Simplify algebraic fractions

1 Simplify:

a $\frac{40y}{450}$ **b** $\frac{6n}{9n}$ **c** $\frac{42xy}{24y}$ **d** $\frac{15n^2}{-40n}$

e $\frac{64mn^2}{40m^2n}$ **f** $-\frac{10x^2}{6xy^3}$ **g** $\frac{6p^5}{3p^3}$ **h** $\frac{10a^3b^5}{25a^2b^9}$

2 Simplify:

a $\frac{25x-40}{55}$ **b** $\frac{4uv-20uw}{4u}$ **c** $\frac{5u}{5uv-30uw}$ **d** $\frac{9m^2+24mn}{18m^2}$

3 Simplify:

a $\frac{4x+20y}{x+5y}$ **b** $\frac{2x-6y}{x-3y}$ **c** $\frac{x-2}{2-x}$ **d** $\frac{5y-30}{6-y}$

e $\frac{2x+6}{x^2+3x}$ **f** $\frac{8-2x}{5x^2-20x}$ **g** $\frac{10x-15}{2x^3-3x^2}$ **h** $\frac{8-2xy}{x^2y-4x}$

i $\frac{12m^2+6mn}{18mn+36m^2}$

4 Simplify:

a $\frac{(x+3)(x-2)}{x+3}$ **b** $\frac{(2x-5)(x+4)}{x+4}$ **c** $\frac{x-7}{2(x-7)(x+4)}$ **d** $\frac{(x+6)(x+8)}{(x+6)(x-9)}$

e $\frac{x^2-8x+16}{x-4}$ **f** $\frac{x^2+7x+12}{x+4}$ **g** $\frac{x^2+9x+20}{(x+5)(x+1)}$ **h** $\frac{x^2-3x-18}{(x-6)(x+2)}$

5 Simplify:

a $\frac{m^2-16}{m-4}$ **b** $\frac{m^2-64}{8+m}$ **c** $\frac{x-10}{x^2-100}$ **d** $\frac{a^2-121}{11-a}$

6 Simplify:

a $\frac{8(p-q)^2}{8p^2-8q^2}$ **b** $\frac{x+10}{x^2+20x+100}$ **c** $\frac{x^2-4x+4}{7(x-2)}$ **d** $\frac{x^2+8x+15}{x+5}$

e $\frac{x+5}{x^2+2x-15}$ **f** $\frac{3x^2-18x-216}{4(x+6)}$ **g** $\frac{3y+6}{y^2-4}$ **h** $\frac{m^2-16}{5m+20}$

i $\frac{x^2+6x+9}{x^2-9}$ **j** $\frac{x^2-3x-10}{x^2-5x}$ **k** $\frac{x^2+5x+6}{x^2-2x-8}$ **l** $\frac{8(x+3)}{5x^2+30x+45}$

m $\frac{(x+3)(x-11)}{x^2-18x+77}$ **n** $\frac{4x^2+8x-192}{2x+16}$

Multiplication of algebraic fractions

1 Simplify the following expressions:

a $\frac{1}{a} \times \frac{1}{b}$

b $\frac{a}{3} \times \frac{a}{7}$

c $\frac{b}{q} \times \frac{k}{u}$

d $\frac{c}{3} \times \frac{d}{2}$

e $\frac{3y}{8} \times \frac{4y}{9}$

f $\frac{9x}{2} \times \frac{5y}{7}$

g $\frac{4x}{5} \times \frac{3y}{7}$

h $\frac{5u}{3a} \times \frac{3v}{5b}$

i $\frac{11s}{7t} \times \frac{3r}{5q}$

j $\frac{9a}{10b} \times \frac{3b}{8a}$

k $\frac{8x}{7y} \times \frac{9y}{5x}$

l $\frac{9u}{7v} \times \frac{8w}{11y}$

m $\frac{14u}{15v} \times \frac{40v}{24q}$

n $\frac{9u}{77v} \times \frac{110v}{24q}$.

o $\frac{x^2}{3} \times \frac{6}{x}$

p $\frac{p^2}{q} \times \frac{q^2}{p}$

q $\frac{r^2}{n} \times \frac{n^2}{r}$

r $\frac{5y^2}{9} \times 27y^2$

s $\frac{5y}{2} \times 6y^5$.

t $\frac{4m^2}{20} \times \frac{3y^2}{7m}$

u $\frac{7a^2}{15b^2} \times \frac{10b}{11a}$.

v $\frac{11a^2}{4b^2} \times \frac{10b}{3a}$

w $\left(-\frac{10}{21x}\right) \times \frac{3x^2}{110}$

x $\frac{2n^2}{6} \times \frac{-2y^2}{3n}$

2 Simplify:

a $\frac{m}{3} \times \frac{5m}{9} \times 81q$

b $\frac{2n}{3m} \times \frac{21q}{4n} \times \frac{10m}{49p}$

3 Simplify $\left(\frac{x}{3y}\right)^2 \times \frac{5x^2}{2y}$.

4 Complete the following:

$$\frac{\blacksquare}{3a^2} \times \frac{7b}{\blacksquare} = \frac{14bc}{9a^2d}$$

5 The product of two fractions is 1. If one of the fractions is $\frac{3x}{4yz}$, what is the other fraction?

Addition and subtraction of algebraic fractions

1 Simplify:

a $\frac{2x}{7} + \frac{7x}{7}$ **b** $\frac{3x}{4} + \frac{2x}{4}$ **c** $\frac{2x}{3} + \frac{10x}{3}$ **d** $\frac{3x}{12} + \frac{12x}{12}$

e $\frac{-8x}{9} + \frac{4x}{9}$ **f** $\frac{-3x}{14} + \frac{-2x}{14}$ **g** $\frac{8x}{2} - \frac{3x}{2}$ **h** $\frac{13x}{3} - \frac{8x}{3}$

i $\frac{7x}{5} - \frac{10x}{5}$ **j** $\frac{31x}{6} - \frac{7x}{6}$ **k** $\frac{25x}{24} - \frac{5x}{24}$ **l** $-\frac{4x}{7} - \frac{5x}{7}$

2 Simplify:

a $\frac{2x}{3} + \frac{x+3}{3}$ **b** $\frac{7x}{8} + \frac{5x+2}{8}$

c $\frac{-2x}{7} + \frac{4x-3}{7}$ **d** $\frac{5x-1}{3} - \frac{2x+3}{3}$

e $\frac{4x-10}{3} + \frac{-5x+10}{3}$ **f** $\frac{-6x-8}{5} + \frac{-3x-5}{5}$

g $\frac{4x^2+11}{6} - \frac{3x^2-4x}{6}$ **h** $\frac{2x^2-3}{5} + \frac{3x^2+6}{5}$

i $\frac{-5x^2-2x}{3} + \frac{-3x^2-2x}{3}$ **j** $\frac{2x-7x^3}{4} - \frac{-x^2-3x^3+1}{4}$

3 Find the lowest common denominator for the following pairs of algebraic fractions:

a $\frac{w}{2}$ and $\frac{w}{6}$ **b** $\frac{s}{3}$ and $\frac{s}{6}$ **c** $\frac{x}{2}$ and $\frac{y}{3}$ **d** $\frac{r}{5}$ and $\frac{s}{3}$

4 Consider the algebraic fractions $\frac{2m}{5}$ and $\frac{3m}{6}$.

a Find the lowest common denominator. **b** Hence, simplify $\frac{2m}{5} + \frac{3m}{6}$.

5 Complete the following:

$$\frac{3x}{5} + \frac{2x}{\blacksquare} = \frac{\blacksquare}{35}$$

6 Simplify:

a $\frac{2x}{5} + \frac{4x}{7}$ **b** $\frac{2x}{14} + \frac{14x}{21}$ **c** $\frac{4x}{5} - \frac{x}{7}$ **d** $\frac{4x}{45} - \frac{x}{18}$

e $-9x - \frac{5x}{7}$ **f** $\frac{3x}{5} - \frac{2x}{7}$ **g** $-3x - \frac{3x}{8}$ **h** $\frac{11x}{14} + \frac{7x}{21}$

i $\frac{x}{15} + \frac{3x}{35}$ **j** $\frac{4x^2}{3} - \frac{5x^2}{2}$

7 Simplify:

a $\frac{2x}{3} + \frac{2x+5}{9}$

b $\frac{8x}{13} + \frac{3x+4}{39}$

c $\frac{3x}{10} - \frac{4x+3}{16}$

d $\frac{4x}{9} + \frac{2x-3}{36}$

e $\frac{8x}{11} - \frac{2x-3}{44}$

f $\frac{y}{10} - \frac{5y+1}{25}$

g $\frac{7x}{8} - \frac{5x+6}{20}$

h $\frac{x+3}{3} + \frac{3x-2}{12}$

i $\frac{x+3}{3} - \frac{5x-7}{12}$

j $\frac{2x+3}{4} - \frac{2x+2}{12}$

k $\frac{5x+8}{6} + \frac{x+2}{2}$

l $\frac{3x+4}{6} + \frac{3x+4}{5}$

m $\frac{2x+2}{4} - \frac{x+3}{8}$

n $\frac{6x+7}{3} - \frac{3x-5}{9}$

o $\frac{6x^2}{7} + \frac{x^2+5x}{21}$

p $\frac{2x^2}{7} + \frac{3x^2+2x}{28}$

8 Simplify:

a $\frac{3}{x} + \frac{5}{y}$

b $\frac{c}{ab} + \frac{2}{bc^2}$

c $\frac{2x^2+4y}{y} - \frac{x-3}{x}$

d $\frac{4}{x} + \frac{2}{x-2}$

1. **WE10a** Solve the following equations.

   a. $a + 61 = 85$

   b. $k - 75 = 46$

   c. $g + 9.3 = 12.2$

   d. $r - 2.3 = 0.7$

2. **WE10b** Solve the following equations.

   a. $\frac{f}{4} = 3$

   b. $\frac{i}{10} = -6$

   c. $6z = -42$

7. **WE11a** Solve the following.

   a. $5a + 6 = 26$

   b. $6b + 8 = 44$

   c. $8i - 9 = 15$

8. Solve the following.

   a. $\frac{f}{4} + 6 = 16$

   b. $\frac{g}{6} + 4 = 9$

   c. $\frac{r}{10} + 6 = 5$

9. Solve the following.

   a. $6(x + 8) = 56$

   b. $7(y - 4) = 35$

   c. $5(m - 3) = 7$

10. **WE11b** Solve the following.

    a. $\frac{3k}{5} = 15$

    b. $\frac{9m}{8} = 18$

    c. $\frac{7p}{10} = -8$

11. Solve the following.

    a. $\frac{x - 5}{3} = 7$

    b. $\frac{2m + 1}{3} = -3$

    c. $\frac{3w - 1}{4} = 6$

Equations with brackets

1 Solve the following equations:

a $4(x+4)=24$

b $5(3x+6)=78$

c $5(2x-4)=-70$

d $2(x+5)=16$

e $6(2x-4)=-84$

f $3(x+1)+2=-1$

g $-2(x+2)-4=4$

h $3(x-1)-5=-11$

2 Solve the following equations:

a $3(6x-7)-5x+8=26$

b $2(3x-5)+3(4x+6)=62$

c $-3(2x-5)+5(3x+4)=53$

d $6(5x-8)-3x+7=40$

e $2(4x-5)+3(2x+6)=36$

f $3(x+6)+3(x+24)=12$

g $3(x-1)-5=-8.6$

h $3.8(x+4)-2=1.8$

i $3\left(x+\frac{3}{2}\right)+3=-\frac{15}{2}$

j $2.2(x+3)-4=-8.4$

Equations with fractions

3 Solve the following equations:

a $\frac{x}{6}=\frac{5}{3}$

b $\frac{x}{9}=2\frac{2}{3}$

c $\frac{2}{2.5}=\frac{7}{x}$

d $x+\frac{2x-4}{4}=3$

e $x+\frac{4x+5}{6}=2$

f $\frac{7}{9}=\frac{8}{x}$

g $x+\frac{5x-5}{6}=1$

h $x+\frac{4x+2}{6}=5$

Equations with pronumerals on both sides

5 Solve the following equations:

a $3x=x+10$

b $3x=-4x-35$

c $12x-8=-4x$

d $27+7x=-2x$

e $10+4x=-11x-20$

f $28-3x=11+14x$

g $5x+22=8-2x$

h $4x-3=-8x+57$

Solve inequalities

3 State the largest integer value x can have if $x \leq -2$.

4 State the smallest integer value x can have if $x > 19$.

5 State whether the following are solutions of $k \leq 15$:

a $k = 14$ b $k = -15$ c $k = 15$ d $k = 16$

6 Consider the inequality: $a + 9 < 12$.

a Find the value of the left-hand side of the inequality when $a = 1$.

b Is $a = 1$ a solution of $a + 9 < 12$?

12 Solve the following inequalities:

a $7y > 42$

b $8x \leq -48$

c $-9x < 63$

d $-4x \leq -16$

e $x - 1 < 15$

f $7 - x < 2$

g $2 \leq x - 6$

h $8x + 16 > 64$

i $7x - 49 \geq 42$

j $7x + 5 > x + 35$

k $-3(x + 9) \geq -6$

l $-9(x + 6) \geq -27$

m $23 - 3k \leq 2k + 3$

n $-4 - 4x > -7x + 2$

o $\frac{x}{3} \geq 7$

p $\frac{x}{-4} \geq 8$

q $\frac{x + 9}{8} \geq 9$

r $2 - \frac{1}{2}x > 5$

s $\frac{x}{2} + \frac{x}{3} \geq 5$

t $\frac{x}{9} - \frac{x}{7} < 0$

u $\frac{a}{6} < 11$

v $\frac{a}{5} + 9 > 9$

w $91 - 4k \leq 5k + 10$

17 Consider the inequality: $30 + 3x > 30$.

a Solve the inequality.

b State whether the following are solutions of: $30 + 3x > 30$.

i $x = 1$ ii $x = 0$ iii $x = 3$ iv $x = 6$

2. Plot the linear graphs defined by the following rules for the given range of x-values.

	Rule	x-values
a	$y = -3x + 2$	

x	-6	-4	-2	0	2	4	6
y							

	Rule	x-values
b	$y = -x + 3$	

x	-3	-2	-1	0	1	2	3
y							

3. **WE2** Sketch graphs of the following linear equations by finding the x and $y$intercepts.

   a. $5x - 3y = 10$

   b. $5x + 3y = 10$

   c. $-5x + 3y = 10$

4. **WE3** Sketch graphs of the following linear equations using the gradient–intercept method.

   a. $y = 4x + 1$

   b. $y = 3x - 7$

   c. $y = -2x + 3$

6. **WE5** Sketch the graphs of the following linear equations.

   a. $y = 10$

   b. $y = -10$

   c. $x = 10$

7. Transpose each of the equations to standard form (that is, $y = mx + c$). State the x- and y-intercept for each.

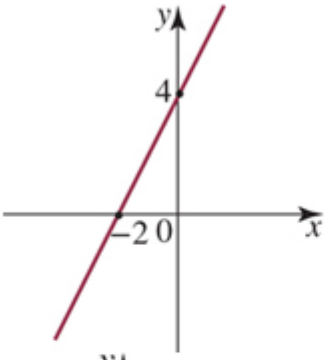
   a. $5(y + 2) = 4(x + 3)$

   b. $5(y - 2) = 4(x - 3)$

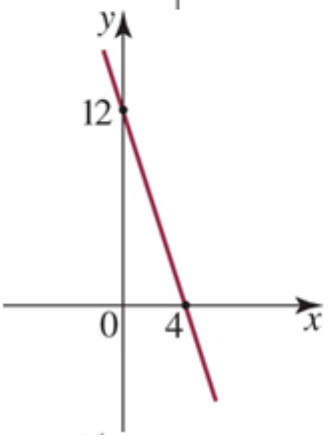
   c. $2(y + 3) = 3(x + 2)$

1. **WE6** Determine the equation for each of the straight lines shown.

a.

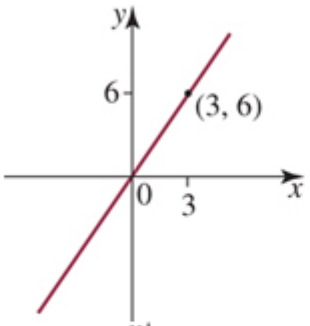


b.

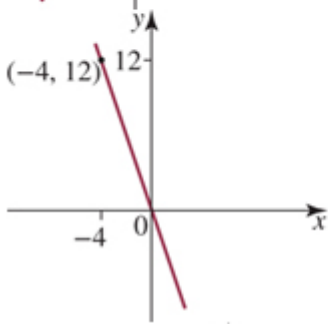


2. **WE7** Determine the equation of each of the straight lines shown.

a.



b.



3. **WE8** Find the equation of the straight line that passes through each pair of points.

   a. $(1, 4)$ and $(3, 6)$

   b. $(0, -1)$ and $(3, 5)$

   c. $(-1, 4)$ and $(3, 2)$

4. **WE9** Find the linear equation given the information in each case below.

   a. Gradient = 3, y-intercept = 3

   b. Gradient = -3, y–intercept = 4

   c. Gradient = -4, y–intercept = 2

5. **WE10, 11** For each of the following, find the equation of the straight line with the given gradient and passing through the given point.

   a. Gradient = 5, point = $(5, 6)$

   b. Gradient = -5, point = $(5, 6)$

   c. Gradient = -4, point = $(-2, 7)$