. The length can be found using Pythagoras' theorem and the midpoint can be found by considering the midpoints of the horizontal and vertical components of the line segment.

Using coordinates to define locations and calculate distances are widely applied procedures, including by spatial engineers, geodetic engineers, surveyors, cartographers, navigators, geologists, archaeologists and biologists.

LESSON STARTER Developing the rules

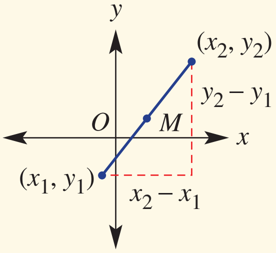
The line segment shown has endpoints (x_1, y_1) and (x_2, y_2).

- Length: Use your knowledge of Pythagoras' theorem to find the rule for the length of the segment.
- Midpoint: State the coordinates of M (the midpoint) in terms of x_1, y_1, x_2 and y_2. Give reasons for your answer.



KEY IDEAS

- The **length of a line segment** (or distance between two points (x_1, y_1) and (x_2, y_2)) d is given by the rule:
  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
  - This rule comes from Pythagoras' theorem where the distance d is the length of the hypotenuse of the right-angled triangle formed.



- The **midpoint** M of a line segment between (x_1, y_1) and (x_2, y_2) is given by:
  $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

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BUILDING UNDERSTANDING

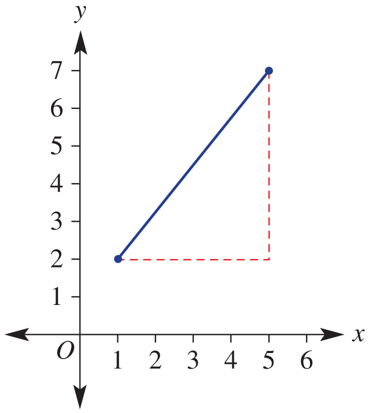
1 The endpoints for the given line segment are (1, 2) and (5, 7).

a What is the horizontal distance between the two endpoints?

b What is the vertical distance between the two endpoints?

c Use Pythagoras' theorem ($c^2 = a^2 + b^2$) to find the exact length of the segment.

d State the midpoint of the segment.



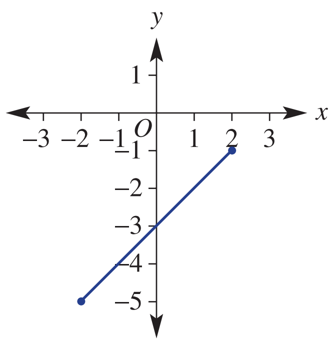
2 The endpoints for the given line segment are (−2, −5) and (2, −1).

a What is the horizontal distance between the two endpoints?

b What is the vertical distance between the two endpoints?

c Use Pythagoras' theorem ($c^2 = a^2 + b^2$) to find the exact length of the segment.

d State the midpoint of the segment.



3 Simplify the following.

a $\dfrac{2 + 4}{2}$

b $\dfrac{3 + 8}{2}$

c $\dfrac{-4 + 10}{2}$

d $\dfrac{-6 + (-2)}{2}$

Example 22 Finding the distance between two points

Find the exact distance between each pair of points.

a (0, 2) and (1, 7)

b (−3, 8) and (4, −1)

SOLUTION

a
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(1 - 0)^2 + (7 - 2)^2}$$
$$= \sqrt{1^2 + 5^2}$$
$$= \sqrt{26}$$

EXPLANATION

Let $(x_1, y_1) = (0, 2)$ and $(x_2, y_2) = (1, 7)$.
Alternatively, sketch the points and use Pythagoras' theorem.

Simplify and express your answer exactly, using a surd.

b
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - (-3))^2 + (-1 - 8)^2}$$
$$= \sqrt{7^2 + (-9)^2}$$
$$= \sqrt{49 + 81}$$
$$= \sqrt{130}$$

Let $(x_1, y_1) = (-3, 8)$ and $(x_2, y_2) = (4, -1)$.
Alternatively, let $(x_1, y_1) = (4, -1)$ and $(x_2, y_2) = (-3, 8)$.
Either way the answers will be the same.

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Now you try

Find the exact distance between each pair of points.

a (0, 3) and (1, 5) **b** (−2, 7) and (3, −1)

Example 23 Finding the midpoint of a line segment joining two points

Find the midpoint of the line segment joining (−3, −5) and (2, 8).

SOLUTION

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-3 + 2}{2}, \frac{-5 + 8}{2}\right)$$
$$= \left(-\frac{1}{2}, \frac{3}{2}\right)$$

EXPLANATION

Let $(x_1, y_1) = (-3, -5)$ and $(x_2, y_2) = (2, 8)$.

This is equivalent to finding the average of the x-coordinates and the average of the y-coordinates of the two points.

Now you try

Find the midpoint of the line segment joining (−2, −6) and (3, −2).

Example 24 Using a given distance to find coordinates

Find the values of a if the distance between $(2, a)$ and $(4, 9)$ is $\sqrt{5}$.

SOLUTION

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\sqrt{5} = \sqrt{(4 - 2)^2 + (9 - a)^2}$$
$$\sqrt{5} = \sqrt{2^2 + (9 - a)^2}$$
$$5 = 4 + (9 - a)^2$$
$$1 = (9 - a)^2$$
$$\pm 1 = 9 - a$$

So $9 - a = 1$ or $9 - a = -1$.

$\therefore a = 8$ or $a = 10$

EXPLANATION

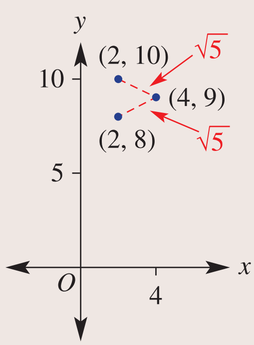
Substitute all the given information into the rule for the distance between two points.

Simplify and then square both sides to eliminate the square roots.

Subtract 4 from both sides and take the square root of each side.

Remember, if $x^2 = 1$ then $x = \pm 1$.

Solve for a. You can see there are two solutions.



Now you try

Find the values of a if the distance between $(1, a)$ and $(3, 7)$ is $\sqrt{13}$.

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Exercise 1H

FLUENCY	1, 2–3(1/2)	2–3(1/2)	2–3(1/3)

1 Find the exact distance between:

Example 22a
a **i** (0, 1) and (1, 3) **ii** (0, 3) and (2, 7)

Example 22b
b **i** (−3, 7) and (4, −2) **ii** (−2, 8) and (1, −1)

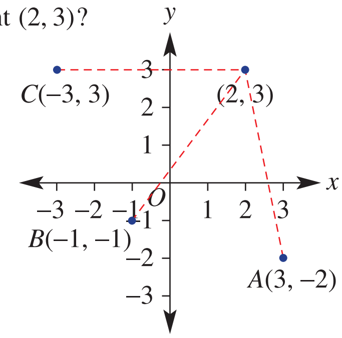
Example 22
2 Find the exact distance between these pairs of points.

a (0, 4) and (2, 9) **b** (0, −1) and (3, 6) **c** (−1, 4) and (0, −2)
d (−3, 8) and (1, 1) **e** (−2, −1) and (4, −2) **f** (−8, 9) and (1, −3)
g (−8, −1) and (2, 0) **h** (−4, 6) and (8, −1) **i** (−10, 11) and (−4, 10)

Example 23
3 Find the midpoint of the line segment joining the given points in Question **2**.

PROBLEM-SOLVING	4, 5–6(1/2)	4, 5–6(1/2)	5–6(1/2), 7

4 Which of the points A, B or C shown on these axes is closest to the point (2, 3)?



5 Find the value of a and b when:

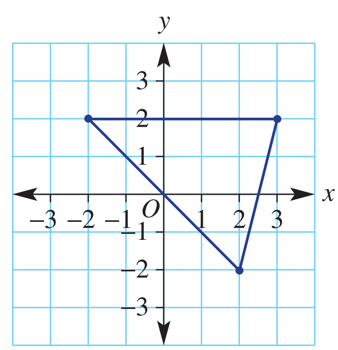
a The midpoint of $(a, 3)$ and $(7, b)$ is (5, 4).
b The midpoint of $(a, -1)$ and $(2, b)$ is (−1, 2).
c The midpoint of $(-3, a)$ and $(b, 2)$ is $\left(-\frac{1}{2}, 0\right)$.
d The midpoint of $(-5, a)$ and $(b, -4)$ is $\left(-\frac{3}{2}, \frac{7}{2}\right)$.

Example 24
6 Find the values of a when:

a The distance between $(1, a)$ and $(3, 5)$ is $\sqrt{8}$.
b The distance between $(2, a)$ and $(5, 1)$ is $\sqrt{13}$.
c The distance between $(a, -1)$ and $(4, -3)$ is $\sqrt{29}$.
d The distance between $(-3, -5)$ and $(a, -9)$ is 5.

7 A block of land is illustrated on this simple map, which uses the ratio 1 : 100 (i.e. 1 unit represents 100 m).

a Find the perimeter of the block, correct to the nearest metre.
b The block is to be split up into four triangular areas by building three fences that join the three midpoints of the sides of the block. Find the perimeter of the inside triangular area.



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REASONING	8	8, 9	9, 10(½)

8 A line segment has endpoints $(-2, 3)$ and $(1, -1)$.

- **a** Find the midpoint using $(x_1, y_1) = (-2, 3)$ and $(x_2, y_2) = (1, -1)$.
- **b** Find the midpoint using $(x_1, y_1) = (1, -1)$ and $(x_2, y_2) = (-2, 3)$.
- **c** Give a reason why the answers to parts **a** and **b** are the same.
- **d** Find the length of the segment using $(x_1, y_1) = (-2, 3)$ and $(x_2, y_2) = (1, -1)$.
- **e** Find the length of the segment using $(x_1, y_1) = (1, -1)$ and $(x_2, y_2) = (-2, 3)$.
- **f** What do you notice about your answers to parts **d** and **e**? Give an explanation for this.

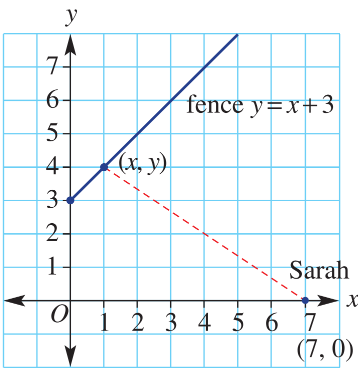
9 The distance between the points $(-2, -1)$ and $(a, 3)$ is $\sqrt{20}$. Find the values of a and use a Cartesian plane to illustrate why there are two solutions for a.

10 Find the coordinates of the point that divides the segment joining $(-2, 0)$ and $(3, 4)$ in the given ratio. Ratios are to be considered from left to right.

a 1 : 1 **b** 1 : 2 **c** 2 : 1 **d** 4 : 1 **e** 1 : 3 **f** 2 : 3

ENRICHMENT: Shortest distance	–	–	11

11 Sarah pinpoints her position on a map as $(7, 0)$ and wishes to hike towards a fence line that follows the path $y = x + 3$, as shown. (Note: 1 unit = 100 m).



- **a** Using the points $(7, 0)$ and (x, y), write a rule in terms of x and y for the distance between Sarah and the fence.
- **b** Use the equation of the fence line to write the rule in part **a** in terms of x only.
- **c** Use your rule from part **b** to find the distance between Sarah and the fence line to the nearest metre when:
  i $x = 1$ **ii** $x = 2$ **iii** $x = 3$ **iv** $x = 4$
- **d** Which x-value from part **c** gives the shortest distance?
- **e** Consider any point on the fence line and find the coordinates of the point such that the distance will be a minimum. Give reasons.

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

1I Perpendicular and parallel lines

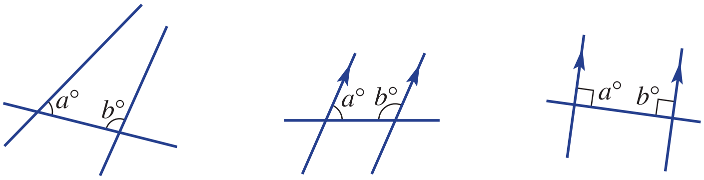
Learning intentions

- To know what it means for lines to be parallel or perpendicular
- To know that parallel lines have the same gradient
- To know that the gradients of perpendicular lines multiply to –1
- To be able to determine if lines are parallel or perpendicular using their gradients
- To be able to find the equation of a parallel or perpendicular line given a point on the line

Euclid of Alexandria (300 BC) was a Greek mathematician and is known as the 'father of geometry'. In his texts, known as *Euclid's Elements*, his work is based on five simple axioms. The fifth axiom, called the 'Parallel Postulate', states: 'It is true that if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, intersect on that side on which are the angles less than the two right angles.'

The mathematician Euclid of Alexandria

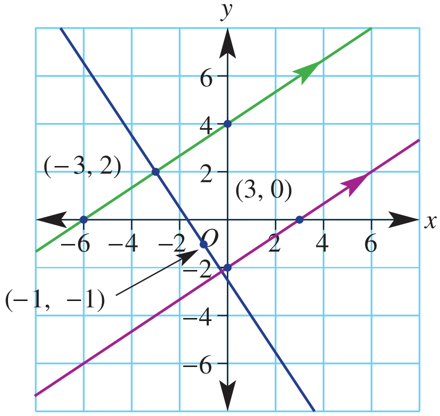
In simple terms, the Parallel Postulate says that if cointerior angles do not sum to 180°, then the two lines are not parallel. Furthermore, if the two interior angles are equal and also sum to 180°, then the third line must be perpendicular to the other two.



LESSON STARTER Gradient connection

Shown here is a pair of parallel lines and a third line that is perpendicular to the other two lines.

- Find the equation of each line, using the coordinates shown on the graph.
- What is common about the rules for the two parallel lines?
- Is there any connection between the rules of the parallel lines and the perpendicular line? Can you write down this connection as a formula?



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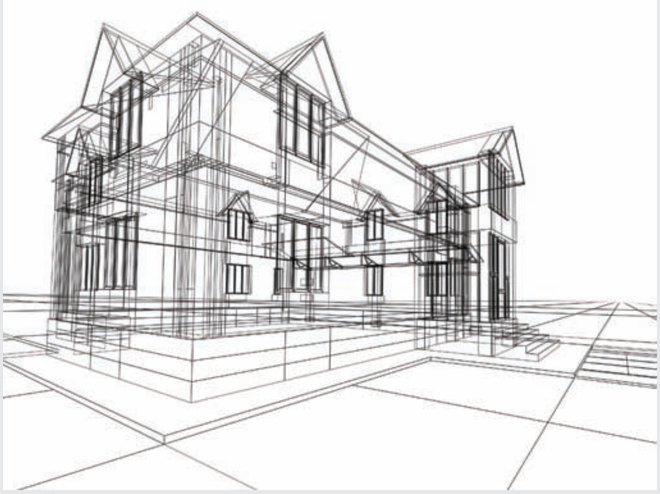
KEY IDEAS

- Two **parallel lines** have the same gradient.
  For example, $y = 3x - 1$ and $y = 3x + 8$ have the same gradient of 3.
- Two **perpendicular lines** with gradients m_1 and m_2 satisfy the following rule:
  $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$ (i.e. m_2 is the negative reciprocal of m_1).
- Equations of parallel or perpendicular lines can be found by:
  - first finding the gradient (m)
  - then substituting a point to find c in $y = mx + c$.



Two planes, such as walls, intersect along a straight line. In a house design, linear equations in 3D can model the many parallel and perpendicular lines. Solving simultaneously finds the precise location of the intersection points.

BUILDING UNDERSTANDING

1 What is the gradient of the line that is parallel to the graph of these equations?

a $y = 4x - 6$ **b** $y = -7x - 1$ **c** $y = -\frac{3}{4}x + 2$ **d** $y = \frac{8}{7}x - \frac{1}{2}$

2 Use $m_2 = -\frac{1}{m_1}$ to find the gradient of the line that is perpendicular to the graphs of the following equations.

a $y = 3x - 1$ **b** $y = -2x + 6$ **c** $y = \frac{7}{8}x - \frac{2}{3}$ **d** $y = -\frac{4}{9}x - \frac{4}{7}$

3 A line is parallel to the graph of the rule $y = 5x - 2$ and its y-intercept is 4. The rule for the line is of the form $y = mx + c$.

a State the value of m. **b** State the value of c. **c** Find the rule.

4 Answer true or false.

a The lines $y = 2x$ and $y = 2x + 3$ are parallel.
b The lines $y = 3x$ and $y = -3x + 2$ are perpendicular.

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Example 25 Deciding if lines are parallel or perpendicular

Decide if the graph of each pair of rules will be parallel, perpendicular or neither.

a $y = -3x - 8$ and $y = \frac{1}{3}x + 1$

b $y = \frac{1}{2}x + 2$ and $2y - x = 5$

c $3x + 2y = -5$ and $x - y = 2$

SOLUTION / EXPLANATION

a $y = -3x - 8,\ m = -3$ (1)

Both equations are in the form $y = mx + c$.

$y = \frac{1}{3}x + 1,\ m = \frac{1}{3}$ (2)

$-3 \times \frac{1}{3} = -1$

Test: $m_1 \times m_2 = -1$.

So the lines are perpendicular.

b $y = \frac{1}{2}x + 2,\ m = \frac{1}{2}$ (1)

Write both equations in the form $y = mx + c$.

$2y - x = 5$

$2y = x + 5$

$y = \frac{1}{2}x + \frac{5}{2},\ m = \frac{1}{2}$ (2)

Both lines have a gradient of $\frac{1}{2}$, so the lines are parallel.

So the lines are parallel.

c $3x + 2y = -5$

Write both equations in the form $y = mx + c$.

$2y = -3x - 5$

Note: $\frac{-3x}{2} = -\frac{3x}{2}$

$y = -\frac{3}{2}x - \frac{5}{2},\ m = -\frac{3}{2}$ (1)

$x - y = 2$

Subtract x from both sides, then divide both sides by -1.

$-y = -x + 2$

$y = x - 2, m = 1$ (2)

$-\frac{3}{2} \times 1 \neq -1$

Test: $m_1 \times m_2 = -1$.

So the lines are neither parallel nor perpendicular.

The gradients are not equal and $m_1 \times m_2 \neq -1$.

Now you try

Decide if the graph of each pair of rules will be parallel, perpendicular or neither.

a $y = -4x - 1$ and $y = \frac{1}{4}x + 3$

b $y = \frac{1}{2}x + 1$ and $2y + x = 7$

c $5x - 3y = -10$ and $5x = 3y + 2$

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Example 26 Finding the equation of a parallel or perpendicular line

Find the equation of the line that is:

a parallel to $y = -2x - 7$ and passes through (1, 9)

b perpendicular to $y = \frac{3}{4}x - 1$ and passes through (3, −2)

SOLUTION

a $y = mx + c$

$m = -2$

$y = -2x + c$

Substitute (1, 9):

$9 = -2(1) + c$

$11 = c$

$\therefore y = -2x + 11$

b $y = mx + c$

$m = \frac{-1}{\left(\frac{3}{4}\right)}$

$= -\frac{4}{3}$

$y = -\frac{4}{3}x + c$

Substitute (3,−2):

$-2 = -\frac{4}{3}(3) + c$

$-2 = -4 + c$

$c = 2$

$\therefore y = -\frac{4}{3}x + 2$

EXPLANATION

a Write the general equation of a line.

Since the line is parallel to $y = -2x - 7$, $m = -2$.

Substitute the given point (1, 9), where $x = 1$ and $y = 9$, and solve for c.

b The perpendicular gradient is the negative reciprocal of $\frac{3}{4}$.

Substitute (3, −2) and solve for c.

Now you try

Find the equation of the line that is:

a parallel to $y = -3x - 5$ and passes through (1, 5)

b perpendicular to $y = \frac{2}{3}x - 3$ and passes through (2, −1)

Exercise 1I

FLUENCY	1, 2–3(1/2)	2–3(1/2)	2–3(1/2)

1 Decide if the graph of each pair of rules will be parallel, perpendicular or neither.

Example 25a **a** $y = -2x - 3$ and $y = \frac{1}{2}x + 2$

Example 25b **b** $y = \frac{1}{2}x + 1$ and $2y - x = 4$

Example 25c **c** $4x + 2y = -4$ and $x - y = 3$

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Example 25

2 Decide if the line graphs of each pair of rules will be parallel, perpendicular or neither.

a $y = 3x - 1$ and $y = 3x + 7$

b $y = \frac{1}{2}x - 6$ and $y = \frac{1}{2}x - 4$

c $y = -\frac{2}{3}x + 1$ and $y = \frac{2}{3}x - 3$

d $y = -4x - 2$ and $y = x - 7$

e $y = -\frac{3}{7}x - \frac{1}{2}$ and $y = \frac{7}{3}x + 2$

f $y = -8x + 4$ and $y = \frac{1}{8}x - 2$

g $2y + x = 2$ and $y = -\frac{1}{2}x - 3$

h $x - y = 4$ and $y = x + \frac{1}{2}$

i $8y + 2x = 3$ and $y = 4x + 1$

j $3x - y = 2$ and $x + 3y = 5$

Example 26

3 Find the equation of the line that is:

a parallel to $y = x + 3$ and passes through $(1, 5)$

b parallel to $y = -x - 5$ and passes through $(1, -7)$

c parallel to $y = -4x - 1$ and passes through $(-1, 3)$

d parallel to $y = \frac{2}{3}x + 1$ and passes through $(3, -4)$

e parallel to $y = -\frac{4}{5}x + \frac{1}{2}$ and passes through $(5, 3)$

f perpendicular to $y = 2x + 3$ and passes through $(2, 5)$

g perpendicular to $y = -4x + 1$ and passes through $(-4, -3)$

h perpendicular to $y = \frac{2}{3}x - 4$ and passes through $(4, -1)$

i perpendicular to $y = \frac{4}{3}x + \frac{1}{2}$ and passes through $(-4, -2)$

j perpendicular to $y = -\frac{2}{7}x - \frac{3}{4}$ and passes through $(-8, 3)$.

PROBLEM-SOLVING	4(½)	4–6(½)	4–6(½), 7

4 This question involves vertical and horizontal lines. Find the equation of the line that is:

a parallel to $x = 3$ and passes through $(6, 1)$

b parallel to $x = -1$ and passes through $(0, 0)$

c parallel to $y = -3$ and passes through $(8, 11)$

d parallel to $y = 7.2$ and passes through $(1.5, 8.4)$

e perpendicular to $x = 7$ and passes through $(0, 3)$

f perpendicular to $x = -4.8$ and passes through $(2.7, -3)$

g perpendicular to $y = -\frac{3}{7}$ and passes through $\left(\frac{2}{3}, \frac{1}{2}\right)$

h perpendicular to $y = \frac{8}{13}$ and passes through $\left(-\frac{4}{11}, \frac{3}{7}\right)$.

5 Find the equation of the line that is parallel to these equations and passes through the given points.

a $y = \frac{2x - 1}{3}, (0, 5)$

b $y = \frac{3 - 5x}{7}, (1, 7)$

c $3y - 2x = 3, (-2, 4)$

d $7x - y = -1, (-3, -1)$

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Cambridge University Press
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6 Find the equation of the line that is perpendicular to the equations given in Question 5 and passes through the same given points.

7 A line with equation $3x - 2y = 12$ intersects a second line at the point where $x = 2$. If the second line is perpendicular to the first line, find where the second line cuts the x-axis.

REASONING	8	8, 9	8–10

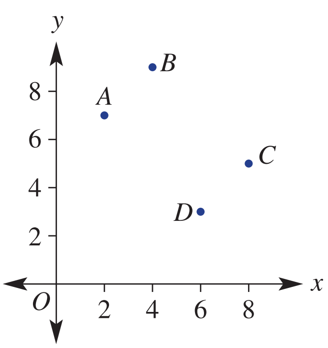
8 Find an expression for the gradient of a line if it is:
- a parallel to $y = mx + 8$
- b parallel to $ax + by = 4$
- c perpendicular to $y = mx - 1$
- d perpendicular to $ax + by = -3$.

9
- a Find the value of a if $y = \frac{a}{7}x + c$ is parallel to $y = 2x - 4$.
- b Find the value of a if $y = \left(\frac{2a + 1}{3}\right)x + c$ is parallel to $y = -x - 3$.
- c Find the value of a if $y = \left(\frac{1 - a}{2}\right)x + c$ is perpendicular to $y = \frac{1}{2}x - \frac{3}{5}$.
- d Find the value of a if $ay = 3x + c$ is perpendicular to $y = -\frac{3}{7}x - 1$.

10 Find the equation of a line that is:
- a parallel to $y = 2x + c$ and passes through (a, b)
- b parallel to $y = mx + c$ and passes through (a, b)
- c perpendicular to $y = -x + c$ and passes through (a, b)
- d perpendicular to $y = mx + c$ and passes through (a, b).

ENRICHMENT: Perpendicular and parallel geometry	–	–	11–13

11 A quadrilateral, $ABCD$, has vertex coordinates $A(2, 7)$, $B(4, 9)$, $C(8, 5)$ and $D(6, 3)$.
- a Find the gradient of these line segments.
  - i AB ii BC iii CD iv DA
- b What do you notice about the gradients of opposite and adjacent sides?
- c What type of quadrilateral is $ABCD$?



12 The vertices of triangle ABC are $A(0, 0)$, $B(3, 4)$ and $C\left(\frac{25}{3}, 0\right)$.
- a Find the gradient of these line segments.
  - i AB ii BC iii CA
- b What type of triangle is $\triangle ABC$?
- c Find the perimeter of $\triangle ABC$.

13 Find the equation of the perpendicular bisector of the line segment joining (1, 1) with (3, 5) and find where this bisector cuts the x-axis.

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Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Business profit

1 Abby runs an online business making and selling Christmas stockings. Over time she has worked out that the cost to make and deliver 3 Christmas stockings is $125, while the cost to make and deliver 7 stockings is $185. The cost to produce each Christmas stocking is the same. Costs involved also include the purchase of the tools to make the stockings.

Abby is interested in exploring the relationship between her profit, costs and selling price. She wants to determine the 'break-even' point and look at how this is impacted if the selling price is adjusted.

- **a** Draw a graph of cost versus the number of stockings made using the information given and assuming a linear relationship.
- **b** From your graph, determine a rule for the cost, C dollars, of making and delivering s stockings.

Abby sells each stocking for $25.

- **c** Write a rule for the amount she would receive (revenue), R dollars, from selling s stockings and sketch its graph on the same axes, as in part **a**.
- **d** What are the coordinates of the point where the graphs intersect?
- **e** Profit is defined as revenue – cost.
  - **i** Give a rule for the profit, P dollars, from selling s stockings.
  - **ii** Use your equation from part **i** to find how many stockings must be sold to break even.
- **f** In the lead up to Christmas each year, Abby finds that she sells on average t stockings. She considers adjusting the selling price of the stockings at this time of year.
  - **i** Determine the minimum price, p dollars, she should sell her stockings for, in terms of t, to break even.
  - **ii** Use your result from part **i** to determine the minimum selling price if $t = 5$ or if $t = 25$.

Comparing speeds

2 To solve problems involving distance, speed and time we use the following well-known rule:

$$\text{distance} = \text{speed} \times \text{time}$$

We will explore how we can use this simple rule to solve common problems. These include problems where objects travel towards each other, follow behind or chase one another, and problems where speed is altered mid-journey.

- **a** Two cars travel towards each other on a 100 km stretch of road. One car travels at 80 km/h and the other at 70 km/h.
  - **i** How far does each car travel in 1 hour?

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Cambridge University Press
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ii Complete the table below to determine how far each car has travelled after t hours.

	Speed (km/h)	Time (hours)	Distance (km)
Car A		t	
Car B		t	

iii Hence, if the cars set off at the same time, how long will it be before the cars meet (i.e. cover the 100 km between them)? Answer in minutes.

iv If two cars travel at x km/h and y km/h respectively and there is d km between them, determine after how long they will meet in terms of x, y and d.

b Ed's younger brother leaves the house on his bicycle and rides at 2 km/h. Ed sets out after his brother on his bike x hours later, travelling at 7 km/h.

i Use an approach like in part **a** to find a rule for the time taken for Ed to catch up to his younger brother in terms of x.

ii What is this time if $x = 1$?

c Meanwhile, Sam is driving from city A to city B. After 2 hours of driving she noticed that she covered 80 km and calculated that, if she continued driving at the same speed, she would end up being 15 minutes late. She therefore increased her speed by 10 km/h and she arrived at city B 36 minutes earlier than she planned. Find the distance between cities A and B.

Crossing the road

3 Coordinate geometry provides a connection between geometry and algebra where points and lines can be explored precisely using coordinates and equations.

We will investigate the shortest path between sets of points positioned on parallel lines to find the shortest distance to cross the road.

Two parallel lines with equations $y = 2x + 2$ and $y = 2x + 12$ form the sides of a road. A chicken is positioned at (2, 6) along one of the sides of the road. Three bags of grain are positioned on the other side of the road at (0, 12), (−2, 8) and (3, 18).

a What is the shortest distance the chicken would have to cover to get to one of the bags of grain?

b If the chicken crosses the road to get directly to the closest bag of grain, give the equation of the direct line the chicken walks along.

c By considering your equation in part **b**, explain why this is the shortest possible distance the chicken could walk to cross the road.

d Using the idea from part **c**, find the distance between the parallel lines with equations $y = 3x + 1$ and $y = 3x + 11$ using the point (3, 10) on the first line.

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

1J Simultaneous equations using substitution

Learning intentions

- To understand that a single linear equation in two variables has an infinite number of solutions
- To know that two simultaneous linear equations (straight lines) can have 0 or 1 solution (points of intersection)
- To understand that the solution of two simultaneous equations satisfies both equations and lies on both straight line graphs
- To know how to substitute one algebraic expression for another to obtain an equation in one unknown
- To be able to solve simultaneous equations using the substitution method

When we try to find a solution to a set of equations rather than just a single equation, we say that we are solving simultaneous equations. For two linear simultaneous equations we are interested in finding the point where the graphs of the two equations meet. The point, for example, at the intersection of a company's cost equation and revenue equation is the 'break-even point'. This determines the point at which a company will start making a profit.

Simultaneous equations can solve personal finance questions such as: finding the best deal for renting a house or buying a car; the job where you will earn the most money over time; and the most profitable investment account.

LESSON STARTER Give up and do the algebra

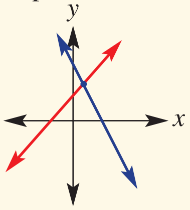
The two simultaneous equations $y = 2x - 3$ and $4x - y = 5\frac{1}{2}$ have a single solution.

- Use a guess and check (i.e. trial and error) technique to try to find the solution.
- Try a graphical technique to find the solution. Is this helpful?
- Now find the exact solution using the algebraic method of substitution.
- Which method is better? Discuss.

KEY IDEAS

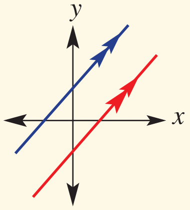
- Solving two **simultaneous equations** involves finding a solution that satisfies both equations.
  - When two straight lines are not parallel, there will be a single (**unique**) solution.

Non-parallel lines



1 point of intersection

Parallel lines (same gradient)



0 points of intersection

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Cambridge University Press
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- The **substitution** method is usually used when at least one of the equations has one pronumeral as the subject. For example, $y = 3x + 2$ or $x = 3y - 1$.
  - By substituting one equation into the other, a single equation in terms of one pronumeral is formed and can then be solved.
    For example:
    $x + y = 8$ (1)
    $y = 3x + 4$ (2)
    Substitute (2) in (1): $x + (3x + 4) = 8$
    $4x + 4 = 8$
    $\therefore x = 1$
    Find y: $y = 3(1) + 4 = 7$
    So the solution is $x = 1$ and $y = 7$. The point $(1, 7)$ is the intersection point of the graphs of the two relations.

BUILDING UNDERSTANDING

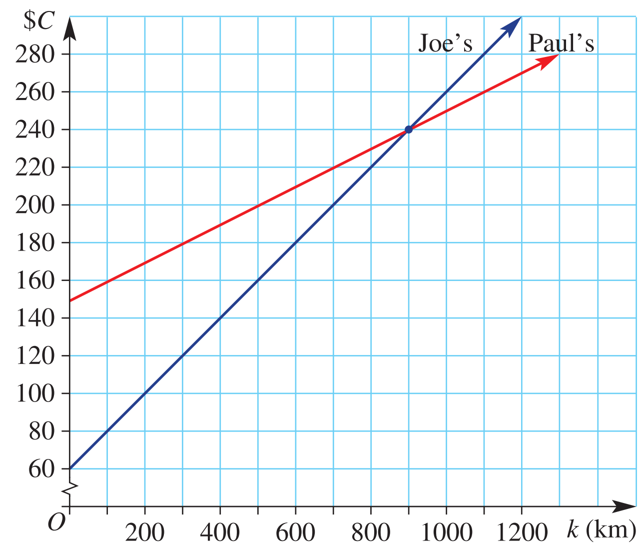
1 By substituting the given values of x and y into both equations, decide whether it is the solution to these simultaneous equations.

a $x + y = 5$ and $x - y = -1$; $x = 2, y = 3$
b $3x - y = 2$ and $x + 2y = 10$; $x = 2, y = 4$
c $3x + y = -1$ and $x - y = 0$; $x = -1, y = 2$
d $2y = x + 2$ and $x - y = 4$; $x = -2, y = -6$
e $2(x + y) = -20$ and $3x - 2y = -20$; $x = -8, y = -2$

2 This graph represents the rental cost, $\$C$, after k kilometres of a new car from two car rental firms called Paul's Motor Mart and Joe's Car Rental.

a **i** Determine the initial rental cost from each company.
ii Find the cost per kilometre when renting from each company.
iii Find the linear equations for the total rental cost from each company.
iv Determine the number of kilometres for which the cost is the same from both rental firms.

b If you had to travel 300 km, which company would you choose?



c If you had \$260 to spend on travel, which firm would give you the most kilometres?

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Example 27 Solving simultaneous equations using substitution

Solve these pairs of simultaneous equations using the method of substitution.

a $2x + y = -7$ and $y = x + 2$

b $2x - 3y = -8$ and $y = x + 3$

SOLUTION

a $2x + y = -7$ (1)

$y = x + 2$ (2)

Substitute equation (2) into equation (1).

$2x + (x + 2) = -7$

$3x + 2 = -7$

$3x = -9$

$x = -3$

Substitute $x = -3$ into equation (2).

$y = -3 + 2$

$= -1$

Solution is $x = -3, y = -1$.

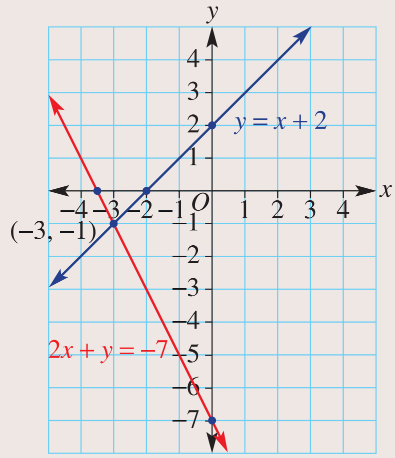
EXPLANATION

Label your equations.

Substitute equation (2) into equation (1) since equation (2) has a pronumeral as the subject.

Solve the resulting equation for x.

Substitute to find y.



SOLUTION

b $2x - 3y = -8$ (1)

$y = x + 3$ (2)

Substitute equation (2) into equation (1).

$2x - 3(x + 3) = -8$

$2x - 3x - 9 = -8$

$-x - 9 = -8$

$-x = 1$

$x = -1$

Substitute $x = -1$ into equation (2).

$y = -1 + 3$

$= 2$

Solution is $x = -1, y = 2$.

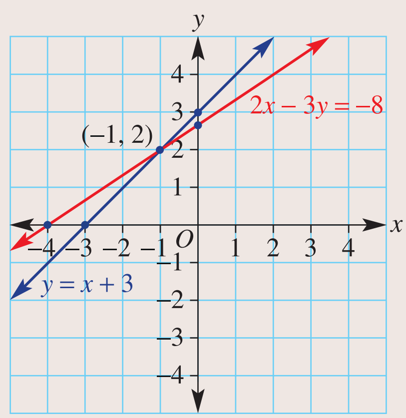
EXPLANATION

Label your equations.

Substitute equation (2) into equation (1).

Expand and simplify then solve the equation for x.

Substitute $x = -1$ into either equation to find y.



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Now you try

Solve these pairs of simultaneous equations using the method of substitution.

a $3x + y = 4$ and $y = x - 4$

b $x - 2y = -7$ and $y = x + 4$

Example 28 Solving with both equations in the form $y = mx + c$

Solve the pair of simultaneous equations: $y = -3x + 2$ and $y = 7x - 8$.

SOLUTION

$y = -3x + 2$ (1)

$y = 7x - 8$ (2)

Substitute equation (2) into equation (1).

$7x - 8 = -3x + 2$

$10x = 10$

$x = 1$

Substitute $x = 1$ into equation (1).

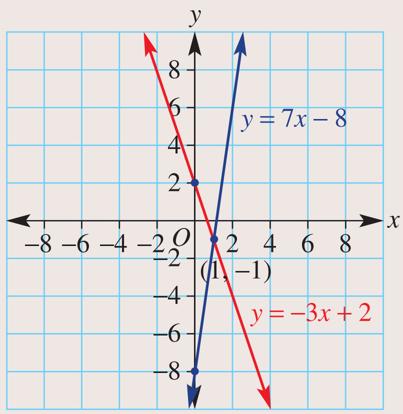
$y = -3(1) + 2$

$= -1$

Solution is $x = 1, y = -1$.

EXPLANATION

Write down and label each equation.



Now you try

Solve the pair of simultaneous equations: $y = -4x + 7$ and $y = 5x - 11$

Exercise 1J

FLUENCY	1, 2–3(1/2)	2–3(1/2)	2–3(1/3)

1 Solve these pairs of simultaneous equations using the method of substitution.

Example 27a **a** $2x + y = 1$ and $y = x - 5$

Example 27b **b** $3x - 2y = -9$ and $y = x + 4$

Example 27 **2** Solve the following pairs of simultaneous equations, using the method of substitution. You can check your solution graphically by sketching the pair of graphs and locating the intersection point.

a $y = x + 5$ and $3x + y = 13$

b $y = x + 3$ and $6x + y = 17$

c $y = x - 2$ and $3x - 2y = 7$

d $y = x - 1$ and $3x + 2y = 8$

e $y = x$ and $4x + 3y = 7$

f $y = x$ and $7x + 3y = 10$

g $x = 2y + 3$ and $11y - 5x = -14$

h $x = 3y - 2$ and $7y - 2x = 8$

i $x = 3y - 5$ and $3y + 5x = 11$

j $x = 4y + 1$ and $2y - 3x = -23$

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Updated July 2021

Example 28

3 Solve the following pairs of simultaneous equations, using the method of substitution. Check your solution graphically if you wish.

- **a** $y = 4x + 2$ and $y = x + 8$
- **b** $y = -2x - 3$ and $y = -x - 4$
- **c** $x = y - 6$ and $x = -2y + 3$
- **d** $x = -7y - 1$ and $x = -y + 11$
- **e** $y = 4 - x$ and $y = x - 2$
- **f** $y = 5 - 2x$ and $y = \frac{3}{2}x - 2$
- **g** $y = 5x - 1$ and $y = \frac{11 - 3x}{2}$
- **h** $y = 8x - 5$ and $y = \frac{5x + 13}{6}$

PROBLEM-SOLVING	4, 6	4–6	5–7

4 The salary structures for companies A and B are given by:
Company A: \$20 per hour
Company B: \$45 plus \$15 per hour

- **a** Find a rule for $\$E$ earned for t hours for:
  - **i** company A
  - **ii** company B.
- **b** Solve your two simultaneous equations from part **a**.
- **c** **i** State the number of hours worked for which the earnings are the same for the two companies.
  - **ii** State the amount earned when the earnings are the same for the two companies.

5 The value of two cars is depreciating (i.e. decreasing) at a constant rate according to the information in this table.

Car	Initial value	Annual depreciation
Luxury sports coupe	\$62 000	\$5000
Family sedan	\$40 000	\$3000

- **a** Write rules for the value, $\$V$, after t years for:
  - **i** the luxury sports coupe
  - **ii** the family sedan.
- **b** Solve your two simultaneous equations from part **a**.
- **c** **i** State the time taken for the cars to have the same value.
  - **ii** State the value of the cars when they have the same value.

6 The sum of the ages of a boy and his mother is 48. If the mother is three more than twice the boy's age, find the difference in the ages of the boy and his mother.

7 The perimeter of a rectangular farm is 1800 m and its length is 140 m longer than its width. Find the area of the farm.

REASONING	8(½)	8(½), 9	8(½), 9, 10

8 When two lines have the same gradient, there will be no intersection point. Use this idea to decide if these pairs of simultaneous equations will have a solution.

- **a** $y = 3x - 1$ and $y = 3x + 2$
- **b** $y = 7x + 2$ and $y = 7x + 6$
- **c** $y = 2x - 6$ and $y = -2x + 1$
- **d** $y = -x + 7$ and $y = 2x - 1$
- **e** $2x - y = 4$ and $y = 2x + 1$
- **f** $7x - y = 1$ and $y = -7x + 2$
- **g** $2y - 3x = 0$ and $y = -\frac{3x}{2} - 1$
- **h** $\frac{x - 2y}{4} = 0$ and $y = \frac{x}{2} + 3$

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Cambridge University Press
Updated July 2021

9 For what value of k will these pairs of simultaneous equations have no solution?

a $y = -4x - 7$ and $y = kx + 2$

b $y = kx + 4$ and $3x - 2y = 5$

c $kx - 3y = k$ and $y = 4x + 1$

10 Solve these simple equations for x and y. Your solution should contain the pronumeral k.

a $x + y = k$ and $y = 2x$

b $x - y = k$ and $y = -x$

c $2x - y = -k$ and $y = x - 1$

d $y - 4x = 2k$ and $x = y + 1$

ENRICHMENT: Factorise to solve	–	–	11(½), 12

11 Factorisation can be used to help solve harder literal equations (i.e. equations including other pronumerals).

For example: $ax + y = b$ and $y = bx$

Substituting $y = bx$ into $ax + y = b$ gives:

$ax + bx = b$

$x(a + b) = b$ (Factor out x.)

$$x = \frac{b}{a+b}$$

Substitute to find y.

$$\therefore y = b \times \left(\frac{b}{a+b}\right) = \frac{b^2}{a+b}$$

Now solve these literal equations.

a $ax - y = b$ and $y = bx$

b $ax + by = 0$ and $y = x + 1$

c $x - by = a$ and $y = -x$

d $y = ax + b$ and $y = bx$

e $y = (a - b)x$ and $y = bx + 1$

f $ax + by = c$ and $y = ax + c$

g $\frac{x}{a} + \frac{y}{b} = 1$ and $y = ax$

h $ax + y = b$ and $y = \frac{x}{a}$

12 Make up your own literal equation like the ones in Question **11**. Solve it and then test it on a classmate.

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

1K Simultaneous equations using elimination

Learning intentions

- To be able to identify and form equations involving a matching pair of terms
- To be able to use the process of elimination to solve simultaneous equations

The elimination method for the solution of simultaneous equations is commonly used when the equations are written in the same form. One pronumeral is eliminated using addition or subtraction and the value of the other pronumeral can then be found using one of the original equations.

Using the initial cost of machinery and the production cost per item, financial analysts working for manufacturing companies, e.g. biscuit makers, can use simultaneous equations to determine the most profitable equipment to invest in.

LESSON STARTER Which operation?

Below are four sets of simultaneous equations.

- For each set discuss whether addition or subtraction would be used to eliminate one pronumeral.
- State which pronumeral might be eliminated first in each case.
- Describe how you would first deal with parts **c** and **d** so that elimination can be used.

a $x + y = 5$, $2x + y = 7$
b $-4x - 2y = -8$, $4x + 3y = 10$
c $5x - y = 1$, $3x - 2y = -5$
d $3x + 2y = -5$, $4x - 3y = -1$

KEY IDEAS

- The method of **elimination** is generally used when both equations are in the form $ax + by = d$.

  For example: $2x - y = 6$, $3x + y = 10$ or $-5x + y = -2$, $6x + 3y = 5$

  - When there is no matching pair (as in the second example above) one or both of the equations can be multiplied by a chosen factor. This is shown in **Example 30a** and **b**.

BUILDING UNDERSTANDING

1 Find the answer.
a $2x$ subtract $2x$ **b** $5y$ add $-5y$ **c** $-2x$ add $2x$ **d** $-3y$ subtract $-3y$

2 Decide if you would add or subtract the two given terms to give a result of zero.
a $7x, 7x$ **b** $-5y, 5y$ **c** $2y, -2y$ **d** $-7y, -7y$

3 What is the resulting equation when $2x - 3y = 4$ is multiplied on both sides by the following?
a 2 **b** 3 **c** 4 **d** 10

Example 29 Using the elimination method with a matching pair of terms

Solve the following pair of simultaneous equations using the elimination method.
$x + y = 6$ and $3x - y = 10$

SOLUTION

$$\begin{aligned} x + y &= 6 \quad (1) \\ 3x - y &= 10 \quad (2) \\ (1) + (2):\ 4x &= 16 \\ x &= 4 \end{aligned}$$

Substitute $x = 4$ into equation (1).

$$\begin{aligned} (4) + y &= 6 \\ \therefore\ y &= 2 \end{aligned}$$

Solution is $x = 4, y = 2$.

EXPLANATION

Label your equations to help you refer to them.

Add the two equations to eliminate y. Then solve for the remaining pronumeral x.

Substitute $x = 4$ into one of the equations to find y.

State the solution and check by substituting the solution into the original equations.

Now you try

Solve the following pair of simultaneous equations using the elimination method.
$x + y = 4$ and $5x - y = 14$

Example 30 Using the elimination method to solve simultaneous equations

Solve the following pairs of simultaneous equations using the elimination method.

a $y - 3x = 1$ and $2y + 5x = 13$

b $3x + 2y = 6$ and $5x + 3y = 11$

SOLUTION

a

$$\begin{aligned} y - 3x &= 1 \quad (1) \\ 2y + 5x &= 13 \quad (2) \\ (1) \times 2\quad 2y - 6x &= 2 \quad (3) \\ (2) - (3):\quad 11x &= 11 \\ x &= 1 \end{aligned}$$

Substitute $x = 1$ into equation (1).

$$\begin{aligned} y - 3(1) &= 1 \\ \therefore\ y &= 4 \end{aligned}$$

Solution is $x = 1, y = 4$.

EXPLANATION

Label your equations.

Multiply equation (1) by 2 so that there is a matching pair ($2y$).

$2y - 2y = 0$ and $5x - (-6x) = 11x$.
Solve for x.

Substitute into one of the equations to find y.

Continued on next page

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

b

$$\begin{aligned} & 3x + 2y = 6 && (1) \\ & 5x + 3y = 11 && (2) \\ (1) \times 3 \quad & 9x + 6y = 18 && (3) \\ (2) \times 2 \quad & 10x + 6y = 22 && (4) \\ (4) - (3): \quad & x \quad = 4 && \end{aligned}$$

Multiply equation (1) by 3 and equation (2) by 2 to generate $6y$ in each equation. (Alternatively, multiply (1) by 5 and (2) by 3 to obtain matching x coefficients.)

Subtract to eliminate y.

Substitute $x = 4$ into equation (1).

Substitute $x = 4$ into one of the equations to find y.

$$\begin{aligned} 3(4) + 2y &= 6 \\ 2y &= -6 \\ y &= -3 \end{aligned}$$

State and check the solution.

Solution is $x = 4, y = -3$.

Now you try

Solve the following pairs of simultaneous equations using the elimination method.

a $y - 2x = 1$ and $4y + 3x = 15$
b $5x + 2y = 11$ and $3x + 5y = -1$

Exercise 1K

FLUENCY	1, 2–4(½)	2–4(½)	2–3(⅓), 4(½)

Example 29

1 Solve the pairs of simultaneous equations using the elimination method.

a $x + y = 5$ and $2x - y = 4$
b $x + y = 7$ and $5x - y = 17$

Example 29

2 Solve the following pairs of simultaneous equations using the elimination method.

a $x + y = 7$ and $5x - y = 5$
b $x + y = 5$ and $3x - y = 3$
c $x - y = 2$ and $2x + y = 10$
d $x - y = 0$ and $4x + y = 10$
e $3x + 4y = 7$ and $2x + 4y = 6$
f $x + 3y = 5$ and $4x + 3y = 11$
g $2x + 3y = 1$ and $2x + 5y = -1$
h $4x + y = 10$ and $4x + 4y = 16$
i $2x + 3y = 8$ and $2x - 4y = -6$
j $3x + 2y = 8$ and $3x - y = 5$
k $-3x + 2y = -4$ and $5x - 2y = 8$
l $-2x + 3y = 8$ and $-4x - 3y = -2$

Example 30a

3 Solve the following pairs of simultaneous equations using the elimination method.

a $3x + 5y = 8$ and $x - 2y = -1$
b $2x + y = 10$ and $3x - 2y = 8$
c $x + 2y = 4$ and $3x - y = 5$
d $3x - 4y = 24$ and $x - 2y = 10$
e $y - 3x = -\frac{1}{2}$ and $x + 2y = \frac{5}{2}$
f $7x - 2y = -\frac{5}{2}$ and $3x + y = -2$

Example 30b

4 Solve the following pairs of simultaneous equations using the elimination method.

a $3x + 2y = 6$ and $5x + 3y = 11$
b $3x + 2y = 5$ and $2x + 3y = 5$
c $4x - 3y = 0$ and $3x + 4y = 25$
d $2x + 3y = 10$ and $3x - 4y = -2$
e $-2y - 4x = 0$ and $3y + 2x = -2$
f $-7x + 3y = 22$ and $3x - 6y = -11$

PROBLEM-SOLVING	5	5, 6	6, 7

5 The sum of two numbers is 1633 and their difference is 35. Find the two numbers.

6 The cost of one apple and one banana at the school canteen is \$1 and the cost of 3 apples and 2 bananas is \$2.40. Find the cost of a single banana.

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7 A group of 5 adults and 3 children paid a total of $108 for their concert tickets. Another group of 3 adults and 10 children paid $155. Find the cost of an adult ticket and the cost of a child's ticket.

REASONING	8	8, 9(½)	9, 10

8 Describe the error made in this working and then correct the error to find the correct solution.

$$3x - 2y = 5 \quad (1)$$
$$-4x - 2y = -2 \quad (2)$$
$$(1) + (2): \quad -x = 3$$
$$\therefore\ x = -3$$
$$3(-3) - 2y = 5$$
$$-9 - 2y = 5$$
$$-2y = 14$$
$$y = -7$$

Solution is $x = -3$ and $y = -7$.

9 Solve these literal simultaneous equations for x and y.

a $ax + y = 0$ and $ax - y = 2$

b $x - by = 4$ and $2x + by = 9$

c $ax + by = 0$ and $ax - by = -4$

d $ax + by = a$ and $ax - by = b$

e $ax + by = c$ and $bx + ay = c$

10 Explain why there is no solution to the set of equations $3x - 7y = 5$ and $3x - 7y = -4$.

ENRICHMENT: Partial fractions	–	–	11(½)

11 Writing $\frac{6}{(x-1)(x+1)}$ as a sum of two 'smaller' fractions $\frac{a}{x-1} + \frac{b}{x+1}$, known as partial fractions, involves a process of finding the values of a and b for which the two expressions are equal. Here is the process.

$$\frac{6}{(x-1)(x+1)} = \frac{a}{x-1} + \frac{b}{x+1}$$
$$= \frac{a(x+1) + b(x-1)}{(x-1)(x+1)}$$
$$\therefore\ a(x+1) + b(x-1) = 6$$
$$ax + a + bx - b = 6$$
$$ax + bx + a - b = 6$$
$$x(a+b) + (a-b) = 0x + 6$$

By equating coefficients: $a + b = 0 \quad (1)$

$a - b = 6 \quad (2)$

$(1) + (2)$ gives $2a = 6$

$\therefore a = 3.$

and so $b = -3$.

$$\therefore\ \frac{6}{(x-1)(x+1)} = \frac{3}{x-1} - \frac{3}{x+1}$$

Use this technique to write the following as the sum of two fractions.

a $\frac{4}{(x-1)(x+1)}$

b $\frac{7}{(x+2)(2x-3)}$

c $\frac{-5}{(2x-1)(3x+1)}$

d $\frac{9x+4}{(3x-1)(x+2)}$

e $\frac{2x-1}{(x+3)(x-4)}$

f $\frac{1-x}{(2x-1)(4-x)}$

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

1L Further applications of simultaneous equations

Learning intentions
- To know the steps involved in solving a word problem with two unknowns
- To be able to form linear equations in two unknowns from a word problem
- To be able to choose an appropriate method to solve two equations simultaneously

When a problem involves two unknown variables, simultaneous equations can be used to find the solution to the problem, provided that the two pronumerals can be identified and two equations can be written from the problem description.

Simultaneous equations can be used by farmers, home gardeners, nurses and pharmacists to accurately calculate required volumes when mixing solutions of different concentrations to get a desired final concentration.

LESSON STARTER 19 scores but how many goals?

Nathan heard on the news that his AFL team scored 19 times during a game and the total score was 79 points. He wondered how many goals (worth 6 points each) and how many behinds (worth 1 point each) were scored in the game. Nathan looked up simultaneous equations in his maths book and it said to follow these steps:

1 Define two variables.
2 Write two equations.
3 Solve the equations.
4 Answer the question in words.

Can you help Nathan with the four steps to find out what he wants to know?

KEY IDEAS

- When solving problems with two unknowns:
  - Define a variable for each unknown.
  - Write down two equations from the information given.
  - Solve the equations simultaneously to find the solution.
  - Interpret the solution and answer the question in words.

ISBN 978-1-108-77290-7

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Updated July 2021

BUILDING UNDERSTANDING

1 Let x and y be two numbers that satisfy the following statements. State two linear equations according to the information.

a They sum to 16 but their difference is 2.

b They sum to 30 but their difference is 10.

c They sum to 7 and twice the larger number plus the smaller number is 12.

d The sum of twice the first plus three times the second is 11 and the difference between four times the first and three times the second is 13.

2 The perimeter of a rectangle is 56 cm. If the length, l, of the rectangle is three times its width, w, state two simultaneous equations that would allow you to solve to determine the dimensions.

3 State expressions for the following.

a the cost of 5 tickets at \$$x$ each

b the cost of y pizzas at \$15 each

c the cost of 3 drinks at \$$d$ each and 4 pies at \$$p$ each

Example 31 Setting up and solving simultaneous equations

The sum of the ages of two children is 17 and the difference in their ages is 5. If Kara is the older sister of Ben, determine their ages.

SOLUTION	EXPLANATION
Let k be Kara's age and b be Ben's age. $k + b = 17$ (1) $k - b = 5$ (2) (1) + (2): $2k = 22$ $\therefore k = 11$	Define the unknowns and use these to write two equations from the information in the question. • The sum of their ages is 17. • The difference in their ages is 5. Add the equations to eliminate b and then solve to find k.
Substitute $k = 11$ into equation (1). $11 + b = 17$ $b = 6$	Substitute $k = 11$ into one of the equations to find the value of b.
$\therefore$ Kara is 11 years old and Ben is 6 years old.	Answer the question in words.

Now you try

The sum of the ages of two children is 20 and the difference in their ages is 8. If Tim is the older brother of Tina, determine their ages.

ISBN 978-1-108-77290-7

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Example 32 Solving further applications with two variables

John buys 3 daffodils and 5 petunias from the nursery and pays \$25. Julia buys 4 daffodils and 3 petunias for \$26. Determine the cost of each type of flower.

SOLUTION

Let $\$d$ be the cost of a daffodil and $\$p$ be the cost of a petunia.

$$3d + 5p = 25 \quad (1)$$
$$4d + 3p = 26 \quad (2)$$

$$(1) \times 4 \quad 12d + 20p = 100 \quad (3)$$
$$(2) \times 3 \quad 12d + 9p = 78 \quad (4)$$
$$(3) - (4): \quad 11p = 22$$
$$\therefore\ p = 2$$

Substitute $p = 2$ into equation (1).

$$3d + 5(2) = 25$$
$$3d + 10 = 25$$
$$3d = 15$$
$$\therefore\ d = 5$$

A petunia costs \$2 and a daffodil costs \$5.

EXPLANATION

Define the unknowns and set up two equations from the question.

If 1 daffodil costs d dollars then 3 will cost $3 \times d = 3d$.

- 3 daffodils and 5 petunias cost \$25.
- 4 daffodils and 3 petunias cost \$26.

Multiply equation (1) by 4 and equation (2) by 3 to generate $12d$ in each equation.

Subtract the equations to eliminate d and then solve for p.

Substitute $p = 2$ into one of the equations to find the value of d.

Answer the question in words.

Now you try

Georgie buys 2 coffees and 3 muffins for \$17 and Rick buys 4 coffees and 2 muffins for \$22 from the same shop. Determine the cost of each coffee and muffin.

Exercise 1L

FLUENCY	1–4	2–5	3–6

Example 31

1 The sum of the ages of two children is 24 and the difference between their ages is 8. If Nikki is the older sister of Travis, determine their ages by setting up and solving a pair of simultaneous equations.

Example 31

2 Cam is 3 years older than Lara. If their combined age is 63, determine their ages by solving an appropriate pair of equations.

Example 32

3 Luke buys 4 bolts and 6 washers for \$2.20 and Holly spends \$1.80 on 3 bolts and 5 washers at the same local hardware store. Determine the costs of a bolt and a washer.

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4 It costs \$3 for children and \$7 for adults to attend a school basketball game. If 5000 people attended the game and the total takings at the door was \$25 000, determine the number of children and the number of adults that attended the game.

5 A vanilla thickshake is \$2 more than a fruit juice. If 3 vanilla thickshakes and 5 fruit juices cost \$30, determine their individual prices.

6 A paddock contains ducks and sheep. There are a total of 42 heads and 96 feet in the paddock. How many ducks and how many sheep are in the paddock?

PROBLEM-SOLVING	7, 8	7, 8	8, 9

7 James has \$10 in 5-cent and 10-cent coins in his change jar and counts 157 coins in total. How many 10-cent coins does he have?

8 Connor the fruiterer sells two fruit packs.
Pack 1: 10 apples and 5 mangoes (\$12)
Pack 2: 15 apples and 4 mangoes (\$14.15)
Determine the cost of 1 apple and 5 mangoes.

9 Five years ago I was 5 times older than my son. In 8 years' time I will be 3 times older than my son. How old am I today?

REASONING	10	10, 11	11, 12

10 Erin goes off on a long bike ride, averaging 10 km/h. One hour later her brother Alistair starts chasing after her at 16 km/h. How long will it take Alistair to catch up to Erin? (*Hint*: Use the rule $d = s \times t$.)

11 Two ancient armies are 1 km apart and begin walking towards each other. The Vikons walk at a pace of 3 km/h and the Mohicas walk at a pace of 4 km/h. How long will they walk for before the battle begins?

12 A river is flowing downstream at a rate of 2 metres per second. Brendan, who has an average swimming speed of 3 metres per second, decides to go for a swim in the river. He dives into the river and swims downstream to a certain point, then swims back upstream to the starting point. The total time taken is 4 minutes. How far did Brendan swim downstream?

ENRICHMENT: Concentration time	–	–	13, 14

13 Molly has a bottle of 15% strength cordial and wants to make it stronger. She adds an amount of 100% strength cordial to her bottle to make a total volume of 2 litres of cordial drink. If the final strength of the drink is 25% cordial, find the amount of 100% strength cordial that Molly added. (*Hint*: Use Concentration = Volume (cordial) ÷ Total volume.)

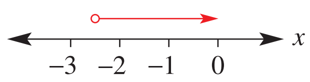
14 A fruit grower accidentally made a 5% strength chemical mixture to spray his grape vines. The strength of spray should be 8%. He then adds pure chemical until the strength reaches 8% by which time the volume is 350 litres. How much pure chemical did he have to add?

ISBN 978-1-108-77290-7

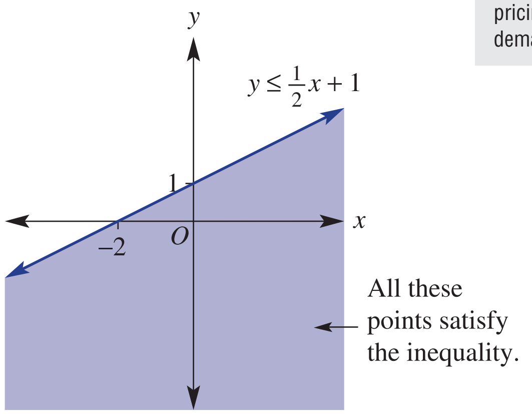
1M Half planes EXTENDING

Learning intentions
- To know the meaning of the term half plane
- To know how to determine which side of a line to shade to sketch a half plane
- To understand that if a pair of coordinates satisfy an inequality then the point is in the required region
- To be able to find the intersecting region of two or more half planes

You will remember that an inequality is a mathematical statement that contains one of these symbols: $<, \leqslant, >$ or $\geqslant$. The linear inequality with one pronumeral, for example $2x - 5 > -10$, has the solution $x > -2.5$.



Linear inequalities can also have two variables: $2x - 3y \geqslant 5$ and $y < 3 - x$ are two examples. The solutions to such inequalities will be an infinite set of points on a plane sitting on one side of a line. This region is called a half plane.



Operations research analysts use half-plane graph calculations to optimise profit within certain limitations. For example, for airline companies to find the most economical combination of flight routes, seat pricing and pilot scheduling that aligns with customer demand.

LESSON STARTER Which side do I shade?

You are asked to shade all the points on a graph that satisfy the inequality $4x - 3y \geqslant 12$.

- First, graph the equation $4x - 3y = 12$.
- Substitute the point $(-2, 3)$ into the inequality $4x - 3y \geqslant 12$. Does the point satisfy the inequality? Plot the point on your graph.
- Now test these points:
  a $(3, -2)$ **b** $(3, -1)$ **c** $(3, 0)$ **d** $(3, 1)$
- Can you now decide which side of the line is to be shaded to represent all the solutions to the inequality? Should the line itself be included in the solution?

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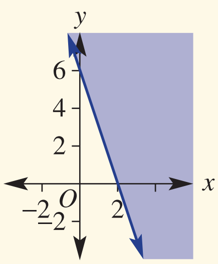
Cambridge University Press
Updated July 2021

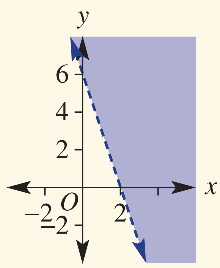
KEY IDEAS

- The solution to a linear inequality with two variables is illustrated using a shaded region called a **half plane**.
- When y is the subject of the inequality, follow these simple rules.
  - $y \geqslant mx + c$ Draw a solid line (as it is included in the region) and shade above.
  - $y > mx + c$ Draw a broken line (as it is not included in the region) and shade above.
  - $y \leqslant mx + c$ Draw a solid line (as it is included in the region) and shade below.
  - $y < mx + c$ Draw a broken line (as it is not included in the region) and shade below.

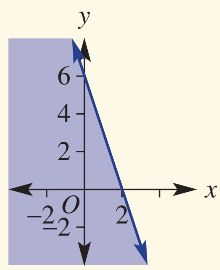
  Here are examples of each.



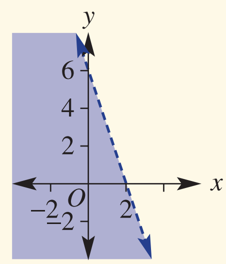
$y \geqslant -3x + 6$



$y > -3x + 6$

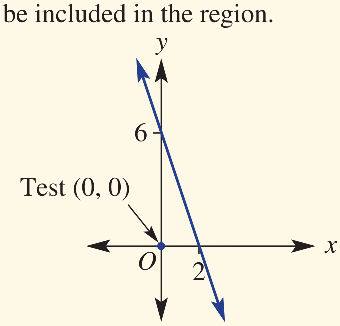


$y \leqslant -3x + 6$



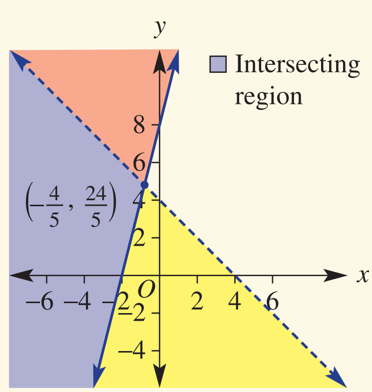
$y < -3x + 6$

- If the equation is of the form $ax + by = d$, it is usually simpler to test a point, for example $(0, 0)$, to see which side of the line is to be included in the region.



- When two or more half planes are sketched on the same set of axes, the half planes overlap and form an **intersecting region**. The set of points inside the intersecting region will be the solution to the simultaneous inequalities.

  For example, $y \geqslant 4x + 8$ (pink)
  $y < -x + 4$ (yellow)



  - To help define the intersecting region correctly, you should determine and label the point of intersection.

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BUILDING UNDERSTANDING

1 Substitute the point (0, 0) into these inequalities to decide if the point satisfies the inequality; i.e. is the inequality true for $x = 0$ and $y = 0$?

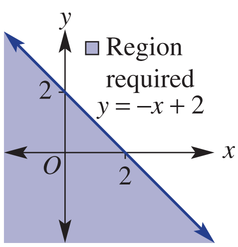
a $y < 3x - 1$ **b** $y > -\frac{x}{2} - 3$ **c** $y \geqslant 1 - 7x$

d $3x - 2y < -1$ **e** $x - y > 0$ **f** $2x - 3y \leqslant 0$

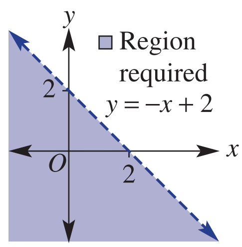
2 Match the rules with the graphs (**A**, **B** and **C**) below.

a $x + y < 2$ **b** $y \geqslant -x + 2$ **c** $y \leqslant -x + 2$

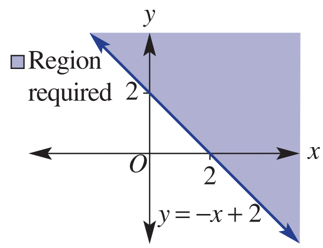
A



B



C



3 **a** Sketch the vertical line $x = -1$ and the horizontal line $y = 4$ on the same set of axes.

b Shade the region $x \geqslant -1$ (i.e. all points with an x-coordinate greater than or equal to -1).

c Shade the region $y \leqslant 4$ (i.e. all points with a y-coordinate less than or equal to 4).

d Now use a different colour to shade all the points that satisfy both $x \geqslant -1$ and $y \leqslant 4$ simultaneously.

Example 33 Sketching half planes

Sketch the half planes for the following linear inequalities.

a $y > 1.5x - 3$ **b** $y + 2x \leqslant 4$

SOLUTION

a $y = 1.5x - 3$

y-intercept ($x = 0$):

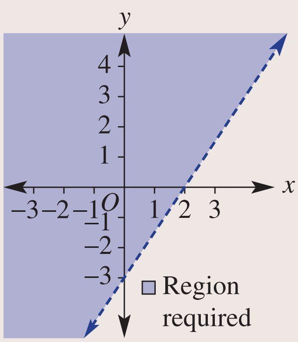
$y = -3$

x-intercept ($y = 0$):

$0 = 1.5x - 3$

$1.5x = 3$

$\therefore\ x = 2$



EXPLANATION

First, sketch $y = 1.5x - 3$ by finding the x- and y-intercepts.

Sketch a dotted line (since the sign is $>$ not $\geqslant$) joining the intercepts and shade above the line, since y is greater than $1.5x - 3$.

ISBN 978-1-108-77290-7

Cambridge University Press
Updated July 2021

b $y + 2x = 4$

y-intercept $(x = 0)$:

$y = 4$

x-intercept $(y = 0)$:

$2x = 4$

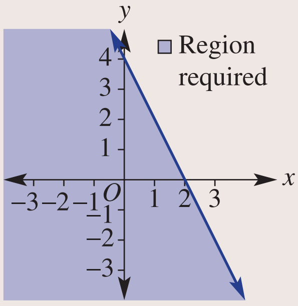
$x = 2$

Shading: Test $(0, 0)$.

$0 + 2(0) \leqslant 4$

$0 \leqslant 4$ (True)

$\therefore$ $(0, 0)$ is included.



First, sketch $y + 2x = 4$ by finding the x- and y-intercepts.

Decide which side to shade by testing the point $(0, 0)$; i.e. substitute $x = 0$ and $y = 0$. Since $0 \leqslant 4$, the point $(0, 0)$ should sit inside the shaded region.

Sketch a solid line since the inequality sign is $\leqslant$, and shade the region that includes $(0, 0)$.

Now you try

Sketch the half planes for the following linear inequalities.

a $y > 2.5x - 5$ **b** $y + 3x \leqslant 6$

Example 34 Finding the intersecting region

Sketch both the inequalities $4x + y \leqslant 12$ and $3x - 2y < -2$ on the same set of axes, show the region of intersection and find the point of intersection of the two lines.

SOLUTION

$4x + y = 12$

y-intercept $(x = 0)$:

$y = 12$

x-intercept $(y = 0)$:

$4x = 12$

$x = 3$

Shading: Test $(0, 0)$.

$4(0) + 0 \leqslant 12$

$0 \leqslant 12$ (True)

So $(0, 0)$ is included.

EXPLANATION

First sketch $4x + y = 12$ by finding the x- and y-intercepts.

Test $(0, 0)$ to see if it is in the included region.

Continued on next page

$3x - 2y = -2$

y-intercept ($x = 0$):

$-2y = -2$

$y = 1$

x-intercept ($y = 0$):

$3x = -2$

$x = -\frac{2}{3}$

Sketch $3x - 2y = -2$ by finding x- and y-intercepts.

Shading: Test $(0, 0)$.

$3(0) + 2(0) < -2$

$0 < -2$ (False)

So $(0, 0)$ is not included.

Test $(0, 0)$ to see if it is in the included region.

Point of intersection:

$$\begin{aligned} 4x + y &= 12 \quad (1) \\ 3x - 2y &= -2 \quad (2) \\ (1) \times 2 \quad 8x + 2y &= 24 \quad (3) \\ (2) + (3): \quad 11x &= 22 \\ x &= 2 \end{aligned}$$

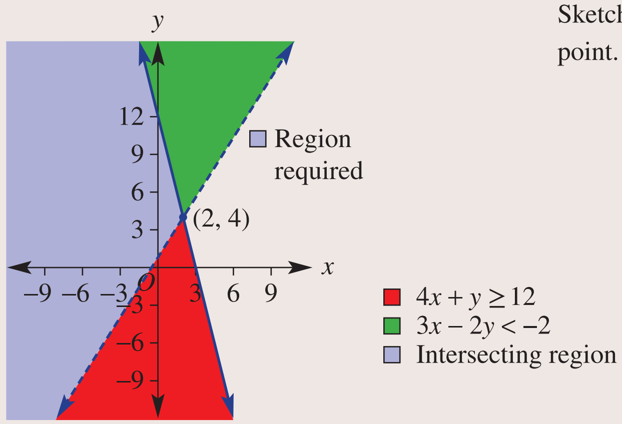
Find the point of intersection by solving the equations simultaneously using the method of elimination.

Substitute $x = 2$ into equation (1).

$4(2) + y = 12$

$y = 4$

The point of intersection is $(2, 4)$.



Sketch both regions and label the intersection point. Also label the intersecting region.

Now you try

Sketch both the inqualities $3x + y \leqslant 6$ and $2x - 3y < -7$ on the same set of axes. Show the region of intersection and find the point of intersection of the two lines.

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Exercise 1M

FLUENCY	1, 2–3(½), 5(½)	2–5(½)	2–5(½)

1 Sketch the half planes for the following inequalities.

Example 33a
a i $y > 2x - 1$ ii $y < \frac{1}{2}x + 2$

Example 33b
b i $y + x \leqslant 3$ ii $y + 2x \geqslant 6$

Example 33a
2 Sketch the half planes for the following linear inequalities.

a $y \geqslant x + 4$ b $y < 3x - 6$ c $y > 2x - 8$ d $y \leqslant 3x - 5$

e $y < 2 - 4x$ f $y \leqslant 2x + 7$ g $y < 4x$ h $y > 6 - 3x$

i $y \leqslant -x$ j $x > 3$ k $x < -2$ l $y \geqslant 2$

3 Decide whether the following points are in the region defined by $2x - 3y > 8$.

a $(5, 0)$ b $(2.5, -1)$ c $(0, -1)$ d $(2, -5)$

4 Decide whether the following points are in the region defined by $4x + 3y \leqslant -2$.

a $\left(-\frac{1}{2}, \frac{1}{2}\right)$ b $(-5, 6)$ c $(2, -3)$ d $(-3, 4)$

Example 33b
5 Sketch the half planes for the following linear inequalities.

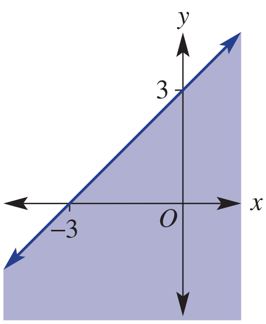
a $x + 3y < 9$ b $3x - y \geqslant 3$ c $4x + 2y \geqslant 8$ d $2x - 3y > 18$

e $-2x + y \leqslant 5$ f $-2x + 4y \leqslant 6$ g $2x + 5y > -10$ h $4x + 9y < -36$

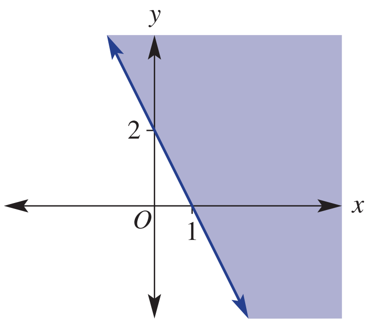
PROBLEM-SOLVING	6	6, 7(½)	6–7(½)

6 Write down the inequalities that give these half planes.

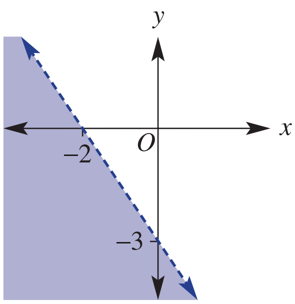
a



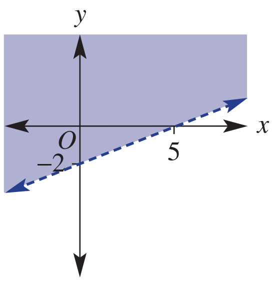
b



c



d



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Example 34

7 Sketch both inequalities on the same set of axes, shade the region of intersection and find the point of intersection of the two lines.

a $x + 2y \geqslant 4$
$2x + 2y < 8$

b $3x + 4y \leqslant 12$
$3x + y > 3$

c $2x - 3y > 6$
$y < x - 2$

d $3x - 5y \leqslant 15$
$y - 3x > -3$

e $y \geqslant -x + 4$
$2x + 3y \geqslant 6$

f $2y - x \leqslant 5$
$y < 10 - x$

g $3x + 2y \leqslant 18$
$4y - x < 8$

h $2y \geqslant 5 + x$
$y < 6 - 3x$

REASONING	8(1/2)	8(1/2)	8(1/2), 9

8 Sketch the following systems of inequalities on the same axes. Show the intersecting region and label the points of intersection. The result should be a triangle in each case.

a $x \geqslant 0$
$y \geqslant 0$
$3x + 6y \leqslant 6$

b $x \geqslant 0$
$y \leqslant 0$
$2x - y \leqslant 4$

c $x \geqslant 0$
$5x + 2y \leqslant 30$
$4y - x \geqslant 16$

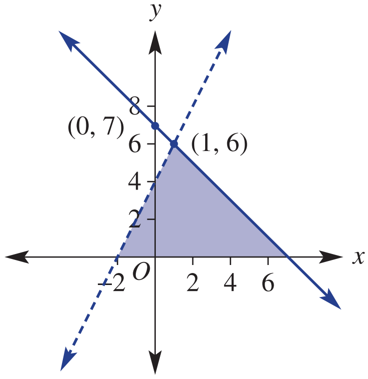
d $x < 2$
$y < 3$
$2x + 5y > 10$

e $x \leqslant 0$
$y < x + 7$
$2x + 3y \geqslant -6$

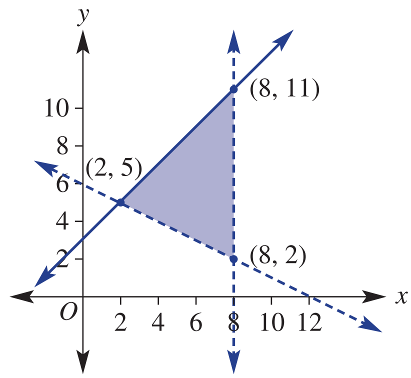
f $x + y \leqslant 9$
$2y - x \geqslant 6$
$3x + y \geqslant -2$

9 Determine the original inequalities that would give the following regions of intersection.

a



b



ENRICHMENT: Areas of regions	–	–	10, 11

10 Find the area of the triangles formed in Question **8** parts **a** to **d**.

11 **a** Find the exact area bound by:

i $x < 0$
$y > 0$
$x + 2y < 6$
$x - y > -7$

ii $y < 7$
$x + y > 5$
$3x - 2y < 14$

b Make up your own set of inequalities that gives an area of 6 square units.

ISBN 978-1-108-77290-7

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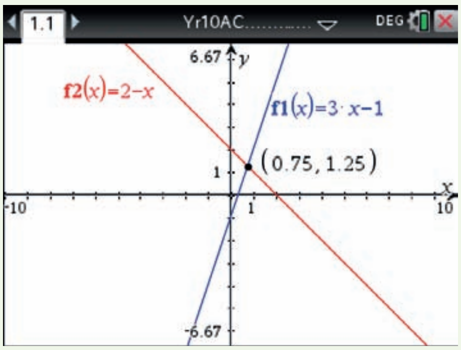
Updated July 2021

Using calculators to find intersection points

1. Sketch a graph of $y = 3x - 1$ and $y = 2 - x$ and locate the intersection point.
2. Sketch the intersecting region of $y < 3x - 1$ and $y > 2 - x$.

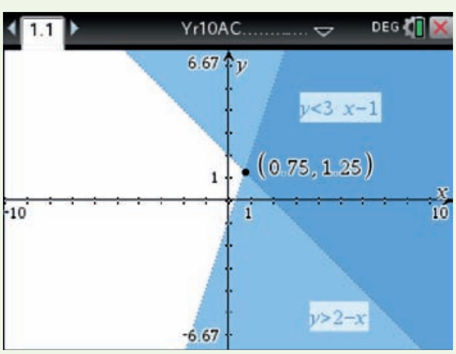
Using the TI-Nspire:

1. In a **Graphs** page enter the rules $f1(x) = 3x - 1$ and $f2(x) = 2 - x$. Select menu **>Analyze Graph>Intersection** and select the lower and upper bounds containing the intersection point. Press enter to paste the coordinates to the graph.



Hint: if multiple graphs are being entered use the down arrow to enter subsequent graphs.
Hint: if the graph entry line is not showing, press tab or double click in an open area.

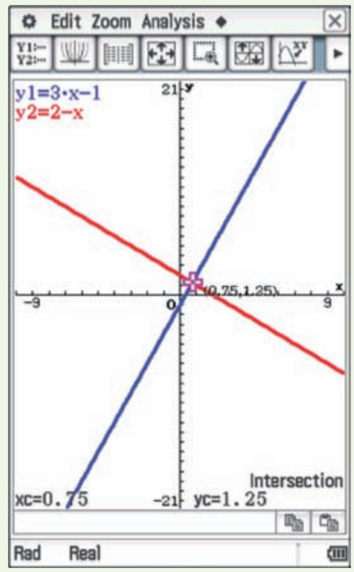
2. In a **Graphs** page press del and select the required inequality from the list and edit $f1(x)$ to $y < 3x - 1$ and $f2(x)$ to $y > 2 - x$.



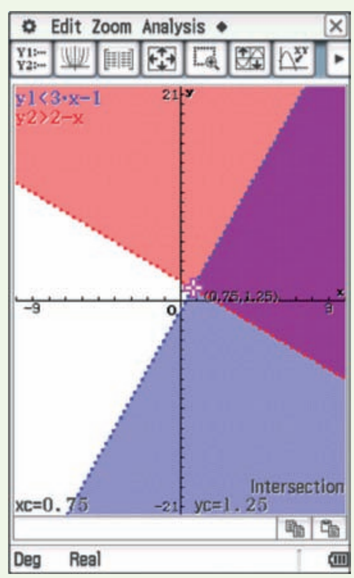
Find the intersection point as shown above.
Hint: if multiple graphs are being entered use the down arrow to enter subsequent graphs.
Hint: if the graph entry line is not showing, press tab or double click in an open area.

Using the ClassPad:

1. In the **Graph&Table** application enter the rules $y1 = 3x - 1$ and $y2 = 2 - x$ then tap [graph icon]. Tap **Zoom Quick**, **Quick Standard** to adjust the window. Tap **Analysis**, **G-Solve Intersect**.



2. Tap [y1: y2: icon] and clear all functions. With the cursor in $y1$ tap y=, select y<, enter the rule $3x - 1$ and press **EXE**. With the cursor in $y2$ tap y=, select y>, enter the rule $2 - x$ and press **EXE**. Tap [graph icon].



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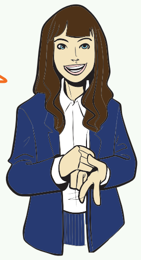
Problems and challenges

Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.



1 Tom walks at 4 km/h and runs at 6 km/h. He can save $3\frac{3}{4}$ minutes by running from his house to the train station instead of walking. How many kilometres is it from his house to the station?

2 A fraction is such that when its numerator is increased by 1 and its denominator is decreased by 1, it equals 1 and when its numerator is doubled and its denominator increased by 4 it is also equal to 1. What is the fraction?

3 Show that the following sets of points are collinear (i.e. in a straight line).
   a $(2, 12)$, $(-2, 0)$ and $(-5, -9)$
   b $(a, 2b)$, $(2a, b)$ and $(-a, 4b)$

4 Use two different methods from this chapter to prove that triangle ABC with vertices $A(1, 6)$, $B(4, 1)$ and $C(-4, 3)$ is a right-angled triangle.

5 Two missiles 2420 km apart are launched at the same time and are headed towards each other. They pass after 1.5 hours. The average speed of one missile is twice that of the other. What is the average speed of each missile?

6 Show that the points $(7, 5)$ and $(-1, 9)$ lie on a circle centred at $(2, 5)$ with radius 5 units.

7 A quadrilateral whose diagonals bisect each other at right angles will always be a rhombus. Prove that the points $A(0, 0)$, $B(4, 3)$, $C(0, 6)$ and $D(-4, 3)$ are the vertices of a rhombus. Is it also a square?

8 Solve this set of simultaneous equations:

$$\begin{aligned} x - 2y - z &= 9 \\ 2x - 3y + 3z &= 10 \\ 3x + y - z &= 4 \end{aligned}$$

9 A triangle, PQR, has $P(8, 0)$, $Q(0, -8)$ and point R is on the line $y = x - 2$. Find the area of the triangle PQR.

10 The average age of players at a ten pin bowling alley increases by 1 when either four 10-year olds leave or, alternatively, if four 22-year olds arrive. How many players were there originally and what was their average age?

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Updated July 2021

Linear relations

Midpoint and length of a line segment

Line segment joining (x_1, y_1) and (x_2, y_2) has:

Midpoint $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Length/distance

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Parallel and perpendicular lines

Parallel lines have the same gradient; e.g. $y = 2x + 7$ and $y = 2x - 4$

For perpendicular lines with gradients m_1 and m_2:

$m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$

e.g. $y = 3x + 4$ and $y = -\frac{1}{3}x + 2$

Finding a rule for a graph

For points (x_1, y_1) and (x_2, y_2)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

To find line equation $y = mx + c$:

- find gradient, m
- substitute a point if c is unknown using $y = mx + c$ or $y - y_1 = m(x - x_1)$

Review of algebra

Add/subtract like terms only.

e.g. $2x + 4x + y = 6x + y$

Multiply: $3x \times 2y = 6xy$

Divide: $4ab \div (2b) = \frac{^2\cancel{4ab}}{_1\cancel{2b}}$

$= 2a$

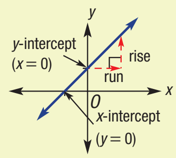
Expand: $a(b + c) = ab + ac$

$a(b - c) = ab - ac$

Graphs of linear relations

In the form $y = mx + c$, m is the gradient, c is the y-intercept.

$m = \frac{\text{rise}}{\text{run}}$



To graph:

a use gradient and y-intercept or

b locate x-intercept ($y = 0$) and y-intercept ($x = 0$)

Algebraic fractions

Add/subtract: find LCD.

e.g. $\frac{2}{a} + \frac{3}{5} = \frac{10}{5a} + \frac{3a}{5a} = \frac{10 + 3a}{5a}$

Multiply: cancel common factors first.

e.g. $\frac{\cancel{2}^1}{x+2} \times \frac{3x+6}{\cancel{8}_4} = \frac{1}{\cancel{x+2}_1} \times \frac{3\cancel{(x+2)}^1}{4} = \frac{3}{4}$

Divide: multiply by the reciprocal of the fraction following the ÷.

Solving linear equations

Some steps include:

- using inverse operations
- collecting like terms on one side
- multiplying by the LCD.

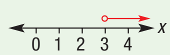
Linear inequalities

Equation contains '=' sign.

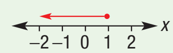
Inequalities have $>, \geq, <$ or $\leq$.

On a number line:

$x > 3$



x ° 1



Solve as per linear equations except when multiplying or dividing by a negative, reverse the inequality sign.

Simultaneous equations

Solve two equations in two pronumerals using:

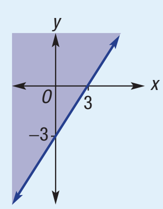
- substitution when one pronumeral is the subject; e.g. $y = x + 4$.
- elimination when adding or subtracting multiples of equations eliminates one variable.

Half planes (Ext)

Inequalities with two variables represent the region above or below the line.

For $y \geq x - 3$, shade above the line.

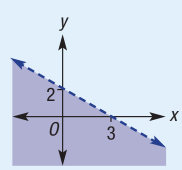
For $y > x - 3$, use a dashed line.



For $2x + 3y < 6$, test (0, 0) to decide which region to shade.

$2(0) + 3(0) < 6$ (true)

Include (0, 0); i.e. shade below dashed line.



Two or more half planes form an intersecting region.

Applications of simultaneous equations

- Define unknowns using pronumerals.
- Form two equations from the information in the problem.
- Solve simultaneously.
- Answer the question in words.

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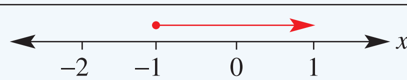
Chapter checklist

Chapter checklist: Success criteria

		✔
1A	**1. I can simplify expressions using the four operations +, −, ×, ÷.** e.g. Simplify $3x \times 2y - 4x^2y + 2xy + 3x^2y$.	
1A	**2. I can expand brackets using the distributive law.** e.g. Expand and simplify $-4y(2y + 3)$.	
1A	**3. I can factorise simple algebraic expressions with a common factor.** e.g. Factorise $6x^2 - 15x$.	
1A	**4. I can substitute numbers for pronumerals and evaluate.** e.g. Given $a = -3$ and $b = 4$, evaluate $ab + a^2$.	
1B	**5. I can cancel common factors in algebraic fractions.** e.g. Simplify $\frac{4x - 2}{2}$.	
1B	**6. I can multiply and divide algebraic fractions.** e.g. Simplify $\frac{3x - 9}{20} \div \frac{2x - 6}{5}$.	
1C	**7. I can add and subtract simple algebraic fractions using the lowest common denominator.** e.g. Simplify $\frac{4}{x} + \frac{5}{6}$.	
1C	**8. I can add and subtract algebraic fractions with binomial numerators or denominators.** e.g. Simplify $\frac{x + 3}{3} - \frac{x - 2}{7}$ and $\frac{4}{x - 2} + \frac{3}{x + 4}$.	
1D	**9. I can solve linear equations with brackets and variables on both sides.** e.g. Solve $4(3x - 5) = 7x$.	
1D	**10. I can solve linear equations involving algebraic fractions.** e.g. Solve $\frac{x + 1}{3} + \frac{x - 2}{5} = 2$ and $\frac{1 - 2x}{3} = \frac{x + 1}{2}$.	
1E	**11. I can interpret number lines to write inequalities.** e.g. Write as an inequality. 	
1E	**12. I can represent a set of solutions on a number line.** e.g. Graph the inequality $x > 3$ on a number line.	
1F	**13. I can solve linear inequalities.** e.g. Solve $4 - \frac{x}{3} > 8$.	
1F	**14. I can determine if a point is on a straight line.** e.g. Decide if the point $(3, -1)$ is on the line $3x + 2y = 7$.	
1F	**15. I can find the gradient and *y*-intercept from a straight line equation.** e.g. State the gradient and y-intercept of $3y - 2x = 6$.	
1F	**16. I can use the gradient and *y*-intercept to sketch a graph.** e.g. Find the gradient and y-intercept of $y = -2x + 7$ and sketch its graph.	
1F	**17. I can find the *x*- and *y*-intercepts of a linear graph.** e.g. Find the x- and y-intercepts and sketch the graph of $3x + y = 9$	

ISBN 978-1-108-77290-7

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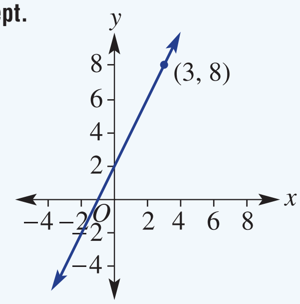
Chapter checklist

		✔
1F	**18. I can sketch a horizontal or vertical line.** e.g. Sketch $y = 3$.	
1F	**19. I can sketch a line of the form $y = mx$.** e.g. Sketch $y = 2x$ labelling the axis intercept and one other point.	
1G	**20. I can find the gradient of a line joining two points.** e.g. Determine the gradient of the line joining the points $(-2, 4)$ and $(3, 1)$.	
1G	**21. I can find the equation of a line using a point and the y-intercept.** e.g. Find the equation of the straight line shown. 	
1G	**22. I can find the equation of a line given two points.** e.g. Find the equation of the straight line joining the points $(-2, -2)$ and $(2, 3)$.	
1H	**23. I can find the distance between two points.** e.g. Find the exact distance between the points $(2, 4)$ and $(5, 2)$.	
1H	**24. I can find the midpoint of a line segment joining two points.** e.g. Find the midpoint of the line segment joining $(-1, 5)$ and $(5, 2)$.	
1H	**25. I can use a given distance to find coordinates.** e.g. Find the values of a if the distance between $(3, a)$ and $(6, 10)$ is $\sqrt{34}$.	
1I	**26. I can decide if lines are parallel, perpendicular or neither.** e.g. Decide if the graph of the lines $y = 2x + 5$ and $2y + x = 3$ will be parallel, perpendicular or neither.	
1I	**27. I can find the equation of a parallel or perpendicular line.** e.g. Find the equation of the line that is parallel to $y = 3x - 4$ and passes through $(2, 4)$.	
1J	**28. I can solve simultaneous equations using substitution.** e.g. Solve the pair of simultaneous equations $x - 2y = 4$ and $y = x - 3$ using the method of substitution.	
1K	**29. I can solve simultaneous equations by adding or subtracting them.** e.g. Solve the simultaneous equations $x - 2y = 10$ and $x + y = 4$ using elimination.	
1K	**30. I can use the elimination method to solve simultaneous equations.** e.g. Solve the pair of simultaneous equations $2x + 3y = 5$ and $3x - 4y = -18$ using the elimination method.	
1L	**31. I can set up and solve simultaneous equations.** e.g. A teacher buys 5 of the same chocolate bars and 2 of the same ice-creams for \$18 while another teacher buys 4 of the same chocolate bar and 5 of the same ice-creams for \$28. Determine the individual costs of these chocolate bars and ice-creams.	
1M	**32. I can sketch a half plane.** e.g. Sketch the half plane $y > 2x - 5$. Ext	
1M	**33. I can find the intersecting region.** e.g. Sketch both the inequalities $2x + y \geqslant -2$ and $2x - 3y < 6$ on the same set of axes, showing the point of intersection of the two lines and the intersecting region. Ext	

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Short-answer questions

1A **1** Simplify the following. You may need to expand the brackets first.

a $8xy + 5x - 3xy + x$ **b** $3a \times 4ab$
c $18xy \div (12y)$ **d** $3(b + 5) + 6$
e $-3m(2m - 4) + 4m^2$ **f** $3(2x + 4) - 5(x + 2)$

1B **2** Simplify these algebraic fractions by first looking to cancel common factors.

a $\dfrac{12x - 4}{4}$ **b** $\dfrac{5}{3x} \times \dfrac{6x}{5x + 10}$ **c** $\dfrac{3x - 3}{28} \div \dfrac{x - 1}{7}$

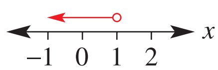
1C **3** Simplify the following algebraic fractions.

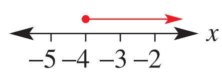
a $\dfrac{3}{7} - \dfrac{a}{2}$ **b** $\dfrac{5}{6} + \dfrac{3}{a}$
c $\dfrac{x + 4}{6} + \dfrac{x + 3}{15}$ **d** $\dfrac{2}{x - 3} - \dfrac{3}{x + 1}$

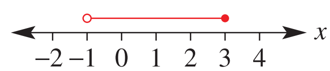
1D **4** Solve these linear equations.

a $3 - 2x = 9$ **b** $3(2x + 1) = 7 - 2(x + 5)$
c $\dfrac{5x - 9}{4} = -2$ **d** $\dfrac{2x - 1}{3} = \dfrac{x + 2}{4}$

1E **5** Write each of the following as an inequality.

a 

b 

c 

1E **6** Solve the following inequalities.

a $4x - 3 > 17$ **b** $3x + 2 \leqslant 4(x - 2)$
c $1 - \dfrac{x}{3} < 2$ **d** $-2x \geqslant -4(1 - 3x)$

1D **7** Marie's watering can is initially filled with 2 litres of water. However, the watering can has a small hole in the base and is leaking at a rate of 0.4 litres per minute.

a Write a rule for the volume of water, V litres, in the can after t minutes.
b What volume of water remains after 90 seconds?
c How long would it take for all the water to leak out?
d If Marie fills the can with 2 litres of water at her kitchen sink, what is the maximum amount of time she can take to get to her garden if she needs at least 600 mL to water her roses?

1F **8** Sketch graphs of the following linear relations, labelling the points where the graph cuts the axes.

a $y = 3x - 9$ **b** $y = 5 - 2x$
c $y = 3$ **d** $x = 5$
e $y = 2x$ **f** $y = -5x$
g $x + 2y = 8$ **h** $3x + 8y = 24$

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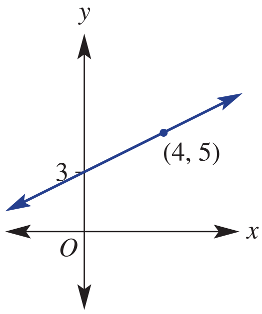
1G

9 Find the equation of these straight lines.

a



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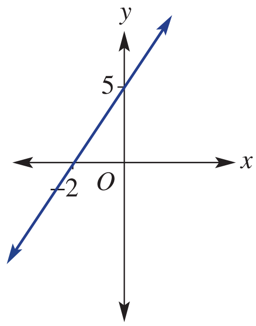

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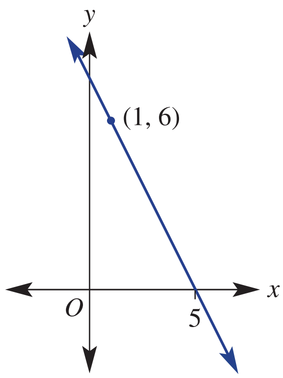
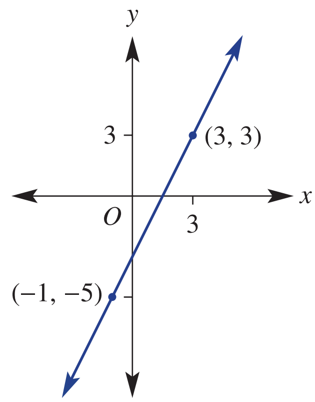


d



1G

10 For the line that passes through the points $(-2, 8)$ and $(3, 5)$, determine:

a the gradient of the line

b the equation of the line.

1H

11 Find the midpoint and the exact length of the line segment joining these points.

a $(2, 5)$ and $(6, 11)$

b $(3, -2)$ and $(8, 4)$

c $(-1, -4)$ and $(2, -1)$

1I

12 Determine the equation of the line that is:

a parallel to the line $y = 3x + 8$ and passes through the point $(2, 4)$

b parallel to the line with equation $y = 4$ and passes through the point $(3, -1)$

c perpendicular to the line $y = 2x - 4$ and has a y-intercept with coordinates $(0, 5)$

d perpendicular to the line with equation $x + 3y = 5$ and passes through the point $(2, 5)$.

1H/I

13 Find the value(s) of the pronumeral in each situation below.

a The gradient of the line joining the points $(2, -5)$ and $(6, a)$ is 3.

b The line $bx + 2y = 7$ is parallel to the line $y = 4x + 3$.

c The distance between $(c, -1)$ and $(2, 2)$ is $\sqrt{13}$.

1J

14 Solve the following simultaneous equations, using the substitution method.

a $y = 5x + 14$
$y = 2x + 5$

b $3x - 2y = 18$
$y = 2x - 5$

1K

15 Solve these simultaneous equations by elimination.

a $3x + 2y = -11$
$2x - y = -5$

b $2y - 5x = 4$
$3y - 2x = 6$

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1L **16** At the movies Jodie buys three regular popcorns and five small drinks for her friends at a cost of \$24.50. Her friend Renee buys four regular popcorns and three small drinks for her friends at a cost of \$23.50. Find the individual costs of a regular popcorn and a small drink.

1M Ext **17** Sketch these half planes.

a $y \geqslant 3x - 4$

b $2x - 3y > -8$

1M Ext **18** Shade the intersecting region of the inequalities $x + 2y \geqslant 4$ and $3x - 2y < 12$ by sketching their half planes on the same axes and finding their point of intersection.

Multiple-choice questions

1A **1** The simplified form of $3x + 2x \times 7y - 3xy + 5x$ is:

A $10x + 4y$ **B** $35xy - 6x^2y$ **C** $\dfrac{35y - 6xy}{7}$ **D** $5xy - 6x^2$ **E** $8x + 11xy$

1B **2** $\dfrac{3x - 6}{2} \times \dfrac{8}{x - 2}$ simplifies to:

A 9 **B** -6 **C** $-4(x - 3)$ **D** 12 **E** $4(x - 2)$

1C **3** $\dfrac{4}{x - 1} - \dfrac{5}{2x - 3}$ simplifies to:

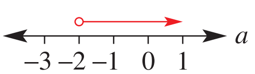
A $\dfrac{-1}{(x - 1)(2x - 3)}$ **B** $\dfrac{3x - 7}{(x - 1)(2x - 3)}$ **C** $\dfrac{3x - 17}{(2x - 3)(x - 1)}$

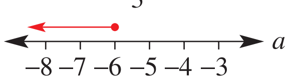
D $\dfrac{11 - 6x}{(x - 1)(2x - 3)}$ **E** $\dfrac{x - 7}{(2x - 3)(x - 1)}$

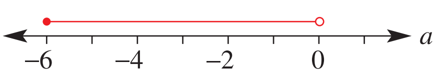
1D **4** The solution to $-4(2x - 6) = 10x$ is:

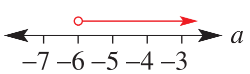
A $x = \dfrac{3}{2}$ **B** $x = 12$ **C** $x = \dfrac{4}{3}$ **D** $x = -12$ **E** $x = -\dfrac{4}{3}$

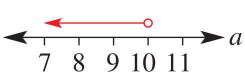
1E **5** The number line that represents the solution to the inequality $2 - \dfrac{a}{3} < 4$ is:

A


B


C


D


E


1F **6** If $(-1, 2)$ is a point on the line $ax - 4y + 11 = 0$, the value of a is:

A -19 **B** 3 **C** $-\dfrac{15}{2}$ **D** 5 **E** -1

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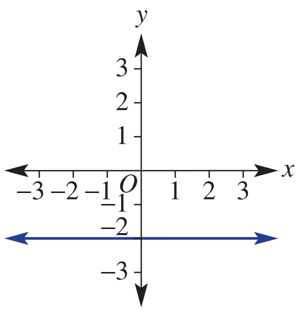
Chapter review

1F **7** The graph shown has equation:

A $x = -2$
B $y = -2x$
C $y = -2$
D $x + y = -2$
E $y = x - 2$



1F **8** The gradient and the y-intercept, respectively, of the graph of $3x + 8y = 2$ are:

A $-\frac{3}{8}, \frac{1}{4}$ **B** $3, 2$ **C** $\frac{2}{3}, \frac{1}{4}$ **D** $-3, 2$ **E** $\frac{3}{8}, 2$

1G **9** The equation of the line joining the points $(-1, 3)$ and $(1, -1)$ is:

A $2y - x = 1$ **B** $y = 2x - 1$ **C** $y = -2x + 1$
D $y - 2x = 1$ **E** $y = \frac{1}{2}x + 1$

1H **10** The midpoint of the line segment joining the points $(a, -6)$ and $(7, b)$ is $(4.5, -1)$. The values of the pronumerals are:

A $a = 2, b = 8$ **B** $a = 3, b = -11$ **C** $a = 9, b = 5$
D $a = 2, b = 4$ **E** $a = 2.5, b = 5$

1I **11** The line that is perpendicular to the line with equation $y = -3x + 7$ is:

A $y = -3x + 2$ **B** $3x + y = -1$ **C** $y = 3x - 3$ **D** $3y = 4 - x$ **E** $3y - x = 4$

1I **12** The line that is parallel to the line with equation $y = 2x + 3$ and passes through the point $(-3, 2)$ has the equation:

A $2x + y = 5$ **B** $y = 2x + 8$ **C** $y = -\frac{1}{2}x + \frac{1}{2}$ **D** $y = 2x - 4$ **E** $y - 2x = -7$

1J **13** The solution to the simultaneous equations $2x - 3y = -1$ and $y = 2x + 3$ is:

A $x = -2, y = -1$ **B** $x = \frac{5}{2}, y = 8$ **C** $x = 2, y = 7$
D $x = -\frac{2}{3}, y = -\frac{1}{9}$ **E** $x = -3, y = 3$

1L **14** A community fundraising concert raises \$3540 from ticket sales to 250 people. Children's tickets were sold for \$12 and adult tickets sold for \$18. If x adults and y children attended the concert, the two equations that represent this problem are:

A $x + y = 250$
$18x + 12y = 3540$

B $x + y = 3540$
$216xy = 3540$

C $x + y = 250$
$12x + 18y = 3540$

D $x + y = 3540$
$18x + 12y = 250$

E $3x + 2y = 3540$
$x + y = 250$

1M Ext **15** The point that is *not* in the region defined by $2x - 3y \leqslant 5$ is:

A $(0, 0)$ **B** $(1, -1)$ **C** $(-3, 2)$
D $(2, -1)$ **E** $\left(\frac{5}{2}, 3\right)$

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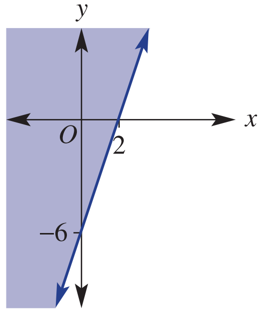
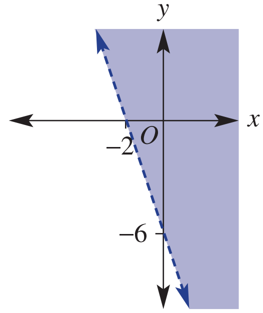
1M Ext

16 The half plane that represents the inequality $y < 3x - 6$ is:

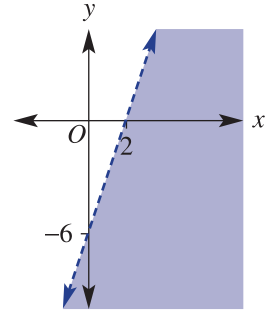
A



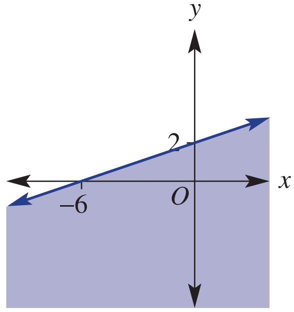
B



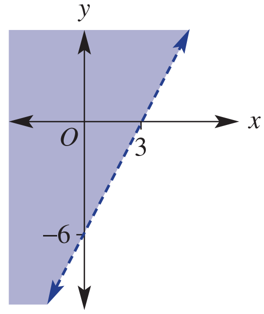
C



D



E



Extended-response questions

1 There are two shrubs in Chen's backyard that grow at a constant rate. Shrub A had an initial height of 25 cm and has grown to 33 cm after 2 months. Shrub B was 28 cm high after 2 months and 46 cm high after 5 months.

a Write a rule for the height, h cm, after t months for:
  i shrub A
  ii shrub B

b What was the initial height (i.e. at $t = 0$) of shrub B?

c Refer to your rules in part **a** to explain which shrub is growing at a faster rate.

d Graph each of your rules from part **a** on the same set of axes for $0 \leqslant t \leqslant 12$.

e Determine graphically after how many months the height of shrub B will overtake the height of shrub A.

f i Shrub B reaches its maximum height after 18 months. What is this height?
  ii Shrub A has a maximum expected height of 1.3 m. After how many months will it reach this height?
  iii Chen will prune shrub A when it is between 60 cm and 70 cm. Within what range of months after it is planted will Chen need to prune the shrub?

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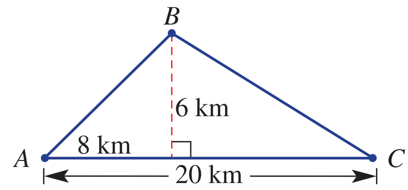
2 A triangular course has been roped off for a cross-country run. The run starts and ends at A and goes via checkpoints B and C, as shown.



a Draw the area of land onto a set of axes, taking point A to be the origin $(0, 0)$. Label the coordinates of B and C.

b Find the length of the course, to one decimal place, by calculating the distance of legs AB, BC and CA.

c A drink station is located at the midpoint of BC. Label the coordinates of the drink station on your axes.

d Find the equation of each leg of the course:

i AB ii BC iii CA

Ext e Write a set of three inequalities that would overlap to form an intersecting region equal to the area occupied by the course.

f A fence line runs beyond the course. The fence line passes through point C and would intersect AB at right angles if AB was extended to the fence line. Find the equation of the fence line.