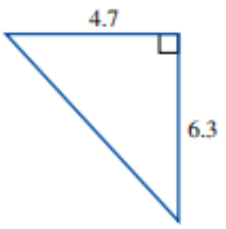
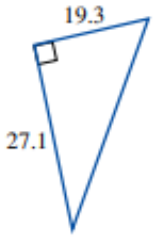
Exercise 5.2 Pythagoras' theorem

1 WE1 For each of the following triangles, calculate the length of the hypotenuse, giving answers correct to 2 decimal places.

a

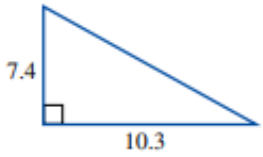


b



c

d



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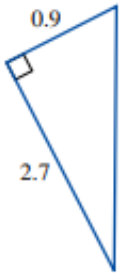
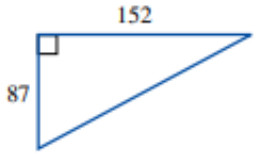
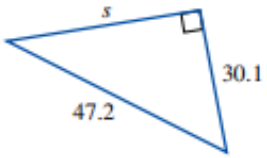


f

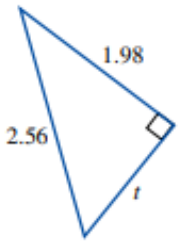


2 WE2 Find the value of the pronumeral, correct to 2 decimal places.

a



b



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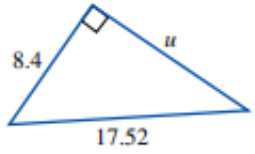
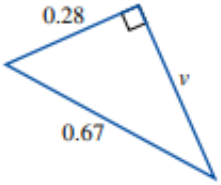
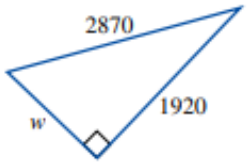


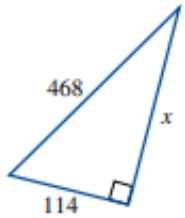
d



e



f



3 WE3 The diagonal of the rectangular sign at right is 34 cm. If the height of this sign is 25 cm, find the width.

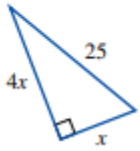
4 A right-angled triangle has a base of 4 cm and a height of 12 cm. Calculate the length of the hypotenuse to 2 decimal places.

6 The diagonal of a rectangle is 90 cm. One side has a length of 50 cm. Determine:

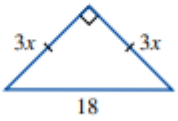
- **a** the length of the other side
- **b** the perimeter of the rectangle
- **c** the area of the rectangle.

7 WE4 Find the value of the pronumeral, correct to 2 decimal places for each of the following.

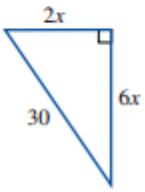
a



b



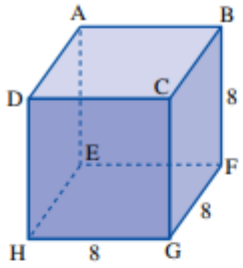
c



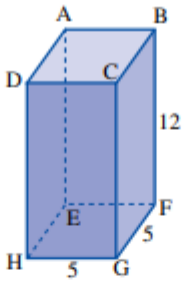
Exercise 5.3 Pythagoras' theorem in three dimensions

1 WE5 Calculate the length of AG in each of the following figures.

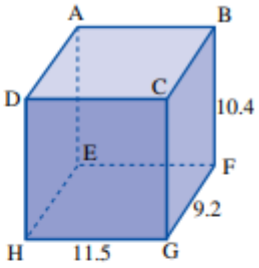
a



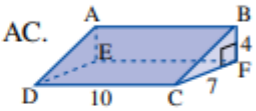
b



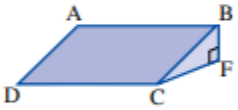
c



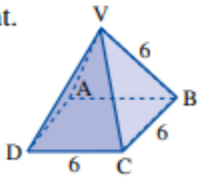
2 Calculate the length of CE in the wedge at right and, hence, obtain AC.



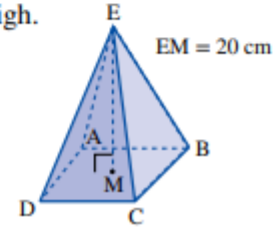
3 If DC = 3.2 m, AC = 5.8 m, and CF = 4.5 m in the figure at right, calculate the length of AD and BF.



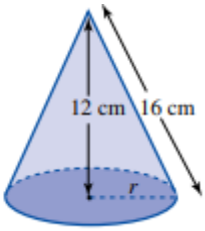
4 Calculate the length of BD and, hence, the height of the pyramid at right.



5 The pyramid ABCDE has a square base. The pyramid is 20 cm high. Each sloping edge measures 30 cm. Calculate the length of the sides of the base.

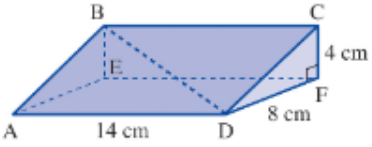


6 The sloping side of a cone is 16 cm and the height is 12 cm. What is the length of the radius of the base?



UNDERSTANDING

7. WE6 A piece of cheese in the shape of a right-angled wedge sits on a table. It has a base measuring 20 mm by 10 mm, and is 4 mm high at the thickest point, as shown in the figure. A fly crawls diagonally across the sloping face. How far, to the nearest millimetre, does the fly walk?



Exercise 5.4 Trigonometric ratios

1 Calculate each of the following, correct to 4 decimal places.

a $\sin 30°$ **b** $\cos 45°$ **c** $\tan 25°$

d $\sin 57°$ **e** $\tan 83°$ **f** $\cos 44°$

2 WE7 Calculate each of the following, correct to 4 decimal places.

a $\sin 40°30'$ **b** $\cos 53°57'$ **c** $\tan 27°34'$

d $\tan 123°40'$ **e** $\sin 92°32'$ **f** $\sin 42°8'$

g $\cos 35°42'35''$ **h** $\tan 27°42'50''$ **i** $\cos 143°25'23''$

j $\sin 23°58'21''$ **k** $\cos 8°54'2''$ **l** $\sin 286°$

m $\tan 420°$ **n** $\cos 845°$ **o** $\sin 367°35'$

3 WE8 Find the size of angle θ, correct to the nearest degree, for each of the following.

a $\sin\theta = 0.763$ **b** $\cos\theta = 0.912$ **c** $\tan\theta = 1.351$

d $\cos\theta = 0.321$ **e** $\tan\theta = 12.86$ **f** $\cos\theta = 0.756$

4 WE9a Find the size of the angle θ, correct to the nearest minute.

a $\sin\theta = 0.814$ **b** $\sin\theta = 0.110$ **c** $\tan\theta = 0.015$

d $\cos\theta = 0.296$ **e** $\tan\theta = 0.993$ **f** $\sin\theta = 0.450$

5 WE9b Find the size of the angle θ, correct to the nearest second.

a $\tan\theta = 0.5$ **b** $\cos\theta = 0.438$ **c** $\sin\theta = 0.9047$

d $\tan\theta = 1.1141$ **e** $\cos\theta = 0.8$ **f** $\tan\theta = 43.76$

6 Find the value of each expression, correct to 3 decimal places.

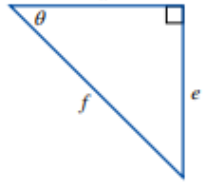
a $3.8\cos 42°$ **b** $118\sin 37°$ **c** $2.5\tan 83°$

d $\dfrac{2}{\sin 45°}$ **e** $\dfrac{220}{\cos 14°}$ **f** $\dfrac{2\cos 23°}{5\sin 18°}$

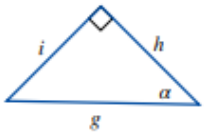
7 WE10 For each labelled angle in the following triangles, write an expression for:

i sine **ii** cosine **iii** tangent.

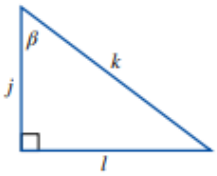
a



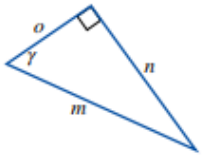
b



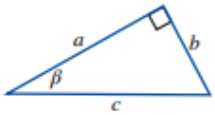
c



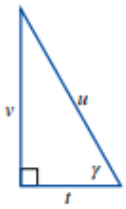
d



e

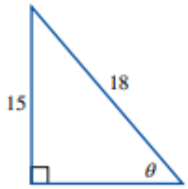


f

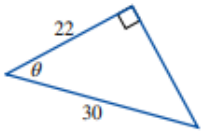


8 WE11 Write the equation that relates the two marked sides and the marked angle in each of the following triangles.

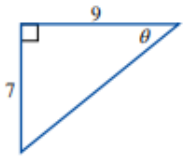
a



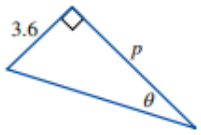
b



c



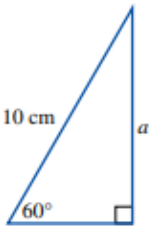
d



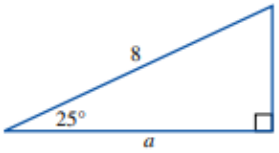
Exercise 5.5 Using trigonometry to calculate side lengths

1 WE12 Find the length of the unknown side in each of the following, correct to 3 decimal places.

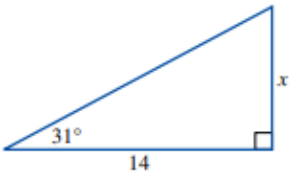
a



b

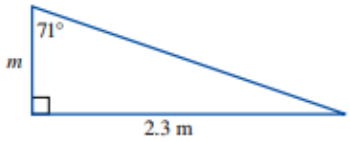


c



2 WE13 Find the length of the unknown side in each of the following triangles, correct to 2 decimal places.

a



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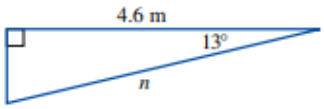
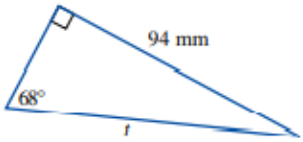
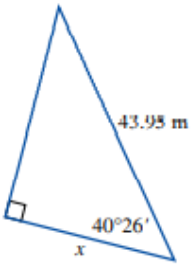


c

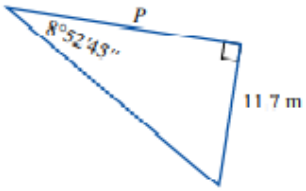


3 Find the length of the unknown side in each of the following, correct to 2 decimal places.

a



b



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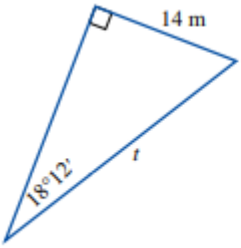
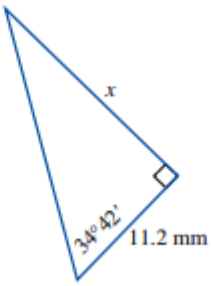
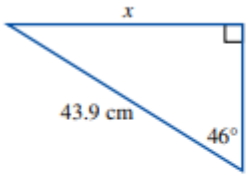


d



4 Find the value of the pronumeral in each of the following, correct to 2 decimal places.

a



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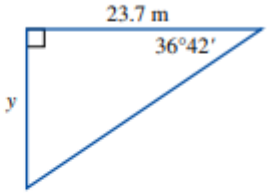
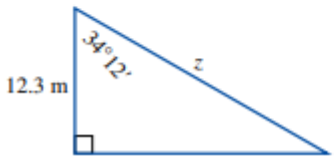
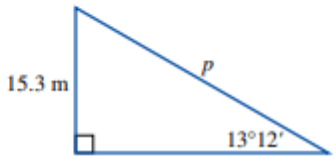


c



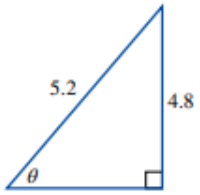
d



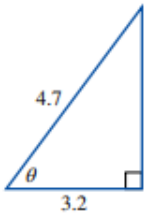
Exercise 5.6 Using trigonometry to calculate angle size

1 WE14 Find the size of the angle, θ, in each of the following. Give your answer correct to the nearest degree.

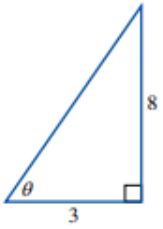
a



b

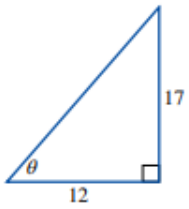


c

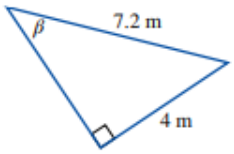


2 WE15b Find the size of the angle marked with the pronumeral in each of the following. Give your answer correct to the nearest minute.

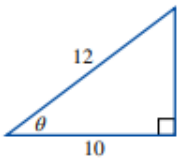
a



b



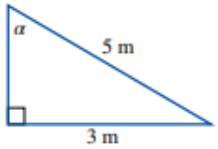
c



3 WE15a Find the size of the angle marked with the pronumeral in each of the following. Give your answer correct to the nearest second.

a

b



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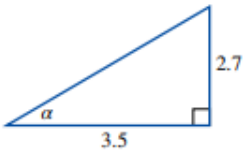
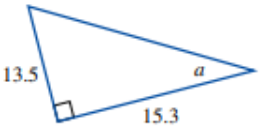
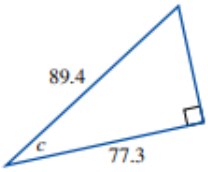


4 Find the size of the angle marked with the pronumeral in each of the following, giving your answer correct to the nearest degree.

a



b



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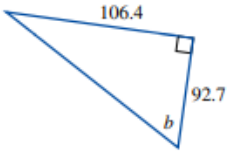

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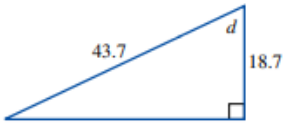

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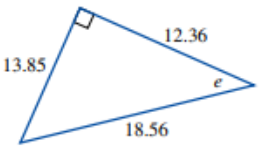

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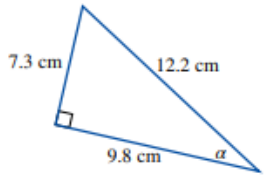
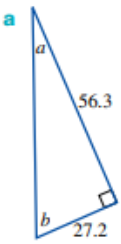


5 Find the size of each of the angles in the following, giving your answers correct to the nearest minute.

a



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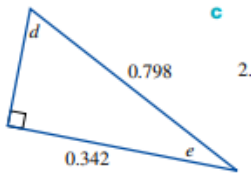

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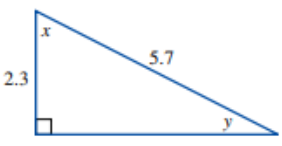


Exercise 5.7 Angles of elevation and depression

1 WE16 From a point P on the ground the angle of elevation from an observer to the top of a tree is 54°22′. If the tree is known to be 12.19 m high, how far is P from the tree (measured horizontally)?

2 From the top of a cliff 112 m high, the angle of depression to a boat is 9°15′. How far is the boat from the foot of the cliff?

3 A person on a ship observes a lighthouse on the cliff, which is 830 metres away from the ship. The angle of elevation of the top of the lighthouse is 12°.

a How far above sea level is the top of the lighthouse?

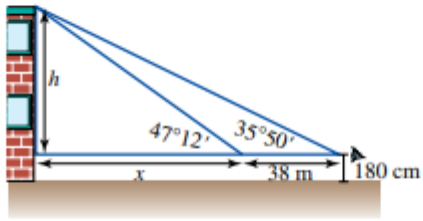
b If the height of the lighthouse is 24 m, how high is the cliff?

4 At a certain time of the day a post, 4 m tall, casts a shadow of 1.8 m. What is the angle of elevation of the sun at that time?

5 An observer who is standing 47 m from a building measures the angle of elevation of the top of the building as 17°. If the observer's eye is 167 cm from the ground, what is the height of the building?

UNDERSTANDING

6 A surveyor needs to determine the height of a building. She measures the angle of elevation of the top of the building from two points, 38 m apart. The surveyor's eye level is 180 cm above the ground.

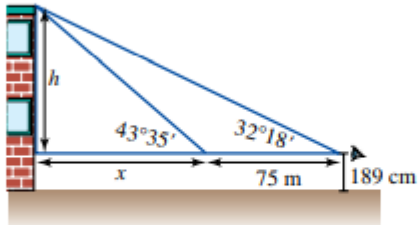


a Find two expressions for the height of the building, h, in terms of x using the two angles.

b Solve for x by equating the two expressions obtained in a.

c Find the height of the building.

7 The height of another building needs to be determined but cannot be found directly. The surveyor decides to measure the angle of elevation of the top of the building from different sites, which are 75 m apart. The surveyor's eye level is 189 cm above the ground.



a Find two expressions for the height of the building, h, in terms of x using the two angles.

b Solve for x.

c Find the height of the building.

Exercise 5.8 Bearings

1 Change each of the following compass bearings to true bearings.

a N20°E **b** N20°W **c** S35°W

d S28°E **e** N34°E **f** S42°W

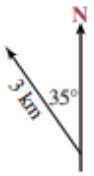
2 Change each of the following true bearings to compass bearings.

a 049°T **b** 132°T **c** 267°T

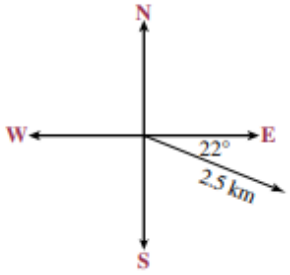
d 330°T **e** 086°T **f** 234°T

3 Describe the following paths using true bearings.

a



b



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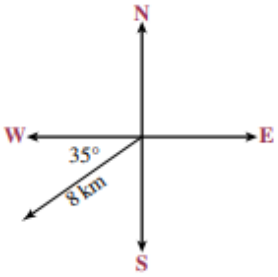

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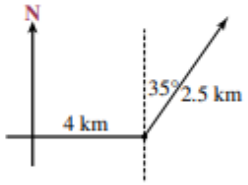

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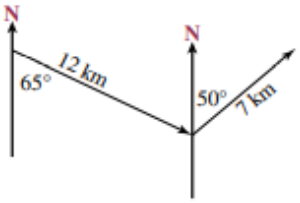

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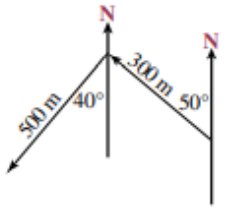


4 Show each of the following journeys as a diagram.

a A ship travels 040°T for 40 km and then 100°T for 30 km.

b A plane flies for 230 km in a direction 135°T and a further 140 km in a direction 240°T.

c A bushwalker travels in a direction 260°T for 0.8 km, then changes direction to 120°T for 1.3 km, and finally travels in a direction of 32° for 2.1 km.

d A boat travels N40°W for 8 km, then changes direction to S30°W for 5 km and then S50°E for 7 km.

e A plane travels N20°E for 320 km, N70°E for 180 km and S30°E for 220 km.

5 WE17 **a** A yacht travels 20 km from A to B on a bearing of 042°T:

i how far east of A is B?

ii how far north of A is B?

iii what is the bearing of A from B?

b The yacht then sails 80 km from B to C on a bearing of 130°T.

i Show the journey using a diagram.

ii How far south of B is C?

iii How far east of B is C?

iv What is the bearing of B from C?

6 If a farmhouse is situated 220 m N35°E from a shed, what is the true bearing of the shed from the house?

TOPIC 5 TRIGONOMETRY I

REVIEW QUESTIONS

FLUENCY

1 The most accurate measure for the length of the third side in the triangle below is:

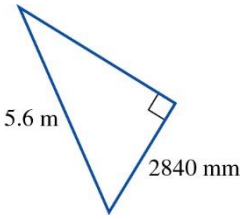
A 4.83 m

B 23.3 cm

C 3.94 m

D 2330 mm

E 4826 mm



2 What is the value of x in this figure?

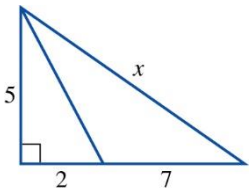
A 5.4

B 7.5

C 10.1

D 10.3

E 4



3 What is the closest length of AG of the cube below?

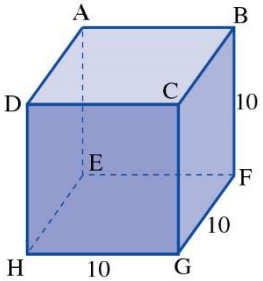
A 10

B 30

C 20

D 14

E 17



4 If sin 38° = 0.6157, which of the following will also give this result?

A sin 218° **B** sin 322° **C** sin 578° **D** sin 682° **E** sin 142°

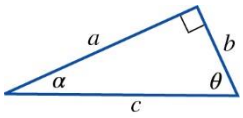
5 The angle 118°52′34″ is also equal to:

A 118.5234° **B** $118\frac{52}{34}°$ **C** 118.861° **D** 118.876° **E** 118.786°

6 Which trigonometric ratio for the triangle shown below is incorrect?

A $\sin\alpha = \frac{b}{c}$ **B** $\sin\alpha = \frac{a}{c}$ **C** $\cos\alpha = \frac{a}{c}$

D $\tan\alpha = \frac{b}{a}$ **E** $\tan\theta = \frac{a}{b}$

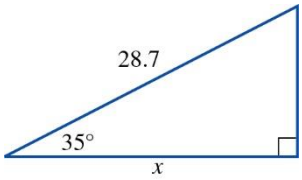


7 Which of the following statements is correct?

A $\sin 55° = \cos 55°$ **B** $\sin 45° = \cos 35°$ **C** $\cos 15° = \sin 85°$

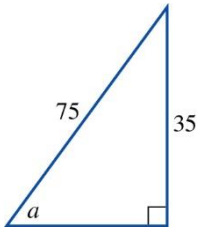
D $\sin 30° = \sin 60°$ **E** $\sin 42° = \cos 48°$

8 Which of the following can be used to find the value of x in the diagram below?



A $28.7 \sin 35°$ **B** $28.7 \cos 35°$ **C** $28.7 \tan 35°$ **D** $\frac{28.7}{\sin 35°}$ **E** $\frac{28.7}{\cos 35°}$

9 Which of the following expressions can be used to find the value of a in the triangle shown?



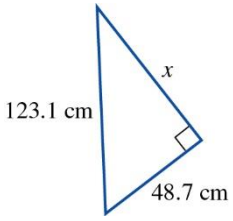
A $35 \sin 75°$ **B** $\sin^{-1}\frac{35}{75}$ **C** $\sin^{-1}\frac{75}{35}$ **D** $\cos^{-1}\frac{35}{75}$ **E** $\cos^{-1}\frac{75}{35}$

10 If a school is 320 m S42°W from the police station, what is the true bearing of the police station from the school?

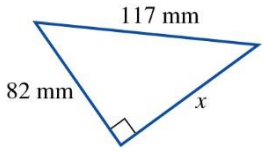
A 042°T **B** 048°T **C** 222°T **D** 228°T **E** 312°T

11 Calculate x, correct to 2 decimal places.

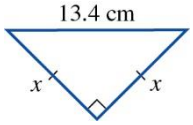
a



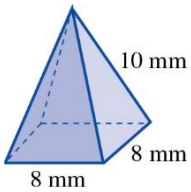
b



12 Calculate the value of the pronumeral, correct to 2 decimal places.



13 Calculate the height of this pyramid.



14 A person standing 23 m away from a tree observes the top of the tree at an angle of elevation of 35°. If the person's eye level is 1.5 m from the ground, what is the height of the tree?

15 A man with an eye level height of 1.8 m stands at the window of a tall building. He observes his young daughter in the playground below. If the angle of depression from the man to the girl is 47° and the floor on which the man stands is 27 m above the ground, how far from the bottom of the building is the child?

16 A plane flies 780 km in a direction of 185°T. How far west has it travelled from the starting point?

17 A hiker travels 3.2 km on a bearing of 250°T and then 1.8 km on a bearing of 320°T. How far west has she travelled from the starting point?

18 If a 4 m ladder is placed against a wall and the foot of the ladder is 2.6 m from the wall, what angle (in degrees and minutes) does the ladder make with the wall?

PROBLEM SOLVING

1 The height of a right square-based pyramid is 13 cm. If the angle the face makes with the base is 67°, find:

- **a** the length of the edge of the square base
- **b** the length of the diagonal of the base
- **c** the angle the slanted edge makes with the base in degrees and minutes.

2 A car is travelling northwards on an elevated expressway 6 m above ground at a speed of 72 km/h. At noon another car passes under the expressway, at ground level, travelling west, at a speed of 90 km/h.

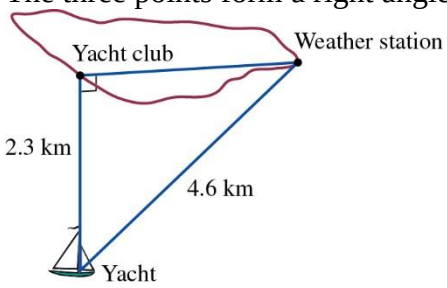
- **a** How far apart, in metres, are the two cars 40 seconds after noon?
- **b** At this time the first car stops, while the second car keeps going. At what time will they be 3.5 km apart? Write your answer correct to the nearest tenth of a second.

3 Two towers face each other separated by a distance, d, of 20 metres. As seen from the top of the first tower, the angle of depression of the second tower's base is 59° and that of the top is 31°. What is the height, in metres correct to 2 decimal places, of each of the towers?

4 A piece of flat pastry is cut in the shape of a right-angled triangle. The longest side is $6b$ cm and the shortest is $2b$ cm

- **a** Find the length of the third side. Give your answer in exact form.
- **b** Find the sizes of the angles in the triangle.
- **c** Prove that the area of the triangle is equal to $4\sqrt{2}b^2$ cm^2.

5 A yacht is anchored off an island. It is 2.3 km from the yacht club and 4.6 km from a weather station. The three points form a right angled triangle at the yacht club.



- **a** Calculate the angle at the yacht between the yacht club and the weather station.
- **b** Calculate the distance between the yacht club and the weather station.

The next day the yacht travels directly towards the yacht club, but is prevented from reaching the club because of dense fog. The weather station notifies the yacht that it is now 4.2 km from the station.

- **c** Calculate the new angle at the yacht between the yacht club and the weather station.
- **d** Determine how far the yacht is now from the yacht club.