

SKILLSHEET

Like terms

Terms containing exactly the same pronumeral or a group of pronumerals are called *like terms*.

Note that if the pronumerals are multiplied together, order is not important. For example, mnp and npm are like terms, although m , n and p are written in a different order in the two terms.

WORKED EXAMPLE

Select the like terms from the following list:

ab , $3b$, $5ba$, $5a$, $5abc$, 5 , $\frac{2}{3}ab$.

THINK

Like terms must contain exactly the same pronumeral(s). The first, third and last terms all contain pronumerals a and b and so are considered to be like terms.

WRITE

ab , $5ba$ and $\frac{2}{3}ab$

Try these

Select the like terms from each of the following lists.

- | | | |
|----|--|-------------------|
| 1 | ab , bc , $2ac$, $3ca$, $4b$ | Like terms: |
| 2 | b , $3b$, $\frac{1}{4}b$, $4ab$, $5a$ | Like terms: |
| 3 | mp , $2m$, $3p$, $4mpn$, p | Like terms: |
| 4 | $5abc$, $3bc$, $4cb$, $9ac$, $2acd$ | Like terms: |
| 5 | $\frac{2}{5}xy$, $4y$, yx , $3x$, 4 | Like terms: |
| 6 | $9gh$, $2fhg$, $31hg$, $2.5h$ | Like terms: |
| 7 | $9am$, $9m$, $9a$, $9amn$, 9 , $9nma$ | Like terms: |
| 8 | $5cd$, $5c$, $17c$, $5d$, 5 | Like terms: |
| 9 | $3yz$, $12zy$, $4z$, $10y$, zy | Like terms: |
| 10 | $\frac{1}{2}rs$, $2st$, $9tr$, $9rs$, $\frac{1}{2}r$ | Like terms: |

SKILLSHEET

Collecting like terms

Terms that contain exactly the same pronumeral(s) are called *like terms*. Like terms can be *collected* by adding (or subtracting) their coefficients. (A coefficient is a number in front of the term. Note that if there is no number, then the coefficient is 1.)

WORKED EXAMPLE

Simplify each of the following expressions.

a $x + 3y - 2x - 5y$ **b** $3x + 6 + 4x - 1$

THINK

- a**
- 1 Write the expression.
 - 2 Collect the terms containing x by adding their coefficients (that is, $1 - 2 = -1$).
 - 3 Collect the terms containing y by adding their coefficients (that is, $3 - 5 = -2$).
- b**
- 1 Write the expression.
 - 2 Collect the terms containing x by adding their coefficients (that is, $3 + 4 = 7$).
 - 3 Simplify further by subtracting 1 from 6.

WRITE

a $x + 3y - 2x - 5y$
 $= -x + 3y - 5y$
 $= -x - 2y$

b $3x + 6 + 4x - 1$
 $= 7x + 6 - 1$
 $= 7x + 5$

Try these

Simplify each of the following expressions.

- 1 $5x + 7x = \dots\dots\dots$
- 2 $11x - 7x = \dots\dots\dots$
- 3 $4x + 3y + 8x = \dots\dots\dots + 3y$
- 4 $6x - 2y - x - y = \dots\dots\dots - 2y - y$
 $= \dots\dots\dots - \dots\dots\dots$
- 5 $9x - 5y + 2x + 4y = \dots\dots\dots$
 $= \dots\dots\dots$
- 6 $x + 4x^2 - 3x = \dots\dots\dots$
 $= \dots\dots\dots$
- 7 $3x + 4 - 7x - 9 = \dots\dots\dots$
 $= \dots\dots\dots$
- 8 $15x - 8 - 7x + 3 = \dots\dots\dots$
 $= \dots\dots\dots$
- 9 $4x + 12 - x = \dots\dots\dots$
 $= \dots\dots\dots$
- 10 $2x + 5 + 3x - 2 = \dots\dots\dots$
 $= \dots\dots\dots$
- 11 $7x + 8 - 2x + 7 = \dots\dots\dots$
 $= \dots\dots\dots$
- 12 $4x^2 - 2x + 6 - x - 5 = \dots\dots\dots$
 $= \dots\dots\dots$
- 13 $-2a + 6ab + 4a - 3ab = \dots\dots\dots$
 $= \dots\dots\dots$
- 14 $3a^2 + 5ab - ab - 4 = \dots\dots\dots$
 $= \dots\dots\dots$
- 15 $7 - 3x + 6xy - 8 = \dots\dots\dots$
 $= \dots\dots\dots$

SKILLSHEET

Finding the highest common factor (HCF)

Factors that are the same for two or more numbers are called *common factors*. The largest of the common factors is called the *highest common factor*, or HCF.

WORKED EXAMPLE

Find the highest common factor for the numbers 12 and 20.

THINK

- List all factors of 12.
- List all factors of 20.
- Select the numbers that appear on both lists (that is, the common factors).
- State the largest of the common factors.

WRITE

1, 2, 3, 4, 6, 12
 1, 2, 4, 5, 10, 20
 Common factors: 1, 2, 4
 HCF = 4

Try these

Find the highest common factor for the following pairs of numbers.

1 4 and 6

Factors of 4: 1, 2, 4

Factors of 6: 1, 2, _____, _____

Common factors: _____, _____

HCF:

2 12 and 16

Factors of 12:

Factors of 16:

Common factors:

HCF:

3 9 and 27

Factors of 9:

Factors of 27:

Common factors:

HCF:

4 10 and 18

Factors of 10:

Factors of 18:

Common factors:

HCF:

5 4 and 12

Factors of 4:

Factors of 12:

Common factors:

HCF:

6 8 and 20

Factors of 8:

Factors of 20:

Common factors:

HCF:

7 15 and 18

Factors of 15:

Factors of 18:

Common factors:

HCF:

8 16 and 24

Factors of 16:

Factors of 24:

Common factors:

HCF:

(continued)

SKILLSHEET

Addition and subtraction of fractions

To add or subtract fractions, follow these steps.

Step 1 Convert (if necessary) to equivalent fractions with a common denominator.

Step 2 Add (subtract) numerators together and leave the denominators unchanged.

Step 3 Simplify (if necessary); if the result is an improper fraction, convert it to a mixed number.

WORKED EXAMPLE

Calculate each of the following.

a $\frac{2}{5} + \frac{7}{8}$ b $\frac{5}{8} - \frac{1}{6}$

THINK

- a 1 The lowest common denominator (LCD) of 5 and 8 is 40. To convert fractions to the denominator of 40, multiply the numerator and denominator of the first fraction by 8 and of the second fraction by 5.
- 2 Add numerators (16 and 35) together and leave the denominator unchanged.
- 3 Convert an improper fraction to a mixed number.
- b 1 The LCD of 8 and 6 is 24. To convert fractions to the denominator of 24, multiply the numerator and denominator of the first fraction by 3 and of the second fraction by 4.
- 2 Subtract the numerators (15 - 4) and leave the denominator unchanged.

WRITE

$$\begin{aligned} \text{a } \frac{2}{5} + \frac{7}{8} &= \frac{2}{5} \times \frac{8}{8} + \frac{7}{8} \times \frac{5}{5} \\ &= \frac{16}{40} + \frac{35}{40} \\ &= \frac{51}{40} \\ &= 1\frac{11}{40} \end{aligned}$$

$$\begin{aligned} \text{b } \frac{5}{8} - \frac{1}{6} &= \frac{5 \times 3}{8 \times 3} - \frac{1 \times 4}{6 \times 4} \\ &= \frac{15}{24} - \frac{4}{24} \\ &= \frac{11}{24} \end{aligned}$$

Try these

Calculate each of the following.

1 $\frac{1}{5} + \frac{3}{8}$

$$= \frac{1}{5} \times \frac{\dots}{\dots} + \frac{3}{8} \times \frac{\dots}{\dots}$$

$$= \frac{\dots}{\dots} + \frac{\dots}{\dots}$$

$$= \frac{\dots}{\dots}$$

2 $\frac{2}{3} + \frac{7}{8}$

$$= \frac{2}{3} \times \frac{\dots}{\dots} + \frac{7}{8} \times \frac{\dots}{\dots}$$

$$= \frac{\dots}{\dots} + \frac{\dots}{\dots}$$

$$= \frac{\dots}{\dots}$$

$$= \dots\dots\dots$$

3 $\frac{3}{4} + \frac{5}{8}$

$$= \frac{3}{4} \times \frac{\dots}{\dots} + \frac{5}{8}$$

$$= \frac{\dots}{\dots} + \frac{5}{8}$$

$$= \frac{\dots}{\dots}$$

$$= \dots\dots\dots$$

(continued)

SKILLSHEET

Multiplication of fractions

To multiply fractions, follow these steps.

- Step 1 Convert all mixed numbers into improper fractions.
 Step 2 Simplify as much as possible.
 Step 3 Multiply the numerators and multiply the denominators.
 Step 4 If the answer is an improper fraction, convert it to a mixed number.

WORKED EXAMPLE

Perform the following multiplications.

a $\frac{3}{5} \times \frac{1}{6}$ b $1\frac{3}{4} \times \frac{6}{7}$

THINK

- a 1 Write the multiplication problem.
- 2 Cross-cancel 3 and 6 by dividing each by 3 (that is, $3 \div 3 = 1$; $6 \div 3 = 2$).
- 3 Multiply the numerators and multiply the denominators.
- b 1 Write the multiplication problem.
- 2 Convert the mixed number into an improper fraction.
- 3 Cross-cancel 7 and 7 by dividing each by 7 (that is, $7 \div 7 = 1$).
 Next cross-cancel 6 and 4 by dividing each by 2
 (that is, $4 \div 2 = 2$; $6 \div 2 = 3$).
- 4 Multiply the numerators and multiply the denominators.
- 5 Convert an improper fraction into a mixed number.

WRITE

a $\frac{3}{5} \times \frac{1}{6}$

$$= \frac{\cancel{3}^1}{5} \times \frac{1}{\cancel{6}_2}$$

$$= \frac{1}{5} \times \frac{1}{2}$$

$$= \frac{1}{10}$$

b $1\frac{3}{4} \times \frac{6}{7}$

$$= \frac{7}{4} \times \frac{6}{7}$$

$$= \frac{\cancel{7}^1}{\cancel{4}_2} \times \frac{\cancel{6}_3}{\cancel{7}_1}$$

$$= \frac{1}{2} \times \frac{3}{1}$$

$$= \frac{3}{2}$$

$$= 1\frac{1}{2}$$

(continued)

SKILLSHEET

Division of fractions

To divide fractions, follow these steps.

Step 1 Convert mixed numbers to improper fractions.

Step 2 Turn the second fraction upside down; replace the + sign with \times and perform multiplication (multiply and tip).

Step 3 If the result is an improper fraction, convert it to a mixed number.

WORKED EXAMPLE

Calculate each of the following.

a $\frac{3}{8} \div \frac{1}{3}$ b $1\frac{1}{4} \div \frac{1}{2}$

THINK

- a
- 1 Turn the second fraction upside down and replace the + sign with \times .
 - 2 Multiply the numerators and multiply the denominators.
 - 3 Convert the improper fraction into a mixed number.
- b
- 1 Convert the mixed number into an improper fraction first.
 - 2 Turn the second fraction upside down and replace the + sign with \times .
 - 3 Cross-cancel 4 and 2 by dividing each by 2 (i.e. $4 \div 2 = 2$; $2 \div 2 = 1$).
 - 4 Multiply the numerators and multiply the denominators.
 - 5 Convert the improper fraction into a mixed number.

WRITE

a $\frac{3}{8} \div \frac{1}{3} = \frac{3}{8} \times \frac{3}{1}$
 $= \frac{9}{8}$
 $= 1\frac{1}{8}$

b $1\frac{1}{4} \div \frac{1}{2} = \frac{5}{4} \div \frac{1}{2}$
 $= \frac{5}{4} \times \frac{2}{1}$
 $= \frac{5}{\cancel{2}} \times \frac{\cancel{2}^1}{1}$
 $= \frac{5}{2} \times \frac{1}{1}$
 $= \frac{5}{2}$
 $= 2\frac{1}{2}$

Try these

Calculate each of the following.

1 $\frac{3}{8} \div \frac{3}{4}$
 $= \frac{3}{8} \times \frac{\dots}{3}$
 $= \frac{\dots}{\dots} \times \frac{\dots}{\dots}$
 $= \frac{\dots}{\dots}$

2 $\frac{3}{5} \div \frac{5}{6}$
 $= \frac{3}{5} \times \frac{\dots}{\dots}$
 $= \frac{\dots}{\dots}$

3 $\frac{1}{8} \div \frac{1}{2}$
 $= \frac{\dots}{\dots} \times \frac{\dots}{\dots}$
 $= \frac{\dots}{\dots} \times \frac{\dots}{\dots}$
 $= \frac{\dots}{\dots}$

(continued)

4 $\frac{4}{9} \div \frac{1}{3}$

=

=

=

5 $1\frac{1}{4} \div \frac{3}{8}$

=

=

=

=

=

6 $\frac{1}{4} \div 1\frac{1}{2}$

=

=

=

=

7 $1\frac{3}{5} \div \frac{4}{5}$

=

=

=

=

=

8 $\frac{2}{3} \div 1\frac{5}{7}$

=

=

=

=

=

9 $2\frac{3}{4} \div \frac{1}{6}$

=

=

=

=

=

SKILLSHEET

Order of operations

If an expression contains more than one operation, the calculations must be performed in the following order:

- 1 brackets
- 2 multiplication and division (from left to right)
- 3 addition and subtraction (from left to right).

WORKED EXAMPLE

Find the value of each of the following, using the order of operations rules.

a $12 + 2 + 4 \times 5$ **b** $15 + (6 + 4) \times 7 + 10$

THINK

- a**
- 1 Write the question.
 - 2 There are three operations to be performed: division, addition and multiplication. According to the order of operations rules, multiplication and division must be done before the addition, from left to right. So, perform the division first, followed by the multiplication and finally the addition.
- b**
- 1 Write the question.
 - 2 The calculation in the brackets must be done first, followed by multiplication and division. These are done in order of appearance (that is, from left to right), so in this case multiplication must be done before division. Addition is done last.

WRITE

a $12 + 2 + 4 \times 5$
 $= 6 + 4 \times 5$
 $= 6 + 20$
 $= 26$

b $15 + (6 + 4) \times 7 + 10$
 $= 15 + 10 \times 7 + 10$
 $= 15 + 70 + 10$
 $= 15 + 7$
 $= 22$

Try these

Find the value of each of the following, using the order of operations rules.

1 $23 - (15 - 6) + 3$
 $= 23 - _ + 3$
 $= 23 - _$
 $= _$

3 $56 - 49 + 7$
 $= 56 - _$
 $= _$

5 $14 - (16 - 5) + 11$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

2 $40 - 8 \times 4 + 2$
 $= 40 - _ + 2$
 $= _ + 2$
 $= _$

4 $12 + (54 + 9) + 3$
 $= 12 + _ + _$
 $= _ + _$
 $= _$

6 $18 + 6 \times 8 + 8 \times 3$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $= \dots\dots\dots$

(continued)

SKILLSHEET

Writing equivalent algebraic fractions with the lowest common denominator (LCD)

To convert algebraic fractions to equivalent fractions with the lowest common denominator (LCD), follow these steps.

- Step 1** Find the LCD. (If the denominators of the fractions are binomial expressions, the LCD is the product of all different expressions.)
- Step 2** To write the equivalent fraction with the LCD, multiply both the numerator and denominator by the factor(s) that is lacking in the denominator of that fraction, as compared to the LCD.

WORKED EXAMPLE

Write the following fractions as equivalent fractions with the lowest common denominator.

a $\frac{3x+1}{4}$ and $\frac{5x-2}{6}$ **b** $\frac{3}{5x}$ and $\frac{1}{20x}$ **c** $\frac{3x}{x+2}$ and $\frac{7}{x+5}$

THINK

- a**
- The LCM of 4 and 6 is 12, so LCD is 12.
 - To change to the LCD of 12, the denominator of the first fraction needs to be multiplied by 3. So to write the equivalent fraction, multiply both the numerator and the denominator of the first fraction by 3 and simplify.
 - To change to the LCD of 12, the denominator of the second fraction needs to be multiplied by 2. So to write the equivalent fraction, multiply both the numerator and the denominator of the second fraction by 2 and simplify.
- b**
- Consider the two denominators. The lowest common multiple of 5 and 20 is 20. Furthermore, both denominators contain the same pronumeral x . So the LCD is $20x$.
 - To change the first fraction to an equivalent fraction with a denominator of $20x$, multiply both the numerator and the denominator by 4 and simplify. The second fraction does not need to be changed, as its denominator is already $20x$.
- c**
- The LCD in this case is the product of the denominators of the two fractions.
 - Compared to the LCD, the denominator of the first fraction is lacking the factor $(x+5)$. So we need to multiply both the numerator and the denominator by $(x+5)$.
 - Compared to the LCD, the denominator of the second fraction is lacking the factor $(x+2)$. So we need to multiply both the numerator and the denominator by $(x+2)$.

WRITE

a LCD = 12

$$\frac{3x+1}{4} = \frac{3 \times (3x+1)}{3 \times 4}$$

$$= \frac{3(3x+1)}{12}$$

$$\frac{5x-2}{6} = \frac{2 \times (5x-2)}{2 \times 6}$$

$$= \frac{2(5x-2)}{12}$$

b LCD = $20x$

$$\frac{3}{5x} = \frac{4 \times 3}{4 \times 5x}$$

$$= \frac{12}{20x}$$

c LCD = $(x+2)(x+5)$

$$\frac{3x}{x+2} = \frac{3x(x+5)}{(x+2)(x+5)}$$

$$\frac{7}{x+5} = \frac{7(x+2)}{(x+2)(x+5)}$$

(continued)

Try these

Write the following fractions as equivalent fractions with the lowest common denominator.

1 $\frac{3x}{8}$ and $\frac{x}{6}$ LCD = 24

$$= \frac{3 \times 3x}{\quad \times 8} \text{ and } \frac{\quad \times x}{\quad \times 6}$$

$$= \frac{\quad}{\quad} \text{ and } \frac{\quad}{\quad}$$

3 $\frac{4x+1}{6}$ and $\frac{3x-2}{9}$ LCD =

$$= \frac{\quad \times \quad}{\quad \times \quad} \text{ and } \frac{\quad \times \quad}{\quad \times \quad}$$

$$= \frac{\quad}{\quad} \text{ and } \frac{\quad}{\quad}$$

5 $\frac{2}{8x}$ and $\frac{3}{10x}$ LCD =

$$= \frac{\quad \times \quad}{\quad \times \quad} \text{ and } \frac{\quad \times \quad}{\quad \times \quad}$$

$$= \frac{\quad}{\quad} \text{ and } \frac{\quad}{\quad}$$

2 $\frac{x+3}{3}$ and $\frac{2x-1}{5}$ LCD =

$$= \frac{\quad \times (x+3)}{5 \times 3} \text{ and } \frac{\quad \times (2x-1)}{\quad \times 5}$$

$$= \frac{\quad}{\quad} \text{ and } \frac{\quad}{\quad}$$

4 $\frac{5}{3x}$ and $\frac{1}{6x}$ LCD = 6x

$$= \frac{2 \times \quad}{2 \times \quad} \text{ and } \frac{1 \times \quad}{1 \times \quad}$$

$$= \frac{\quad}{\quad} \text{ and } \frac{\quad}{\quad}$$

6 $\frac{2}{13x}$ and $\frac{4}{x}$ LCD =

$$= \frac{\quad \times \quad}{\quad \times \quad} \text{ and } \frac{\quad \times \quad}{\quad \times \quad}$$

$$= \frac{\quad}{\quad} \text{ and } \frac{\quad}{\quad}$$

7 $\frac{3}{x-1}$ and $\frac{5x}{x-5}$ LCD = $(x-1)(x-5)$

$$= \frac{(x-5) \times 3}{(x-5)(x-1)} \text{ and } \frac{\quad \times 5x}{\quad \times \quad}$$

$$= \frac{\quad}{\quad} \text{ and } \frac{\quad}{\quad}$$

9 $\frac{x+7}{x+1}$ and $\frac{x-2}{x+2}$ LCD =

$$= \frac{\quad \times \quad}{\quad \times \quad} \text{ and } \frac{\quad \times \quad}{\quad \times \quad}$$

$$= \frac{\quad}{\quad} \text{ and } \frac{\quad}{\quad}$$

8 $\frac{6x}{x+7}$ and $\frac{2x}{x+2}$ LCD =

$$= \frac{\quad \times \quad}{\quad \times \quad} \text{ and } \frac{\quad \times \quad}{\quad \times \quad}$$

$$= \frac{\quad}{\quad} \text{ and } \frac{\quad}{\quad}$$

10 $\frac{3x-1}{2x+4}$ and $\frac{2x+5}{3x-2}$ LCD =

$$= \frac{\quad \times \quad}{\quad \times \quad} \text{ and } \frac{\quad \times \quad}{\quad \times \quad}$$

$$= \frac{\quad}{\quad} \text{ and } \frac{\quad}{\quad}$$

SKILLSHEET

Simplification of algebraic fractions

To simplify a fraction, divide both numerator and denominator by the highest common factor (HCF). Note that the common factor could be a pronumeral or an algebraic expression with two (or more) terms.

WORKED EXAMPLE

Write each of the following fractions in simplest form.

a $\frac{6(a+1)}{9}$ b $\frac{8x}{12x(x-3)}$ c $\frac{4(x+2)}{5(x+2)}$

THINK

- a The highest common factor of 6 and 9 is 3. So divide both numerator and denominator by 3.
- b The highest common factor of 8 and 12 is 4. Also both the numerator and the denominator contain x , so divide them both by $4x$.
- c The highest common factor in this case is $(x+2)$. Divide both numerator and denominator by this factor.

WRITE

a $\frac{6(a+1)}{9} = \frac{2(a+1)}{3}$

b $\frac{8x}{12x(x-3)} = \frac{2}{3(x-3)}$

c $\frac{4(x+2)}{5(x+2)} = \frac{4}{5}$

Try these

Write each of the following fractions in simplest form.

1 $\frac{2(a+1)}{4} = \frac{\dots}{\dots}$

2 $\frac{3(b+4)}{12} = \frac{\dots}{\dots}$

3 $\frac{8(x-4)}{4} = \frac{\dots}{\dots}$

4 $\frac{12}{2(y+3)} = \frac{\dots}{\dots}$

5 $\frac{-y}{54(x-7)} = \frac{\dots}{\dots}$

6 $\frac{2a}{6a(a-1)} = \frac{\dots}{\dots}$

7 $\frac{x(x-3)}{x} = \frac{\dots}{\dots}$

8 $\frac{8x}{10x(x+4)} = \frac{\dots}{\dots}$

9 $\frac{15y}{3y(y-2)} = \frac{\dots}{\dots}$

10 $\frac{a(a-2)}{a(a+3)} = \frac{\dots}{\dots}$

11 $\frac{8x(x-1)}{2x(3+x)} = \frac{\dots}{\dots}$

12 $\frac{24(a+3)}{16(a+3)} = \frac{\dots}{\dots}$

13 $\frac{18(m+1)}{27(m+1)} = \frac{\dots}{\dots}$

14 $\frac{(x+3)(x-2)}{(x-2)(x-7)} = \frac{\dots}{\dots}$

15 $\frac{(x-4)(x+4)}{(x+4)(x-1)} = \frac{\dots}{\dots}$