Irrational numbers

1 State which of the following are rational or irrational:

   a $\sqrt{25}$ b $\sqrt{361}$ c $\sqrt{33}$ d $\sqrt[3]{64}$

   e $\sqrt[3]{47}$ f $\sqrt[3]{125}$ g $\sqrt[3]{1000}$ h $\sqrt[3]{1729}$

2 Evaluate the following, rounding your answers to two decimal places where necessary:

   a $\sqrt{25}$ b $\sqrt[3]{64}$ c $\sqrt{33}$ d $\sqrt[3]{26}$

   e $\sqrt{121}$ f $\sqrt[3]{216}$ g $\sqrt{83}$ h $\sqrt[3]{52}$

3 Is $\sqrt{15}$ an integer?

4 Determine whether each of the following numbers is a surd:

   a $\sqrt{2}$ b $\sqrt{1}$ c $\sqrt{50}$ d $\sqrt[3]{27}$

   e $\sqrt[3]{9}$ f $\sqrt[3]{8}$ g $\sqrt{80}$ h $\sqrt{144}$

Simplifying surds

7 Are the following expressions written in their simplest surd form?:

   a $\sqrt{17}$ b $\sqrt{14}$ c $\sqrt{50}$ d $5\sqrt{50}$

   e $7\sqrt{125}$ f $11\sqrt{21}$ g $\sqrt{63}$ h $\sqrt{112}$

8 Simplify the following:

   a $\sqrt{180}$ b $\sqrt{125}$ c $3\sqrt{54}$ d $7\sqrt{32}$

   e $6\sqrt{100}.$ f $\sqrt{25 \times 6}$ g $\dfrac{1}{2}\sqrt[3]{8 \times 6}.$ h $\sqrt[3]{24 \times 9}.$

9 Simplify the following:

   a $\dfrac{1}{\sqrt{25}}$ b $\dfrac{16}{\sqrt{16}}$ c $\dfrac{\sqrt{64}}{8}$ d $\dfrac{36}{\sqrt[3]{216}}$

Equivalent surd expressions

1 State whether the following equations are true or false:

a $\sqrt{11} + \sqrt{5} = \sqrt{16}$

b $\sqrt{4} + \sqrt{4} = 4$

c $5 - \sqrt{5} = \sqrt{5}$

d $\sqrt{1} + \sqrt{4} = \sqrt{25}$

e $\sqrt{16} + \sqrt{9} = 7$

f $\sqrt{32} + \sqrt{2} = 5\sqrt{2}$

g $\sqrt{25} - \sqrt{9} = \sqrt{16}$

h $\sqrt{100} - \sqrt{64} = 2$

3 Simplify the following:

a $10\sqrt{2} + 14\sqrt{2}$

b $\sqrt{6} + 14\sqrt{6}$

c $\sqrt{10} - 20\sqrt{10}$

d $12\sqrt{2} - 3\sqrt{2}$

e $4\sqrt{3} - 12\sqrt{3}$

f $18\sqrt{5} - 15\sqrt{5}$

g $6\sqrt{7} - 8\sqrt{7}$

h $6\sqrt{2} + 13\sqrt{2}$

4 Simplify the following:

a $5\sqrt{5} + 9\sqrt{5} - 7\sqrt{5}$

b $10\sqrt{6} - 3\sqrt{6} + 20\sqrt{6}$

c $7\sqrt{6} + 18\sqrt{6} - 9\sqrt{6}$

d $19\sqrt{6} - 4\sqrt{6} - 2\sqrt{6}$

e $8\sqrt{5} - 9\sqrt{5} - 14\sqrt{5}$

f $18\sqrt{10} - 9\sqrt{10} - 7\sqrt{10}$

5 Simplify the following:

a $8\sqrt{2} + 2\sqrt{11} + 2\sqrt{2} + 4\sqrt{11}$

b $10\sqrt{2} + 5\sqrt{3} + 4\sqrt{2} - 7\sqrt{3}$

c $6\sqrt{7} + 7\sqrt{5} - 3\sqrt{7} + 8\sqrt{5}$

d $18\sqrt{7} - 9\sqrt{3} + 20\sqrt{7} + 11\sqrt{3}$

e $20\sqrt{7} + 7\sqrt{11} + \sqrt{7} + 25\sqrt{11}$

f $7\sqrt{11} + 28\sqrt{5} - 25\sqrt{11} - 11\sqrt{5}$

g $20\sqrt{11} + 26\sqrt{5} + 22\sqrt{5} - 15\sqrt{11}$

6 Simplify the following:

a $\sqrt{3} + \sqrt{48}$

b $\sqrt{180} + \sqrt{5}$

c $\sqrt{45} + \sqrt{80}$

d $\sqrt{245} - \sqrt{5}$

e $\sqrt{48} - \sqrt{12}$

f $3\sqrt{27} + 2\sqrt{12}$

g $3\sqrt{192} - 2\sqrt{108}$

h $\frac{3\sqrt{2}}{2} + \frac{\sqrt{2}}{6}$.

Multiplication

1 Are the following statements true or false?

  a $\sqrt{8^2} = \left(\sqrt{8}\right)^2$ **b** $\sqrt{5^2} = \left(\sqrt{5 \times 5}\right)^2$

  c $\sqrt{2^2} = \sqrt{2} + 2$ **d** $\sqrt{8^2} = \sqrt{16} \times \sqrt{4}$

2 Complete the following statements by following the example:

$$\sqrt{9 \times 4} = \sqrt{9} \times \sqrt{4} = 3 \times 2 = 6$$

  a $\sqrt{36 \times 25} = \sqrt{\square} \times \sqrt{\square} = \square \times \square = \square$ **b** $\sqrt{9 \times 11} = \sqrt{\square} \times \sqrt{\square} = \square\sqrt{\square}$

  c $\sqrt{49 \times 5} = \sqrt{\square} \times \sqrt{\square} = \square\sqrt{\square}$ **d** $\sqrt{64 \times 3} = \sqrt{\square} \times \sqrt{\square} = \square\sqrt{\square}$

3 Simplify the following:

  a $\sqrt{75}$ **b** $\sqrt{19} \times \sqrt{17}$ **c** $\left(6\sqrt{8}\right)^2$ **d** $\left(6\sqrt{3}\right)^2$

4 Simplify the following:

  a $\sqrt{5} \times \sqrt{7}$ **b** $8 \times 10\sqrt{5}$ **c** $\sqrt{7} \times \sqrt{3} \times \sqrt{11}$ **d** $\sqrt{55} \times \sqrt{11}$

  e $4\sqrt{11} \times 5$ **f** $2\sqrt{5} \times 15\sqrt{11}$ **g** $7\sqrt{22} \times \sqrt{2}$ **h** $\sqrt{180} \times \sqrt{48}$

  i $8\sqrt{15} \times 8\sqrt{5}$ **j** $5\sqrt{17} \times 8\sqrt{3}$ **k** $17\sqrt{35} \times 4\sqrt{5}$ **l** $8\sqrt{51} \times 9\sqrt{3}$

5 Simplify the following:

  a $\sqrt{11}\left(\sqrt{7} + 4\right)$ **b** $\sqrt{7}\left(3 + \sqrt{3}\right)$ **c** $\sqrt{2}\left(\sqrt{11} - 6\right)$ **d** $3\sqrt{3}\left(\sqrt{13} - 5\right)$

  e $\sqrt{3}\left(\sqrt{11} + \sqrt{13}\right)$ **f** $4\sqrt{7}\left(\sqrt{2} - \sqrt{11}\right)$ **g** $3\sqrt{5}\left(\sqrt{55} + \sqrt{11}\right)$ **h** $8\sqrt{2}\left(\sqrt{3} - 3\sqrt{7}\right)$

6 Simplify the following:

  a $\sqrt{15} \div \sqrt{5}$ **b** $\sqrt{55} \div \sqrt{5}$ **c** $\sqrt{51} \div \sqrt{17}$ **d** $\sqrt{21} \div \sqrt{3}$

  e $\sqrt{91} \div \sqrt{7}$ **f** $40\sqrt{7} \div 8$ **g** $10\sqrt{55} \div \sqrt{11}$ **h** $15\sqrt{22} \div \sqrt{11}$

  i $4\sqrt{35} \div 2\sqrt{5}$ **j** $\sqrt{27} \div \sqrt{3}$ **k** $3\sqrt{20} \div \sqrt{5}$ **l** $5\sqrt{8} \div \sqrt{2}$

  m $40\sqrt{96} \div 10\sqrt{6}$ **n** $50\sqrt{24} \div 10\sqrt{6}$ **o** $\sqrt{25} \div \sqrt{81}$ **p** $\sqrt{162} \div \sqrt{8}$

7 Simplify the following:

  a $\sqrt{\dfrac{28}{7}}$ **b** $\sqrt{\dfrac{9}{45}}$ **c** $\sqrt{\dfrac{64}{4}}$ **d** $\sqrt{\dfrac{48}{144}}$

  e $\dfrac{\sqrt{12}}{\sqrt{36}}$ **f** $\dfrac{\sqrt{56}}{\sqrt{14}}$ **g** $\dfrac{\sqrt{36}}{\sqrt{81}}$ **h** $\dfrac{\sqrt{72}}{\sqrt{32}}$

Binomial products

1 Expand the following brackets:

a $\left(5-\sqrt{13}\right)\left(5+\sqrt{13}\right)$

b $\left(\sqrt{11}-11\right)\left(\sqrt{11}+11\right)$

c $\left(8\sqrt{5}-6\right)\left(8\sqrt{5}+6\right)$

d $\left(\sqrt{7}+\sqrt{5}\right)\left(\sqrt{7}-\sqrt{5}\right)$

e $\left(7\sqrt{11}-\sqrt{7}\right)\left(7\sqrt{11}+\sqrt{7}\right)$

f $\left(5\sqrt{3}+3\sqrt{5}\right)\left(5\sqrt{3}-3\sqrt{5}\right)$

2 Expand the following brackets:

a $\left(\sqrt{3}-13\right)^{2}$

b $\left(\sqrt{7}+\sqrt{3}\right)^{2}$

c $\left(3\sqrt{3}+8\right)^{2}$

d $\left(4\sqrt{2}-\sqrt{13}\right)^{2}$

e $\left(3\sqrt{2}+4\sqrt{13}\right)^{2}$

f $\left(5\sqrt{2}-\sqrt{32}\right)^{2}$

3 Expand the following brackets:

a $\left(\sqrt{11}+10\right)\left(\sqrt{3}-9\right)$

b $\left(\sqrt{11}-\sqrt{13}\right)\left(\sqrt{7}-\sqrt{2}\right)$

c $\left(11\sqrt{2}-\sqrt{7}\right)\left(13\sqrt{3}-\sqrt{5}\right)$

d $\left(4\sqrt{2}-\sqrt{7}\right)\left(3\sqrt{3}+\sqrt{8}\right)$

e $\left(17\sqrt{3}-8\sqrt{8}\right)\left(\sqrt{24}-\sqrt{5}\right)$

f $\left(\sqrt{90}-7\sqrt{7}\right)\left(\sqrt{72}-6\sqrt{5}\right)$

Equivalent expressions

4 Consider the following equation:

$$\left(3\sqrt{35}-2\sqrt{7}\right)^{2}=x-y\sqrt{5}$$

a Expand and simplify the left hand side of the equation.

b State the values of x and y.

5 Consider the following equation:

$$\left(\sqrt{m}+n\right)^{2}=16+6\sqrt{7}$$

a Expand and simplify the left hand side of the equation.

b State the values of m and n.

Conjugates

1 Find the conjugate of each of the following:

a $5+\sqrt{5}$ **b** $6-\sqrt{v}$ **c** $\sqrt{6}+\sqrt{3}$ **d** $\sqrt{n}-\sqrt{5}$

e $\sqrt{5}+y$ **f** $2-9\sqrt{3}$ **g** $3+6\sqrt{8}$ **h** $7-2\sqrt{9}$

i $3\sqrt{7}+4\sqrt{3}$ **j** $9\sqrt{2}-3\sqrt{r}$ **k** $4\sqrt{s}+8\sqrt{t}$ **l** $2\sqrt{w}-9\sqrt{x}$

Rationalise monomial denominators

3 Evaluate the following:

a $(\sqrt{3}+\sqrt{10})\times(\sqrt{3}-\sqrt{10})$ **b** $(\sqrt{3}+\sqrt{10})\times(\sqrt{3}+\sqrt{10})$

c $(-(\sqrt{3}+5))\times\sqrt{3}$ **d** $(\sqrt{5}+\sqrt{12})\times(\sqrt{5}-\sqrt{12})$

e $(-2\sqrt{5}-\sqrt{12})\times\sqrt{12}$ **f** $(2\sqrt{5}-\sqrt{12})\times(5\sqrt{5}-\sqrt{12})$

4 Rationalise the denominator of the given expressions. Express your answer in simplest surd form:

a $\dfrac{1}{\sqrt{7}}$ **b** $\dfrac{2}{\sqrt{6}}$ **c** $\dfrac{3}{\sqrt{13}}$ **d** $\dfrac{\sqrt{13}}{\sqrt{2}}$

e $\dfrac{\sqrt{21}}{\sqrt{7}}$ **f** $\dfrac{\sqrt{5}}{\sqrt{30}}$ **g** $\dfrac{4\sqrt{30}}{\sqrt{6}}$ **h** $\dfrac{11\sqrt{7}}{13\sqrt{3}}$

i $-\dfrac{6\sqrt{22}}{5\sqrt{11}}$ **j** $-\dfrac{14\sqrt{10}}{6\sqrt{5}}$ **k** $\dfrac{8\sqrt{32}}{20\sqrt{7}}$ **l** $-\dfrac{15\sqrt{18}}{19\sqrt{8}}$

5 Rationalise the denominator of the given expressions. Express your answer in simplest surd form:

a $\dfrac{\sqrt{5}+9}{\sqrt{7}}$ **b** $\dfrac{\sqrt{5}+3}{\sqrt{5}}$ **c** $\dfrac{4-\sqrt{12}}{\sqrt{10}}$ **d** $\dfrac{\sqrt{7}-3}{\sqrt{3}}$

e $\dfrac{10\sqrt{2}+7}{\sqrt{11}}$ **f** $\dfrac{15-2\sqrt{3}}{\sqrt{2}}$ **g** $\dfrac{3\sqrt{5}+12}{\sqrt{20}}$ **h** $\dfrac{6\sqrt{14}-11}{\sqrt{5}}$

i $\dfrac{\sqrt{39}+\sqrt{6}}{\sqrt{3}}$ **j** $\dfrac{\sqrt{7}-\sqrt{13}}{\sqrt{15}}$ **k** $\dfrac{6\sqrt{2}+10\sqrt{10}}{\sqrt{12}}$ **l** $\dfrac{-20\sqrt{5}+7\sqrt{11}}{\sqrt{6}}$

6 Rationalise the denominator of each fraction and then find the sum. Express your answer in simplest surd form:

a $\dfrac{1}{\sqrt{3}}+\dfrac{3}{\sqrt{3}}$ **b** $\dfrac{2}{\sqrt{7}}+\dfrac{1}{\sqrt{7}}$ **c** $\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{3}}$ **d** $\dfrac{1}{\sqrt{5}}-\dfrac{1}{\sqrt{10}}$

e $\dfrac{2}{\sqrt{14}}+\dfrac{4}{\sqrt{7}}$ **f** $\dfrac{6}{\sqrt{3}}-\dfrac{3}{\sqrt{6}}$ **g** $\dfrac{3}{\sqrt{11}}-\dfrac{5}{\sqrt{22}}$ **h** $\dfrac{\sqrt{2}}{\sqrt{7}}+\dfrac{\sqrt{7}}{\sqrt{2}}$

Index laws

1 Write the following expressions in simplest index form:

a $2^{12} \times 2^9$ b $2^8 \times 11^8$ c $11^{12} \div 11^8$ d $21^5 \div 3^5$

e $\left(5^{12}\right)^4$ f $15^{17} \div 15^8 \div 15^5$ g $\left(23^8\right)^9 \times 23^7$ h $\dfrac{\left(17^5\right)^8}{17^{32}}$

i $\dfrac{19^9 \times 19^4}{19^8}$ j $\dfrac{12^6}{12^4} \times 12^5$ k $\dfrac{\left(13^5\right)^2 \times 13^3}{13^5}$ l $\dfrac{\left(15^9\right)^5 \times 15^7}{15^{25}}$

3 Evaluate the following expressions:

a $6^5 \times 6^3$ b $7^3 \times 3^3$ c $4^8 \div 4^3$ d $\left(5^4\right)^2$

e $35^5 \div 5^5$ f $2^4 \times 4^4$ g $11^{18} \div 11^9 \div 11^7$ h $\left(3^3\right)^2$

i $\dfrac{6^5 \times 6^9}{6^{12}}$ j $7^{27} \div 7^{30} \div 7^3$ k $\dfrac{12^{10} \times 12^4}{12^{11}}$ l $\dfrac{\left(6^8\right)^6}{6^{46}}$

Negative bases

4 Write the following expressions in simplest index form:

a $(-11)^{10} \times (-11)^3$ b $(-7)^8 \times 3^8$

c $(-5)^2 \times 3^2$ d $(-3)^{12} \div (-3)^5$

e $(-12)^{20} \div (-12)^{19}$ f $(-30)^{50} \div (-30)^{47}$

g $(-48)^3 \div (-6)^3$ h $(-33)^{11} \div (-3)^{11}$

i $(-35)^5 \div 5^5$ j $(-42)^2 \div 7^2$

6 Evaluate the following expressions:

a $(-4)^{11} \div (-4)^7$ b $(-2)^3 \times (-2)^3$ c $(-3)^3 \times (-3)^2$ d $4^3 \times (-5)^3$

e $(-3)^8 \div (-3)^5$ f $15^5 \div (-3)^5$ g $2^3 \times (-3)^3$ h $(-14)^{11} \div 2^{11}$

i $(-7)^2 \times 5^2$ j $(-9)^4 \times (-3)^4$ k $(-100)^6 \div 50^6$ l $60^3 \div (-3)^3$

Fractional bases

7 Write the following in simplest index form:

a $\left(\dfrac{1}{3}\right)^4$ b $\left(\dfrac{3}{8}\right)^3$ c $\left(\dfrac{4}{16}\right)^8$ d $\left(\dfrac{15}{6}\right)^2$

e $\left(\dfrac{10}{33}\right)^5$ f $\left(\dfrac{2}{35}\right)^6$ g $\left(\dfrac{5}{18}\right)^3$ h $\left(\dfrac{29}{41}\right)^7$

i $\left(\dfrac{11}{13}\right)^9$ j $\left(\dfrac{20}{3}\right)^2$ k $\left(\dfrac{17}{4}\right)^4$ l $\left(\dfrac{31}{50}\right)^5$

2 Evaluate the following expressions:

a 6^0
b $3^3 \times 3^0$
c $7^2 \div 7^2$
d $(6 \times 19)^0$

e $(-3)^0$
f -4^0
g $\left(\dfrac{2}{3}\right)^0$
h $7\,(14 \times 18)^0$

Negative indices

4 Consider the following expressions:

i   Identify the base.
ii   Identify the power.

a 10^{-7}
b 2^{-4}
c 13^{-10}
d $(-5)^{-8}$

5 Complete the following tables:

a

2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}
32	16					

b

10^5	10^4	10^3	10^2	10^1	10^0	10^{-1}
100 000	10 000					

6 Express the following expressions with a positive index:

a 6^{-10}
b 73^{-14}
c $(-9)^{-7}$
d 9^{-1}

e 17^{-6}
f 55^{-1}
g $(-12)^{-8}$
h -45^{-5}

i -8^{-11}
j $(-20)^{-3}$
k 7^{-6}
l $(-5)^{-1}$

7 Express the following expressions with a negative index:

a $\dfrac{1}{3}$
b $\dfrac{1}{37}$
c $\dfrac{1}{5}$
d $\dfrac{1}{4^7}$

e $\dfrac{1}{-15^3}$
f $\dfrac{1}{10^5}$
g $\dfrac{1}{(-24)^{10}}$
h $\dfrac{1}{25^3}$

i $\dfrac{1}{13^{11}}$
j $\dfrac{1}{7^8}$
k $\dfrac{1}{16^{12}}$
l $\dfrac{1}{(-45)^7}$

Scientific notation

1 Express the following numbers as basic numerals:

a 2×10^7 b 4.13×10^4 c 9×10^{-3} d 8.97×10^5

e 5.03×10^5 f 3.014×10^3 g 8×10^6 h 1.3008×10^7

2 Express the following numbers in scientific notation:

a 45 b 128 c 2000 d 884 000

e 84 626 000 f 23 456 g 6.14 h 9 700 000 000

3 Express the following numbers in scientific notation:

a 0.6 b 0.002 c 0.000 74 d $\dfrac{1}{10\,000}$

e 0.000 347 f $\dfrac{3}{100\,000}$ g 0.000 059 32 h $\dfrac{92}{1000}$

4 Express the following numbers as decimals:

a 8.29×10^0 b 7×10^{-3} c 5.28×10^{-2} d 4.7×10^{-6}

e 2.13×10^{-7} f 3.62×10^{-4} g 6.4×10^{-5} h 9.72×10^{-2}

Scientific notation and calculators

1 Write the output on your calculator when you enter the following:

a 1.6×10^6 b 1.8×10^{-4} c 2.7×10^{-2} d 3.32×10^5

e 7.45×10^{-7} f 6.2×10^3 g 4.35×10^8 h 8.16×10^{-5}

2 Write the following in scientific notation:

a 254 b 0.0019 c 7870 d 0.000 314

e 0.000 009 3 f 12 345 g 5 600 000 000 h 0.000 017

3 Find the value of the following in scientific notation:

a $82.97 \times 7.1 \times 10^4$ b $81\,000^2 \times 4\,100\,000$

c $3.808 \times 10^{15} \div (5.6 \times 10^8)$ d $(8.3 \times 10^{10}) \times (7.9 \times 10^6)$

Positive fractional indices

1 Write the following in surd form:

a $12^{\frac{1}{2}}$
b $15^{\frac{1}{2}}$
c $7^{\frac{1}{3}}$
d $35^{\frac{1}{4}}$

e $21^{\frac{1}{3}}$
f $31^{\frac{1}{6}}$
g $10^{\frac{1}{5}}$
h $24^{\frac{1}{2}}$

2 For each of the following expressions:

i Write in surd form.
ii Evaluate the expression.

a $36^{\frac{1}{2}}$
b $64^{\frac{1}{2}}$
c $81^{\frac{1}{2}}$
d $25^{\frac{1}{2}}$

e $49^{\frac{1}{2}}$
f $100^{\frac{1}{2}}$
g $121^{\frac{1}{2}}$
h $256^{\frac{1}{2}}$

7 Evaluate:

a $1000^{\frac{1}{3}}$
b $64^{\frac{1}{3}}$
c $256^{\frac{1}{4}}$
d $32^{\frac{1}{5}}$

e $64^{\frac{1}{6}}$
f $1^{\frac{1}{6}}$
g $27^{\frac{1}{3}}$
h $16^{\frac{1}{4}}$

i $64^{\frac{1}{3}}$
j $125^{\frac{1}{3}}$
k $128^{\frac{1}{7}}$
l $216^{\frac{1}{3}}$

8 Write each of the following with a fractional index:

a $\sqrt[3]{5}$
b $\sqrt[8]{23}$
c $\sqrt[6]{6}$
d $\sqrt[3]{10}$

e $\sqrt[4]{15}$
f $\sqrt[7]{18}$
g $\sqrt[5]{31}$
h $\sqrt[8]{57}$

9 Write each of the following with a fractional index:

a $\left(\sqrt[3]{19}\right)^{7}$
b $\left(\sqrt[3]{7}\right)^{8}$
c $\left(\sqrt[5]{12}\right)^{6}$
d $\left(\sqrt[6]{45}\right)^{5}$

e $\sqrt[4]{5^{3}}$
f $\sqrt[7]{13^{4}}$
g $\sqrt[9]{21^{5}}$
h $\sqrt[5]{11^{12}}$

10 Write the following in surd form:

a $19^{\frac{5}{3}}$
b $7^{\frac{3}{2}}$
c $11^{\frac{6}{5}}$
d $24^{\frac{8}{7}}$

Negative fractional indices

13 For each of the following expressions:

i Write in surd form.
ii Evaluate the expression.

a $49^{-\frac{1}{2}}$
b $121^{-\frac{1}{2}}$
c $81^{-\frac{1}{4}}$
d $8^{-\frac{1}{3}}$

e $64^{-\frac{1}{3}}$
f $128^{-\frac{1}{7}}$
g $125^{-\frac{1}{3}}$
h $256^{-\frac{1}{4}}$

1 Solve for x in each of the following.

   a **i** $5^x = 25$ **ii** $2^x = 8$

   b **i** $2^x = \dfrac{1}{4}$ **ii** $5^x = \dfrac{1}{125}$

   c **i** $9^x = 27$ **ii** $4^x = 8$

2 Solve for x in each of the following.

   a $3^x = 27$ **b** $2^x = 8$ **c** $6^x = 36$

   d $9^x = 81$ **e** $5^x = 125$ **f** $4^x = 64$

   g $3^x = 81$ **h** $6^x = 216$ **i** $5^x = 625$

   j $2^x = 32$ **k** $10^x = 10000$ **l** $7^x = 343$

3 Solve for x in each of the following.

   a $7^x = \dfrac{1}{49}$ **b** $9^x = \dfrac{1}{81}$ **c** $11^x = \dfrac{1}{121}$

   d $4^x = \dfrac{1}{256}$ **e** $3^x = \dfrac{1}{243}$ **f** $5^{-x} = \dfrac{1}{125}$

   g $3^{-x} = \dfrac{1}{9}$ **h** $2^{-x} = \dfrac{1}{64}$ **i** $7^{-x} = \dfrac{1}{343}$

4 Solve for x in each of the following.

   a $9^x = 27$ **b** $8^x = 16$ **c** $25^x = 125$

   d $16^x = 64$ **e** $81^x = 9$ **f** $216^x = 6$

   g $32^x = 2$ **h** $10000^x = 10$ **i** $7^{-x} = 49$

   j $4^{-x} = 256$ **k** $16^{-x} = 64$ **l** $25^{-x} = 125$

6 Solve for x in each of the following.

   a $2^{x+1} = 8^x$ **b** $3^{2x+1} = 27^x$ **c** $7^{x+9} = 49^{2x}$

   d $5^{x+3} = 25^{2x}$ **e** $6^{2x+3} = 216^{2x}$ **f** $9^{x+12} = 81^{x+5}$

   g $27^{x+3} = 9^{2x}$ **h** $25^{x+3} = 125^{3x}$ **i** $32^{2x+3} = 128^{2x}$
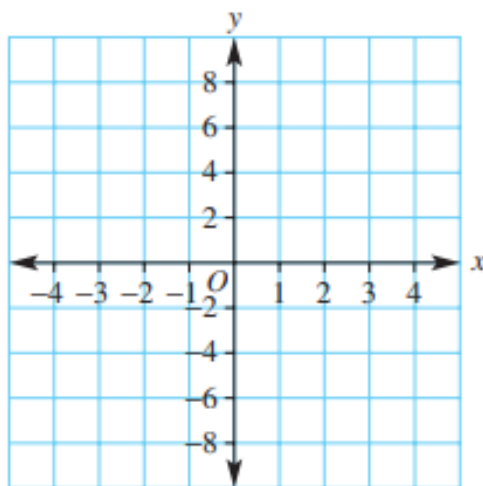
   j $27^{2x+3} = 9^{2x-1}$ **k** $9^{x-1} = 27^{2x-6}$ **l** $49^{2x-3} = 343^{2x-1}$

Complete this table and graph all three relations on the same set of axes before discussing the points below.

x	−3	−2	−1	0	1	2	3
$y_1 = 2^x$	$\dfrac{1}{8}$			1		4	
$y_2 = -2^x$							
$y_3 = 2^{-x}$							

- Discuss the shape of each graph.
- Where does each graph cut the y-axis?
- Do the graphs have x-intercepts? Why not?
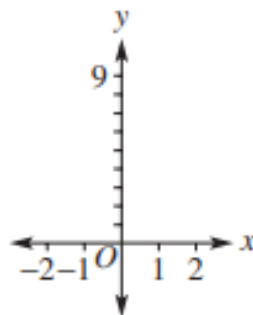- What is the one feature they all have in common?

1 Consider the exponential rule $y = 3^x$.

  a Complete this table.

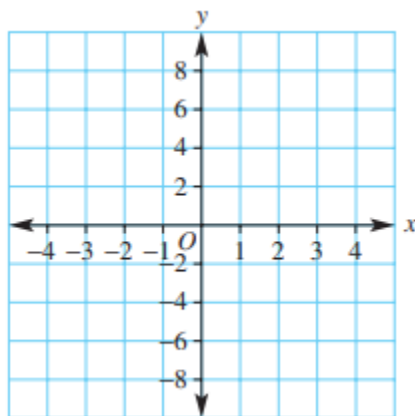| x	−2	−1	0	1	2
y		$\dfrac{1}{3}$	1		

  b Plot the points in the table to form the graph of $y = 3^x$.

1 Sketch the graph of the following on the same set of axes, labelling the y-intercept and the point where $x = 1$.

  a $y = 2^x$ **b** $y = 6^x$

6 a Find the coordinates on the graph of $y = 3^x$, where:

    i $x = 0$ **ii** $x = -1$ **iii** $y = 1$ **iv** $y = 9$

11

Imagine you have an antique car valued at $100 000 and you hope that it will increase in value at 10% p.a. The 10% increase is to be added to the value of the car each year.

- Complete this table.

Year	0	1	2	3
Value ($)	100 000	$100\,000 \times 1.1$ = _____	$100\,000 \times 1.1 \times$ ___ = _____	_____ = _____

BUILDING UNDERSTANDING

1 An antique ring is purchased for $1000 and is expected to grow in value by 5% per year. Round your answers to the nearest cent.

 a Find the increase in value in the first year.
 b Find the value of the ring at the end of the first year.
 c Find the increase in value in the second year.
 d Find the increase in value in the third year.
 e Find the value of the ring at the end of the fifth year.

2 The mass of a limestone 5 kg rock exposed to the weather is decreasing at a rate of 2% per annum.

 a Find the mass of the rock at the end of the first year.
 b State the missing numbers for the mass of the rock (M kg) after t years.
 $$M = 5(1 - \underline{\quad})^t$$
 $$= 5 \times \underline{\quad}^t$$
 c Use your rule to calculate the mass of the rock after 5 years, correct to two decimal places.

3 Decide if the following represent exponential *growth* or exponential *decay*.

 a $A = 1000 \times 1.3^t$ b $A = 350 \times 0.9^t$
 c $P = P_0\left(1 + \dfrac{3}{100}\right)^t$ d $T = T_0\left(1 - \dfrac{7}{100}\right)^t$

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1 Form exponential rules for the following situations.

 a Lara has a necklace that is valued at $6000 and it is expected to increase in value by 12% per annum.
 b A village's initial population of 2000 is decreasing by 8% per year.

3 The value of a house purchased for $500 000 is expected to grow by 10% per year. Let $A be the value of the house after t years.

 a Write the missing number in the rule connecting A and t.
 $$A = 500000 \times \underline{\quad}^t$$
 b Use your rule to find the expected value of the house after the following number of years. Round your answer to the nearest cent.
 i 3 years   ii 10 years   iii 20 years

1 Consider $500 invested at 10% p.a., compounded annually.
    a How much interest is earned in the first year?
    b What is the balance of the account once the first year's interest is added?
    c How much interest is earned in the second year?
    d What is the balance of the account at the end of the second year?
    e Use your calculator to work out $500(1.1)^2$.

2 By considering an investment of $4000 at 5% p.a., compounded annually, calculate the missing values in the table below.

Year	Amount ($)	Interest ($)	New amount ($)
1	4000	200	4200
2	4200		
3			
4			
5			

4 State the missing numbers.
    a $700 invested at 8% p.a., compounded annually for 2 years.
$$A = \boxed{}(1.08)^{\boxed{}}$$
    b $1000 invested at 15% p.a., compounded annually for 6 years.
$$A = 1000\left(\boxed{}\right)^6$$
    c $850 invested at 6% p.a., compounded annually for 4 years.
$$A = 850\left(\boxed{}\right)^{\boxed{}}$$

1 Determine the amount after 4 years if $5000 is compounded annually at 6%. Round to the nearest cent.

2 Determine the amount after 5 years if:
    a $4000 is compounded annually at 5%
    b $8000 is compounded annually at 8.35%
    c $6500 is compounded annually at 16%
    d $6500 is compounded annually at 8%.

3 Determine the amount if $100000 is compounded annually at 6% for:
    a 1 year
    b 2 years
    c 3 years
    d 5 years
    e 10 years
    f 15 years.

4 Calculate the number of periods (n) and the rates of interest (r) offered per period for the following. (Round the interest rate to three decimal places where necessary.)
    a 6% p.a. over 3 years, paid bi-annually
    b 12% p.a. over 5 years, paid monthly
    c 4.5% p.a. over 2 years, paid fortnightly
    d 10.5% p.a. over 3.5 years, paid quarterly

1 Which is better on an investment of $100 for 2 years:

   A simple interest calculated at 20% p.a. or

   B compound interest calculated at 20% p.a. and paid annually?

2 State the values of P, r and n for an investment of $750 at 7.5% p.a., compounded annually for 5 years.

3 State the values of I, P, r and t for an investment of $300 at 3% p.a. simple interest over 300 months.

4 Use the simple interest formula $I = \dfrac{Prt}{100}$ to find the simple interest on an investment of $2000 at 4% p.a. over 3 years.

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1 Find the total amount of the following investments, using technology.

   a $7000 at 4% p.a., compounded annually for 5 years

   b $7000 at 4% p.a., simple interest for 5 years

2 a Find the total amount of the following investments, using technology.

    i $6000 at 6% p.a., compounded annually for 3 years

    ii $6000 at 3% p.a., compounded annually for 5 years

    iii $6000 at 3.4% p.a., compounded annually for 4 years

    iv $6000 at 10% p.a., compounded annually for 2 years

    v $6000 at 5.7% p.a., compounded annually for 5 years

   b Which of the above yields the most interest?

3 a Find the total amount of the following investments, using technology where possible.

    i $6000 at 6% p.a. simple interest for 3 years

    ii $6000 at 3% p.a. simple interest for 6 years

    iii $6000 at 3.4% p.a. simple interest for 7 years

    iv $6000 at 10% p.a. simple interest for 2 years

    v $6000 at 5.7% p.a. simple interest for 5 years

   b Which of the above yields the most interest?

4 a Determine the total simple and compound interest accumulated in the following cases.

    i $4000 at 6% p.a. payable annually for:

      I 1 year   **II** 2 years   **III** 5 years   **IV** 10 years.

    ii $4000 at 6% p.a. payable bi-annually for:

      I 1 year   **II** 2 years   **III** 5 years   **IV** 10 years.