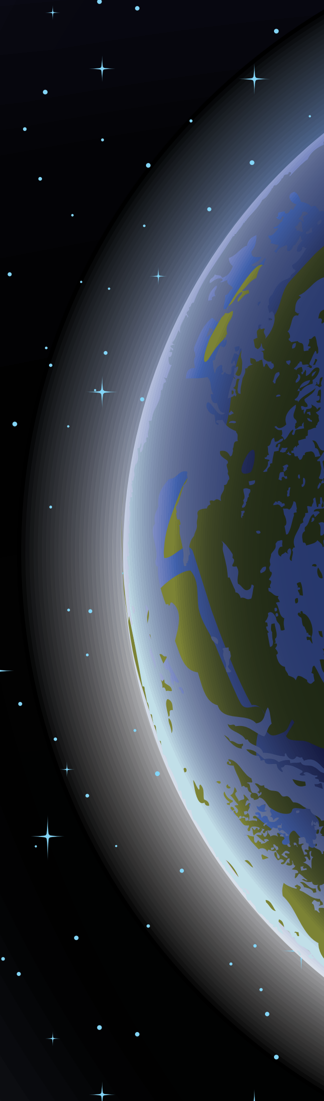
TOPIC 5

Trigonometry I



5.1 Overview

Why learn this?

Nearly 2000 years ago, Ptolemy of Alexandria published the first book of trigonometric tables, which he used to chart the heavens and plot the courses of the Moon, stars and planets. He also created geographical charts and provided instructions on how to create maps. Trigonometry is the branch of mathematics that makes the whole universe more easily understood.

What do you know?

assess on

1 **THINK** List what you know about trigonometry. Use a thinking tool such as a concept map to show your list.

2 **PAIR** Share what you know with a partner and then with a small group.

3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of trigonometry.

Learning sequence

5.1 Overview
5.2 Pythagoras' theorem
5.3 Pythagoras' theorem in three dimensions
5.4 Trigonometric ratios
5.5 Using trigonometry to calculate side lengths
5.6 Using trigonometry to calculate angle size
5.7 Angles of elevation and depression
5.8 Bearings
5.9 Applications
5.10 Review eBook*plus* ONLINE ONLY

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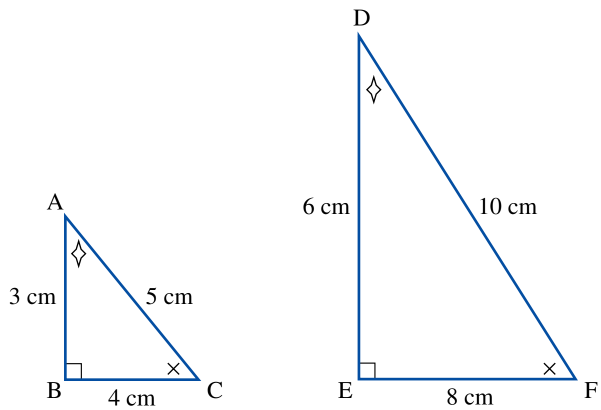
5.2 Pythagoras' theorem

Similar right-angled triangles

In the two similar right-angled triangles shown below, the angles are the same and the corresponding sides are in the same ratio.



The corresponding sides are in the same ratio.

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

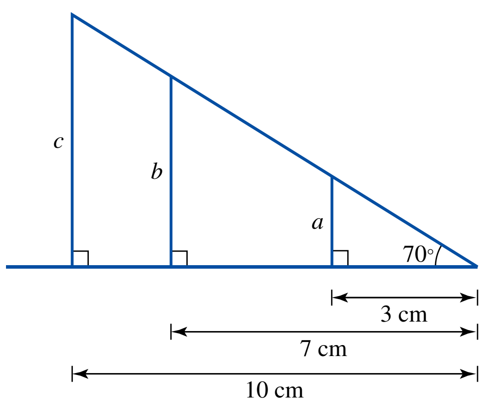
To write this using the side lengths of the triangles gives:

$$\frac{AB}{DE} = \frac{3}{6} = \frac{1}{2}$$
$$\frac{AC}{DF} = \frac{5}{10} = \frac{1}{2}$$
$$\frac{BC}{EF} = \frac{4}{8} = \frac{1}{2}$$

This means that for right-angled triangles, when the angles are fixed, the ratios of the sides in the triangle are constant.

We can examine this idea further by completing the following activity.

Using a protractor and ruler, draw an angle of 70°, measuring horizontal distances of 3 cm, 7 cm and 10 cm as demonstrated in the diagram below.



Note: Diagram not drawn to scale.

Measure the perpendicular heights a, b and c.

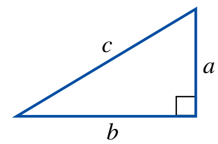
$a \approx 8.24$ cm $\qquad b \approx 19.23$ cm $\qquad c \approx 27.47$ cm

To test if the theory for right-angled triangles, that when the angles are fixed the ratios of the sides in the triangle are constant, is correct, calculate the ratios of the side lengths.

$$\frac{a}{3} \approx \frac{8.24}{3} \approx 2.75 \qquad \frac{b}{7} \approx \frac{19.23}{7} \approx 2.75 \qquad \frac{c}{10} \approx \frac{27.47}{10} \approx 2.75$$

The ratios are the same because the triangles are similar.
This important concept forms the basis of trigonometry.

Review of Pythagoras' theorem



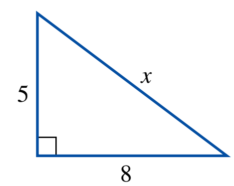
- The **hypotenuse** is the longest side of a right-angled triangle and is always the side that is opposite the right angle.
- **Pythagoras' theorem** states that in any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. The rule is written as $c^2 = a^2 + b^2$ where a and b are the two shorter sides and c is the hypotenuse.
- Pythagoras' theorem gives us a way of finding the length of the third side in a triangle, if we know the lengths of the two other sides.

Finding the hypotenuse

- To calculate the length of the hypotenuse when given the length of the two shorter sides, substitute the known values into the formula for Pythagoras' theorem, $c^2 = a^2 + b^2$.

WORKED EXAMPLE 1

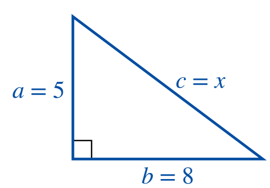
For the triangle at right, calculate the length of the hypotenuse, x, correct to 1 decimal place.



THINK

WRITE/DRAW

1. Copy the diagram and label the sides a, b and c. Remember to label the hypotenuse as c.



2. Write Pythagoras' theorem.

$c^2 = a^2 + b^2$

3. Substitute the values of a, b and c into this rule and simplify.

$$\begin{aligned} x^2 &= 5^2 + 8^2 \\ &= 25 + 64 \\ &= 89 \end{aligned}$$

4. Take the square root of both sides. Round the positive answer correct to 1 decimal place, since $x > 0$.

$x = \pm\sqrt{89}$
$x \approx 9.4$

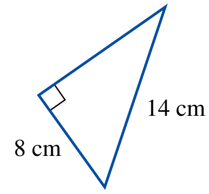
Finding a shorter side

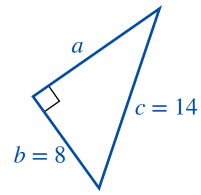
- Sometimes a question will give you the length of the hypotenuse and ask you to find one of the shorter sides. In such examples, we need to rearrange Pythagoras' formula. Given that $c^2 = a^2 + b^2$, we can rewrite this as:

$$a^2 = c^2 - b^2$$
$$\text{or} \quad b^2 = c^2 - a^2.$$

WORKED EXAMPLE 2 TI CASIO

Calculate the length, correct to 1 decimal place, of the unmarked side of the triangle at right.

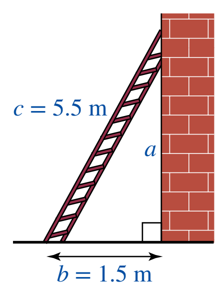


THINK	WRITE/DRAW
1 Copy the diagram and label the sides a, b and c. Remember to label the hypotenuse as c; it does not matter which side is a and which side is b.	
2 Write Pythagoras' theorem.	$c^2 = a^2 + b^2$
3 Substitute the values of a, b and c into this rule and solve for a.	$14^2 = a^2 + 8^2$ $196 = a^2 + 64$ $a^2 = 196 - 64$ $= 132$
4 Find a by taking the square root of both sides and round to 1 decimal place $(a > 0)$.	$a = \pm\sqrt{132}$ ≈ 11.5 cm

- Pythagoras' theorem can be used to solve many practical problems.
  First model the problem by drawing a diagram, then use Pythagoras' theorem to solve the right-angled triangle. Use the result to give a worded answer.

WORKED EXAMPLE 3

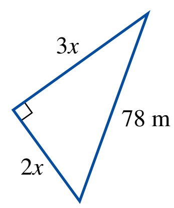
A ladder that is 5.5 m long leans up against a vertical wall. The foot of the ladder is 1.5 m from the wall. How far up the wall does the ladder reach? Give your answer correct to 1 decimal place.

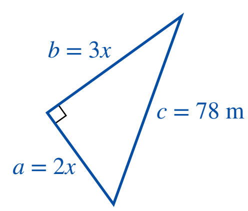
THINK	WRITE/DRAW
1 Draw a diagram and label the sides a, b and c. Remember to label the hypotenuse as c.	

2 Write Pythagoras' theorem.	$c^2 = a^2 + b^2$
3 Substitute the values of a, b and c into this rule and simplify.	$5.5^2 = a^2 + 1.5^2$ $30.25 = a^2 + 2.25$ $a^2 = 30.25 - 2.25$ $= 28$
4 Find a by taking the square root of 28. Round to 1 decimal place, $a > 0$.	$a = \pm\sqrt{28}$ ≈ 5.3
5 Answer the question in a sentence using words.	The ladder reaches 5.3 m up the wall.

WORKED EXAMPLE 4

Determine the unknown side lengths of the triangle, correct to 2 decimal places.



THINK	WRITE/DRAW
1 Copy the diagram and label the sides a, b and c.	
2 Write Pythagoras' theorem.	$c^2 = a^2 + b^2$
3 Substitute the values of a, b and c into this rule and simplify.	$78^2 = (3x)^2 + (2x)^2$ $6084 = 9x^2 + 4x^2$ $6084 = 13x^2$
4 Rearrange the equation so that the pronumeral is on the left-hand side of the equation.	$13x^2 = 6084$
5 Divide both sides of the equation by 13.	$\frac{13x^2}{13} = \frac{6084}{13}$ $x^2 = 468$
6 Find x by taking the square root of both sides. Round the answer correct to 2 decimal places.	$x = \pm\sqrt{468}$ ≈ 21.6333
7 Substitute the value of x into $2x$ and $3x$ to find the length of the unknown sides.	$2x \approx 43.27$ m $3x \approx 64.90$ m

assess on

Exercise 5.2 Pythagoras' theorem

REFLECTION
The square root of a number usually gives us both a positive and negative answer. Why do we take only the positive answer when using Pythagoras' theorem?

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–4, 6, 12–15, 17, 20	Questions: 1–3, 5–8, 12, 15–18, 20, 22	Questions: 1, 2, 5, 7, 9–11, 19–23

■■■ **Individual pathway interactivity** int-4585 eBook plus

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Rounding to a given number of decimal places
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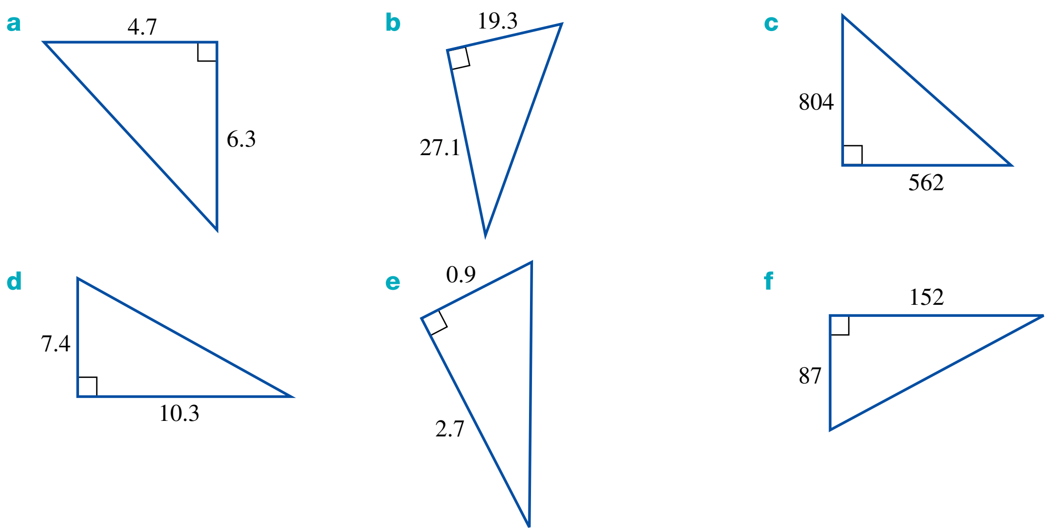
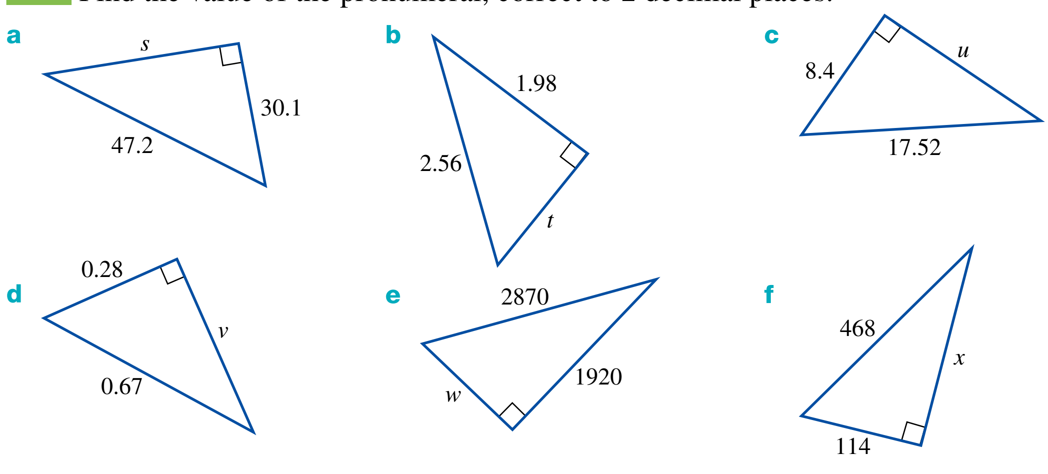
FLUENCY

1 **WE1** For each of the following triangles, calculate the length of the hypotenuse, giving answers correct to 2 decimal places.



2 **WE2** Find the value of the pronumeral, correct to 2 decimal places.



3 **WE3** The diagonal of the rectangular sign at right is 34 cm. If the height of this sign is 25 cm, find the width.

4 A right-angled triangle has a base of 4 cm and a height of 12 cm. Calculate the length of the hypotenuse to 2 decimal places.

5 Calculate the lengths of the diagonals (to 2 decimal places) of squares that have side lengths of:

a 10 cm

b 17 cm

c 3.2 cm.

6 The diagonal of a rectangle is 90 cm. One side has a length of 50 cm. Determine:

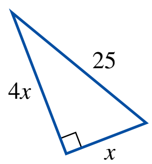
a the length of the other side

b the perimeter of the rectangle

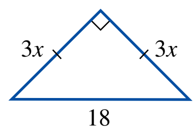
c the area of the rectangle.

7 **WE4** Find the value of the pronumeral, correct to 2 decimal places for each of the following.

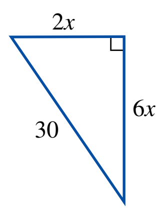
a



b



c



UNDERSTANDING

8 An isosceles triangle has a base of 25 cm and a height of 8 cm. Calculate the length of the two equal sides.

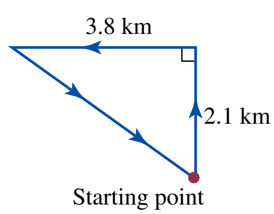
9 An equilateral triangle has sides of length 18 cm. Find the height of the triangle.

10 A right-angled triangle has a height of 17.2 cm, and a base that is half the height. Calculate the length of the hypotenuse, correct to 2 decimal places.

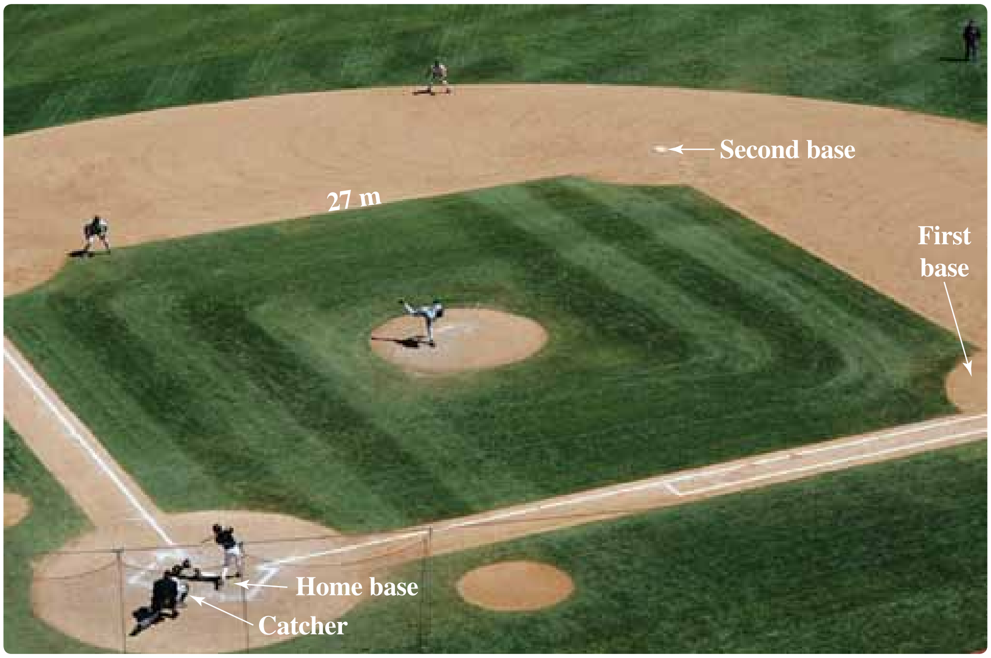
11 The road sign shown below is based on an equilateral triangle. Find the height of the sign and, hence, find its area.

12 A flagpole, 12 m high, is supported by three wires, attached from the top of the pole to the ground. Each wire is pegged into the ground 5 m from the pole. How much wire is needed to support the pole?

13 Sarah goes canoeing in a large lake. She paddles 2.1 km to the north, then 3.8 km to the west. Use the triangle at right to find out how far she must then paddle to get back to her starting point in the shortest possible way.



14 A baseball diamond is a square of side length 27 m. When a runner on first base tries to steal second base, the catcher has to throw the ball from home base to second base. How far is that throw?



15 A rectangle measures 56 mm by 2.9 cm. Calculate the length of its diagonal in millimetres to 2 decimal places.

16 A rectangular envelope has a length of 24 cm and a diagonal measuring 40 cm. Calculate:

a the width of the envelope

b the area of the envelope.

17 A swimming pool is 50 m by 25 m. Peter is bored by his usual training routine, and decides to swim the diagonal of the pool. How many diagonals must he swim to complete his normal distance of 1500 m? Give your answer to 2 decimal places.

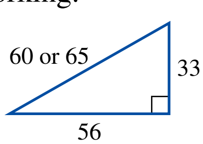
18 A hiker walks 2.9 km north, then 3.7 km east. How far in metres is she from her starting point? Give your answer to 2 decimal places.

19 A square has a diagonal of 14 cm. What is the length of each side?

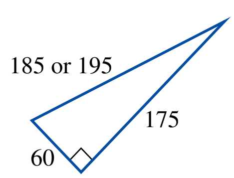
REASONING

20 The triangles below are right-angled triangles. Two possible measurements have been suggested for the hypotenuse in each case. For each triangle, complete calculations to determine which of the lengths is correct for the hypotenuse in each case. Show your working.

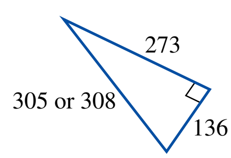
a



b



c



21 Four possible side length measurements are 105, 208, 230 and 233. Three of them together produce a right-angled triangle.

a Which of the measurements could not be the hypotenuse of the triangle? Explain.

b Complete as few calculations as possible to calculate which combination of side lengths will produce a right-angled triangle.

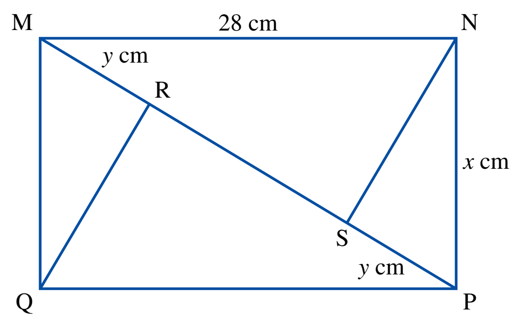
PROBLEM SOLVING

22 The area of the rectangle MNPQ is 588 cm^2. Angles MRQ and NSP are right angles.

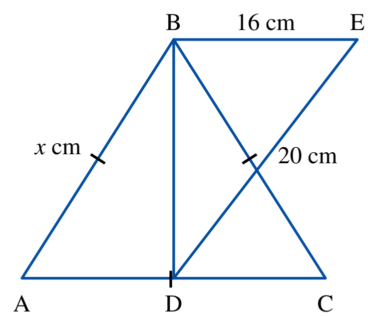
a Find the integer value of x.

b Find the length of MP.

c Find the value of y and hence determine the length of RS.

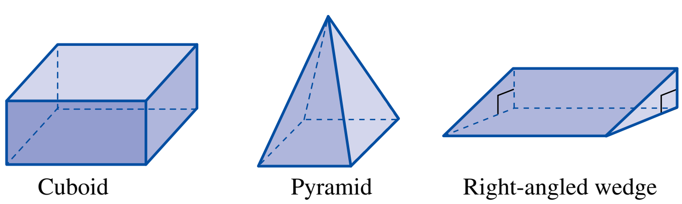


23 Triangle ABC is an equilateral triangle of side length x cm. Angles ADB and DBE are right angles. Find the value of x, correct to 2 decimal places.



5.3 Pythagoras' theorem in three dimensions

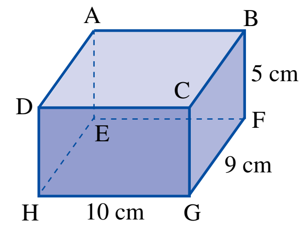
- Many real-life situations involve **3-dimensional** (3-D) objects: objects with length, width and height. Some common 3-D objects used in this section include cuboids, pyramids and right-angled wedges.



- In diagrams of 3-D objects, right angles may not look like right angles, so it is important to redraw sections of the diagram in two dimensions, where the right angles can be seen accurately.

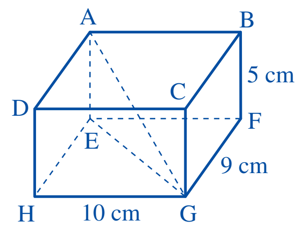
WORKED EXAMPLE 5

Determine the length AG in this rectangular prism (cuboid), correct to two decimal places.

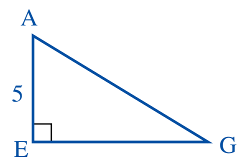


THINK | **WRITE/DRAW**

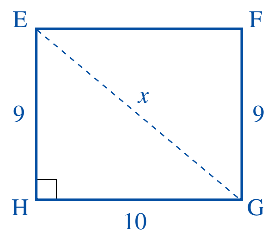
1. Draw the diagram in three dimensions. Draw the lines AG and EG. $\angle$AEG is a right angle.



2. Draw ΔAEG, showing the right angle. Only 1 side is known, so EG must be found.



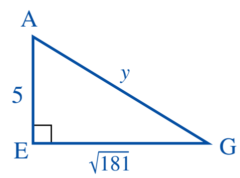
3. Draw EFGH in two dimensions and label the diagonal EG as x.



4. Use Pythagoras' theorem to calculate x. ($c^2 = a^2 + b^2$)

$$\begin{aligned} x^2 &= 9^2 + 10^2 \\ &= 81 + 100 \\ &= 181 \\ x &= \sqrt{181} \end{aligned}$$

5. Place this information on triangle AEG. Label the side AG as y.



6. Use Pythagoras' theorem to find y. ($c^2 = a^2 + b^2$)

$$\begin{aligned} y^2 &= 5^2 + (\sqrt{181})^2 \\ &= 25 + 181 \\ &= 206 \\ y &= \sqrt{206} \\ &\approx 14.35 \end{aligned}$$

7. Answer the question in a sentence.

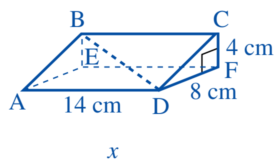
The length of AG is 14.35 cm.

WORKED EXAMPLE 6

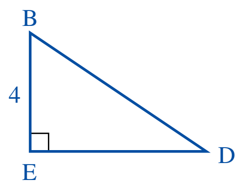
A piece of cheese in the shape of a right-angled wedge sits on a table. It has a rectangular base measuring 14 cm by 8 cm, and is 4 cm high at the thickest point. An ant crawls diagonally across the sloping face. How far, to the nearest millimetre, does the ant walk?

THINK | **WRITE/DRAW**

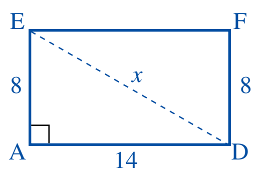
1. Draw a diagram in three dimensions and label the vertices. Mark BD, the path taken by the ant, with a dotted line. ∠BED is a right angle.



2. Draw ΔBED, showing the right angle. Only one side is known, so ED must be found.



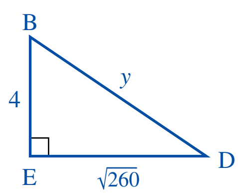
3. Draw EFDA in two dimensions, and label the diagonal ED. Label the side ED as x in both diagrams.



4. Use Pythagoras' theorem to calculate x.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ x^2 &= 8^2 + 14^2 \\ &= 64 + 196 \\ &= 260 \\ x &= \sqrt{260} \end{aligned}$$

5. Place this information on triangle BED. Label the side BD as y.



6. Solve this triangle for y.

$$\begin{aligned} y^2 &= 4^2 + (\sqrt{260})^2 \\ &= 16 + 260 \\ &= 276 \\ y &= \sqrt{276} \\ &\approx 16.61 \text{ cm} \\ &\approx 166.1 \text{ mm} \end{aligned}$$

7. Answer the question in a sentence.

The ant walks 166 mm, correct to the nearest millimetre.

assess on

Exercise 5.3 Pythagoras' theorem in three dimensions

INDIVIDUAL PATHWAYS

REFLECTION
The diagonal distance across a rectangle of dimensions *x* by *y* is $\sqrt{x^2 + y^2}$. What would be the rule to find the length of a diagonal across a cuboid of dimensions *x* by *y* by *z*? Use your rule to check your answers to question 1.

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–b, 2, 6, 7, 8a, 10	Questions: 1, 3, 4, 6, 7, 8, 10, 11, 13, 14	Questions: 1, 3–5, 8–16

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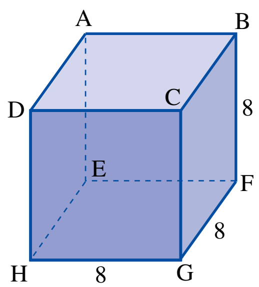
Where appropriate in this exercise, give answers correct to 2 decimal places.

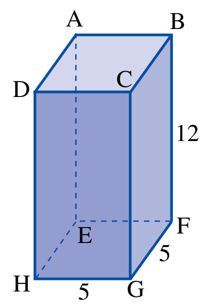
FLUENCY

1 WE5 Calculate the length of AG in each of the following figures.

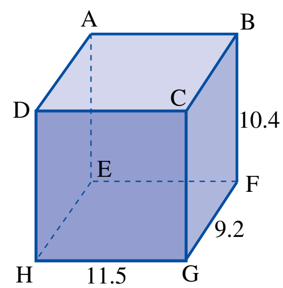
a



b



c



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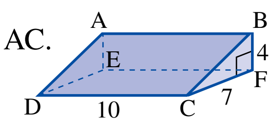
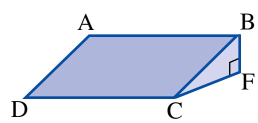
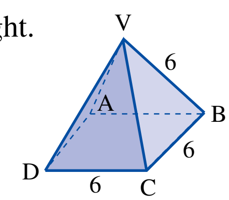
2 Calculate the length of CE in the wedge at right and, hence, obtain AC.



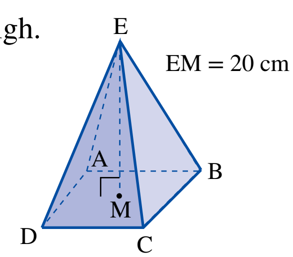
3 If DC = 3.2 m, AC = 5.8 m, and CF = 4.5 m in the figure at right, calculate the length of AD and BF.



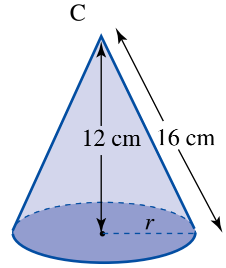
4 Calculate the length of BD and, hence, the height of the pyramid at right.



5 The pyramid ABCDE has a square base. The pyramid is 20 cm high. Each sloping edge measures 30 cm. Calculate the length of the sides of the base.

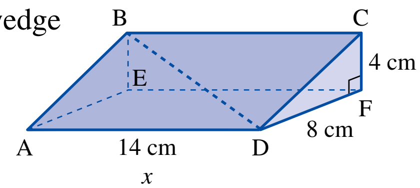


6 The sloping side of a cone is 16 cm and the height is 12 cm. What is the length of the radius of the base?

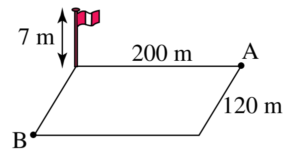


UNDERSTANDING

7 WE6 A piece of cheese in the shape of a right-angled wedge sits on a table. It has a base measuring 20 mm by 10 mm, and is 4 mm high at the thickest point, as shown in the figure. A fly crawls diagonally across the sloping face. How far, to the nearest millimetre, does the fly walk?



8 A 7 m high flagpole is in the corner of a rectangular park that measures 200 m by 120 m.



a Calculate:

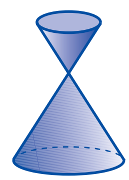
i the length of the diagonal of the park

ii the distance from A to the top of the pole

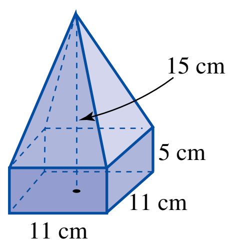
iii the distance from B to the top of the pole.

b A bird flies from the top of the pole to the centre of the park. How far does it fly?

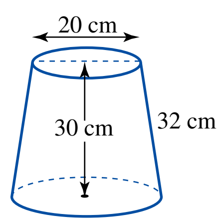
9 A candlestick is in the shape of two cones, joined at the vertices as shown. The smaller cone has a diameter and sloping side of 7 cm, and the larger one has a diameter and sloping side of 10 cm. How tall is the candlestick?



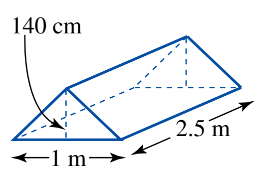
10 The total height of the shape below is 15 cm. Calculate the length of the sloping side of the pyramid.



11 A sandcastle is in the shape of a truncated cone as shown. Calculate the length of the diameter of the base.



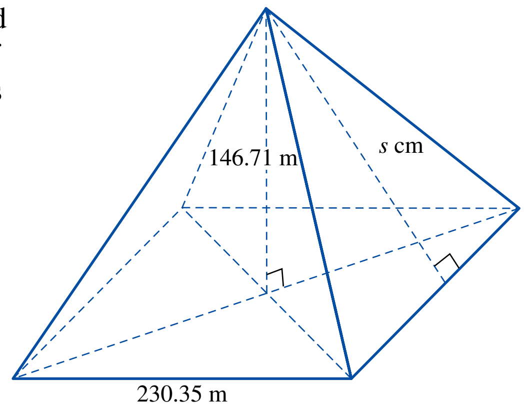
12 A tent is in the shape of a triangular prism, with a height of 140 cm as shown in the following diagram. The width across the base of the door is 1 m and the tent is 2.5 m long. Calculate the length of each sloping side, in metres. Then calculate the area of fabric used in the construction of the sloping rectangles which form the sides.



REASONING

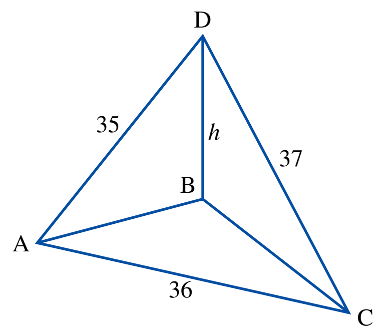
13 Stephano is renovating his apartment, which he accesses through two corridors. The corridors of the apartment building are 2 m wide with 2 m high ceilings, and the first corridor is at right angles to the second. Show that he can carry lengths of timber up to 6 m long to his apartment.

14 The Great Pyramid in Egypt is a square-based pyramid. The square base has a side length of 230.35 metres and the perpendicular height is 146.71 metres. Find the slant height, s cm, of the great pyramid. Give your answer correct to 1 decimal place.



PROBLEM SOLVING

15 Angles ABD, CBD and ABC are right angles. Find the value of h, correct to 3 decimal places.



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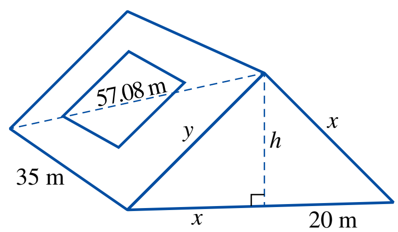
Digital doc
WorkSHEET 5.1
doc-5230

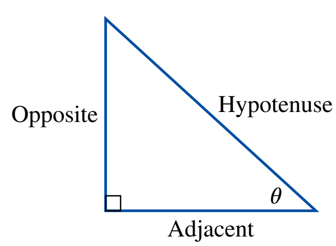
16 The roof of a squash centre is constructed to allow for maximum use of sunlight. Find the value of h, giving your answer correct to 1 decimal place.



5.4 Trigonometric ratios

Naming the sides in a right-angled triangle

- In a right-angled triangle, the longest side is called the hypotenuse.
- If one of the two acute angles is named (say θ), then the other two sides can also be given names, as shown in the diagram.



Three basic definitions

- Using the diagram opposite, the following three **trigonometric ratios** can be defined:
  - the **sine ratio**, $\text{sine } \theta = \dfrac{\text{length of opposite side}}{\text{length of hypotenuse}}$
  - the **cosine ratio**, $\text{cosine } \theta = \dfrac{\text{length of adjacent side}}{\text{length of hypotenuse}}$
  - the **tangent ratio**, $\text{tangent } \theta = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$.
- The names of the three ratios are usually shortened to $\sin \theta$, $\cos \theta$ and $\tan \theta$.
- The three ratios are often remembered using the mnemonic SOHCAHTOA, where SOH means Sin θ = Opposite over Hypotenuse and so on.

Finding values using a calculator

- The sine, cosine and tangent of an angle have numerical values that can be found using a calculator.
- Traditionally angles were measured in degrees, minutes and seconds, where 60 seconds = 1 minute and 60 minutes = 1 degree.
  For example, $50° 33' 48''$ means 50 degrees, 33 minutes and 48 seconds.

WORKED EXAMPLE 7 | TI | CASIO

Calculate the value of each of the following, correct to 4 decimal places, using a calculator. (Remember to first work to 5 decimal places before rounding.)

a $\cos 65°57'$ **b** $\tan 56°45'30''$

THINK	WRITE
a Write your answer to the required number of decimal places.	**a** $\cos 65°57' \approx 0.40753$ ≈ 0.4075
b Write your answer to the correct number of decimal places.	**b** $\tan 56°45'30'' \approx 1.52573$ ≈ 1.5257

WORKED EXAMPLE 8

Calculate the size of angle θ, correct to the nearest degree, given $\sin \theta = 0.7854$.

THINK	WRITE
1 Write the given equation.	$\sin \theta = 0.7854$
2 To find the size of the angle, we need to undo sine with its inverse, $\sin^{-1}$. (Ensure your calculator is in degrees mode.)	$\theta = \sin^{-1} 0.7854$ $\approx 51.8°$
3 Write your answer to the nearest degree.	$\theta \approx 52°$

WORKED EXAMPLE 9 TI CASIO

Calculate the value of θ:

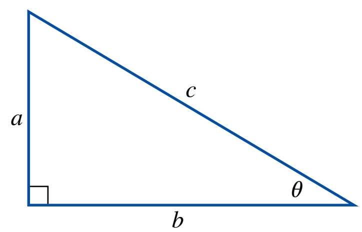
a **correct to the nearest minute, given that $\cos\theta = 0.2547$**

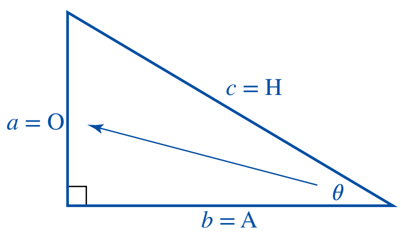
b **correct to the nearest second, given that $\tan\theta = 2.364$.**

THINK		WRITE
a 1	Write the equation.	**a** $\cos\theta = 0.2547$
2	Write your answer, including seconds. There are 60 seconds in 1 minute. Round to the nearest minute. (Remember $60'' = 1'$, so $39''$ is rounded up.)	$\cos^{-1} 0.2547 \approx 75°14'39''$ $\approx 75°15'$
b 1	Write the equation.	**b** $\tan\theta = 2.364$
2	Write the answer, rounding to the nearest second.	$\tan^{-1} 2.364 \approx 67°4'15.8''$ $\approx 67°4'16''$

WORKED EXAMPLE 10

For the triangle shown, write the rules for the sine, cosine and tangent ratios of the given angle.

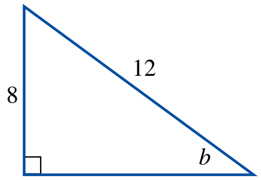


THINK		WRITE/DRAW
1	Label the diagram using the symbols O, A, H with respect to the given angle (angle θ).	
2	From the diagram, identify the values of O (opposite side), A (adjacent side) and H (the hypotenuse).	$O = a, A = b, H = c$
3	Write the rule for each of the sine, cosine and tangent ratios.	$\sin\theta = \frac{O}{H}, \cos\theta = \frac{A}{H}, \tan\theta = \frac{O}{A}$
4	Substitute the values of A, O and H into each rule.	$\sin\theta = \frac{a}{c}, \cos\theta = \frac{b}{c}, \tan\theta = \frac{a}{b}$

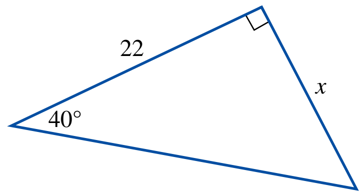
WORKED EXAMPLE 11

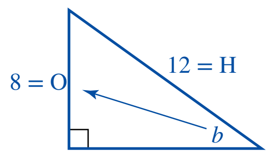
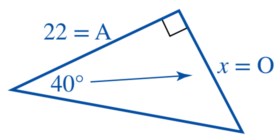
Write the equation that relates the two marked sides and the marked angle.

a



b



THINK	WRITE/DRAW
a **1** Label the given sides of the triangle.	**a** 
2 Write the ratio that contains O and H.	$\sin\theta = \dfrac{\text{O}}{\text{H}}$
3 Identify the values of the pronumerals.	$\text{O} = 8, \text{H} = 12$
4 Substitute the values of the pronumerals into the ratio. (Since the given angle is denoted with the letter *b*, replace θ with *b*.)	$\sin b = \dfrac{8}{12} = \dfrac{2}{3}$
b **1** Label the given sides of the triangle.	**b** 
2 Write the ratio that contains O and A.	$\tan\theta = \dfrac{\text{O}}{\text{A}}$
3 Identify the values of the pronumerals.	$\text{O} = x, \text{A} = 22, \theta = 40°$
4 Substitute the values of the pronumerals into the ratio.	$\tan\ 40° = \dfrac{x}{22}$

Exercise 5.4 Trigonometric ratios

assess on

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1, 3, 6a–f, 7, 8	Questions: 2–4, 6a–f, 7–9, 11	Questions: 2, 3, 4, 5, 6c–l, 7–12

■ ■ ■ **Individual pathway interactivity** int-4587 eBook*plus*

REFLECTION
How do we determine which of sin, cos or tan to use in a trigonometry question?

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Digital docs
SkillSHEET
Labelling the sides of a right-angled triangle
doc-5226
SkillSHEET
Selecting an appropriate trigonometric ratio based on the given information
doc-5231

FLUENCY

1 Calculate each of the following, correct to 4 decimal places.

a $\sin 30°$ **b** $\cos 45°$ **c** $\tan 25°$
d $\sin 57°$ **e** $\tan 83°$ **f** $\cos 44°$

2 **WE7** Calculate each of the following, correct to 4 decimal places.

a $\sin 40°30'$ **b** $\cos 53°57'$ **c** $\tan 27°34'$
d $\tan 123°40'$ **e** $\sin 92°32'$ **f** $\sin 42°8'$
g $\cos 35°42'35''$ **h** $\tan 27°42'50''$ **i** $\cos 143°25'23''$
j $\sin 23°58'21''$ **k** $\cos 8°54'2''$ **l** $\sin 286°$
m $\tan 420°$ **n** $\cos 845°$ **o** $\sin 367°35'$

3 **WE8** Find the size of angle θ, correct to the nearest degree, for each of the following.

a $\sin \theta = 0.763$ **b** $\cos \theta = 0.912$ **c** $\tan \theta = 1.351$
d $\cos \theta = 0.321$ **e** $\tan \theta = 12.86$ **f** $\cos \theta = 0.756$

4 **WE9a** Find the size of the angle θ, correct to the nearest minute.

a $\sin \theta = 0.814$ **b** $\sin \theta = 0.110$ **c** $\tan \theta = 0.015$
d $\cos \theta = 0.296$ **e** $\tan \theta = 0.993$ **f** $\sin \theta = 0.450$

5 **WE9b** Find the size of the angle θ, correct to the nearest second.

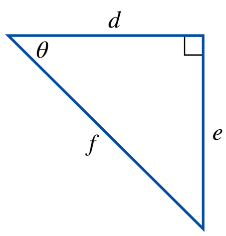
a $\tan \theta = 0.5$ **b** $\cos \theta = 0.438$ **c** $\sin \theta = 0.9047$
d $\tan \theta = 1.1141$ **e** $\cos \theta = 0.8$ **f** $\tan \theta = 43.76$

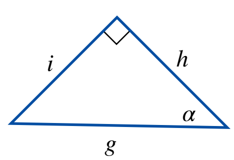
6 Find the value of each expression, correct to 3 decimal places.

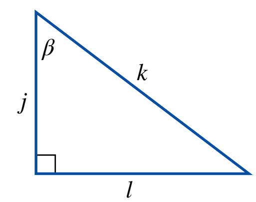
a $3.8 \cos 42°$ **b** $118 \sin 37°$ **c** $2.5 \tan 83°$
d $\dfrac{2}{\sin 45°}$ **e** $\dfrac{220}{\cos 14°}$ **f** $\dfrac{2 \cos 23°}{5 \sin 18°}$
g $\dfrac{12.8}{\tan 60°32'}$ **h** $\dfrac{18.7}{\sin 35°25'42''}$ **i** $\dfrac{55.7}{\cos 89°21'}$
j $\dfrac{3.8 \tan 1°51'44''}{4.5 \sin 25°45'}$ **k** $\dfrac{2.5 \sin 27°8'}{10.4 \cos 83°2'}$ **l** $\dfrac{3.2 \cos 34°52'}{0.8 \sin 12°48'}$

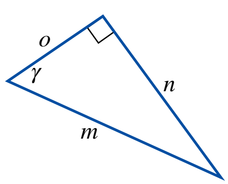
7 **WE10** For each labelled angle in the following triangles, write an expression for:

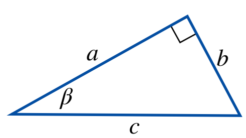
i sine **ii** cosine **iii** tangent.

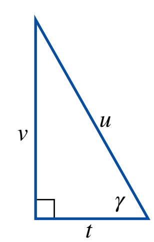
a


b


c


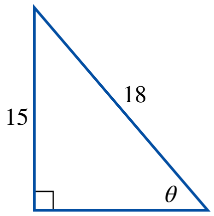
d


e


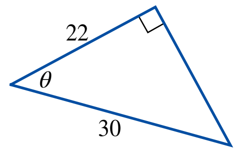
f


8 WE11 Write the equation that relates the two marked sides and the marked angle in each of the following triangles.

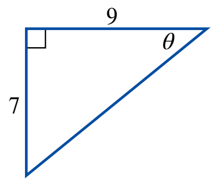
a



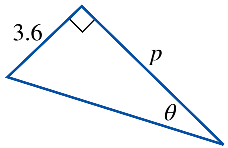
b



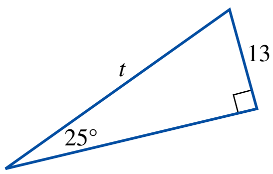
c



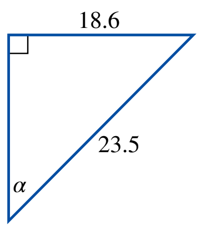
d



e

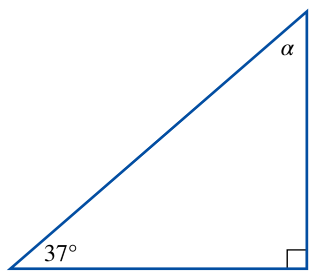


f



REASONING

9 Consider the right-angled triangle shown at right.



a Label each of the sides using the letters O, A and H with respect to the 37° angle.

b Determine the value of each trigonometric ratio. (Where applicable, answers should be given correct to 2 decimal places.)

i sin 37°

ii cos 37°

iii tan 37°

c What is the value of the unknown angle, α?

d Determine the value of each of these trigonometric ratios, correct to 2 decimal places.

i sin α

ii cos α

iii tan α

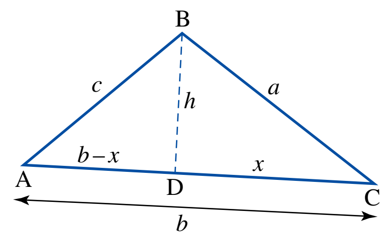
(*Hint:* First relabel the sides of the triangle with respect to angle α.)

e What do you notice about the relationship between sin 37° and cos α?
f What do you notice about the relationship between sin α and cos 37°?
g Make a general statement about the two angles.

10 Using a triangle labelled with a, h and o and algebra, show that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. (*Hint:* Write all the sides in terms of the hypotenuse.)

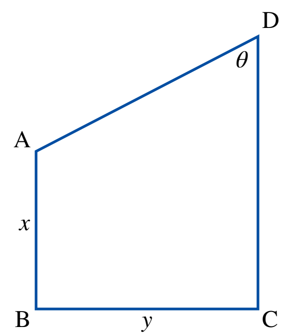
PROBLEM SOLVING

11 ABC is a scalene triangle with side lengths a, b and c as shown. Angles BDA and BDC are right angles.



a Express h^2 in terms of a and x.
b Express h^2 in terms of b, c and x.
c Equate the two equations for h^2 to show that $c^2 = a^2 + b^2 - 2bx$.
d Use your knowledge of trigonometry to produce the equation $c^2 = a^2 + b^2 - 2ab\cos C$, which is known as the cosine rule for non-right-angled triangles.

12 Find the length of the side DC in terms of x, y and θ.



5.5 Using trigonometry to calculate side lengths

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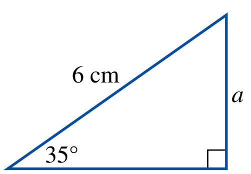
Interactivity
Using trigonometry
int-1146

In a right-angled triangle if one side length and one acute angle are known, the lengths of the other sides can be found by applying trigonometric ratios.

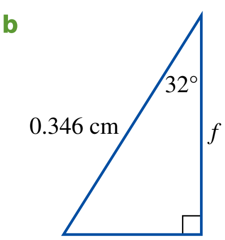
WORKED EXAMPLE 12

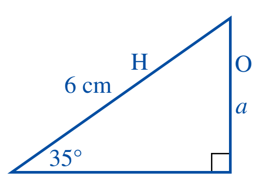
Find the value of each pronumeral giving answers correct to 3 decimal places.

a



b



THINK	WRITE/DRAW
a 1 Label the marked sides of the triangle.	**a** 

2 Identify the appropriate trigonometric ratio to use. — $\sin\ \theta = \dfrac{O}{H}$

3 Substitute O = a, H = 6 and θ = 35°. — $\sin 35° = \dfrac{a}{6}$

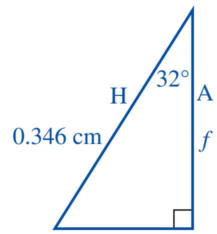
4 Make a the subject of the equation. — $6 \sin 35° = a$

$a = 6 \sin 35°$

5 Calculate and round the answer, correct to 3 decimal places. — $a \approx 3.441$ cm

b **1** Label the marked sides of the triangle.

b



2 Identify the appropriate trigonometric ratio to use. — $\cos\ \theta = \dfrac{A}{H}$

3 Substitute A = f, H = 0.346 and θ = 32°. — $\cos 32° = \dfrac{f}{0.346}$

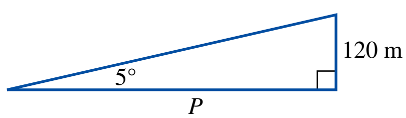
4 Make f the subject of the equation. — $0.346 \cos 32° = f$

$f = 0.346 \cos\ 32°$

5 Calculate and round the answer, correct to 3 decimal places. — ≈ 0.293 cm

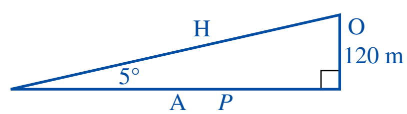
WORKED EXAMPLE 13 | TI | CASIO

Find the value of the pronumeral in the triangle shown. Give the answer correct to 2 decimal places.



THINK | **WRITE/DRAW**

1 Label the marked sides of the triangle.



2 Identify the appropriate trigonometric ratio to use. — $\tan \theta = \dfrac{O}{A}$

3 Substitute O = 120, A = P and θ = 5°. — $\tan 5° = \dfrac{120}{P}$

4 Make P the subject of the equation.
(i) Multiply both sides of the equation by P. — $P \times \tan 5° = 120$
(ii) Divide both sides of the equation by tan 5°. — $P = \dfrac{120}{\tan 5°}$

5 Calculate and round the answer, correct to 2 decimal places. — $P \approx 1371.61$ m

assess on

Exercise 5.5 Using trigonometry to calculate side lengths

REFLECTION
How does solving a trigonometric equation differ when we are finding the length of the hypotenuse side compared to when finding the length of a shorter side?

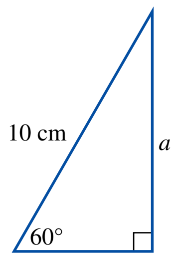
INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 8	Questions: 1–6, 8, 9	Questions: 1–10

Individual pathway interactivity int-4588 eBookplus

FLUENCY

1 WE12 Find the length of the unknown side in each of the following, correct to 3 decimal places.

a


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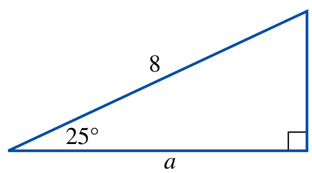

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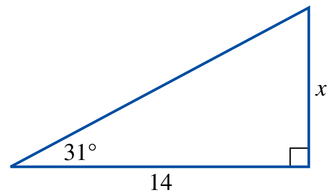
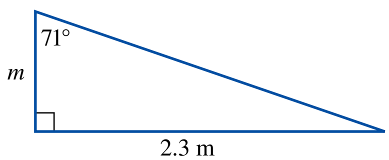


2 WE13 Find the length of the unknown side in each of the following triangles, correct to 2 decimal places.

a


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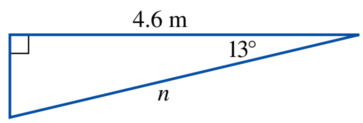

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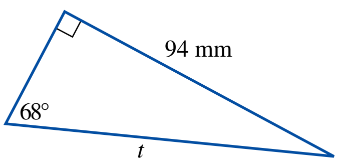
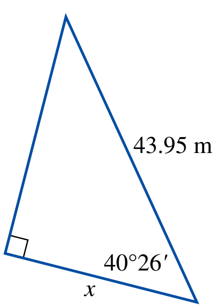


3 Find the length of the unknown side in each of the following, correct to 2 decimal places.

a


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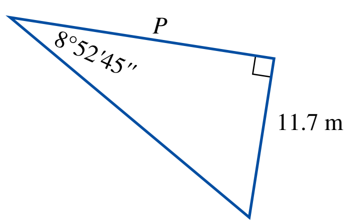

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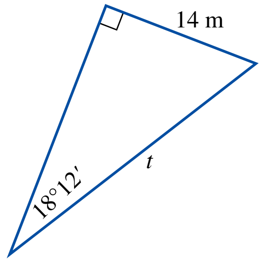
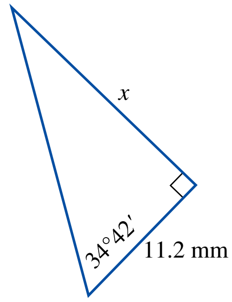


d



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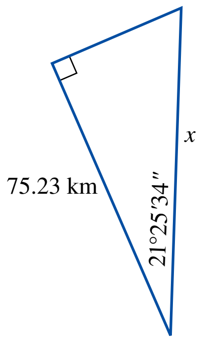
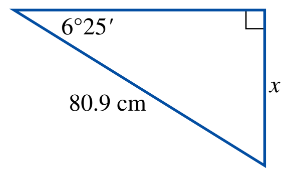
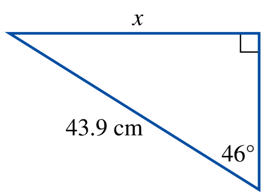


f

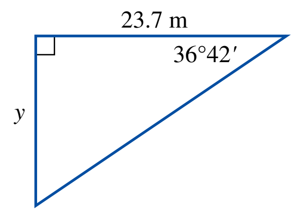


4 Find the value of the pronumeral in each of the following, correct to 2 decimal places.

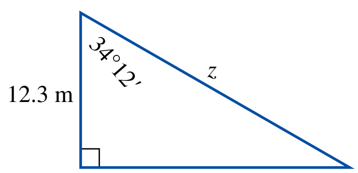
a



b



c



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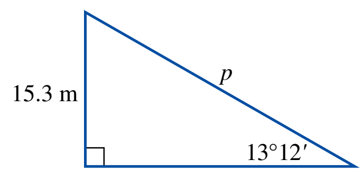

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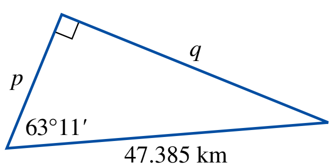

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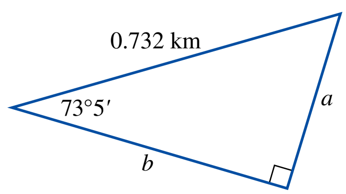


UNDERSTANDING

5 Given that the angle θ is 42° and the length of the hypotenuse is 8.95 m in a right-angled triangle, find the length of:

a the opposite side

b the adjacent side.

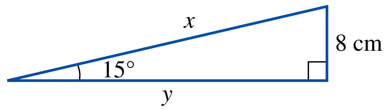
Give each answer correct to 1 decimal point.

6 A ladder rests against a wall. If the angle between the ladder and the ground is 35° and the foot of the ladder is 1.5 m from the wall, how high up the wall does the ladder reach?

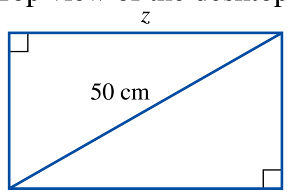
REASONING

7 Tran is going to construct an enclosed rectangular desktop that is at an incline of 15°. The diagonal length of the desktop is 50 cm. At the high end, the desktop, including top, bottom and sides, will be raised 8 cm. The desktop will be made of wood. The diagram below represents this information.

Side view of the desktop



Top view of the desktop



a Determine the values (in centimetres) of x, y and z of the desktop. Write your answers correct to 2 decimal places.

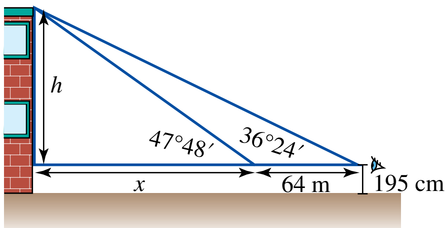
b Using your answer from part **a** determine the minimum area of wood, in cm^2, Tran needs to construct his desktop including top, bottom and sides. Write your answer correct to 2 decimal places.

8 a In a right-angled triangle, under what circumstances will the opposite side and the adjacent side have the same length?

b In a right-angled triangle, for what values of θ (the reference angle) will the adjacent side be longer than the opposite side?

PROBLEM SOLVING

9 A surveyor needs to determine the height of a building. She measures the angle of elevation of the top of the building from two points, 64 m apart. The surveyor's eye level is 195 cm above the ground.

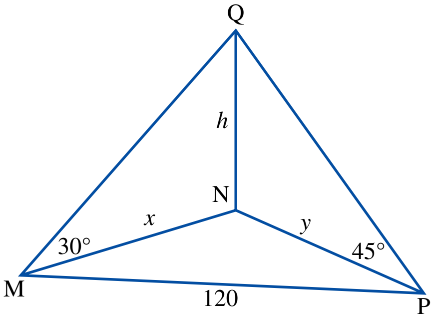


a Find the expressions for the height of the building, h, in terms of x using the two angles.

b Solve for x by equating the two expressions obtained in part **a**. Give your answer to 2 decimal places.

c Find the height of the building correct to 2 decimal places.

10 If angles QNM, QNP and MNP are right angles, find the length of NQ.



5.6 Using trigonometry to calculate angle size

- Just as inverse operations are used to solve equations, inverse trigonometric ratios are used to solve trigonometric equations for the value of the angle.
  – Inverse sine ($\sin^{-1}$) is the inverse of sine.
  – Inverse cosine ($\cos^{-1}$) is the inverse of cosine.
  – Inverse tangent ($\tan^{-1}$) is the inverse of tangent.
  For example, since $\sin(30°) = 0.5$, then $\sin^{-1}(0.5) = 30°$; this is read as 'inverse sine of 0.5 is 30 degrees'.

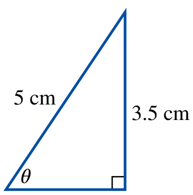
If $\sin\theta = a$, then $\sin^{-1} a = \theta$.
If $\cos\theta = a$, then $\cos^{-1} a = \theta$.
If $\tan\theta = a$, then $\tan^{-1} a = \theta$.

- A calculator can be used to calculate the values of inverse trigonometric ratios.
- The size of any angle in a right-angled triangle can be found if:
  – the lengths of any two sides are known
  – an appropriate trigonometric ratio is identified from the given lengths
  – a calculator is used to evaluate the inverse trigonometric ratio.

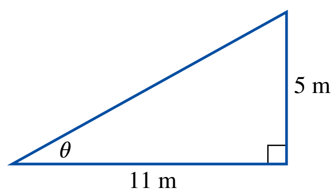
WORKED EXAMPLE 14

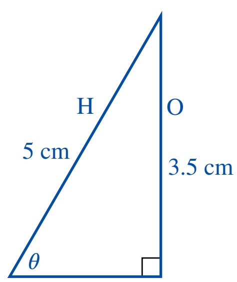
For each of the following, find the size of the angle, θ, correct to the nearest degree.

a



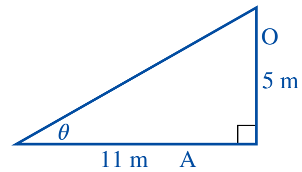
b



THINK	WRITE/DRAW
a 1 Label the given sides of the triangle.	**a** 
2 Identify the appropriate trigonometric ratio to use. We are given O and H.	$\sin\theta = \dfrac{O}{H}$
3 Substitute O = 3.5 and H = 5 and evaluate the expression.	$\sin\theta = \dfrac{3.5}{5}$ $= 0.7$
4 Make θ the subject of the equation using inverse sine.	$\theta = \sin^{-1} 0.7$ $= 44.427\,004°$
5 Evaluate θ and round the answer, correct to the nearest degree.	$\theta \approx 44°$

b **1** Label the given sides of the triangle.

b



2 Identify the appropriate trigonometric ratio to use. Given O and A.

$\tan\theta = \dfrac{O}{A}$

3 Substitute O = 5 and A = 11.

$\tan\theta = \dfrac{5}{11}$

4 Make θ the subject of the equation using inverse tangent.

$\theta = \tan^{-1}\left(\dfrac{5}{11}\right)$

$= 24.443\,954\,78°$

5 Evaluate θ and round the answer, correct to the nearest degree.

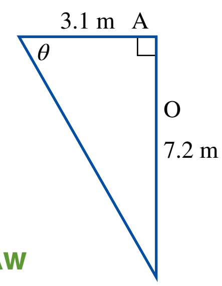
$\theta \approx 24°$

WORKED EXAMPLE 15 | TI | CASIO

Find the size of angle θ:

a correct to the nearest second

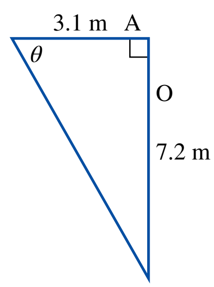
b correct to the nearest minute.



THINK | **WRITE/DRAW**

a **1** Label the given sides of the triangle.

a



2 Identify the appropriate trigonometric ratio to use.

$\tan\theta = \dfrac{O}{A}$

3 Substitute O = 7.2 and A = 3.1.

$\tan\theta = \dfrac{7.2}{3.1}$

4 Make θ the subject of the equation using inverse tangent.

$\theta = \tan^{-1}\left(\dfrac{7.2}{3.1}\right)$

5 Evaluate θ and write the calculator display.

$\theta = 66.705\,436\,75°$

6 Use the calculator to convert the answer to degrees, minutes and seconds.

$= 66°42'19.572''$

7 Round the answer to the nearest second.

$\theta \approx 66°\,42'20''$

b Round the answer to the nearest minute.

b $\theta \approx 66°\,42'$

Exercise 5.6 Using trigonometry to calculate angle size

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3–6, 8	Questions: 1–6, 8, 10	Questions: 1–11

Individual pathway interactivity int-4589 eBook*plus*

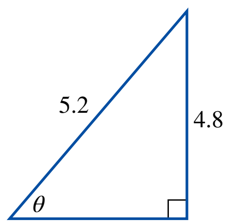
REFLECTION
How is finding the angle of a right-angled triangle different to finding a side length?

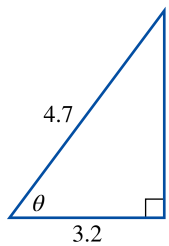
eBook*plus*

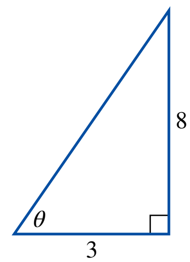
Digital doc
SkillSHEET
Rounding angles to the nearest degree
doc-5232

FLUENCY

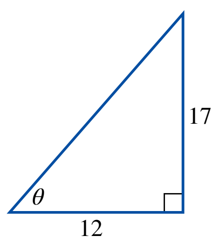
1 WE14 Find the size of the angle, θ, in each of the following. Give your answer correct to the nearest degree.

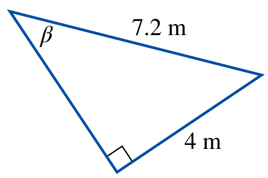
a


b


c


2 WE15b Find the size of the angle marked with the pronumeral in each of the following. Give your answer correct to the nearest minute.

a


b


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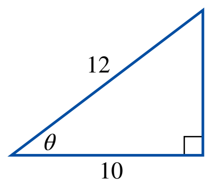
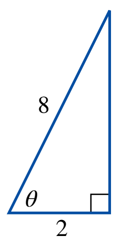


3 WE15a Find the size of the angle marked with the pronumeral in each of the following. Give your answer correct to the nearest second.

a


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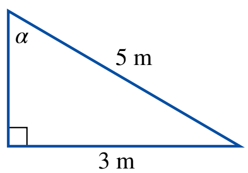

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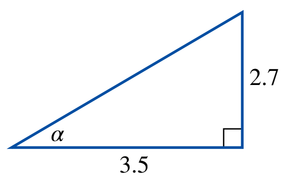
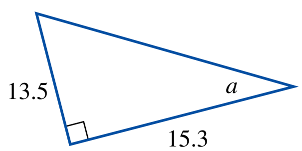
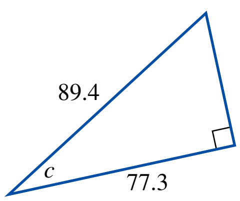


4 Find the size of the angle marked with the pronumeral in each of the following, giving your answer correct to the nearest degree.

a


b


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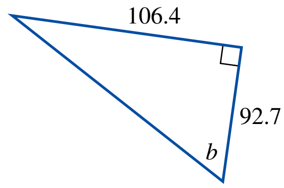
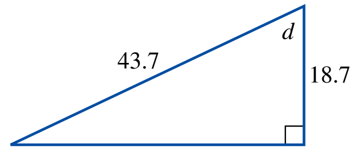
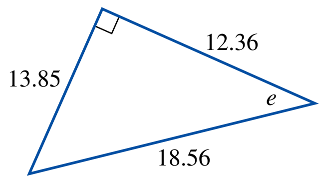


d



e



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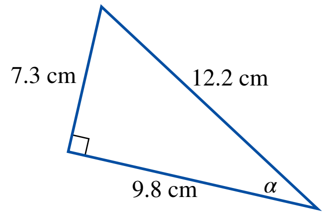
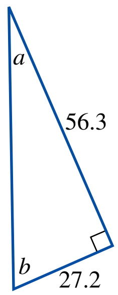
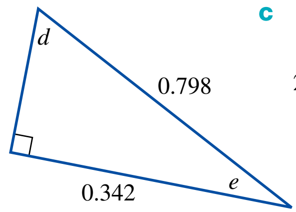


5 Find the size of each of the angles in the following, giving your answers correct to the nearest minute.

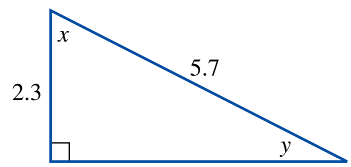
a



b



c

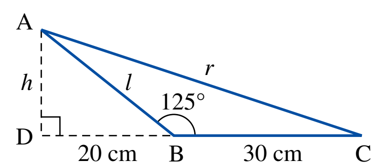


UNDERSTANDING

6 **a** Calculate the length of the sides r, l and h. Write your answers correct to 2 decimal places.

b Calculate the area of ABC, correct to the nearest square centimetre.

c Calculate ∠BCA.



7 In the sport of air racing, small planes have to travel between two large towers (or pylons). The gap between a pair of pylons is smaller than the wing-span of the plane, so the plane has to go through on an angle with one wing 'above' the other. The wing-span of a competition airplane is 8 metres.

a Determine the angle, correct to 1 decimal place, that the plane has to tilt if the gap between pylons is:

i 7 metres **ii** 6 metres **iii** 5 metres.

b Because the plane has rolled away from the horizontal as it travels between the pylons it loses speed. If the plane's speed is below 96 km/h it will stall and possibly crash. For each degree of 'tilt' the speed of the plane is reduced by 0.98 km/h. What is the minimum speed the plane must go through each of the pylons in part **a**? Write your answer correct to 2 decimal places.

REASONING

8 There are two important triangles commonly used in trigonometry. Complete the following steps and answer the questions to create these triangles.

Triangle 1

– Sketch an equilateral triangle with side length 2 units.
– Calculate the size of the internal angles.
– Bisect the triangle to form two right-angled triangles.
– Redraw one of the triangles formed.
– Calculate the side lengths of this right-angled triangle as exact values.
– Fully label your diagram showing all side lengths and angles.

Triangle 2

– Draw a right-angled isosceles triangle.
– Calculate the sizes of the internal angles.
– Let the sides of equal length be 1 unit long.
– Calculate the length of the third side.
– Fully label your diagram showing all side lengths and angles.

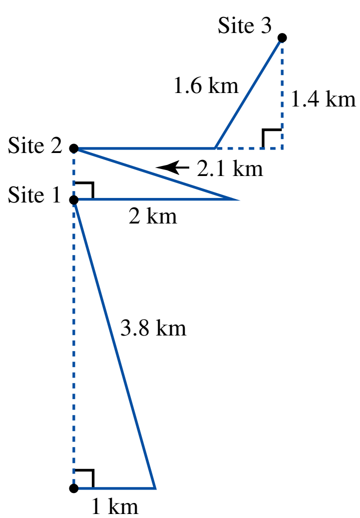
9 **a** Use the triangles formed in question **8** to calculate exact values for sin 30°, cos 30° and tan 30°. Justify your answers.

b Use the exact values for sin 30°, cos 30° and tan 30° to show that $\tan 30° = \dfrac{\sin 30°}{\cos 30°}$.

c Use the formulas $\sin\theta = \dfrac{o}{h}$ and $\cos\theta = \dfrac{a}{h}$ to prove that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

PROBLEM SOLVING

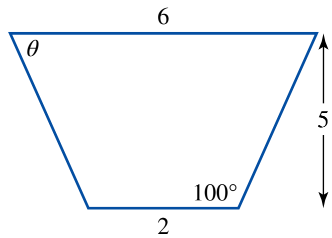
10 During a Science excursion, your class visited an underground cave to observe rock formations. You were required to walk along a series of paths and steps as shown in the diagram below.



a Calculate the angle of the incline (slope) you have to travel down between each site. Give your answers to the nearest whole number.

b Determine which path would have been the most challenging; that is, which path had the steepest slope.

11 Find the angle θ in degrees and minutes.



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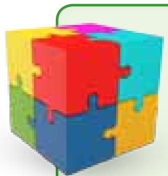
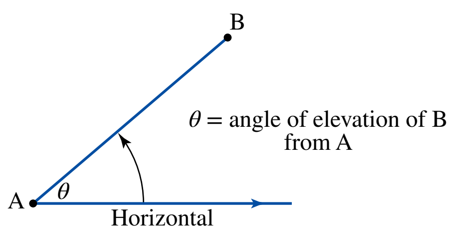


CHALLENGE 5.1

At midday, the hour hand and the minute hand on a standard clock are both pointing at the twelve. Calculate the angles the minute hand and the hour hand have moved 24.5 minutes later. Express both answers in degrees and minutes.

5.7 Angles of elevation and depression

- Consider the points A and B, where B is at a higher elevation than A.



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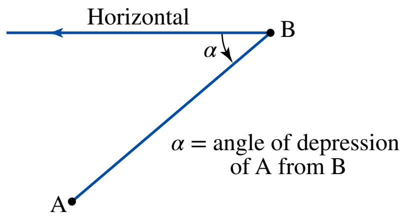
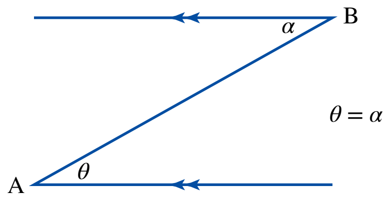
If a horizontal line is drawn from A as shown, forming the angle θ, then θ is called the **angle of elevation** of B *from* A.

- If a horizontal line is drawn from B, forming the angle α, then α is called the **angle of depression** of A *from* B.

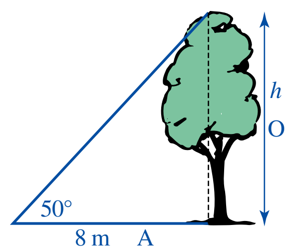


- Because the horizontal lines are parallel, θ and α have the same size (alternate angles).



WORKED EXAMPLE 16

From a point P, on the ground, the angle of elevation of the top of a tree is 50°. If P is 8 metres from the tree, find the height of the tree correct to 2 decimal places.

THINK	WRITE/DRAW
1 Let the height of the tree be h. Sketch a diagram and show the relevant information.	
2 Identify the appropriate trigonometric ratio.	$\tan\theta = \frac{O}{A}$
3 Substitute O = h, A = 8 and θ = 50°.	$\tan 50° = \frac{h}{8}$
4 Rearrange to make h the subject.	$h = 8\tan 50°$
5 Calculate and round the answer to 2 decimal places.	≈ 9.53
6 Give a worded answer.	The height of the tree is 9.53 m.

Exercise 5.7 Angles of elevation and depression

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 8, 10	Questions: 1–6, 9, 10, 14, 15	Questions: 1–7, 9, 11–16

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REFLECTION
What is the difference between an angle of elevation and an angle of depression?

FLUENCY

1 WE16 From a point P on the ground the angle of elevation from an observer to the top of a tree is 54°22′. If the tree is known to be 12.19 m high, how far is P from the tree (measured horizontally)?

2 From the top of a cliff 112 m high, the angle of depression to a boat is 9°15′. How far is the boat from the foot of the cliff?

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Drawing a diagram from given directions
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3 A person on a ship observes a lighthouse on the cliff, which is 830 metres away from the ship. The angle of elevation of the top of the lighthouse is 12°.

a How far above sea level is the top of the lighthouse?

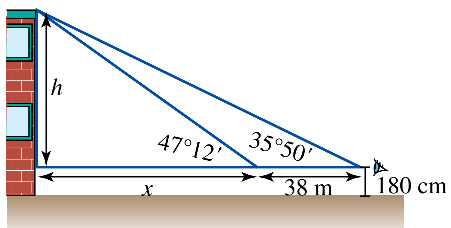
b If the height of the lighthouse is 24 m, how high is the cliff?

4 At a certain time of the day a post, 4 m tall, casts a shadow of 1.8 m. What is the angle of elevation of the sun at that time?

5 An observer who is standing 47 m from a building measures the angle of elevation of the top of the building as 17°. If the observer's eye is 167 cm from the ground, what is the height of the building?

UNDERSTANDING

6 A surveyor needs to determine the height of a building. She measures the angle of elevation of the top of the building from two points, 38 m apart. The surveyor's eye level is 180 cm above the ground.

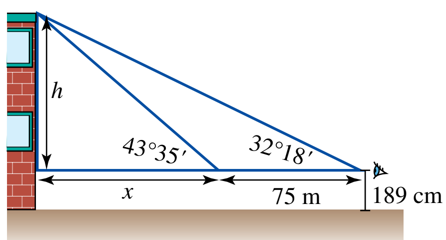


a Find two expressions for the height of the building, h, in terms of x using the two angles.

b Solve for x by equating the two expressions obtained in **a**.

c Find the height of the building.

7 The height of another building needs to be determined but cannot be found directly. The surveyor decides to measure the angle of elevation of the top of the building from different sites, which are 75 m apart. The surveyor's eye level is 189 cm above the ground.

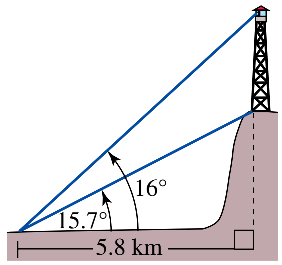


a Find two expressions for the height of the building, h, in terms of x using the two angles.

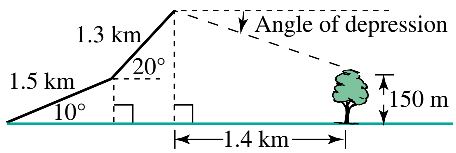
b Solve for x.

c Find the height of the building.

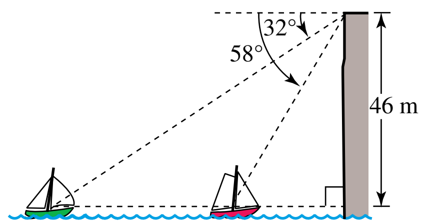
8 A lookout tower has been erected on top of a cliff. At a distance of 5.8 km from the foot of the cliff, the angle of elevation to the base of the tower is 15.7° and to the observation deck at the top of the tower is 16° respectively, as shown in the figure below. How high from the top of the cliff is the observation deck?



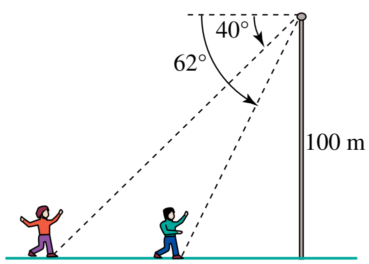
9 Elena and Sonja were on a camping trip to the Grampians, where they spent their first day hiking. They first walked 1.5 km along a path inclined at an angle of 10° to the horizontal. Then they had to follow another path, which was at an angle of 20° to the horizontal. They walked along this path for 1.3 km, which brought them to the edge of the cliff. Here Elena spotted a large gum tree 1.4 km away. If the gum tree is 150 m high, what is the angle of depression from the top of the cliff to the top of the gum tree?



10 From a point on top of a cliff, two boats are observed. If the angles of depression are 58° and 32° and the cliff is 46 m above sea level, how far apart are the boats?



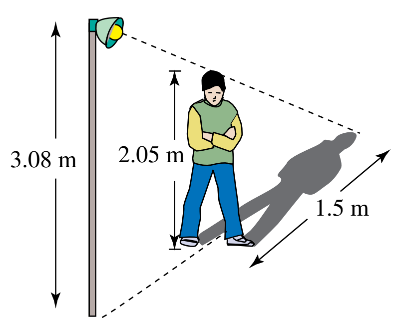
11 The competitors of a cross-country run are nearing the finish line. From a lookout 100 m above the track, the angles of depression to the two leaders, Nathan and Rachel, are 40° and 62° respectively. How far apart are the two competitors?



12 A 2.05 m tall man, standing in front of a street light 3.08 m high, casts a 1.5 m shadow.

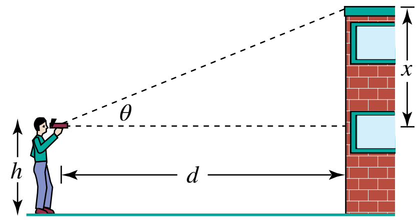
a What is the angle of elevation from the ground to the source of light?

b How far is the man from the bottom of the light pole?



REASONING

13 Joseph is asked to obtain an estimate of the height of his house using any mathematical technique. He decides to use an inclinometer and basic trigonometry. Using the inclinometer, Joseph determines the angle of elevation, θ, from his eye level to the top of his house to be 42°.



The point from which Joseph measures the angle of elevation is 15 m away from his house and the distance from Joseph's eyes to the ground is 1.76 m.

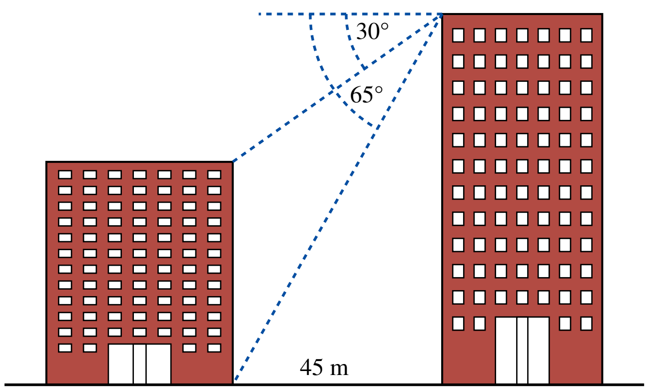
- **a** Fill in the given information on the diagram provided (substitute values for the pronumerals).
- **b** Determine the height of Joseph's house.

14 The angle of elevation of a vertically rising hot air balloon changes from 27° at 7.00 am to 61° at 7.03 am, according to an observer who is 300 m away from the take-off point.

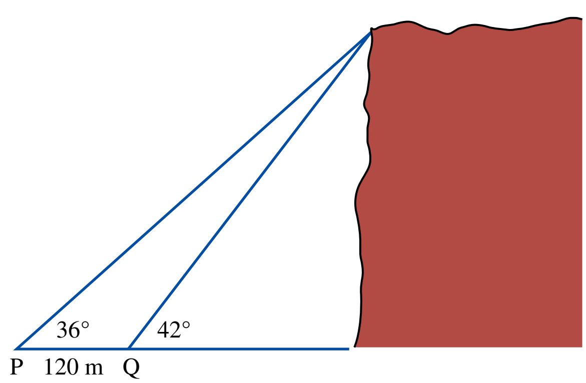
- **a** Assuming a constant speed, calculate that speed (in m/s and km/h) at which the balloon is rising, correct to 2 decimal places.
- **b** The balloon then falls 120 metres. What is the angle of elevation now? Write your answer correct to 1 decimal place.

PROBLEM SOLVING

15 The angle of depression from the top of one building to the foot of another building across the same street and 45 metres horizontally away is 65°. The angle of depression to the roof of the same building is 30°. Calculate the height of the shorter building.



16 P and Q are two points on a horizontal line that are 120 metres apart. The angles of elevation from P and Q to the top of a mountain are 36° and 42° respectively. Find the height of the mountain correct to 1 decimal place.



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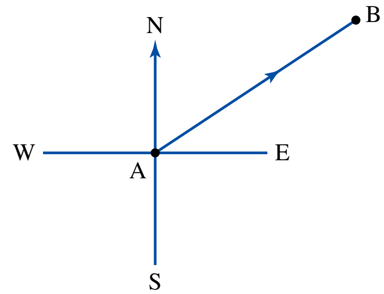
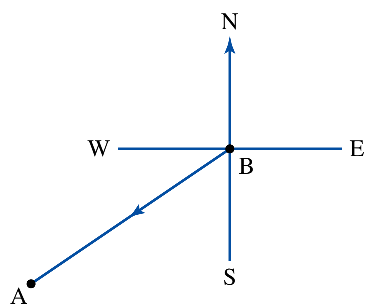
5.8 Bearings

- A bearing gives the direction of travel from one point or object to another.
- The bearing of B from A tells how to get to B *from* A. A compass rose would be drawn at A.



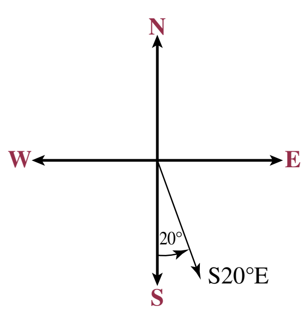
To illustrate the bearing of A *from* B, a compass rose would be drawn at B.

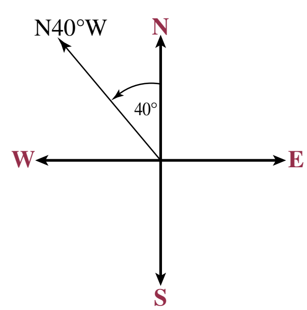


- There are two ways in which bearings are commonly written. They are compass bearings and true bearings.

Compass bearings

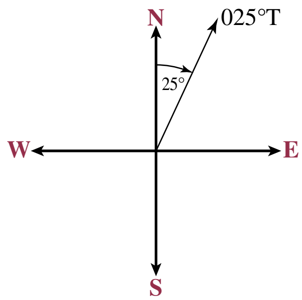
- A **compass bearing** (for example N40°E or S72°W) has three parts.
  – The first part is either N or S (for north or south).
  – The second part is an acute angle.
  – The third part is either E or W (for east or west).
- For example, the compass bearing S20°E means start by facing south and then turn 20° towards the east. This is the direction of travel.
  N40°W means start by facing north and then turn 40° towards the west.

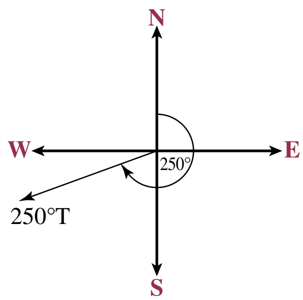




True bearings

- **True bearings** are measured from north in a clockwise direction and are expressed in 3 digits.
- The diagrams below show the bearings of 025° true and 250° true respectively. (These true bearings are more commonly written as 025°T and 250°T.)





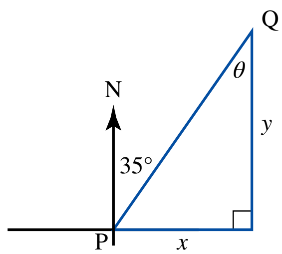
WORKED EXAMPLE 17

A boat travels a distance of 5 km from P to Q in a direction of 035°T.

a How far east of P is Q?

b How far north of P is Q?

c What is the true bearing of P from Q?

THINK		WRITE/DRAW
a	1 Draw a diagram showing the distance and bearing of Q from P. Complete a right-angled triangle travelling x km due east from P and then y km due north to Q.	a 
	2 To determine how far Q is east of P, we need to find the value of x. We are given the length of the hypotenuse (H) and need to find the length of the opposite side (O). Write the sine ratio.	$\sin\theta = \dfrac{O}{H}$
	3 Substitute O = x, H = 5 and θ = 35°.	$\sin 35° = \dfrac{x}{5}$
	4 Make x the subject of the equation.	$x = 5\sin 35°$
	5 Evaluate and round the answer, correct to 2 decimal places.	≈ 2.87
	6 Write the answer in words.	Point Q is 2.87 km east of P.
b	1 To determine how far Q is north of P, we need to find the value of y. This can be done in several ways, namely: using the cosine ratio, the tangent ratio, or Pythagoras' theorem. Write the cosine ratio.	b $\cos\theta = \dfrac{A}{H}$

2 Substitute A = y, H = 5 and θ = 35°. $\cos 35° = \dfrac{y}{5}$

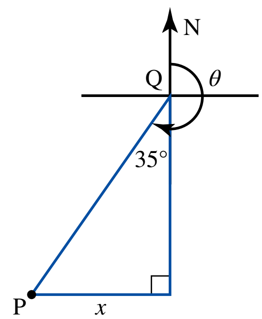
3 Make y the subject of the equation. $y = 5\cos 35°$

4 Evaluate and round the answer, correct to 2 decimal places. ≈ 4.10

5 Write the answer in words. Point B is 4.10 km north of A.

c **1** To find the bearing of P from Q, draw a compass rose at Q. The true bearing is given by $\angle\theta$.

c



2 The value of θ is the sum of 180° (from north to south) and 35°. Write the value of θ.

$$\begin{aligned}\text{True bearing} &= 180° + \alpha\\ \alpha &= 35°\\ \text{True bearing} &= 180° + 35°\\ &= 215°\end{aligned}$$

3 Write the answer in words. The bearing of P from Q is 215°T.

- Sometimes a journey includes a change in directions.
  In such cases, each section of the journey should be dealt with separately.

WORKED EXAMPLE 18

A boy walks 2 km on a true bearing of 090° and then 3 km on a true bearing of 130°.

a **How far east of the starting point is the boy at the completion of his walk? (Answer correct to 1 decimal place.)**

b **How far south of the starting point is the boy at the completion of his walk? (Answer correct to 1 decimal place.)**

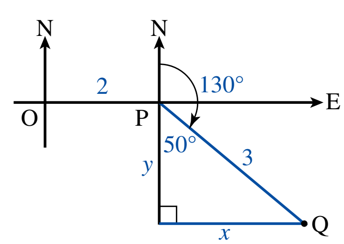
c **To return directly to his starting point, how far must the boy walk and on what bearing?**

THINK | **WRITE/DRAW**

a **1** Draw a diagram of the boy's journey.

The first leg of the journey is due east. Label the easterly component x and the southerly component y.

a



2 Write the ratio to find x. $\sin\theta = \dfrac{O}{H}$

3 Substitute O = x, H = 3 and θ = 50°. $\sin 50° = \dfrac{x}{3}$

4 Make x the subject of the equation.

$x = 3 \sin 50°$

5 Evaluate and round correct to 1 decimal place.

≈ 2.3 km

6 Add to this the 2 km east that was walked in the first leg of the journey and give a worded answer.

$$\begin{aligned}\text{Total distance east} &= 2 + 2.3 \\ &= 4.3 \text{ km}\end{aligned}$$

The boy is 4.3 km east of the starting point.

b 1 To find y (see the diagram in part **a**) we can use Pythagoras' theorem, as we know the lengths of two out of three sides in the right-angled triangle. Round the answer correct to 1 decimal place.
Note: Alternatively, the cosine ratio could have been used.

b Distance south $= y$ km

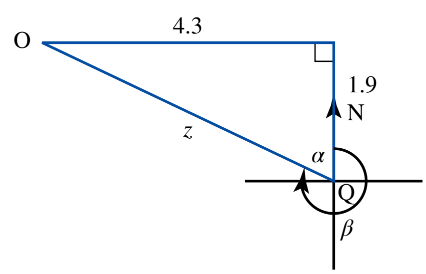
$$\begin{aligned}a^2 &= c^2 - b^2 \\ y^2 &= 3^2 - 2.3^2 \\ &= 9 - 5.29 \\ &= 3.71 \\ y &= \sqrt{3.71} \\ &= 1.9 \text{ km}\end{aligned}$$

2 Write the answer in words.

The boy is 1.9 km south of the starting point.

c 1 Draw a diagram of the journey and write in the results found in parts **a** and **b**. Draw a compass rose at Q.

c



2 Find z using Pythagoras' theorem.

$$\begin{aligned}z^2 &= 1.9^2 + 4.3^2 \\ &= 22.1 \\ z &= \sqrt{22.1} \\ &\approx 4.70\end{aligned}$$

3 Find α using trigonometry.

$$\tan \alpha = \frac{4.3}{1.9}$$

4 Make α the subject of the equation using the inverse tangent function.

$$\alpha = \tan^{-1}\left(\frac{4.3}{1.9}\right)$$

5 Evaluate and round to the nearest minute.

$$\begin{aligned}&= 66.161259\,82° \\ &= 66°9'40.535'' \\ &= 66°10'\end{aligned}$$

6 The angle β gives the bearing.

$$\begin{aligned}\beta &= 360° - 66°10' \\ &= 293°50'\end{aligned}$$

7 Write the answer in words.

The boy travels 4.70 km on a bearing of 293°50′T.

Exercise 5.8 Bearings

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3a–d, 4a–b, 5–7, 11	Questions: 1, 2, 3, 4a–c, 5–8, 11, 13	Questions: 1–6, 8–14

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REFLECTION
What is the difference between true bearings and compass directions?

FLUENCY

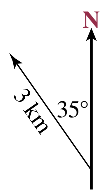
1 Change each of the following compass bearings to true bearings.

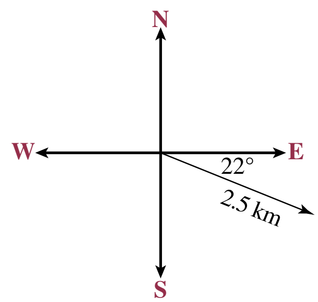
a N20°E **b** N20°W **c** S35°W
d S28°E **e** N34°E **f** S42°W

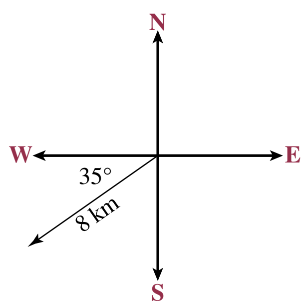
2 Change each of the following true bearings to compass bearings.

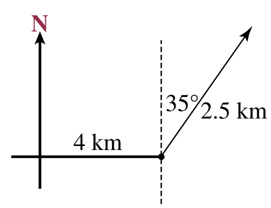
a 049°T **b** 132°T **c** 267°T
d 330°T **e** 086°T **f** 234°T

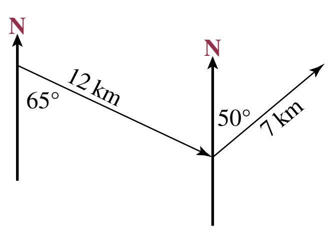
3 Describe the following paths using true bearings.

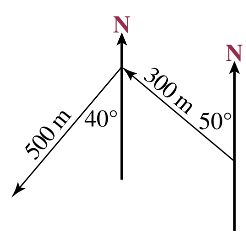
a


b


c


d


e


f


4 Show each of the following journeys as a diagram.

a A ship travels 040°T for 40 km and then 100°T for 30 km.
b A plane flies for 230 km in a direction 135°T and a further 140 km in a direction 240°T.
c A bushwalker travels in a direction 260°T for 0.8 km, then changes direction to 120°T for 1.3 km, and finally travels in a direction of 32° for 2.1 km.
d A boat travels N40°W for 8 km, then changes direction to S30°W for 5 km and then S50°E for 7 km.
e A plane travels N20°E for 320 km, N70°E for 180 km and S30°E for 220 km.

5 WE17 **a** A yacht travels 20 km from A to B on a bearing of 042°T:

   i how far east of A is B?
   ii how far north of A is B?
   iii what is the bearing of A from B?

b The yacht then sails 80 km from B to C on a bearing of 130°T.

   i Show the journey using a diagram.
   ii How far south of B is C?
   iii How far east of B is C?
   iv What is the bearing of B from C?

6 If a farmhouse is situated 220 m N35°E from a shed, what is the true bearing of the shed from the house?

UNDERSTANDING

7 A pair of hikers travel 0.7 km on a true bearing of 240° and then 1.3 km on a true bearing of 300°. How far west have they travelled from their starting point?

8 WE18 A boat travels 6 km on a true bearing of 120° and then 4 km on a true bearing of 080°.

a How far east is the boat from the starting point on the completion of its journey?
b How far south is the boat from the starting point on the completion of its journey?
c What is the bearing of the boat from the starting point on the completion of its journey?

9 A plane flies on a true bearing of 320° for 450 km. It then flies on a true bearing of 350° for 130 km and finally on a true bearing of 050° for 330 km. How far north of its starting point is the plane?

REASONING

10 A bushwalker leaves her tent and walks due east for 4.12 km, then walks a further 3.31 km on a bearing of N20°E. If she wishes to return directly to her tent, how far must she walk and what bearing should she take? (Answer to the nearest degree.)

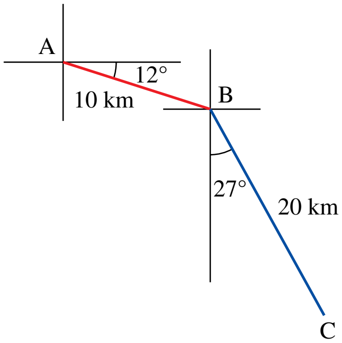
11 A car travels due south for 3 km and then due east for 8 km. What is the bearing of the car from its starting point? (Answer to the nearest degree.)

12 If the bearing of A from O is θ°T, then what is the bearing of O from A:

a if $0° < \theta° < 180°$ **b** if $180° < \theta° < 360°$?

PROBLEM SOLVING

13 A boat sails on a compass direction of E12°S for 10 km then changes direction to S27°E for another 20 km. The boat then decides to return to its starting point.



a How far, correct to 2 decimal places, is the boat from its starting point?

b On what bearing should the boat travel to return to its starting point? Write the angle correct to the nearest degree.

14 Samantha and Tim set off early from the car park of a national park to hike for the day. Initially they walk N 60º E for 12 km to see a spectacular waterfall. They then change direction and walk in a south-easterly direction for 6 km, then stop for lunch. Give all answers correct to 2 decimal places.

a Make a scale diagram of the hiking path they completed.

b How far north of the car park are they at the lunch stop?

c How far east of the car park are they at the lunch stop?

d What is the bearing of the lunch stop from the car park?

Samantha and Tim then walk directly back to the car park.

e Calculate the distance they have covered after lunch.

CHALLENGE 5.2

Starting from their base in the national park, a group of bushwalkers travel 1.5 km at a true bearing of 030°, then 3.5 km at a true bearing of 160°, and then 6.25 km at a true bearing of 300°. How far, and at what true bearing, should the group walk to return to its base?

5.9 Applications

- When applying trigonometry to practical situations, it is essential to draw good mathematical diagrams using points, lines and angles.
- Several diagrams may be required to show all the necessary right-angled triangles.

eBook*plus*

Interactivity
Drafting problems
int-2781

WORKED EXAMPLE 19 TI CASIO

A ladder of length 3 m makes an angle of 32° with the wall.

a How far is the foot of the ladder from the wall?

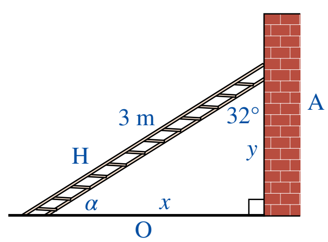
b How far up the wall does the ladder reach?

c What angle does the ladder make with the ground?

THINK

Sketch a diagram and label the sides of the right-angled triangle with respect to the given angle.

WRITE/DRAW



a **1** We need to find the distance of the foot of the ladder from the wall (O) and are given the length of the ladder (H). Write the sine ratio.

a $\sin\theta = \dfrac{O}{H}$

2 Substitute O = x, H = 3 and $\theta = 32°$.

$\sin 32° = \dfrac{x}{3}$

3 Make x the subject of the equation.

$x = 3\sin 32°$

4 Evaluate and round the answer to 2 decimal places.

≈ 1.59 m

5 Write the answer in words.

The foot of the ladder is 1.59 m from the wall.

b **1** We need to find the height the ladder reaches up the wall (A) and are given the hypotenuse (H). Write the cosine ratio.

b $\cos\theta = \dfrac{A}{H}$

2 Substitute A = y, H = 3 and $\theta = 32°$.

$\cos 32° = \dfrac{y}{3}$

3 Make y the subject of the equation.

$y = 3\cos 32°$

4 Evaluate and round the answer to 2 decimal places.

$y \approx 2.54$ m

5 Write the answer in words.

The ladder reaches 2.54 m up the wall.

c **1** To find the angle that the ladder makes with the ground, we could use any of the trigonometric ratios, as the lengths of all three sides are known. However, it is quicker to use the angle sum of a triangle.

c
$$\begin{aligned}\alpha + 90° + 32° &= 180°\\ \alpha + 122° &= 180°\\ \alpha &= 180° - 122°\\ \alpha &= 58°\end{aligned}$$

2 Write the answer in words.

The ladder makes a 58° angle with the ground.

assess on

Exercise 5.9 Applications

INDIVIDUAL PATHWAYS

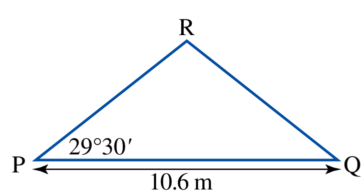
REFLECTION
What are some real-life applications of trigonometry?

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 8, 10, 15	Questions: 1–5, 8, 11, 13, 14, 16	Questions: 1, 3, 4, 6, 7, 9, 12–17

Individual pathway interactivity int-4592 eBook*plus*

FLUENCY

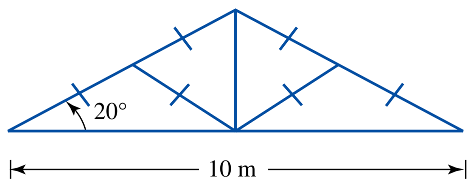
1 A carpenter wants to make a roof pitched at 29°30′, as shown in the diagram. How long should he cut the beam, PR?



2 The mast of a boat is 7.7 m high. A guy wire from the top of the mast is fixed to the deck 4 m from the base of the mast. Determine the angle the wire makes with the horizontal.

UNDERSTANDING

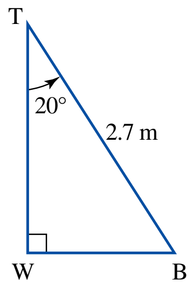
3 A steel roof truss is to be made to the following design.



a How high is the truss?

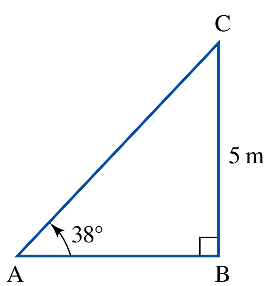
b What is the total length of steel required to make the truss?

4 **WE19** A ladder that is 2.7 m long is leaning against a wall at an angle of 20° as shown.



If the base of the ladder is moved 50 cm further away from the wall, what angle will the ladder make with the wall?

5 A wooden framework is built as shown.



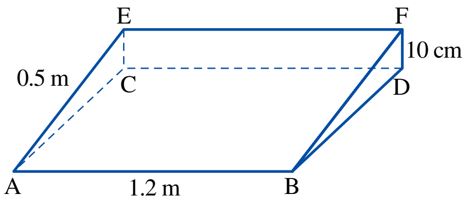
Bella plans to reinforce the framework by adding a strut from C to the midpoint of AB. What will be the length of the strut?

6 Atlanta is standing due south of a 20 m flagpole at a point where the angle of elevation of the top of the pole is 35°. Ginger is standing due east of the flagpole at a point where the angle of elevation of the top of the pole is 27°. How far is Ginger from Atlanta?

7 From a point at ground level, Henry measures the angle of elevation of the top of a tall building to be 41°. After walking directly towards the building, he finds the angle of elevation to be 75°. If the building is 220 m tall, how far did Henry walk between measurements?

8 Sailing towards a mountain peak of height 893 m, Imogen measured the angle of elevation to be 14°. A short time later the angle of elevation was 27°. How far had Imogen sailed in that time?

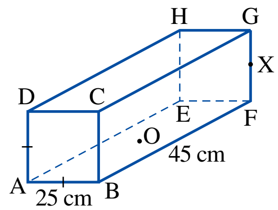
9 A desk top of length 1.2 m and width 0.5 m rises to 10 cm.



Calculate:

a ∠DBF **b** ∠CBE.

10 A cuboid has a square end.



a If the length of the cuboid is 45 cm and its height and width are 25 cm each, calculate:

i the length of BD **ii** the length of BG **iii** the length of BE

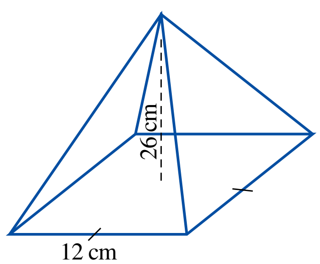
iv the length of BH **v** ∠FBG **vi** ∠EBH.

b If the midpoint of FG is X and the centre of the rectangle ABFE is O calculate:

i the length OF **ii** the length FX

iii ∠FOX **iv** the length OX.

11 In a right square-based pyramid, the length of the side of the base is 12 cm and the height is 26 cm.



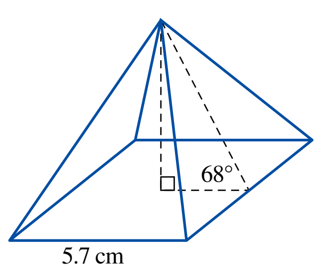
Determine:

a the angle the triangular face makes with the base

b the angle the sloping edge makes with the base

c the length of the sloping edge.

12 In a right square-based pyramid, the length of the side of the square base is 5.7 cm.



If the angle between the triangular face and the base is 68°, determine:

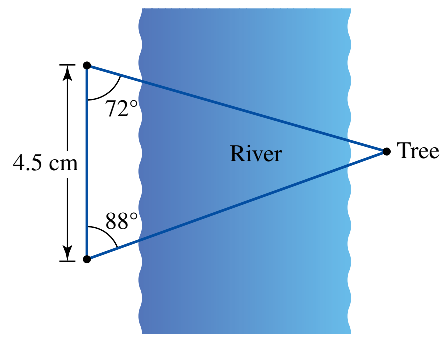
a the height of the pyramid
b the angle the sloping edge makes with the base
c the length of the sloping edge.

13 In a right square-based pyramid, the height is 47 cm. If the angle between a triangular face and the base is 73°, calculate:

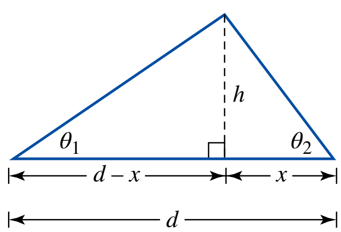
a the length of the side of the square base
b the length of the diagonal of the base
c the angle the sloping edge makes with the base.

REASONING

14 Aldo the carpenter is lost in a rainforest. He comes across a large river and he knows that he can not swim across it. Aldo intends to build a bridge across the river. He draws some plans to calculate the distance across the river as shown in the diagram below.



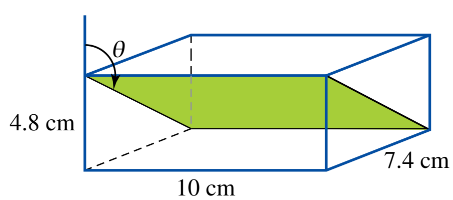
a Aldo used a scale of 1 cm to represent 20 m. Find the real-life distance represented by 4.5 cm in Aldo's plans.
b Use the diagram below to write an equation for h in terms of d and the two angles.



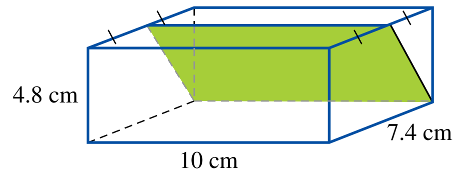
c Use your equation from **b** to find the distance across the river, correct to the nearest metre.

15 A block of cheese is in the shape of a rectangular prism as shown. The cheese is to be sliced with a wide blade that can slice it in one go. Calculate the angle (to the vertical) that the blade must be inclined if:

a the block is to be sliced diagonally into two identical triangular wedges

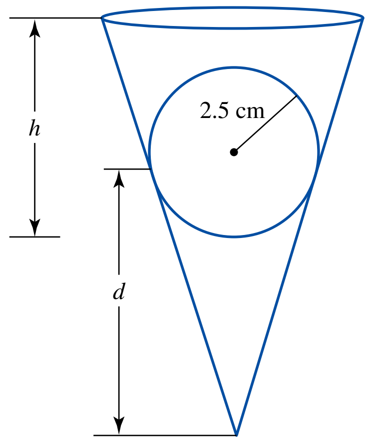


b the blade is to be placed in the middle of the block and sliced through to the bottom corner, as shown.



PROBLEM SOLVING

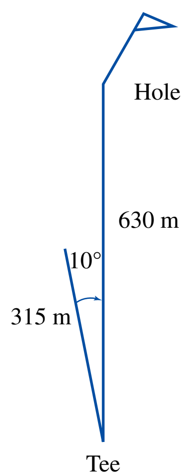
16 A sphere of radius length 2.5 cm rests in a hollow inverted cone as shown. The height of the cone is 12.5 cm and its vertical angle is equal to 36º.



a Find the distance, *d*, from the tip of the cone to the point of contact with the sphere.

b Find the distance, *h*, from the open end of the cone to the bottom of the ball.

17 The ninth hole on a municipal golf course is 630 m from the tee. A golfer drives a ball from the tee a distance of 315 m at a 10º angle off the direct line as shown.



Find how far the ball is from the hole and state the angle of the direct line that the ball must be hit along to go directly to the hole. Give your answers correct to 1 decimal place.

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5.10 Review

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The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

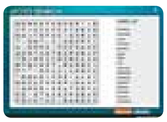
Review questions

Download the Review questions document from the links found in your eBookPLUS.

eBookplus

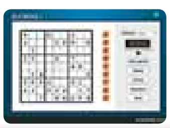
Interactivities

Word search
int-2838



Crossword
int-2839

Sudoku
int-3592



Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

adjacent	dimensions	pyramid
angle of depression	equilateral	Pythagoras' theorem
angle of elevation	horizontal	ratio
bearing	hypotenuse	second
compass rose	inverse	sine
cosine	isosceles	tangent
cuboid	minute	true bearing
degree	opposite	wedge

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

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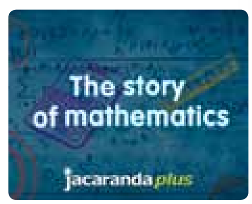
The story of mathematics is an exclusive Jacaranda video series that explores the history of mathematics and how it helped shape the world we live in today.



Hypatia (eles-1844) tells the fascinating and tragic story of an Egyptian woman who used her mathematical talents to make remarkably accurate astronomical observations and calculations in a male-dominated academic world.

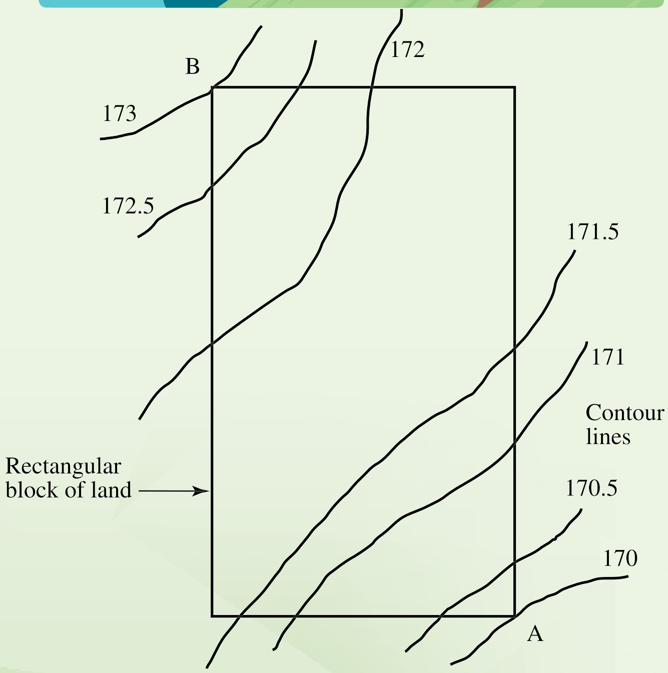
INVESTIGATION

RICH TASK

How steep is the land?

When buying a block of land on which to build a house, the slope of the land is often not very obvious. The slab of a house built on the ground must be level, so it is frequently necessary to remove or build up soil to obtain a flat area. The gradient of the land can be determined from a contour map of the area.

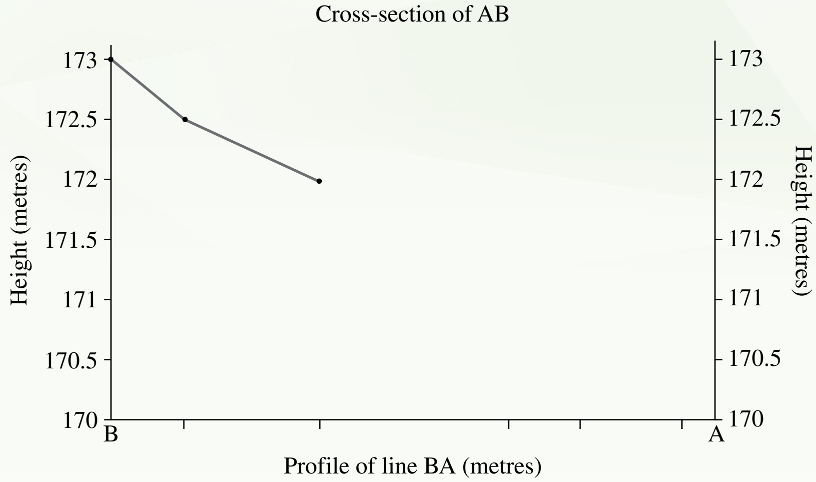
Consider the building block at right. The contour lines join points having the same height above sea level. Their measurements are in metres. The plan clearly shows that the land rises from A to B. The task is to determine the angle of this slope.



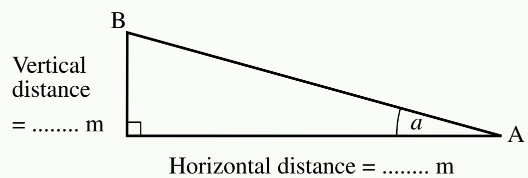
Scale 1 : 500

1 A cross-section shows a profile of the surface of the ground. Let us look at the cross-section of the ground between A and B. The technique used is as follows.
- Place the edge of a piece of paper on the line joining A and B.
- Mark the edge of the paper at the points where the contour lines intersect the paper.
- Transfer this paper edge to the horizontal scale of the profile and mark these points.
- Choose a vertical scale within the range of the heights of the contour lines.
- Plot the height at each point where a contour line crosses the paper.
- Join the points with a smooth curve.

The cross-section has been started for you. Complete the profile of the line AB.
You can now see a visual picture of the profile of the soil between A and B.

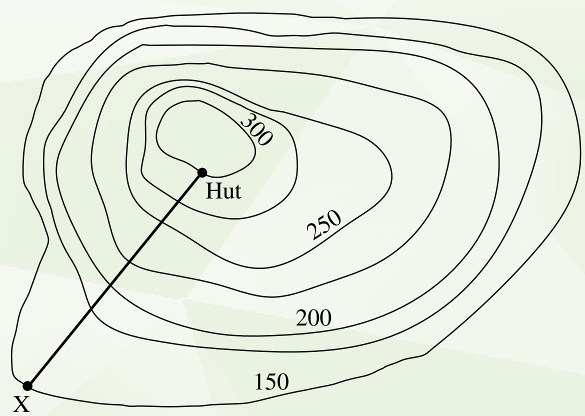


2 We now need to determine the horizontal distance between A and B.
  a Measure the map distance between A and B using a ruler. What is the map length?
  b Using the scale of 1:500, calculate the actual horizontal distance AB (in metres).
3 The vertical difference in height between A and B is indicated by the contour lines. What is this vertical distance?
4 Complete the measurements on this diagram.



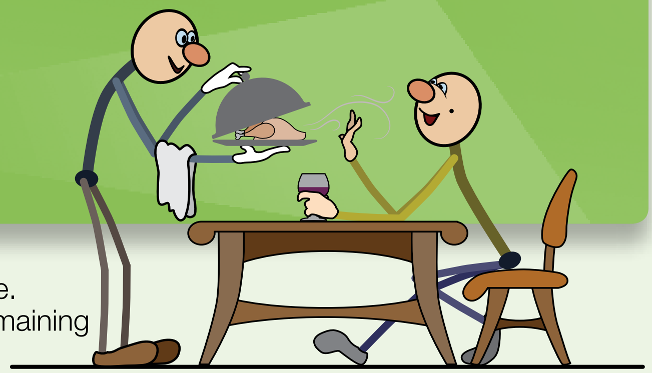
5 The angle *a* represents the angle of the average slope of the land from A to B. Use the tangent ratio to calculate this angle (to the nearest minute).
6 In general terms, an angle less than 5° can be considered a gradual to moderate rise. An angle between 5° and 15° is regarded as moderate to steep while more than 15° is a steep rise. How would you describe this block of land?
7 Imagine that you are going on a bush walk this weekend with a group of friends. At right is a contour map of the area. Starting at *X*, the plan is to walk directly to the hut.

  Draw a cross-section profile of the walk and calculate the average slope of the land.
  How would you describe the walk?



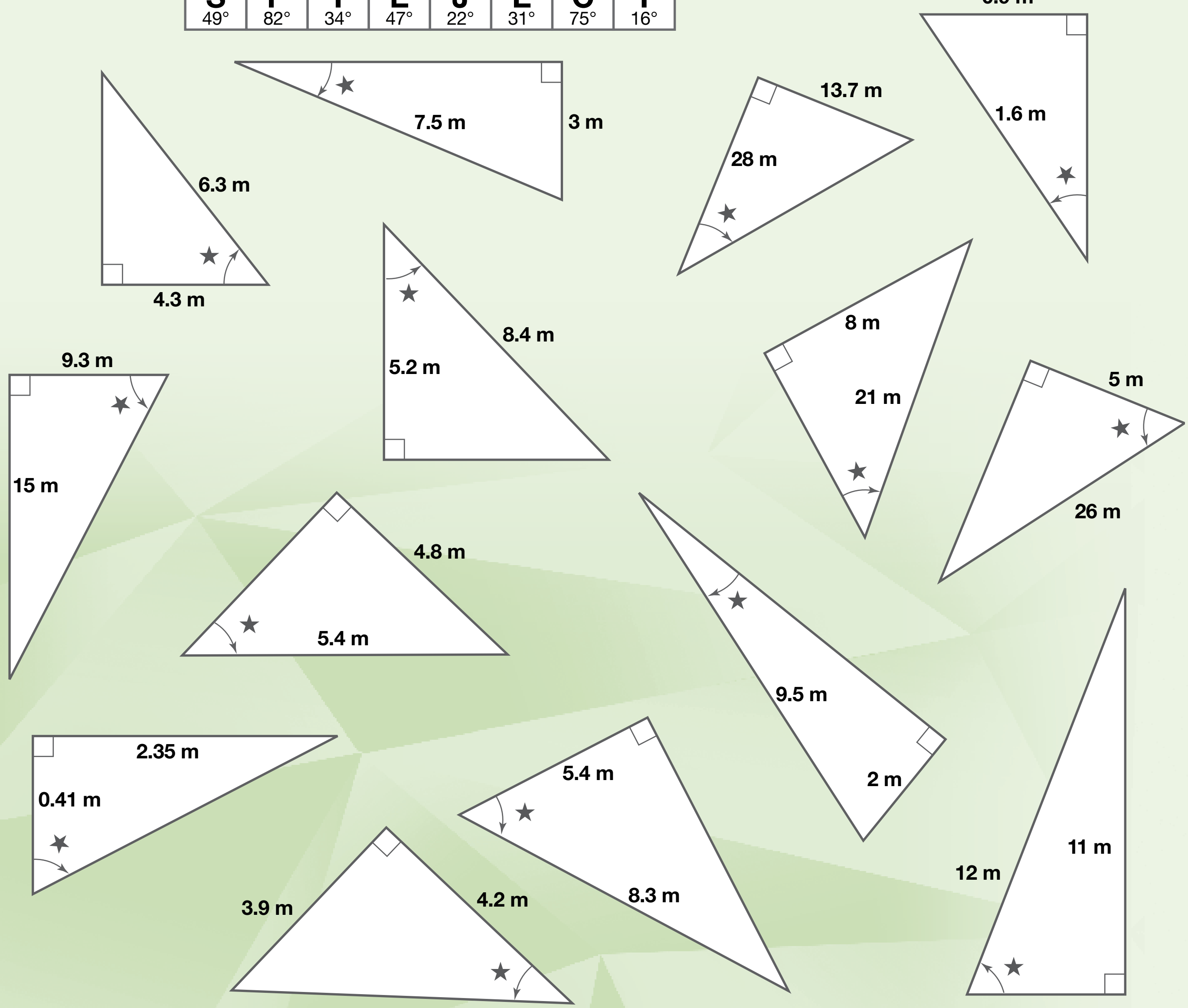
CODE PUZZLE

What will Sir have to follow the chicken?



Find the size of the angles marked ★ to the nearest degree. Shade in the box containing each of these angles. The remaining boxes contain letters that spell out the customer's reply.

T 43°	I 72°	N 29°	Y 52°	D 8°	R 58°	I 70°	P 26°	G 77°	U 80°	E 14°	S 69°	T 61°
L 12°	I 54°	O 45°	N 84°	I 19°	N 63°	T 66°	E 37°	X 60°	A 79°	S 24°		
S 49°	P 82°	Y 34°	E 47°	J 22°	E 31°	C 75°	T 16°					



eBookplus Activities

5.1 Overview

Video

- The story of mathematics (eles-1844)

5.2 Pythagoras' theorem

Interactivity

- IP interactivity 5.2 (int-4585): Pythagoras' theorem

Digital doc

- SkillSHEET (doc-5224): Rounding to a given number of decimal places

5.3 Pythagoras' theorem in three dimensions

Interactivity

- IP interactivity 5.3 (int-4586): Pythagoras' theorem in three dimensions

Digital docs

- SkillSHEET (doc-5229): Drawing 3-D shapes
- WorkSHEET 5.1 (doc-5230): Pythagoras' theorem

5.4 Trigonometric ratios

Interactivity

- IP interactivity 5.4 (int-4587): Trigonometric ratios

Digital docs

- SkillSHEET (doc-5226): Labelling the sides of a right-angled triangle
- SkillSHEET (doc-5231): Selecting an appropriate trigonometric ratio based on the given information

5.5 Using trigonometry to calculate side lengths

Interactivities

- Using trigonometry (int-1146)
- IP interactivity 5.5 (int-4588): Using trigonometry to calculate side lengths

5.6 Using trigonometry to calculate angle size

Interactivity

- IP interactivity 5.6 (int-4589): Using trigonometry to calculate angle size

Digital docs

- SkillSHEET (doc-5232): Rounding angles to the nearest degree
- WorkSHEET 5.2 (doc-5233): Using trigonometry

5.7 Angles of elevation and depression

eLesson

- Height of a satellite (eles-0173)

Interactivity

- IP interactivity 5.7 (int-4590): Angles of elevation and depression

Digital docs

- SkillSHEET (doc-5228): Drawing a diagram from given directions
- WorkSHEET 5.3 (doc-5234): Elevation and depression

5.8 Bearings

Interactivity

- IP interactivity 5.8 (int-4591): Bearings

5.9 Applications

Interactivities

- Drafting problems (int-2781)
- IP interactivity 5.9 (int-4592): Applications

5.10 Review

Interactivities

- Word search (int-2838)
- Crossword (int-2839)
- Sudoku (int-3592)

Digital docs

- Topic summary (doc-13719)
- Concept map (doc-13720)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 5 Trigonometry I

Exercise 5.2 — Pythagoras' theorem

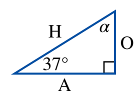
1 a 7.86 **b** 33.27 **c** 980.95
d 12.68 **e** 2.85 **f** 175.14
2 a 36.36 **b** 1.62 **c** 15.37
d 0.61 **e** 2133.19 **f** 453.90
3 23.04 cm
4 12.65 cm
5 a 14.14 cm **b** 24.04 cm **c** 4.53 cm
6 a 74.83 cm **b** 249.67 cm **c** 3741.66 cm^2
7 a 6.06 **b** 4.24 **c** 4.74
8 14.84 cm
9 15.59 cm
10 19.23 cm
11 72.75 cm; 3055.34 cm^2
12 39 m
13 4.34 km
14 38.2 m
15 63.06 mm
16 a 32 cm **b** 768 cm^2
17 26.83 diagonals, so would need to complete 27
18 4701.06 m
19 9.90 cm
20 a 65 **b** 185 **c** 305
21 a Neither 105 nor 208 can be the hypotenuse of the triangle, because they are the two smallest values. The other two values could be the hypotenuse if they enable the creation of a right-angled triangle.
b 105, 208, 233
22 a 21 cm **b** 35 cm
c $y = 12.6$ cm and RS = 9.8 cm
23 13.86 cm

Exercise 5.3 — Pythagoras' theorem in three dimensions

1 a 13.86 **b** 13.93 **c** 18.03
2 12.21, 12.85
3 4.84 m, 1.77 m
4 8.49, 4.24
5 31.62 cm
6 10.58 cm
7 23 cm
8 a i 233.24 cm **ii** 200.12 m **iii** 120.20 m
b 116.83 m
9 14.72 cm
10 12.67 cm
11 42.27 cm
12 1.49 m, 7.43 m^2
13 Students' own working
14 186.5 m
15 25.475
16 28.6 m

Exercise 5.4 — Trigonometric ratios

1 a 0.5000 **b** 0.7071 **c** 0.4663
d 0.8387 **e** 8.1443 **f** 0.7193
2 a 0.6494 **b** 0.5885 **c** 0.5220
d –1.5013 **e** 0.9990 **f** 0.6709
g 0.8120 **h** 0.5253 **i** –0.8031
j 0.4063 **k** 0.9880 **l** –0.9613
m 1.7321 **n** –0.5736 **o** 0.1320
3 a 50° **b** 24° **c** 53°
d 71° **e** 86° **f** 41°
4 a 54°29′ **b** 6°19′ **c** 0°52′
d 72°47′ **e** 44°48′ **f** 26°45′
5 a 26°33′54″ **b** 64°1′25″ **c** 64°46′59″
d 48°5′22″ **e** 36°52′12″ **f** 88°41′27″
6 a 2.824 **b** 71.014 **c** 20.361
d 2.828 **e** 226.735 **f** 1.192
g 7.232 **h** 32.259 **i** 4909.913
j 0.063 **k** 0.904 **l** 14.814
7 a i $\sin(\theta) = \frac{e}{f}$ **ii** $\cos(\theta) = \frac{d}{f}$ **iii** $\tan(\theta) = \frac{e}{d}$
b i $\sin(\alpha) = \frac{i}{g}$ **ii** $\cos(\alpha) = \frac{h}{g}$ **iii** $\tan(\alpha) = \frac{i}{h}$
c i $\sin(\beta) = \frac{l}{k}$ **ii** $\cos(\beta) = \frac{j}{k}$ **iii** $\tan(\beta) = \frac{l}{j}$
d i $\sin(\gamma) = \frac{n}{m}$ **ii** $\cos(\gamma) = \frac{o}{m}$ **iii** $\tan(\gamma) = \frac{n}{o}$
e i $\sin(\beta) = \frac{b}{c}$ **ii** $\cos(\beta) = \frac{a}{c}$ **iii** $\tan(\beta) = \frac{b}{a}$
f i $\sin(\gamma) = \frac{v}{u}$ **ii** $\cos(\gamma) = \frac{t}{u}$ **iii** $\tan(\gamma) = \frac{v}{t}$
8 a $\sin(\theta) = \frac{15}{18}$ **b** $\cos(\theta) = \frac{22}{30}$ **c** $\tan(\theta) = \frac{7}{9}$
d $\tan(\theta) = \frac{3.6}{p}$ **e** $\sin(25°) = \frac{13}{t}$ **f** $\sin(\alpha) = \frac{18.6}{23.5}$
9 a



b i $\sin(37°) = 0.60$ **ii** $\cos(37°) = 0.75$
iii $\tan(37°) = 0.80$
c $\alpha = 53°$
d i $\sin(53°) = 0.80$ **ii** $\cos(53°) = 0.60$
iii $\tan(53°) = 1.33$
e They are equal.
f They are equal.
g The sin of an angle is equal to the cos of its complement angle.
10 $\sin(\theta) = \frac{\text{opp}}{\text{hyp}}, \cos(\theta) = \frac{\text{adj}}{\text{hyp}} \Rightarrow \frac{\sin(\theta)}{\cos(\theta)} = \frac{\text{opp}}{\text{adj}} = \tan(\theta)$
11 a $h^2 = a^2 - x^2$ **b** $h^2 = c^2 - b^2 + 2bx - x^2$
c Teacher to check **d** Teacher to check
12 $\text{DC} = x + \frac{y}{\tan(\theta)}$

Exercise 5.5 — Using trigonometry to calculate side lengths

1 a 8.660 **b** 7.250 **c** 8.412
2 a 0.79 **b** 4.72 **c** 101.38
3 a 33.45 m **b** 74.89 m **c** 44.82 m
d 7.76 mm **e** 80.82 km **f** 9.04 cm
4 a $x = 31.58$ cm **b** $y = 17.67$ m
c $z = 14.87$ m **d** $p = 67.00$ m
e $p = 21.38$ km, $q = 42.29$ km **f** $a = 0.70$ km, $b = 0.21$ km
5 a 6.0 m **b** 6.7 m
6 1.05 m
7 a $x = 30.91$ cm, $y = 29.86$ cm, $z = 39.30$ cm
b 2941.54 cm^2

8 a In an isosceles right-angled triangle
b $\theta < 45°$
9 a $h = \tan(47°48')x$ m
$h = \tan(36°24')\,(x + 64)$ m
b 129.07 m **c** 144.20 m
10 60

Exercise 5.6 — Using trigonometry to calculate angle size

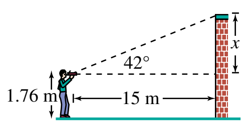
1 a 67° **b** 47° **c** 69°
2 a 54°47′ **b** 33°45′ **c** 33°33′
3 a 75°31′21″ **b** 36°52′12″ **c** 37°38′51″
4 a 41° **b** 30° **c** 49°
d 65° **e** 48° **f** 37°
5 a $a = 25°47'$, $b = 64°13'$ **b** $d = 25°23'$, $e = 64°37'$
c $x = 66°12'$, $y = 23°48'$
6 a $r = 57.58$, $l = 34.87$, $h = 28.56$
b 428 cm^2 **c** 29.7°
7 a i 29.0° **ii** 41.4° **iii** 51.3°
b i 124.42 km/h **ii** 136.57 km/h **iii** 146.27 km/h
8 Answers will vary.
9 a $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{\sqrt{3}}{3}$
b, c Answers will vary.
10 a Between site 3 and site 2: 61°
Between site 2 and site 1: 18°
Between site 1 and bottom: 75°
b Between site 1 and bottom: 75° slope
11 31°57′

Challenge 5.1

147°0′; 12°15′

Exercise 5.7 — Angles of elevation and depression

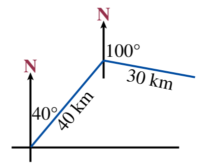
1 8.74 m
2 687.7 m
3 a 176.42 m **b** 152.42 m
4 65°46′
5 16.04 m
6 a $h = x\tan(47°12')$ m; $h = (x + 38)\tan(35°50')$ m
b $x = 76.69$ m **c** 84.62 m
7 a $h = x\tan(43°35')$ m; $h = (x + 75)\tan(32°18')$ m
b 148.40 m **c** 141.1 m
8 0.033 km or 33 m
9 21°
10 44.88 m
11 66 m
12 a 54° **b** 0.75 m
13 a



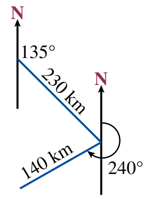
b 15.27 m
14 a 2.16 m/s, 7.77 km/h **b** 54.5°
15 70.522 m
16 451.5 m

Exercise 5.8 — Bearings

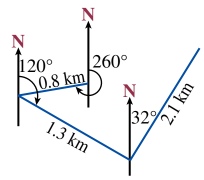
1 a 020°T **b** 340°T **c** 215°T
d 152°T **e** 034°T **f** 222°T
2 a N49°E **b** S48°E **c** S87°W
d N30°W **e** N86°E **f** S54°W
3 a 3 km 325°T
b 2.5 km 112°T
c 8 km 235°T
d 4 km 090°T, then 2.5 km 035°T
e 12 km 115°T, then 7 km 050°T
f 300 m 310°T, then 500 m 220°T
4 a



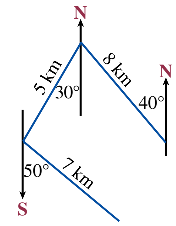
b



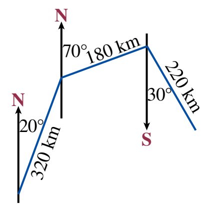
c



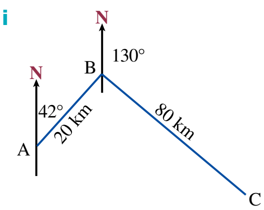
d



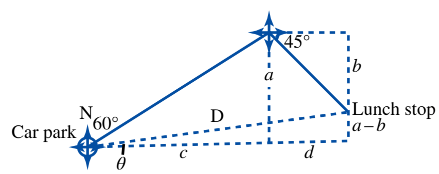
e



5 a i 13.38 km **ii** 14.86 km **iii** 222°T
b i



ii 51.42 km
iii 61.28 km
iv 310°T
6 215°T
7 1.732 km
8 a 9.135 km **b** 2.305 km **c** 104°10′T
9 684.86 km
10 6.10 km and 239°T
11 111°T
12 a $(180 + \theta)$°T **b** $(\theta - 180)$°T
13 a 27.42 km **b** N43°W or 317°T
14 a



b 1.76 km North **c** 14.63 km East
d N83.15° E **e** D = 14.74 km

Challenge 5.2

3.65 km on a bearing of 108°T

Exercise 5.9 — Applications

1 6.09 m
2 62°33′
3 a 1.82 m **b** 27.78 m
4 31°49′
5 5.94 m
6 49 m
7 194 m
8 1.829 km
9 a 11°32′ **b** 4°25′
10 a i 35.36 cm **ii** 51.48 cm **iii** 51.48 cm
iv 57.23 cm **v** 29°3′ **vi** 25°54′

b **i** 25.74 cm **ii** 12.5 cm **iii** 25°54′
iv 28.61 cm

11 a 77° **b** 71°56′ **c** 27.35 cm

12 a 7.05 cm **b** 60°15′ **c** 8.12 cm

13 a 28.74 cm **b** 40.64 cm **c** 66°37′

14 a 90 m

b $h = \dfrac{d \tan \theta_1}{\tan \theta_1 + \tan \theta_2} \times \tan \theta_2$

c 250 m

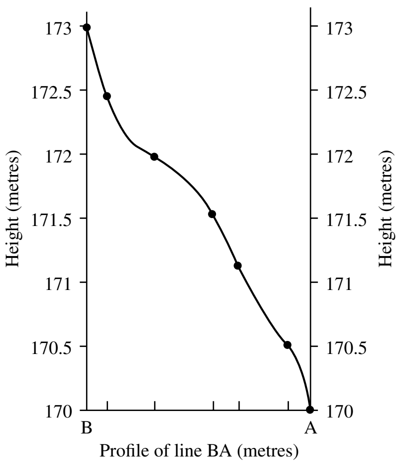
15 a 122.97° **b** 142.37°

16 a 8.09 cm **b** 6.91 cm

17 Golfer must hit the ball 324.4 m at an angle of 9.7° off the direct line.

Investigation — Rich task

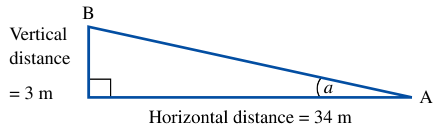
1



2 a 6.8 cm **b** 34 m

3 3 m

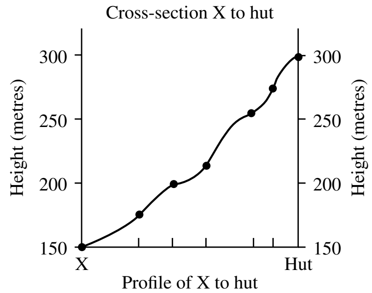
4



5 $a = 5°3'$

6 Moderate to steep

7



The average slope is 14.04° — moderate to steep.

Code puzzle

Indigestion, I expect.