Binomial expansion

1 Expand and simplify:

a $(x+6)(x-12)$

b $(x+2)(x+6)$

c $(v+11)(v+10)$

d $(x-4)(x-7)$

e $(m-5)(m+6)$

f $(10m+9)(m-7)$

g $(3w+5)(7w+6)$

h $(3y-3)(10y+9)$

i $(7y+9)(-4y-4)$

j $3(y+3)(y+5)$

k $8(x-2)(x-6)$

l $-(x+5)(x+6)$

m $-4(y+1)(y+9)$

n $-9(y-7)(y+6)$

o $3(-4x+3)(4x-3)$

p $4(6x-2)(x+2)$

11 Expand and simplify the following expressions:

a $(a+b)^2$

b $(a-b)^2$

c $(x+10)^2$

d $(x^2+1)^2$

e $(m-7)^2$

f $(1-9m)^2$

g $(10x+3)^2$

h $(5x+8y)^2$

Difference of two squares

13 Expand the following expressions:

a $(a+b)(a-b)$

b $(u+5)(u-5)$

c $(x-7)(x+7)$

d $(8-m)(m+8)$

e $(12-p)(12+p)$

f $(7-6p)(7+6p)$

g $(3x-8)(3x+8)$

h $(8y+9)(8y-9)$

i $(-9y-1)(-9y+1)$

j $(-7y-8)(-7y+8)$

k $(9x-2y)(9x+2y)$

l $(0.2x+3)(0.2x-3)$

m $(2x-0.9)(2x+0.9)$

n $(y^2+9)(y^2-9)$

o $\left(v+\frac{1}{3}\right)\left(v-\frac{1}{3}\right)$

p $\left(5x+\frac{1}{6}\right)\left(5x-\frac{1}{6}\right)$

14 Expand and simplify the following expressions:

a $3x(5x-6y)$

b $3(9x-8y)$

c $a^2+(9-a)(9+a)$

d $(t+5)^2$

e $6(8x-9y)$

f $5x(2x-9y)$

Grouping in pairs

1 Factorise:

a $5(a+b)+v(a+b)$

b $x(y-z)-w(y-z)$

c $5y(4w+3x)-z(4w+3x)$

d $2y(2x^2+3z)-(2x^2+3z)$

e $3f(g+h)+(g+h)^2$

f $8y(y-4)+3(4-y)$

g $(2c-d)(c+5d)-3(d-2c)$

h $3x^2(x+4y)-5y(x+4y)$

2 Factorise:

a $8x+xz-16y-2yz$

b $24+3y+8x+xy$

c $7xy+wx+7yz+wz$

d $x^2+2x+5x+10$

e $x^2-3x+8x-24$

f $2mp+6+3p+4m$

g a^3+8a^2+a+8

h $3pq^2-11ypq+3rsq-11yrs$

i $4x+24yz+32xy+3z$

j $2x+18yz+12xy+3z$

Perfect squares

3 Factorise:

a $q^2+2qt+t^2$

b $u^2-2uq+q^2$

c $b^2-2br+r^2$

d x^2+6x+9

e c^2-4c+4

f $64+2e+e^2$

g $49-2p+p^2$

h $16w^2-40w+25$

i $64r^2+48r+9$

j $\frac{1}{4}-3d+9d^2$

4 Factorise:

a x^4+8x^2+16

b a^4-18a^2+81

Difference of two squares

8 Factorise:

a x^2-y^2

b n^2-25

c v^2-1

d $121-v^2$

e $x^2-\frac{1}{4}$

f $x^2-\frac{25}{121}$

g $16-9y^2$

h x^2y^2-49

i $25m^2-49$

j $81x^2-16y^2$

k $7x^2-63$

l $45t^2-20$

Monic quadratic trinomials

1 Factorise:

a $x^2 + 8x + 16$ **b** $x^2 + 16x + 64$

c $x^2 - 4x + 4$ **d** $x^2 - 20x + 100$

e $64 + 16x + x^2$ **f** $36 - 12x + x^2$

2 Complete the following statement: To factorise $x^2 + 9x + 18$, we need to find two numbers whose product is ▮ and whose sum is ▮.

3 Consider the quadratic $x^2 + 11x + 24$. To factorise this quadratic, we need to find two numbers.

a What should their product be? **b** What should their sum be?

4 Given that $p < q$, find the values of p and q in the following pairs of equations:

a $p + q = 19$, $pq = 90$ **b** $p + q = -2$, $pq = -63$ **c** $p + q = -10$, $pq = 24$

7 Factorise:

a $x^2 + 6x + 8$ **b** $x^2 + 11x + 24$ **c** $x^2 + 17x + 72$ **d** $x^2 - 4x + 3$

e $x^2 - 17x + 60$ **f** $x^2 - 19x + 84$ **g** $x^2 - x - 6$ **h** $x^2 - x - 30$

i $x^2 + x - 20$ **j** $x^2 - 3x - 54$ **k** $x^2 - 3x - 70$ **l** $x^2 + 4x - 117$

m $40 + 13x + x^2$ **n** $35 - 12x + x^2$ **o** $-8 - 6x - x^2$ **p** $-12 + 7x - x^2$

8 Factorise:

a $x^4 - x^2 - 12$ **b** $9y^4 - 13y^2 + 4$

Non-monic quadratic trinomials

1 Complete the following factorisations:

a $8x^2 + 11x + 3 = (8x + 3)(x + \square)$

b $2x^2 - 19x + 45 = (\square - 9)(\square - 5)$

c $2x^2 + 3x - 20 = (2x - 5)(x + \square)$

d $10x^2 + 29x + 21 = (\square + 3)(5x + \square)$

2 Factorise the following expressions:

a $2x^2 - 11x - 40$

b $3y^2 + 28y + 9$

c $12t^2 - 13t - 4$

d $81x^2 + 72x + 16$

e $64x^2 - 48x + 9$

f $3x^2 - 25x + 28$

g $8x^2 - 19x + 6$

h $8x^2 - 21x - 9$

i $12x^2 + 7x - 10$

j $56 - 41x - 6x^2$

k $-6x^2 + 5x + 6$

l $-6x^2 + 25x - 14$

3 Factorise the following expressions by first taking out a common factor:

a $3x^2 - 21x - 54$

b $4x^2 + 40x + 100$

c $4x^2 + 24x + 32$

d $10x^2 + 5x - 30$

e $8s^2 + 6s - 54$

f $4x^2 - 4x - 288$

g $2x^3 + 16x^2 + 30x$

h $-4x^2 + 12x + 40$

i $3x^2 - 21x + 30$

j $-3x^2 + 12x - 12.$

Applications

6 A rectangle has an area of $6x^2 + 23x + 20$.
If the length and width are linear factors of $6x^2 + 23x + 20$, what are the dimensions of the rectangle?

Monic complete the square

1 Find the missing coefficient or term so that the following expressions form a perfect square.

a $x^2 - \square x + 81$ **b** $x^2 + 10x + \square$ **c** $x^2 - \square x + 16$ **d** $x^2 - \square x + 121$

2 For each of the following expressions, determine the value of k to make the expression a perfect square:

a $x^2 + x + k$ **b** $x^2 - 2x + k$ **c** $x^2 + 19x + k$ **d** $x^2 - \frac{4}{5}x + k$

3 Complete the following perfect squares:

a $(x + \square)^2 = x^2 + 20x + \square$ **b** $(x - \square)^2 = x^2 - \frac{4}{3}x + \square$

c $x^2 + 4x + \square = (x + \square)^2$ **d** $x^2 - 5x + \square = (x - \square)^2$

e $x^2 - \frac{7}{4}x + \square = (x - \square)^2$ **f** $(x - \square)^2 = x^2 - \frac{3}{2}x + \square$

4 Rewrite the following quadratics in the form $(x + b)^2 + c$ using the method of completing the square:

a $x^2 + 18x$ **b** $x^2 - 8x$ **c** $x^2 + 10x + 31$ **d** $x^2 + 14x + 47$

e $x^2 - 10x + 30$ **f** $x^2 - 18x + 77$ **g** $x^2 + 9x + 16$ **h** $x^2 - 7x + 15$

5 Factorise the following quadratics using the method of completing the square:

a $x^2 + 6x + 4$ **b** $x^2 - 8x + 11$

c $x^2 + 24x + 63$ **d** $x^2 - 20x + 19$

e $x^2 + 42x + 185$ **f** $x^2 - 6x + 5$

g $x^2 - 28x + 115$ **h** $x^2 + 11x + 10$

i $x^2 - 11x + 30$ **j** $(x + 3)(x + 19) - 17$

Quadratic equations

1 The equation $x^2 - 144 = 0$ has a positive integer solution of $x = 12$. Find its other solution.

2 Solve the following equations:

a $x^2 = 2$ **b** $x^2 = 25$ **c** $x^2 = 121$ **d** $x^2 = 294$

e $x^2 - 121 = 0$ **f** $x^2 - 10 = 15$ **g** $\frac{x^2}{16} - 2 = 2$ **h** $\frac{x^2}{25} - 3 = 6$

i $(x+3)^2 = 49$ **j** $(x-3)^2 = 64$ **k** $(x-6)^2 = 2$ **l** $(2-x)^2 = 81$

m $(x-7)^2 = 81$ **n** $(7-x)^2 = 81$ **o** $(8x+9)^2 = 256$ **p** $81x^2 - 16 = 0$

3 Solve the following equations:

a $x(x+7) = 0$ **b** $x(2x-9) = 0$

c $(10x-9)^2 = 0$ **d** $(4x-9)^2 = 0$

e $(-3+7x)^2 = 0$ **f** $(x-4)(x-2) = 0$

g $(x-6)(x+7) = 0$ **h** $(8x-5)(3x-7) = 0$

i $(3x+8)(5x-7) = 0$ **j** $(3x-17)(2x-d) = 0$

4 Solve the following equations:

a $4y^2 = 100$ **b** $25y^2 = 36$

c $-3k^2 = -12$ **d** $81k^2 + 8 = 24$

e $-25v^2 + 64 = 0$ **f** $10(p^2 - 7) = 930$

g $4m(m+5) = 0$ **h** $\frac{m}{2}(m+5) = 0$

Quadratic equations

2 Find the value of a if:

a The nonzero solution of the equation $x(x+a)=0$ is $x=10$.

b The nonzero solution of the equation $x(x+a)=0$ is $x=-1$.

3 Solve the following equations:

a $m^2=14m$

b $m^2=m+20$

c $m^2=7m-6$

d $m^2-11m=-30$

e $m^2-27m=-182$

f $\frac{m}{6}(m-10)=0$

g $-3m(m+2)=0$

h $3\left(n^2+8\right)=-18n$

i $2y-6y^2=0$

j $3y-15y^2=0$

k $(y-3)^2=2y+2$

l $(y+1)^2=4y+4$

m $15-11b-12b^2=0$

n $6k^2=-7+13k$

4 Solve the following equations:

a $x^2-64=0$

b $x^2+12x=0$

c $x^2+13x+42=0$

d $x^2+4x-21=0$

e $x^2-14x+40=0$

f $x^2-5x-50=0$

g $x^2-22x+121=0$

h $-x^2+x+6=0$

i $x(x+18)=-80$

j $x(x+2)=48$

k $x^2+8x+16=36$

l $x^2=13x+114$

m $3x^2-7x-20=0$

n $5x^2+22x+8=0$

o $-4x^2+25x-36=0$

p $-5x^2=-53x+72$

q $-10x+x^2=2-7x-x^2$

r $-8x+x^2=-6-x-x^2$

s $6x^2-27x+28=-3x^2+3x+3$

t $\left(5x^2-13x+6\right)\left(2x^2-13x+20\right)=0$

5H Solving quadratic equations by completing the square

BUILDING UNDERSTANDING

1. What number must be added to the following expressions to form a perfect square?
   a $x^2 + 2x$ b $x^2 + 20x$ c $x^2 - 4x$ d $x^2 + 5x$
2. Factorise using surds.
   a $x^2 - 3 = 0$ b $x^2 - 10 = 0$ c $(x + 1)^2 - 5 = 0$
3. Solve these equations.
   a $(x - \sqrt{2})(x + \sqrt{2}) = 0$ b $(x - \sqrt{7})(x + \sqrt{7}) = 0$
   c $(x - 3 + \sqrt{5})(x - 3 - \sqrt{5}) = 0$ d $(x + 5 + \sqrt{14})(x + 5 - \sqrt{14}) = 0$

1 Solve these quadratic equations by first completing the square.

a i $x^2 - 8x + 3 = 0$ ii $x^2 - 12x + 7 = 0$

b i $x^2 + 4x - 4 = 0$ ii $x^2 + 10x - 7 = 0$

2 Solve by first completing the square.

a $x^2 + 6x + 3 = 0$ b $x^2 + 4x + 2 = 0$ c $x^2 + 10x + 15 = 0$
d $x^2 + 4x - 2 = 0$ e $x^2 + 8x - 3 = 0$ f $x^2 + 6x - 5 = 0$
g $x^2 - 8x - 1 = 0$ h $x^2 - 12x - 3 = 0$ i $x^2 - 2x - 16 = 0$
j $x^2 - 10x + 18 = 0$ k $x^2 - 6x + 4 = 0$ l $x^2 - 8x + 9 = 0$
m $x^2 + 6x - 4 = 0$ n $x^2 + 20x + 13 = 0$ o $x^2 - 14x - 6 = 0$

3 Solve by first completing the square.

a $x^2 + 8x + 4 = 0$ b $x^2 + 6x + 1 = 0$ c $x^2 - 10x + 5 = 0$
d $x^2 - 4x - 14 = 0$ e $x^2 - 10x - 3 = 0$ f $x^2 + 8x - 8 = 0$
g $x^2 - 2x - 31 = 0$ h $x^2 + 12x - 18 = 0$ i $x^2 + 6x - 41 = 0$

4 Solve by first completing the square.

a $x^2 + 5x + 2 = 0$ b $x^2 + 3x + 1 = 0$ c $x^2 + 7x + 5 = 0$
d $x^2 - 3x - 2 = 0$ e $x^2 - x - 3 = 0$ f $x^2 + 5x - 2 = 0$
g $x^2 - 7x + 2 = 0$ h $x^2 - 9x + 5 = 0$ i $x^2 + x - 4 = 0$
j $x^2 + 9x + 9 = 0$ k $x^2 - 3x - \frac{3}{4} = 0$ l $x^2 + 5x + \frac{5}{4} = 0$

The quadratic formula

1 Considering the quadratic formula, find the values of a, b and c in the following quadratic equations:

a $x^2 + 7x + 10 = 0$

b $x^2 - 3x - 4 = 0$

c $2x^2 + 9x = 0$

d $4x^2 + 3x = 5$

e $8x^2 - 3x = 0$

f $-8x^2 + 3x = 0$

g $-2x^2 + 9x + 5 = 0$

h $3x^2 - 8x + 2 = 9x - 7$

2 Solve the following equations using the quadratic formula:

a $x^2 + 5x + 6 = 0$

b $x^2 - 5x + 6 = 0$

c $x^2 - 5x + \frac{9}{4} = 0$

d $x^2 - 8x + \frac{55}{4} = 0$

e $2x^2 + 6x - 8 = 0$

f $4x^2 - 10x + 4 = 0$

g $4x^2 + 7x + 3 = 0$

h $4x^2 - 17x - 15 = 0$

i $-6 + 7x + 5x^2 = 0$

j $-6 - 13x + 5x^2 = 0$

k $-20 - 11x + 3x^2 = 0$

l $-20 + 21x + 5x^2 = 0$

3 Solve the following equations, leaving your answer in surd form:

a $x^2 - 7x + 9 = 0$

b $x^2 - 5x - 2 = 0$

c $-2x^2 - 15x - 4 = 0$

d $3x^2 + 9x - 4 = 0$

e $5x^2 - 15x + 2 = 0$

f $-5x^2 - 15x + 3 = 0$

5 Solve the following equations, rounding your answers to three decimal places:

a $x^2 + 3x - 6 = 0$

b $x^2 + 7x - 3 = 0$

c $4x^2 + 7x + 2 = 0$

d $5x^2 + 9x + 2 = 0$

e $2x(x - 4) = 3x + 1$

f $3x(x + 4) = -3x + 4$