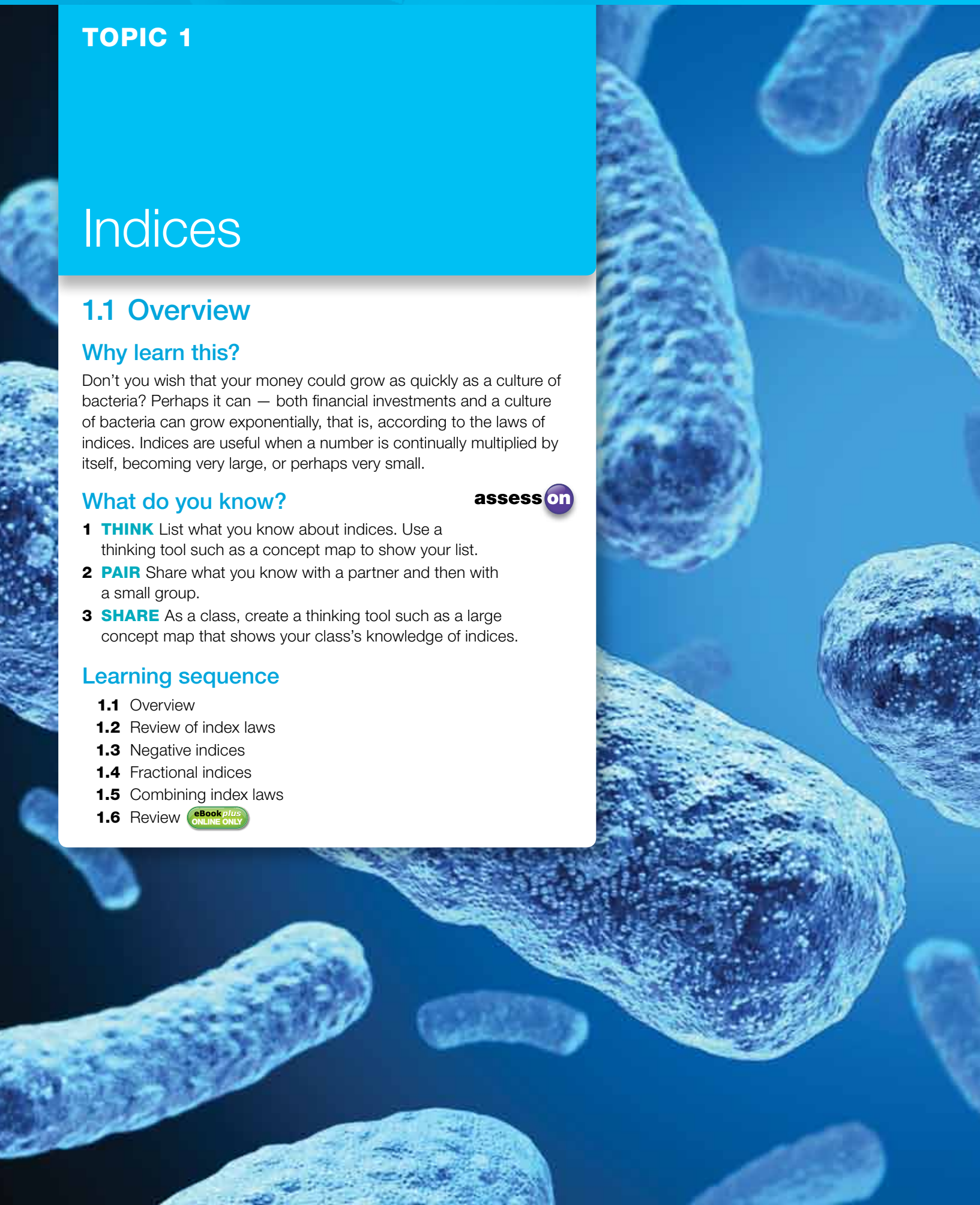
TOPIC 1

Indices



1.1 Overview

Why learn this?

Don't you wish that your money could grow as quickly as a culture of bacteria? Perhaps it can — both financial investments and a culture of bacteria can grow exponentially, that is, according to the laws of indices. Indices are useful when a number is continually multiplied by itself, becoming very large, or perhaps very small.

What do you know?

assessON

1 **THINK** List what you know about indices. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of indices.

Learning sequence

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The story of mathematics: Leibniz

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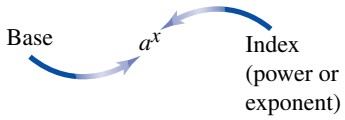
1.2 Review of index laws

- When a number or pronumeral is repeatedly multiplied by itself, it can be written in a shorter form called index form.
- A number written in index form has two parts, the **base** and the **index**, and is written as:



- Another name for an index is an exponent or a power.
- Performing operations on numbers or pronumerals written in index form requires the application of the index laws.

First Index Law: When terms with the same base are multiplied, the indices are added.

$$a^m \times a^n = a^{m+n}$$

Second Index Law: When terms with the same base are divided, the indices are subtracted.

$$a^m \div a^n = a^{m-n}$$

WORKED EXAMPLE 1 TI CASIO

Simplify each of the following.

a $m^4n^3p \times m^2n^5p^3$ **b** $2a^2b^3 \times 3ab^4$ **c** $\dfrac{2x^5y^4}{10x^2y^3}$

THINK

a 1 Write the expression.

2 Multiply the terms with the same base by adding the indices. *Note:* $p = p^1$.

b 1 Write the expression.

2 Simplify by multiplying the coefficients, then multiply the terms with the same base by adding the indices.

c 1 Write the expression.

2 Simplify by dividing both of the coefficients by the same factor, then divide terms with the same base by subtracting the indices.

WRITE

a $m^4n^3p \times m^2n^5p^3$

$= m^{4+2}n^{3+5}p^{1+3}$

$= m^6n^8p^4$

b $2a^2b^3 \times 3ab^4$

$= 2 \times 3 \times a^{2+1} \times b^{3+4}$

$= 6a^3b^7$

c $\dfrac{2x^5y^4}{10x^2y^3}$

$= \dfrac{1x^{5-2}y^{4-3}}{5}$

$= \dfrac{x^3y}{5}$

Third Index Law: Any term (excluding 0) with an index of 0 is equal to 1.

$$a^0 = 1, \ a \neq 0$$

WORKED EXAMPLE 2

Simplify each of the following.

a $(2b^3)^0$ **b** $-4(a^2b^5)^0$

THINK	WRITE
a 1 Write the expression.	**a** $(2b^3)^0$
2 Apply the Third Index Law, which states that any term (excluding 0) with an index of 0 is equal to 1.	$= 1$
b 1 Write the expression.	**b** $-4(a^2b^5)^0$
2 The entire term inside the brackets has an index of 0, so the bracket is equal to 1.	$= -4 \times 1$
3 Simplify.	$= -4$

Fourth Index Law: When a power (a^m) is raised to a power, the indices are multiplied.

$$(a^m)^n = a^{mn}$$

Fifth Index Law: When the base is a product, raise every part of the product to the index outside the brackets.

$$(ab)^m = a^m b^m$$

Sixth Index Law: When the base is a fraction, multiply the indices of both the numerator and denominator by the index outside the brackets.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

WORKED EXAMPLE 3 TI CASIO

Simplify each of the following.

a $(2n^4)^3$ **b** $(3a^2b^7)^3$ **c** $\left(\dfrac{2x^3}{y^4}\right)^4$ **d** $(-4)^3$

THINK	WRITE
a 1 Write the term.	**a** $(2n^4)^3$
2 Apply the Fourth Index Law and simplify.	$= 2^{1 \times 3} \times n^{4 \times 3}$ $= 2^3 n^{12}$ $= 8n^{12}$
b 1 Write the expression.	**b** $(3a^2b^7)^3$
2 Apply the Fifth Index Law and simplify.	$= 3^{1 \times 3} \times a^{2 \times 3} \times b^{7 \times 3}$ $= 3^3 a^6 b^{21}$ $= 27a^6b^{21}$

c 1 Write the expression. **c** $\left(\frac{2x^3}{y^4}\right)^4$

2 Apply the Sixth Index Law and simplify.
$= \frac{2^{1\times4} \times x^{3\times4}}{y^{4\times4}}$
$= \frac{16x^{12}}{y^{16}}$

d 1 Write the expression. **d** $(-4)^3$

2 Write in expanded form. $= -4 \times -4 \times -4$

3 Simplify, taking careful note of the negative sign. $= -64$

assess on

Exercise 1.2 Review of index laws

INDIVIDUAL PATHWAYS

REFLECTION
Why are these laws called index laws?

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–f, 3a–f, 4a–f, 6, 7a–f, 9, 10	Questions: 1d–i, 2d–i, 3a–f, 4e–l, 6, 7a–f, 9–11, 15	Questions: 1d–l, 2d–l, 3, 4d–o, 5, 6, 7d–i, 8–16

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Digital docs
SkillSHEET Index form doc-5168
SkillSHEET Using a calculator to evaluate numbers given in index form doc-5169

FLUENCY

1 WE1a, b Simplify each of the following.

a $a^3 \times a^4$ **b** $a^2 \times a^3 \times a$ **c** $b \times b^5 \times b^2$

d $ab^2 \times a^3b^5$ **e** $m^2n^6 \times m^3n^7$ **f** $a^2b^5c \times a^3b^2c^2$

g $mnp \times m^5n^3p^4$ **h** $2a \times 3ab$ **i** $4a^2b^3 \times 5a^2b \times \frac{1}{2}b^5$

j $3m^3 \times 2mn^2 \times 6m^4n^5$ **k** $4x^2 \times \frac{1}{2}xy^3 \times 6x^3y^3$ **l** $2x^3y^2 \times 4x \times \frac{1}{2}x^4y^4$

2 WE1c Simplify each of the following.

a $a^4 \div a^3$ **b** $a^7 \div a^2$ **c** $b^6 \div b^3$

d $\frac{4a^7}{3a^3}$ **e** $\frac{21b^6}{7b^2}$ **f** $\frac{48m^8}{12m^3}$

g $\frac{m^7n^3}{m^4n^2}$ **h** $\frac{2x^4y^3}{4x^4y}$ **i** $6x^7y \div 8x^4$

j $7ab^5c^4 \div ab^2c^4$ **k** $\frac{20m^5n^3p^4}{16m^3n^3p^2}$ **l** $\frac{14x^3y^4z^2}{28x^2y^2z^2}$

3 WE2 Simplify each of the following.

a a^0 **b** $(2b)^0$ **c** $(3m^2)^0$

d $3x^0$ **e** $4b^0$ **f** $-3 \times (2n)^0$

g $4a^0 - \left(\frac{a}{4}\right)^0$ **h** $5y^0 - 12$ **i** $5x^0 - (5xy^2)^0$

4 WE3 Simplify each of the following.

a $(a^2)^3$ **b** $(2a^5)^4$ **c** $\left(\frac{m^2}{3}\right)^4$

d $\left(\frac{2n^4}{3}\right)^2$ **e** $(a^2b)^3$ **f** $(3a^3b^2)^2$

g $(2m^3n^5)^4$ **h** $\left(\frac{3m^2n}{4}\right)^3$ **i** $\left(\frac{a^2}{b^3}\right)^2$

j $\left(\frac{5m^3}{n^2}\right)^4$ **k** $\left(\frac{7x}{2y^5}\right)^3$ **l** $\left(\frac{3a}{5b^3}\right)^4$

m $(-3)^5$ **n** $(-7)^2$ **o** $(-2)^5$

5 MC **a** $2m^{10}n^5$ is the simplified form of:

A $m^5n^3 \times 2m^4n^2$ **B** $\frac{6m^{10}n^4}{3n}$ **C** $(2m^5n^2)^2$

D $2n(m^5)^2 \times n^4$ **E** $\left(\frac{2m^5}{n^3}\right)^2$

b The value of $4 - (5a)^0$ is:

A –1 **B** 9 **C** 1

D 3 **E** 5

6 MC **a** $4a^3b \times b^4 \times 5a^2b^3$ simplifies to:

A $9a^5b^8$ **B** $20a^5b^7$ **C** $20a^5b^8$

D $9a^5b^7$ **E** $21a^5b^8$

b $\frac{15x^9 \times 3x^6}{9x^{10} \times x^4}$ simplifies to:

A $5x^9$ **B** $9x$ **C** $5x^{29}$

D $9x^9$ **E** $5x$

c $\frac{3p^7 \times 8q^9}{12p^3 \times 4q^5}$ simplifies to:

A $2q^4$ **B** $\frac{p^4q^4}{2}$ **C** $\frac{q^4}{2}$

D $\frac{p^4q^4}{24}$ **E** $\frac{q^4}{24}$

d $\frac{7a^5b^3}{5a^6b^2} \div \frac{7b^3a^2}{5b^5a^4}$ simplifies to:

A $\frac{49a^3b}{25}$ **B** $\frac{25a^3b}{49}$ **C** a^3b

D ab^3 **E** $\frac{25ab^3}{49}$

UNDERSTANDING

7 Evaluate each of the following.

a $2^3 \times 2^2 \times 2$ **b** $2 \times 3^2 \times 2^2$ **c** $(5^2)^2$

d $\frac{3^5 \times 4^6}{3^4 \times 4^4}$ **e** $(2^3 \times 5)^2$ **f** $\left(\frac{3}{5}\right)^3$

g $\frac{4^4 \times 5^6}{4^3 \times 5^5}$ **h** $(3^3 \times 2^4)^0$ **i** $4(5^2 \times 3^5)^0$

8 Simplify each of the following.

a $(x^y)^{3z}$

b $a^b \times (p^q)^0$

c $m^a \times n^b \times (mn)^0$

d $\left(\frac{a^2}{b^3}\right)^x$

e $\frac{n^3m^2}{n^pm^q}$

f $(a^{m+n})^p$

REASONING

9 Explain why $a^3 \times a^2 = a^5$ and not a^6.

10 Is $2x$ ever the same as x^2? Explain your reasoning using examples.

11 Explain the difference between $3x^0$ and $(3x)^0$.

12 **a** In the following table, enter the values of $3a^2$ and $5a$ when $a = 0,\ 1,\ 2$ and 3.

a	0	1	2	3
$3a^2$				
$5a$				
$3a^2 + 5a$				
$3a^2 \times 5a$				

b Enter the values of $3a^2 + 5a$ and $3a^2 \times 5a$ in the table.

c What do you think will happen as a becomes very large?

13 Find algebraically the exact value of x if $4^{x+1} = 2^{x^2}$. Justify your answer.

14 Binary numbers (base 2 numbers) are used in computer operations. As the name implies, binary uses only two types of numbers, 0 and 1, to express all numbers. A binary number such as 101 (read one, zero, one) means $(1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 4 + 0 + 1 = 5$ (in base 10, the base we are most familiar with).

The number 1010 (read one, zero, one, zero) means $(1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) = 8 + 0 + 2 + 0 = 10$.

If we read the binary number from right to left, the index of 2 increases by one each time, beginning with a power of zero.

Using this information, write out the numbers 1 to 10 in binary (base 2) form.

PROBLEM SOLVING

15 Solve for x:

a $\frac{7^x \times 7^{1+2x}}{(7^x)^2} = 16\,807$

b $2^{2x} - 5(2^x) = -4$

16 For the following:

a calculate the correct answer

b identify the error in the solution.

$$\left(\frac{a^2b^3c}{a^2b^2}\right)^3 \times \left(\frac{a^3b^2c^2}{a^2b^3}\right)^2 = \left(\frac{b^3c}{b^2}\right)^3 \times \left(\frac{ab^2c^2}{b^3}\right)^2$$
$$= \left(\frac{bc}{1}\right)^3 \times \left(\frac{ac^2}{b}\right)^2$$
$$= \left(\frac{abc^3}{b}\right)^6$$
$$= \left(\frac{ac^3}{1}\right)^6$$
$$= a^6c^{18}$$

CHALLENGE 1.1

It was estimated that there were 4×10^{10} locusts in the largest swarm ever seen. If each locust can consume 2 grams of grain in a day, how long would it take the swarm to consume 1 tonne of grain?

1.3 Negative indices

- Consider the expression $\frac{a^3}{a^5}$. This expression can be simplified in two different ways.

1. Written in expanded form: $\frac{a^3}{a^5} = \frac{a \times a \times a}{a \times a \times a \times a \times a}$
$$= \frac{1}{a \times a}$$
$$= \frac{1}{a^2}$$

2. Using the Second Index Law: $\frac{a^3}{a^5} = a^{3-5}$
$$= a^{-2}$$

So, $a^{-2} = \frac{1}{a^2}$.

- In general, $\frac{1}{a^n} = \frac{a^0}{a^n}$ $(1 = a^0)$
  $= a^{0-n}$ (using the Second Index Law)
  $= a^{-n}$

  Seventh Index Law: $a^{-n} = \frac{1}{a^n}$

- The convention is that an expression should be written using positive indices and with pronumerals given in alphabetical order.

WORKED EXAMPLE 4

Express each of the following with positive indices.

a x^{-3} **b** $2m^{-4}n^2$ **c** $\frac{4}{a^{-3}}$

THINK	WRITE
a 1 Write the expression.	**a** x^{-3}
2 Apply the Seventh Index Law.	$= \frac{1}{x^3}$
b 1 Write the expression.	**b** $2m^{-4}n^2$
2 Apply the Seventh Index Law to write the expression with positive indices.	$= \frac{2n^2}{m^4}$
c 1 Write the expression and rewrite the fraction, using a division sign.	**c** $\frac{4}{a^{-3}} = 4 \div a^{-3}$
2 Apply the Seventh Index Law to write the expression with positive indices.	$= 4 \div \frac{1}{a^3}$
3 To divide the fraction, change fraction division into multiplication.	$= 4 \times \frac{a^3}{1}$
	$= 4a^3$

- Part **c** from Worked example 4 demonstrates the **converse** of the Seventh Index Law $\frac{1}{a^{-n}} = a^n$.

WORKED EXAMPLE 5 TI CASIO

Simplify each of the following, expressing the answers with positive indices.

a $a^2b^{-3} \times a^{-5}b$ **b** $\frac{2x^4y^2}{3xy^5}$ **c** $\left(\frac{2m^3}{n^{-2}}\right)^{-2}$

THINK	WRITE
a 1 Write the expression.	**a** $a^2b^{-3} \times a^{-5}b$
2 Apply the First Index Law. Multiply terms with the same base by adding the indices.	$= a^{2+-5}b^{-3+1}$ $= a^{-3}b^{-2}$
3 Express the answer with positive indices.	$= \frac{1}{a^3b^2}$

b 1 Write the expression.

$$\frac{2x^4y^2}{3xy^5}$$

2 Apply the Second Index Law. Divide terms with the same base by subtracting the indices.

$$= \frac{2x^{4-1}y^{2-5}}{3}$$

$$= \frac{2x^3y^{-3}}{3}$$

3 Express the answer with positive indices.

$$= \frac{2x^3}{3y^3}$$

c 1 Write the expression.

$$\left(\frac{2m^3}{n^{-2}}\right)^{-2}$$

2 Apply the Sixth Index Law. Multiply the indices of both the numerator and denominator by the index outside the brackets.

$$= \frac{2^{-2}m^{-6}}{n^4}$$

3 Express all terms with positive indices.

$$= \frac{1}{2^2m^6n^4}$$

4 Simplify.

$$= \frac{1}{4m^6n^4}$$

- Numbers in index form can be easily evaluated if they are expressed with positive indices first. Consider the following example.

WORKED EXAMPLE 6

Evaluate 6×3^{-3} without using a calculator.

THINK	WRITE
1 Write the multiplication.	6×3^{-3}
2 Apply the Seventh Index Law to write 3^{-3} with a positive index.	$= 6 \times \frac{1}{3^3}$
3 Multiply the numerator of the fraction by the whole number.	$= \frac{6}{3^3}$
4 Evaluate the denominator.	$= \frac{6}{27}$
5 Cancel by dividing both the numerator and denominator by the highest common factor (3).	$= \frac{2}{9}$

Exercise 1.3 Negative indices

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–i, 2a–i, 3a–f, 4, 5a–e, 6a–b, 8a–c, 9, 11a, 12	Questions: 1a–i, 2a–i, 3c–h, 4, 5a–g, 6, 7, 8b–e, 9, 11a–b, 12, 13, 15, 18	Questions: 1, 2c–o, 3c–l, 4, 5d–j, 6, 7, 8c–f, 9–18

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REFLECTION
Are there any index laws from Section 1.2 that do not apply to negative indices?

FLUENCY

1 **WE4** Express each of the following with positive indices.

a x^{-5} **b** y^{-4} **c** $2a^{-9}$

d $\frac{4}{5}a^{-3}$ **e** $3x^2y^{-3}$ **f** $2^{-2}m^{-3}n^{-4}$

g $6a^3b^{-1}c^{-5}$ **h** $\dfrac{1}{a^{-6}}$ **i** $\dfrac{2}{3a^{-4}}$

j $\dfrac{6a}{3b^{-2}}$ **k** $\dfrac{7a^{-4}}{2b^{-3}}$ **l** $\dfrac{2m^3n^{-5}}{3a^{-2}b^4}$

2 **WE5** Simplify each of the following, expressing the answers with positive indices.

a $a^3b^{-2} \times a^{-5}b^{-1}$ **b** $2x^{-2}y \times 3x^{-4}y^{-2}$ **c** $3m^2n^{-5} \times m^{-2}n^{-3}$

d $4a^3b^2 \div a^5b^7$ **e** $2xy^6 \div 3x^2y^5$ **f** $5x^{-2}y^3 \div 6xy^2$

g $\dfrac{6m^4n}{2n^3m^6}$ **h** $\dfrac{4x^2y^9}{x^7y^{-3}}$ **i** $\dfrac{2m^2n^{-4}}{6m^5n^{-1}}$

j $(2a^3m^4)^{-5}$ **k** $4(p^7q^{-4})^{-2}$ **l** $3(a^{-2}b^{-3})^4$

m $\left(\dfrac{2p^2}{3q^3}\right)^{-3}$ **n** $\left(\dfrac{a^{-4}}{2b^{-3}}\right)^2$ **o** $\left(\dfrac{6a^2}{3b^{-2}}\right)^{-3}$

3 **WE6** Evaluate each of the following without using a calculator.

a 2^{-3} **b** 6^{-2} **c** 3^{-4}

d $3^{-2} \times 2^3$ **e** $4^{-3} \times 2^2$ **f** 5×6^{-2}

g $\dfrac{6}{2^{-3}}$ **h** $\dfrac{4 \times 3^{-3}}{2^{-3}}$ **i** $\frac{1}{3} \times 5^{-2} \times 3^4$

j $\dfrac{16^0 \times 2^4}{8^2 \times 2^{-4}}$ **k** $\dfrac{5^3 \times 25^0}{25^2 \times 5^{-4}}$ **l** $\dfrac{3^4 \times 4^2}{12^3 \times 15^0}$

4 Write each of these numbers as a power of 2.

a 8 **b** $\frac{1}{8}$ **c** 32 **d** $\frac{1}{64}$

5 Complete each statement by writing the correct index.

a $125 = 5^{\cdots}$ **b** $\frac{1}{16} = 4^{\cdots}$ **c** $\frac{1}{7} = 7^{\cdots}$ **d** $216 = 6^{\cdots}$

e $0.01 = 10^{\cdots}$ **f** $1 = 8^{\cdots}$ **g** $64 = 4^{\cdots}$ **h** $\frac{1}{64} = 4^{\cdots}$

i $\frac{1}{64} = 2^{\cdots}$ **j** $\frac{1}{64} = 8^{\cdots}$

6 Evaluate the following expressions.

a $\left(\frac{2}{3}\right)^{-1}$ **b** $\left(\frac{5}{4}\right)^{-1}$ **c** $\left(3\frac{1}{2}\right)^{-1}$ **d** $\left(\frac{1}{5}\right)^{-1}$

7 Write the following expressions with positive indices.

a $\left(\dfrac{a}{b}\right)^{-1}$ **b** $\left(\dfrac{a^2}{b^3}\right)^{-1}$ **c** $\left(\dfrac{a^{-2}}{b^{-3}}\right)^{-1}$ **d** $\left(\dfrac{m^3}{n^{-2}}\right)^{-1}$

8 Evaluate each of the following, using a calculator.

a 3^{-6} **b** 12^{-4} **c** 7^{-5}

d $\left(\frac{1}{2}\right)^{-8}$ **e** $\left(\frac{3}{4}\right)^{-7}$ **f** $(0.04)^{-5}$

9 **MC** **a** x^{-5} is the same as:

A $-x^5$ **B** $-5x$ **C** $5x$ **D** $\frac{1}{x^5}$ **E** $\frac{1}{x^{-5}}$

b $\frac{1}{a^{-4}}$ is the same as:

A $4a$ **B** $-4a$ **C** a^4 **D** $\frac{1}{a^4}$ **E** $-a^4$

c $\frac{1}{8}$ is the same as:

A 2^3 **B** 2^{-3} **C** 3^2 **D** 3^{-2} **E** $\frac{1}{2^{-3}}$

10 **MC** **a** Which of the following, when simplified, gives $\frac{3m^4}{4n^2}$?

A $\frac{3m^{-4}n^{-2}}{4}$ **B** $3 \times 2^{-2} \times m^4 \times n^{-2}$ **C** $\frac{3n^{-2}}{2^{-2}m^{-4}}$

D $\frac{2^2n^{-2}}{3^{-1}m^{-4}}$ **E** $3m^4 \times 2^2n^{-2}$

b When simplified, $3a^{-2}b^{-7} \div \frac{3}{4}a^{-4}b^6$ is equal to:

A $\frac{4}{a^6b^{13}}$ **B** $\frac{9b}{4a^6}$ **C** $\frac{9a^2}{4b}$ **D** $\frac{4a^2}{b^{13}}$ **E** $\frac{4a^2}{b}$

c When $(2x^6y^{-4})^{-3}$ is simplified, it is equal to:

A $\frac{2x^{18}}{y^{12}}$ **B** $\frac{x^{18}}{8y^{12}}$ **C** $\frac{y^{12}}{8x^{18}}$ **D** $\frac{8y^{12}}{x^{18}}$ **E** $\frac{x^{18}}{6y^{12}}$

d If $\left(\frac{2a^x}{b^y}\right)^3$ is equal to $\frac{8b^9}{a^6}$, then x and y (in that order) are:

A -3 and -6 **B** -6 and -3 **C** -3 and 2

D -3 and -2 **E** -2 and -3

UNDERSTANDING

11 Simplify, expressing your answer with positive indices.

a $\frac{m^{-3}n^{-2}}{m^{-5}n^6}$ **b** $\frac{(m^3n^{-2})^{-7}}{(m^{-5}n^3)^4}$ **c** $\frac{5(a^3b^{-3})^2}{(ab^{-4})^{-1}} \div \frac{(5a^{-2}b)^{-1}}{(a^{-4}b)^3}$

12 Simplify, expanding any expressions in brackets.

a $(r^3 + s^3)(r^3 - s^3)$ **b** $(m^5 + n^5)^2$

c $\frac{(x^{a+1})^b \times x^{a+b}}{x^{a(b+1)} \times x^{2b}}$ **d** $\left(\frac{p^{x+1}}{p^{x-1}}\right)^{-4} \times \frac{p^{8(x+1)}}{(p^{2x})^4} \times \frac{p^2}{(p^{12x})^0}$

13 Write $\left(\frac{2^r \times 8^r}{2^{2r} \times 16}\right)$ in the form 2^{ar+b}.

14 Write $2^{-m} \times 3^{-m} \times 6^{2m} \times 3^{2m} \times 2^{2m}$ as a power of 6.

15 Solve for x if $4^x - 4^{x-1} = 48$.

REASONING

16 Explain why each of these statements is false. Illustrate each answer by substituting a value for the pronumeral.

a $5x^0 = 1$ **b** $9x^5 \div 3x^5 = 3x$ **c** $a^5 \div a^7 = a^2$ **d** $2c^{-4} = \frac{1}{2c^4}$

PROBLEM SOLVING

17 Solve for x and y if $5^{x-y} = 625$ and $3^{2x} \times 3^{y} = 243$.

Hence, evaluate $\dfrac{35^{x}}{7^{-2y} \times 5^{-3y}}$.

18 Solve for n. Verify your answers.

a $(2^{n})^{n} \times (2^{n})^{3} \times 4 = 1$

b $\dfrac{(3^{n})^{n} \times (3^{n})^{-1}}{81} = 1$

1.4 Fractional indices

- Terms with fractional indices can be written as surds, using the following laws:

1. $a^{\frac{1}{n}} = \sqrt[n]{a}$
2. $a^{\frac{m}{n}} = \sqrt[n]{a^{m}}$
   $= (\sqrt[n]{a})^{m}$

- To understand how these laws are formed, consider the following numerical examples.

We know $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^{1}$

and that $\sqrt{4} \times \sqrt{4} = \sqrt{16}$
$= 4$

It follows, then, that $4^{\frac{1}{2}} = \sqrt{4}$.

Similarly, we know that $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^{1}$

and that $\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = \sqrt[3]{512}$
$= 8$

It follows, then that $8^{\frac{1}{3}} = \sqrt[3]{8}$.

This observation can be generalised to $a^{\frac{1}{n}} = \sqrt[n]{a}$.

Now consider: $a^{\frac{m}{n}} = a^{m \times \frac{1}{n}}$ or $a^{\frac{m}{n}} = a^{\frac{1}{n} \times m}$

$= (a^{m})^{\frac{1}{n}}$ $= (a^{\frac{1}{n}})^{m}$

$= \sqrt[n]{a^{m}}$ $= (\sqrt[n]{a})^{m}$

Eighth Index Law: $a^{\frac{m}{n}} = \sqrt[n]{a^{m}} = (\sqrt[n]{a})^{m}$

WORKED EXAMPLE 7

Evaluate each of the following without using a calculator.

a $9^{\frac{1}{2}}$ **b** $16^{\frac{3}{2}}$

THINK		WRITE
a 1	Rewrite the number using the Eighth Index Law.	**a** $9^{\frac{1}{2}} = \sqrt{9}$
2	Evaluate.	$= 3$
b 1	Rewrite the number using $a^{\frac{m}{n}} = (\sqrt[n]{a})^{m}$.	**b** $16^{\frac{3}{2}} = (\sqrt{16})^{3}$
2	Simplify and evaluate the result.	$= 4^{3}$
		$= 64$

WORKED EXAMPLE 8 TI CASIO

Simplify each of the following.

a $m^{\frac{1}{5}} \times m^{\frac{2}{5}}$ **b** $(a^2b^3)^{\frac{1}{6}}$ **c** $\left(\dfrac{x^{\frac{2}{3}}}{y^{\frac{3}{4}}}\right)^{\frac{1}{2}}$

THINK

a 1 Write the expression.

2 Apply the First Index Law to multiply terms with the same base by adding the indices.

b 1 Write the expression.

2 Use the Fourth Index Law to multiply each index inside the brackets by the index outside the brackets.

3 Simplify.

c 1 Write the expression.

2 Use the Sixth Index Law to multiply the index in both the numerator and denominator by the index outside the brackets.

WRITE

a $m^{\frac{1}{5}} \times m^{\frac{2}{5}}$
$= m^{\frac{3}{5}}$

b $(a^2b^3)^{\frac{1}{6}}$
$= a^{\frac{2}{6}}b^{\frac{3}{6}}$
$= a^{\frac{1}{3}}b^{\frac{1}{2}}$

c $\left(\dfrac{x^{\frac{2}{3}}}{y^{\frac{3}{4}}}\right)^{\frac{1}{2}}$
$= \dfrac{x^{\frac{1}{3}}}{y^{\frac{3}{8}}}$

Exercise 1.4 Fractional indices

assess on

INDIVIDUAL PATHWAYS

PRACTISE
Questions:
1–5, 6a, d, g, 7a, d, 8a, d, g, 9a, d, 10a, d, g, 11a, d, g, 12, 13, 14a, d, g, 15, 16

CONSOLIDATE
Questions:
1–5, 6a, b, e, h, i, 7a, b, c, f, 8a, b, d, e, g, h, 9a, b, d, e, 10b, e, h, 11b, e, h, 12, 13, 14b, e, h, 15, 16, 17

MASTER
Questions:
1–5, 6c, f, i, 7c, f, 8c, f, i, 9b, c, e, f, 10c, f, i, 11c, f, i, 12–19

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REFLECTION
Why is it easier to perform operations with fractional indices than with expressions using surds?

FLUENCY

1 WE7 Evaluate each of the following without using a calculator.

a $16^{\frac{1}{2}}$ **b** $25^{\frac{1}{2}}$ **c** $81^{\frac{1}{2}}$ **d** $8^{\frac{1}{3}}$ **e** $64^{\frac{1}{3}}$ **f** $81^{\frac{1}{4}}$

2 Write the following in surd form.

a $15^{\frac{1}{2}}$ **b** $m^{\frac{1}{4}}$ **c** $7^{\frac{2}{5}}$ **d** $7^{\frac{5}{2}}$

e $w^{\frac{3}{8}}$ **f** $w^{1.25}$ **g** $5^{3\frac{1}{3}}$ **h** $a^{0.3}$

3 Write the following in index form.

a $\sqrt{t}$ **b** $\sqrt[4]{5^7}$ **c** $\sqrt[6]{6^{11}}$ **d** $\sqrt[7]{x^6}$

e $\sqrt[6]{x^7}$ **f** $\sqrt[5]{w^{10}}$ **g** $\sqrt[10]{w^5}$ **h** $\sqrt[x]{11^n}$

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Addition of fractions
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SkillSHEET
Subtraction of fractions
doc-5177
SkillSHEET
Multiplication of fractions
doc-5178
SkillSHEET
Writing roots as fractional indices
doc-5179

4 Without using a calculator, find the exact value of each of the following.

a $8^{\frac{2}{3}}$ **b** $8^{\frac{4}{3}}$ **c** $32^{\frac{3}{5}}$ **d** $32^{\frac{4}{5}}$

e $25^{\frac{3}{2}}$ **f** $27^{\frac{2}{3}}$ **g** $27^{\frac{-2}{3}}$ **h** $81^{\frac{3}{4}}$

i $10^{\frac{6}{2}}$ **j** $36^{1\frac{1}{2}}$ **k** $7^{\frac{1}{2}}$ **l** $12^{\frac{1}{3}}$

5 Using a calculator, evaluate each of the following. Give the answer correct to 2 decimal places.

a $3^{\frac{1}{3}}$ **b** $5^{\frac{1}{2}}$ **c** $7^{\frac{1}{5}}$

d $8^{\frac{1}{9}}$ **e** $12^{\frac{3}{8}}$ **f** $(0.6)^{\frac{4}{5}}$

g $\left(\frac{2}{3}\right)^{\frac{3}{2}}$ **h** $\left(\frac{3}{4}\right)^{\frac{3}{4}}$ **i** $\left(\frac{4}{5}\right)^{\frac{2}{3}}$

6 **WE8a** Simplify each of the following.

a $4^{\frac{3}{5}} \times 4^{\frac{1}{5}}$ **b** $2^{\frac{1}{8}} \times 2^{\frac{3}{8}}$ **c** $a^{\frac{1}{2}} \times a^{\frac{1}{3}}$

d $x^{\frac{3}{4}} \times x^{\frac{2}{5}}$ **e** $5m^{\frac{1}{3}} \times 2m^{\frac{1}{5}}$ **f** $\frac{1}{2}b^{\frac{3}{7}} \times 4b^{\frac{2}{7}}$

g $-4y^2 \times y^{\frac{2}{9}}$ **h** $\frac{2}{5}a^{\frac{3}{8}} \times 0.05a^{\frac{3}{4}}$ **i** $5x^3 \times x^{\frac{1}{2}}$

7 Simplify each of the following.

a $a^{\frac{2}{3}}b^{\frac{3}{4}} \times a^{\frac{1}{3}}b^{\frac{3}{4}}$ **b** $x^{\frac{3}{5}}y^{\frac{2}{9}} \times x^{\frac{1}{5}}y^{\frac{1}{3}}$ **c** $2ab^{\frac{1}{3}} \times 3a^{\frac{3}{5}}b^{\frac{4}{5}}$

d $6m^{\frac{3}{7}} \times \frac{1}{3}m^{\frac{1}{4}}n^{\frac{2}{5}}$ **e** $x^3y^{\frac{1}{2}}z^{\frac{1}{3}} \times x^{\frac{1}{6}}y^{\frac{1}{3}}z^{\frac{1}{2}}$ **f** $2a^{\frac{2}{5}}b^{\frac{3}{8}}c^{\frac{1}{4}} \times 4b^{\frac{3}{4}}c^{\frac{3}{4}}$

8 Simplify each of the following.

a $3^{\frac{1}{2}} \div 3^{\frac{1}{3}}$ **b** $5^{\frac{2}{3}} \div 5^{\frac{1}{4}}$ **c** $12^2 \div 12^{\frac{3}{2}}$

d $a^{\frac{6}{7}} \div a^{\frac{3}{7}}$ **e** $x^{\frac{3}{2}} \div x^{\frac{1}{4}}$ **f** $\dfrac{m^{\frac{4}{5}}}{m^{\frac{5}{9}}}$

g $\dfrac{2x^{\frac{3}{4}}}{4x^{\frac{3}{5}}}$ **h** $\dfrac{7n^2}{21n^{\frac{4}{3}}}$ **i** $\dfrac{25b^{\frac{3}{5}}}{20b^{\frac{1}{4}}}$

9 Simplify each of the following.

a $x^3y^2 \div x^{\frac{4}{3}}y^{\frac{3}{5}}$ **b** $a^{\frac{5}{9}}b^{\frac{2}{3}} \div a^{\frac{2}{5}}b^{\frac{2}{5}}$ **c** $m^{\frac{3}{8}}n^{\frac{4}{7}} \div 3n^{\frac{3}{8}}$

d $10x^{\frac{4}{5}}y \div 5x^{\frac{2}{3}}y^{\frac{1}{4}}$ **e** $\dfrac{5a^{\frac{3}{4}}b^{\frac{3}{5}}}{20a^{\frac{1}{5}}b^{\frac{1}{4}}}$ **f** $\dfrac{p^{\frac{7}{8}}q^{\frac{1}{4}}}{7p^{\frac{2}{3}}q^{\frac{1}{6}}}$

10 Simplify each of the following.

a $(2^{\frac{3}{4}})^{\frac{3}{5}}$ **b** $(5^{\frac{2}{3}})^{\frac{1}{4}}$ **c** $(7^{\frac{1}{5}})^6$

d $(a^3)^{\frac{1}{10}}$ **e** $(m^{\frac{4}{9}})^{\frac{3}{8}}$ **f** $(2b^{\frac{1}{2}})^{\frac{1}{3}}$

g $4(p^{\frac{3}{7}})^{\frac{14}{15}}$ **h** $(x^{\frac{m}{n}})^{\frac{n}{p}}$ **i** $(3m^{\frac{a}{b}})^{\frac{b}{c}}$

11 **WE8b, c** Simplify each of the following.

a $(a^{\frac{1}{2}}b^{\frac{1}{3}})^{\frac{1}{2}}$ **b** $(a^4b)^{\frac{3}{4}}$ **c** $(x^{\frac{3}{5}}y^{\frac{7}{8}})^2$

d $(3a^{\frac{1}{3}}b^{\frac{3}{5}}c^{\frac{3}{4}})^{\frac{1}{3}}$ **e** $5(x^{\frac{1}{2}}y^{\frac{2}{3}}z^{\frac{2}{5}})^{\frac{1}{2}}$ **f** $\left(\frac{a^{\frac{3}{4}}}{b}\right)^{\frac{2}{3}}$

g $\left(\frac{m^{\frac{4}{5}}}{n^{\frac{7}{8}}}\right)^2$ **h** $\left(\frac{b^{\frac{3}{5}}}{c^{\frac{4}{9}}}\right)^{\frac{1}{2}}$ **i** $\left(\frac{4x^7}{2y^{\frac{3}{4}}}\right)^{\frac{1}{2}}$

12 **MC** **a** $y^{\frac{2}{5}}$ is equal to:

A $(y^{\frac{1}{2}})^5$ **B** $y \times \frac{2}{5}$ **C** $(y^5)^{\frac{1}{2}}$ **D** $2\sqrt[5]{y}$ **E** $(y^{\frac{1}{5}})^2$

b $k^{\frac{2}{3}}$ is not equal to:

A $(k^{\frac{1}{3}})^2$ **B** $\sqrt[3]{k^2}$ **C** $(k^{\frac{1}{2}})^3$ **D** $(\sqrt[3]{k})^2$ **E** $(k^2)^{\frac{1}{3}}$

c $\frac{1}{\sqrt[5]{g^2}}$ is equal to:

A $g^{\frac{2}{5}}$ **B** $g^{-\frac{2}{5}}$ **C** $g^{\frac{5}{2}}$ **D** $g^{-\frac{5}{2}}$ **E** $2g^{\frac{1}{5}}$

13 **MC** **a** If $(a^{\frac{3}{4}})^{\frac{m}{n}}$ is equal to $a^{\frac{1}{4}}$, then m and n could not be:

A 1 and 3 **B** 2 and 6 **C** 3 and 8 **D** 4 and 9 **E** both **C** and **D**

b When simplified, $\left(\frac{a^{\frac{m}{n}}}{b^{\frac{n}{p}}}\right)^{\frac{p}{m}}$ is equal to:

A $\frac{a^{\frac{m}{p}}}{b^{\frac{n}{m}}}$ **B** $\frac{a^{\frac{p}{n}}}{b^{\frac{n}{m}}}$ **C** $\frac{a^{\frac{mp}{n}}}{b^{\frac{n}{m}}}$ **D** $\frac{a^p}{b^m}$ **E** $\frac{a^{\frac{m^2}{np}}}{b^{\frac{nm}{p^2}}}$

14 Simplify each of the following.

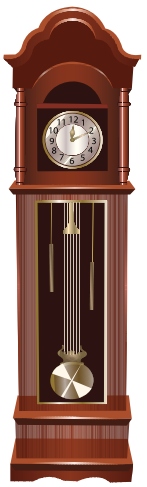
a $\sqrt{a^8}$ **b** $\sqrt[3]{b^9}$ **c** $\sqrt[4]{m^{16}}$ **d** $\sqrt{16x^4}$ **e** $\sqrt[3]{8y^9}$

f $\sqrt[4]{16x^8y^{12}}$ **g** $\sqrt[3]{27m^9n^{15}}$ **h** $\sqrt[5]{32p^5q^{10}}$ **i** $\sqrt[3]{216a^6b^{18}}$

UNDERSTANDING

15 The relationship between the length of a pendulum (L) in a grandfather clock and the time it takes to complete one swing (T) in seconds is given by the following rule. Note that g is the acceleration due to gravity and will be taken as 9.8.

$$T = 2\pi\left(\frac{L}{g}\right)^{\frac{1}{2}}$$



a Calculate the time it takes a 1m long pendulum to complete one swing.

b Calculate the time it takes the pendulum to complete 10 swings.

c How many swings will be completed after 10 seconds?

REASONING

16 Using the index laws, show that $\sqrt[5]{32a^5b^{10}} = 2ab^2$.

17 To rationalise a fraction means to remove all non-rational numbers from the denominator of the fraction. Rationalise $\frac{a^2}{3+\sqrt{b^3}}$ by multiplying the numerator and denominator by $3-\sqrt{b^3}$, and then evaluate if $b = a^2$ and $a = 2$. Show all of your working.

PROBLEM SOLVING

18 Simplify $\frac{m^{\frac{2}{5}} - 2m^{\frac{1}{5}}n^{\frac{1}{5}} + n^{\frac{2}{5}} - p^{\frac{2}{5}}}{m^{\frac{1}{5}} - n^{\frac{1}{5}} - p^{\frac{1}{5}}}$.

19 A scientist has discovered a piece of paper with a complex formula written on it. She thinks that someone has tried to disguise a simpler formula. The formula is:

$$\frac{\sqrt[4]{a^{13}}a^2\sqrt{b^3}}{\sqrt{a^1 b}} \times b^3 \times \left(\frac{\sqrt{a^3}b}{ab^2}\right)^2 \times \left(\frac{b^2}{a^2\sqrt{b}}\right)^3$$

a Simplify the formula using index laws so that it can be worked with.

b From your simplified formula, can a take a negative value? Explain.

c What is the smallest value for a for which the expression will give a rational answer? Consider only integers.

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1.5 Combining index laws

- When several steps are needed to simplify an expression, expand brackets first.
- When fractions are involved, it is usually easier to carry out all multiplications first, leaving one division as the final process.
- Final answers are conventionally written using positive indices.

WORKED EXAMPLE 9

Simplify each of the following.

a $\frac{(2a)^4b^4}{6a^3b^2}$ **b** $\frac{3^{n-2} \times 9^{n+1}}{81^{n-1}}$

THINK

a
1. Write the expression.
2. Apply the Fourth Index Law to remove the bracket.
3. Apply the Second Index Law for each number and pronumeral to simplify.
4. Write the answer.

b
1. Write the expression.
2. Rewrite each term in the expression so that it has a base of 3.
3. Apply the Fourth Index Law to expand the brackets.

WRITE

a $\frac{(2a)^4b^4}{6a^3b^2}$

$= \frac{16a^4b^4}{6a^3b^2}$

$= \frac{8a^{4-3}b^{4-2}}{3}$

$= \frac{8ab^2}{3}$

b $\frac{3^{n-2} \times 9^{n+1}}{81^{n-1}}$

$= \frac{3^{n-2} \times (3^2)^{n+1}}{(3^4)^{n-1}}$

$= \frac{3^{n-2} \times 3^{2n+2}}{3^{4n-4}}$

4	Apply the First and Second Index Laws to simplify and write your answer.	$= \frac{3^{3n}}{3^{4n-4}}$ $= \frac{1}{3^{n-4}}$

WORKED EXAMPLE 10

Simplify each of the following.

a $(2a^3b)^4 \times 4a^2b^3$ **b** $\frac{7xy^3}{(3x^3y^2)^2}$ **c** $\frac{2m^5n \times 3m^7n^4}{7m^3n^3 \times mn^2}$

		THINK	WRITE
a	1	Write the expression.	a $(2a^3b)^4 \times 4a^2b^3$
	2	Apply the Fourth Index Law. Multiply each index inside the brackets by the index outside the brackets.	$= 2^4a^{12}b^4 \times 4a^2b^3$
	3	Evaluate the number.	$= 16a^{12}b^4 \times 4a^2b^3$
	4	Multiply coefficients and multiply pronumerals. Apply the First Index Law to multiply terms with the same base by adding the indices.	$= 16 \times 4 \times a^{12+2}b^{4+3}$ $= 64a^{14}b^7$
b	1	Write the expression.	b $\frac{7xy^3}{(3x^3y^2)^2}$
	2	Apply the Fourth Index Law in the denominator. Multiply each index inside the brackets by the index outside the brackets.	$= \frac{7xy^3}{9x^6y^4}$
	3	Apply the Second Index Law. Divide terms with the same base by subtracting the indices.	$= \frac{7x^{-5}y^{-1}}{9}$
	4	Use $a^{-m} = \frac{1}{a^m}$ to express the answer with positive indices.	$= \frac{7}{9x^5y}$
c	1	Write the expression.	c $\frac{2m^5n \times 3m^7n^4}{7m^3n^3 \times mn^2}$
	2	Simplify each numerator and denominator by multiplying coefficients and then terms with the same base.	$= \frac{6m^{12}n^5}{7m^4n^5}$
	3	Apply the Second Index Law. Divide terms with the same base by subtracting the indices.	$= \frac{6m^8n^0}{7}$
	4	Simplify the numerator using $a^0 = 1$.	$= \frac{6m^8 \times 1}{7}$ $= \frac{6m^8}{7}$

WORKED EXAMPLE 11 TI CASIO

Simplify each of the following.

a $\dfrac{(5a^2b^3)^2}{a^{10}} \times \dfrac{a^2b^5}{(a^3b)^7}$ **b** $\dfrac{8m^3n^{-4}}{(6mn^2)^3} \div \dfrac{4m^{-2}n^{-4}}{6m^{-5}n}$

THINK		WRITE
a 1	Write the expression.	**a** $\dfrac{(5a^2b^3)^2}{a^{10}} \times \dfrac{a^2b^5}{(a^3b)^7}$
2	Remove the brackets in the numerator of the first fraction and in the denominator of the second fraction.	$= \dfrac{25a^4b^6}{a^{10}} \times \dfrac{a^2b^5}{a^{21}b^7}$
3	Multiply the numerators and then multiply the denominators of the fractions. (Simplify across.)	$= \dfrac{25a^6b^{11}}{a^{31}b^7}$
4	Divide terms with the same base by subtracting the indices. (Simplify down.)	$= 25a^{-25}b^4$
5	Express the answer with positive indices.	$= \dfrac{25b^4}{a^{25}}$
b 1	Write the expression.	**b** $\dfrac{8m^3n^{-4}}{(6mn^2)^3} \div \dfrac{4m^{-2}n^{-4}}{6m^{-5}n}$
2	Remove the brackets.	$= \dfrac{8m^3n^{-4}}{216m^3n^6} \div \dfrac{4m^{-2}n^{-4}}{6m^{-5}n}$
3	Change the division to multiplication.	$= \dfrac{8m^3n^{-4}}{216m^3n^6} \times \dfrac{6m^{-5}n}{4m^{-2}n^{-4}}$
4	Multiply the numerators and then multiply the denominators. (Simplify across.)	$= \dfrac{48m^{-2}n^{-3}}{864mn^2}$
5	Cancel common factors and divide pronumerals with the same base. (Simplify down.)	$= \dfrac{m^{-3}n^{-5}}{18}$
6	Simplify and express the answer with positive indices.	$= \dfrac{1}{18m^3n^5}$

Note that the whole numbers in part **b** of Worked example 11 could be cancelled in step 3.

assess on

Exercise 1.5 Combining index laws

INDIVIDUAL PATHWAYS

REFLECTION
Do index laws need to be performed in a certain order?

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–d, 2a–d, 3a–d, 4a–d, 5a–d, 6, 7, 9, 10, 11a, d, 12	Questions: 1c–h, 2c–f, 3c–g, 4b–f, 5c–f, 6–10, 11b–e, 12	Questions: 1f–j, 2e–i, 3f–i, 4d–f, 5e–h, 6–10, 11c–f, 12–15

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FLUENCY

1 WE10a Simplify each of the following.

a $(3a^2b^2)^3 \times 2a^4b^3$

b $(4ab^5)^2 \times 3a^3b^6$

c $2m^3n^{-5} \times (m^2n^{-3})^{-6}$

d $(2pq^3)^2 \times (5p^2q^4)^3$

e $(2a^7b^2)^2 \times (3a^3b^3)^2$

f $5(b^2c^{-2})^3 \times 3(bc^5)^{-4}$

g $6x^{\frac{1}{2}}y^{\frac{1}{3}} \times (4x^{\frac{3}{4}}y^{\frac{4}{5}})^{\frac{1}{2}}$

h $(16m^3n^4)^{\frac{3}{4}} \times (m^{\frac{1}{2}}n^{\frac{1}{4}})^3$

i $2(p^{\frac{2}{3}}q^{\frac{1}{3}})^{-\frac{3}{4}} \times 3(p^{\frac{1}{4}}q^{-\frac{3}{4}})^{-\frac{1}{3}}$

j $(8p^{\frac{1}{5}}q^{\frac{2}{3}})^{-\frac{1}{3}} \times (64p^{\frac{1}{3}}q^{\frac{3}{4}})^{\frac{2}{3}}$

2 WE10b Simplify each of the following.

a $\dfrac{5a^2b^3}{(2a^3b)^3}$

b $\dfrac{4x^5y^6}{(2xy^3)^4}$

c $\dfrac{(3m^2n^3)^3}{(2m^5n^5)^7}$

d $\left(\dfrac{4x^3y^{10}}{2x^7y^4}\right)^6$

e $\dfrac{3a^3b^{-5}}{(2a^7b^4)^{-3}}$

f $\left(\dfrac{3g^2h^5}{2g^4h}\right)^3$

g $\dfrac{(5p^6q^{\frac{1}{3}})^2}{25(p^{\frac{1}{2}}q^{\frac{1}{4}})^{\frac{2}{3}}}$

h $\left(\dfrac{3b^2c^3}{5b^{-3}c^{-4}}\right)^{-4}$

i $\dfrac{(x^{\frac{1}{3}}y^{\frac{1}{4}}z^{\frac{1}{2}})^2}{(x^{\frac{2}{3}}y^{-\frac{1}{4}}z^{\frac{1}{3}})^{-\frac{3}{2}}}$

3 WE10c Simplify each of the following.

a $\dfrac{2a^2b \times 3a^3b^4}{4a^3b^5}$

b $\dfrac{4m^6n^3 \times 12mn^5}{6m^7n^6}$

c $\dfrac{10m^6n^5 \times 2m^2n^3}{12m^4n \times 5m^2n^3}$

d $\dfrac{6x^3y^2 \times 4x^6y}{9xy^5 \times 2x^3y^6}$

e $\dfrac{(6x^3y^2)^4}{9x^5y^2 \times 4xy^7}$

f $\dfrac{5x^2y^3 \times 2xy^5}{10x^3y^4 \times x^4y^2}$

g $\dfrac{a^3b^2 \times 2(ab^5)^3}{6(a^2b^3)^3 \times a^4b}$

h $\dfrac{(p^6q^2)^{-3} \times 3pq}{2p^{-4}q^{-2} \times (5pq^4)^{-2}}$

i $\dfrac{6x^{\frac{3}{2}}y^{\frac{1}{2}} \times x^{\frac{4}{5}}y^{\frac{3}{5}}}{2(x^{\frac{1}{2}}y)^{\frac{1}{5}} \times 3x^{\frac{1}{2}}y^{\frac{1}{5}}}$

4 **WE11a** Simplify each of the following.

a $\dfrac{a^3b^2}{5a^4b^7} \times \dfrac{2a^6b}{a^9b^3}$

b $\dfrac{(2a^6)^2}{10a^7b^3} \times \dfrac{4ab^6}{6a^3}$

c $\dfrac{(m^4n^3)^2}{(m^6n)^4} \times \dfrac{(m^3n^3)^3}{(2mn)^2}$

d $\left(\dfrac{2m^3n^2}{3mn^5}\right)^3 \times \dfrac{6m^2n^4}{4m^3n^{10}}$

e $\left(\dfrac{2xy^2}{3x^3y^5}\right)^4 \times \left(\dfrac{x^3y^9}{2y^{10}}\right)^2$

f $\dfrac{4x^{-5}y^{-3}}{(x^2y^2)^{-2}} \times \dfrac{3x^5y^6}{2^{-2}x^{-7}y}$

g $\dfrac{5p^6q^{-5}}{3q^{-4}} \times \left(\dfrac{5p^6q^4}{3p^5}\right)^{-2}$

h $\dfrac{2a^{\frac{1}{2}}b^{\frac{1}{3}}}{6a^{\frac{1}{3}}b^{\frac{1}{2}}} \times \dfrac{(4a^{\frac{1}{4}}b)^{\frac{1}{2}}}{b^{\frac{1}{4}}a}$

i $\dfrac{3x^{\frac{2}{3}}y^{\frac{1}{5}}}{9x^{\frac{1}{3}}y^{\frac{1}{4}}} \times \dfrac{4x^{\frac{1}{2}}}{x^{\frac{3}{4}}y}$

5 **WE11b** Simplify each of the following.

a $\dfrac{5a^2b^3}{6a^7b^5} \div \dfrac{a^9b^4}{3ab^6}$

b $\dfrac{7a^2b^4}{3a^6b^7} \div \left(\dfrac{3ab}{2a^6b^4}\right)^3$

c $\left(\dfrac{4a^9}{b^6}\right)^3 \div \left(\dfrac{3a^7}{2b^5}\right)^4$

d $\dfrac{5x^2y^6}{(2x^4y^5)^2} \div \dfrac{(4x^6y)^3}{10xy^3}$

e $\left(\dfrac{x^5y^{-3}}{2xy^5}\right)^{-4} \div \dfrac{4x^6y^{-10}}{(3x^{-2}y^2)^{-3}}$

f $\dfrac{3m^3n^4}{2m^{-6}n^{-5}} \div \left(\dfrac{2m^4n^6}{m^{-1}n}\right)^{-2}$

g $4m^{\frac{1}{2}}n^{\frac{3}{4}} \div \dfrac{6m^{\frac{1}{3}}n^{\frac{1}{4}}}{8m^{\frac{3}{4}}n^{\frac{1}{2}}}$

h $\left(\dfrac{4b^3c^{\frac{1}{3}}}{6c^{\frac{1}{5}}b}\right)^{\frac{1}{2}} \div (2b^3c^{-\frac{1}{5}})^{-\frac{3}{2}}$

UNDERSTANDING

6 Evaluate each of the following.

a $(5^2 \times 2)^0 \times (5^{-3} \times 2^0)^5 \div (5^6 \times 2^{-1})^{-3}$

b $(2^3 \times 3^3)^{-2} \div \dfrac{(2^6 \times 3^9)^0}{2^6 \times (3^{-2})^{-3}}$

7 Evaluate the following for $x = 8$. (*Hint:* Simplify first.)

$$(2x)^{-3} \times \left(\frac{x}{2}\right)^2 \div \frac{2x}{(2^3)^4}$$

8 **a** Simplify the following fraction. $\dfrac{a^{2y} \times 9b^y \times (5ab)^y}{(a^y)^3 \times 5(3b^y)^2}$

b Find the value of y if the fraction is equal to 125.

9 **MC** Which of the following is not the same as $(4xy)^{\frac{3}{2}}$?

A $8x^{\frac{3}{2}}y^{\frac{3}{2}}$

B $(\sqrt{4xy})^3$

C $\sqrt{64x^3y^3}$

D $\dfrac{(2x^3y^3)^{\frac{1}{2}}}{(\sqrt{32})^{-1}}$

E $4xy^{\frac{1}{2}} \times (2xy^2)^{\frac{1}{2}}$

10 **MC** The expression $\dfrac{x^2y}{(2xy^2)^3} \div \dfrac{xy}{16x^0}$ is equal to:

A $\dfrac{2}{x^2y^6}$ **B** $\dfrac{2x^2}{b^6}$ **C** $2x^2y^6$ **D** $\dfrac{2}{xy^6}$ **E** $\dfrac{1}{128xy^5}$

11 Simplify the following.

a $\sqrt[3]{m^2n} \div \sqrt{mn^3}$

b $(g^{-2}h)^3 \times \left(\dfrac{1}{n^{-3}}\right)^{\frac{1}{2}}$

c $\dfrac{45^{\frac{1}{3}}}{9^{\frac{3}{4}} \times 15^{\frac{3}{2}}}$

d $2^{\frac{3}{2}} \times 4^{-\frac{1}{4}} \times 16^{-\frac{3}{4}}$

e $\left(\dfrac{a^3b^{-2}}{3^{-3}b^{-3}}\right)^{-2} \div \left(\dfrac{3^{-3}a^{-2}b}{a^4b^{-2}}\right)^2$

f $(\sqrt[5]{d^2})^{\frac{3}{2}} \times (\sqrt[3]{d^5})^{\frac{1}{5}}$

REASONING

12 In a controlled breeding program at the Melbourne Zoo, the population (P) of koalas at t years is modelled by $P = P_0 \times 10^{kt}$. The initial number of koalas is 20 and the population of koalas after 1 year is 40. Given $P_0 = 20$ and $k = 0.3$:

a calculate the number of koalas after 2 years

b determine when the population will be equal to 1000.

13 The decay of uranium is modelled by $D = D_0 \times 2^{-kt}$. If it takes 6 years for the mass of uranium to halve, find the percentage remaining after:

a 2 years

b 5 years

c 10 years.

Give your answers to the nearest whole number.

PROBLEM SOLVING

eBookplus
Digital doc
WorkSHEET 1.2
doc-5181

14 Simplify $\dfrac{7^{2x+1} - 7^{2x-1} - 48}{36 \times 7^{2x} - 252}$.

15 Simplify $\dfrac{z^4 + z^{-4} - 3}{z^2 + z^{-2} - 5^{\frac{1}{2}}}$.

CHALLENGE 1.2

Find an expression for x in terms of y, given that $(\sqrt{a^y})^x = a^x \times a^y$.

1.6 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

Interactivities

Word search
int-2826

Crossword
int-2827

Sudoku
int-3588

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

base
constant
denominator
evaluate
exponent
expression
index
index law
negative
numerator
positive
power indices
pronumeral
simplify
substitute
surd

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

The story of mathematics is an exclusive Jacaranda video series that explores the history of mathematics and how it helped shape the world we live in today.

Leibniz (eles-1840) tells the story of Gottfried Leibniz, a remarkable mathematician who helped refine the binary system that underpins nearly every piece of modern technology in the world today.

INVESTIGATION

RICH TASK

Digital world: 'A bit of this and a byte of that'

'The digital world of today is run by ones and zeros.' What does this mean?

Data is represented on a modern digital computer using a base two (binary) system, that is, using the two digits 1 and 0, thought of as 'on' and 'off'. The smallest unit of data that is transferred on a computer is a **bit** (an abbreviation of binary digit). Computer and storage mechanisms need to hold much larger values than a bit. Units such as bytes, kilobytes (KB), megabytes (MB), gigabytes (GB), and terabytes (TB) are based on the conversion of 8 bits to 1 byte. Your text messages, graphics, music and photos are files stored in sequences of bytes, each byte being 8 bits (8b = 1B).

You may have heard the terms 'meg' and 'gig'. In computer terminology, these refer to gigabytes and megabytes. In the digital world, the prefixes kilo-, mega- and giga- express powers of two, where kilo- means 2^{10}, mega- means $(2^{10})^2$ and so on. Thus the number of bytes in a computer's memory builds in powers of 2, for example 1 kilobyte = 1024 bytes (2^{10} bytes). (This differs from the decimal system, in which the prefixes kilo-, mega- and giga- express powers of ten, with kilo- meaning 10^3, mega- meaning $(10^3)^2$ and so on.)

A byte (8 bits) is used to represent a single character. For example the letter 'A' is represented in binary as 01000001. A book of a thousand pages in print can be stored in millions of bits, but more commonly it would be described as being stored in megabytes with one byte per character.

1 Complete the table below to show the difference in value between the binary and decimal systems.

Unit	Symbol	Power of 2 and value in bytes	Power of 10 and value in bytes
Byte	B	$2^0 = 1$	$10^0 = 1$
Kilobyte	KB	$2^{10} = 1024$	$10^3 = 1000$
Megabyte		$2^{20} =$	
Gigabyte			
Terabyte			

2 The two numbering systems have led to some confusion, with some manufacturers of digital products thinking of a kilobyte as 1000 bytes rather than 1024 bytes. Similar confusion arises with megabytes, gigabytes, terabytes and so on. This means you might not be getting exactly the amount of storage that you think.

If you bought a device quoted as having 16 GB memory, what would be the difference in memory storage if the device had been manufactured using the decimal value of GB as opposed to the binary system?

Many devices allow you to check the availability of storage. On one such device, the iPhone, available storage is found by going to 'General' under the heading 'Settings'.

3 How much storage is left in MB on the following iPhone?

4 If each photo uses 3.2 MB of memory, how many photos can be added?

Have you ever wondered about the capacity of our brain to store information and the speed at which information is transmitted inside it?

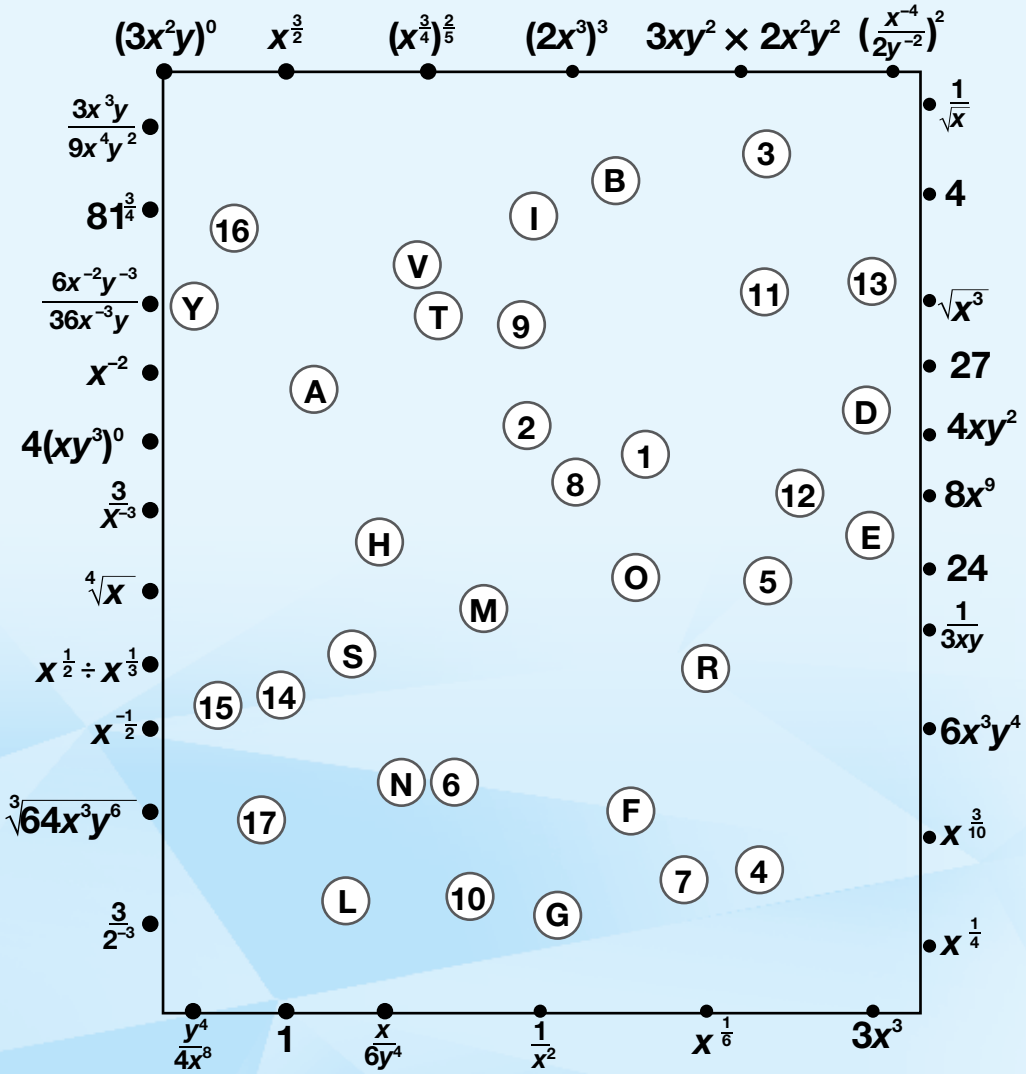
5 Discuss how the storage and speed of our brains compares to our current ability to send and store information in the digital world. The capacity of the human brain is 10–100 terabytes. On average 20 million billion bits of information are transmitted within the brain per second.

6 Investigate which country has the fastest internet speed and compare this to Australia.

CODE PUZZLE

What historical event took place in France in 1783?

Match the expressions across the top and on the left-hand side with the equivalent answer along the bottom or right-hand side by ruling a line between the dots. Each line passes through a number and a letter to reveal the puzzle's code.



1	2	3	4	5	6	7	1	8	9	10	10	3	11	2	12	1	9	5	6
		13	9	14	14	12	12	10	4	14	5	15	2	1	5	10	9		
13	9	14	14	12	12	10	5	10	16	3	10	1	3	11	13	17	1	2	3
8	12	10	1	15	12	14	4	5	3	6	13	6	12	1	2	3	6	7	

eBookplus Activities

1.1 Overview

Video

- The story of mathematics (eles-1840)

1.2 Review of index laws

Digital docs

- SkillSHEET (doc-5168): Index form
- SkillSHEET (doc-5169): Using a calculator to evaluate numbers given in index form

Interactivity

- IP interactivity 1.2 (int-4562): Review of index laws

1.3 Negative indices

Interactivities

- Colour code breaker (int-2777)
- IP interactivity 1.3 (int-4563): Negative indices

1.4 Fractional indices

Digital docs

- SkillSHEET (doc-5176): Addition of fractions
- SkillSHEET (doc-5177): Subtraction of fractions
- SkillSHEET (doc-5178): Multiplication of fractions
- SkillSHEET (doc-5179): Writing roots as fractional indices
- WorkSHEET 1.1 (doc-5180): Fractional indices

Interactivity

- IP interactivity 1.4 (int-4564): Fractional indices

1.5 Combining index laws

Digital doc

- WorkSHEET 1.2 (doc-5181): Combining index laws

Interactivity

- IP interactivity 1.5 (int-4565): Combining index laws

1.6 Review

Interactivities

- Word search (int-2826)
- Crossword (int-2827)
- Sudoku (int-3588)

Digital docs

- Topic summary (doc-13801)
- Concept map (doc-13802)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 1 Indices

Exercise 1.2 — Review of index laws

1 **a** a^7 **b** a^6 **c** b^8 **d** a^4b^7
e m^5n^{13} **f** $a^5b^7c^3$ **g** $m^6n^4p^5$ **h** $6a^2b$
i $10a^4b^9$ **j** $36m^8n^7$ **k** $12x^6y^6$ **l** $4x^8y^6$

2 **a** a **b** a^5 **c** b^3 **d** $\frac{4}{3}a^4$
e $3b^4$ **f** $4m^5$ **g** m^3n **h** $\frac{1}{2}y^2$
i $\frac{3}{4}x^3y$ **j** $7b^3$ **k** $\frac{5}{4}m^2p^2$ **l** $\frac{1}{2}xy^2$

3 **a** 1 **b** 1 **c** 1
d 3 **e** 4 **f** −3
g 3 **h** −7 **i** 4

4 **a** a^6 **b** $16a^{20}$ **c** $\frac{1}{81}m^8$
d $\frac{4}{9}n^8$ **e** a^6b^3 **f** $9a^6b^4$
g $16m^{12}n^{20}$ **h** $\frac{27}{64}m^6n^3$ **i** $\frac{a^4}{b^6}$
j $\frac{625m^{12}}{n^8}$ **k** $\frac{343x^3}{8y^{15}}$ **l** $\frac{81a^4}{625b^{12}}$
m −243 **n** 49 **o** −32

5 **a** D **b** D

6 **a** C **b** E **c** B **d** D

7 **a** 64 **b** 72 **c** 625
d 48 **e** 1600 **f** $\frac{27}{125}$
g 20 **h** 1 **i** 4

8 **a** x^{3yz} **b** a^b **c** m^an^b
d $\frac{a^{2x}}{b^{3x}}$ **e** $n^{3-p}m^{2-q}$ **f** a^{mp+np}

9 $a^3 = a \times a \times a$
$a^2 = a \times a$
$a^3 \times a^2 = a \times a \times a \times a \times a$
$= a^5$, not a^6
Explanations will vary.

10 They are equal when $x = 2$. Explanations will vary.

11 $3x^0 = 3$ and $(3x)^0 = 1$. Explanations will vary.

12 **a, b**

a	0	1	2	3
$3a^2$	0	3	12	27
$5a$	0	5	10	15
$3a^2 + 5a$	0	8	22	42
$3a^2 \times 5a$	0	15	120	405

c $3a^2 \times 5a$ will become much larger than $3a^2 + 5a$.

13 $1 \pm \sqrt{3}$

14 $1 \equiv 1$ $2 \equiv 10$ $3 \equiv 11$ $4 \equiv 100$ $5 \equiv 101$
$6 \equiv 110$ $7 \equiv 111$ $8 \equiv 1000$ $9 \equiv 1001$ $10 \equiv 1010$

15 **a** $x = 4$ **b** $x = 0, 2$

16 **a** a^2bc^7
b The student made a mistake when multiplying the two brackets in line 3. Individual brackets should be expanded first.

Challenge 1.1

1.08 seconds

Exercise 1.3 — Negative indices

1 **a** $\frac{1}{x^5}$ **b** $\frac{1}{y^4}$ **c** $\frac{2}{a^9}$
d $\frac{4}{5a^3}$ **e** $\frac{3x^2}{y^3}$ **f** $\frac{1}{4m^3n^4}$
g $\frac{6a^3}{bc^5}$ **h** a^6 **i** $\frac{2a^4}{3}$
j $2ab^2$ **k** $\frac{7b^3}{2a^4}$ **l** $\frac{2m^3a^2}{3b^4n^5}$

2 **a** $\frac{1}{a^2b^3}$ **b** $\frac{6}{x^6y}$ **c** $\frac{3}{n^8}$ **d** $\frac{4}{a^2b^5}$
e $\frac{2y}{3x}$ **f** $\frac{5y}{6x^3}$ **g** $\frac{3}{m^2n^2}$ **h** $\frac{4y^{12}}{x^5}$
i $\frac{1}{3m^3n^3}$ **j** $\frac{1}{32a^{15}m^{20}}$ **k** $\frac{4q^8}{p^{14}}$ **l** $\frac{3}{a^8b^{12}}$
m $\frac{27q^9}{8p^6}$ **n** $\frac{b^6}{4a^8}$ **o** $\frac{1}{8a^6b^6}$

3 **a** $\frac{1}{8}$ **b** $\frac{1}{36}$ **c** $\frac{1}{81}$ **d** $\frac{8}{9}$
e $\frac{1}{16}$ **f** $\frac{5}{36}$ **g** 48 **h** $\frac{32}{27}$
i $\frac{27}{25} = 1\frac{2}{25}$ **j** 4 **k** 125 **l** $\frac{3}{4}$

4 **a** 2^3 **b** 2^{-3} **c** 2^5 **d** 2^{-6}

5 **a** 3 **b** −2 **c** −1 **d** 3
e −2 **f** 0 **g** 3 **h** −3
i −6 **j** −2

6 **a** $\frac{3}{2}$ **b** $\frac{4}{5}$ **c** $\frac{2}{7}$ **d** 5

7 **a** $\frac{b}{a}$ **b** $\frac{b^3}{a^2}$ **c** $\frac{a^2}{b^3}$ **d** $\frac{1}{m^3n^2}$

8 **a** $\frac{1}{729}$ **b** $\frac{1}{20\,736}$
c 0.000 059 499 or $\frac{1}{16\,807}$ **d** 256
e $\frac{16\,384}{2187}$ **f** 9 765 625

9 **a** D **b** C **c** B

10 **a** B **b** D **c** C **d** E

11 **a** $\frac{m^2}{n^8}$ **b** $\frac{n^2}{m}$ **c** $\frac{25}{a^7b^6}$

12 **a** $r^6 - s^6$ **b** $m^{10} + 2m^5n^5 + n^{10}$
c 1 **d** p^2

13 2^{2r-4} **14** 6^{3m} **15** $x = 3$

16 Answers will vary; check with your teacher.

17 $x = 3, y = -1; 7$

18 **a** $n = -1, -2$ **b** $n = -1, 4$

Exercise 1.4 — Fractional indices

1 **a** 4 **b** 5 **c** 9
d 2 **e** 4 **f** 3

2 **a** $\sqrt{15}$ **b** $\sqrt[4]{m}$ **c** $\sqrt[5]{7^2}$ **d** $\sqrt{7^5}$
e $\sqrt[8]{w^3}$ **f** $\sqrt[4]{w^5}$ **g** $\sqrt[3]{5^{10}}$ **h** $\sqrt[10]{a^3}$

3 **a** $t^{\frac{1}{2}}$ **b** $5^{\frac{7}{4}}$ **c** $6^{\frac{11}{6}}$ **d** $x^{\frac{6}{7}}$

e $x^{\frac{7}{6}}$ **f** w^2 **g** $w^{\frac{1}{2}}$ **h** $11^{\frac{n}{x}}$

4 **a** 4 **b** 16 **c** 8 **d** 16

e 125 **f** 9 **g** $\frac{1}{9}$ **h** 27

i 1000 **j** 216 **k** $\sqrt{7}$ **l** $\sqrt[3]{12}$

5 **a** 1.44 **b** 2.24 **c** 1.48

d 1.26 **e** 2.54 **f** 0.66

g 0.54 **h** 0.81 **i** 0.86

6 **a** $4^{\frac{4}{5}}$ **b** $2^{\frac{1}{2}}$ **c** $a^{\frac{5}{6}}$

d $x^{\frac{23}{20}}$ **e** $10m^{\frac{8}{15}}$ **f** $2b^{\frac{5}{7}}$

g $-4y^{\frac{20}{9}}$ **h** $0.02a^{\frac{9}{8}}$ **i** $5x^{\frac{7}{2}}$

7 **a** $ab^{\frac{3}{2}}$ **b** $x^{\frac{4}{5}}y^{\frac{5}{9}}$ **c** $6a^{\frac{8}{5}}b^{\frac{17}{15}}$

d $2m^{\frac{19}{28}}n^{\frac{2}{5}}$ **e** $x^{\frac{19}{6}}y^{\frac{5}{6}}z^{\frac{5}{6}}$ **f** $8a^{\frac{2}{5}}b^{\frac{9}{8}}c$

8 **a** $3^{\frac{1}{6}}$ **b** $5^{\frac{5}{12}}$ **c** $12^{\frac{1}{2}}$

d $a^{\frac{3}{7}}$ **e** $x^{\frac{5}{4}}$ **f** $m^{\frac{11}{45}}$

g $\frac{1}{2}x^{\frac{3}{20}}$ **h** $\frac{1}{3}n^{\frac{2}{3}}$ **i** $\frac{5}{4}b^{\frac{7}{20}}$

9 **a** $x^{\frac{5}{3}}y^{\frac{7}{5}}$ **b** $a^{\frac{7}{45}}b^{\frac{4}{15}}$ **c** $\frac{1}{3}m^{\frac{3}{8}}n^{\frac{11}{56}}$

d $2x^{\frac{2}{15}}y^{\frac{3}{4}}$ **e** $\frac{1}{4}a^{\frac{11}{20}}b^{\frac{7}{20}}$ **f** $\frac{1}{7}p^{\frac{5}{24}}q^{\frac{1}{12}}$

10 **a** $2^{\frac{9}{20}}$ **b** $5^{\frac{1}{6}}$ **c** $7^{\frac{6}{5}}$

d $a^{\frac{3}{10}}$ **e** $m^{\frac{1}{6}}$ **f** $2^{\frac{1}{3}}b^{\frac{1}{6}}$

g $4p^{\frac{2}{5}}$ **h** $x^{\frac{m}{p}}$ **i** $3^{\frac{b}{c}}m^{\frac{a}{c}}$

11 **a** $a^{\frac{1}{4}}b^{\frac{1}{6}}$ **b** $a^3b^{\frac{3}{4}}$ **c** $x^{\frac{6}{5}}y^{\frac{7}{4}}$

d $3^{\frac{1}{3}}a^{\frac{1}{9}}b^{\frac{1}{5}}c^{\frac{1}{4}}$ **e** $5x^{\frac{1}{4}}y^{\frac{1}{3}}z^{\frac{1}{5}}$ **f** $\frac{a^{\frac{1}{2}}}{b^{\frac{2}{3}}}$

g $\frac{m^{\frac{8}{5}}}{n^{\frac{7}{4}}}$ **h** $\frac{b^{\frac{2}{5}}}{c^{\frac{8}{27}}}$ **i** $\frac{2^{\frac{1}{2}}x^{\frac{7}{2}}}{y^{\frac{3}{8}}}$

12 **a** E **b** C **c** B

13 **a** E **b** B

14 **a** a^4 **b** b^3 **c** m^4

d $4x^2$ **e** $2y^3$ **f** $2x^2y^3$

g $3m^3n^5$ **h** $2pq^2$ **i** $6a^2b^6$

15 **a** 2.007 s **b** 20.07 s **c** 4.98 swings

16 $(2^5a^5b^{10})^{\frac{1}{5}} = 2ab^2$ **17** $\frac{a^2\left(3-\sqrt{b^3}\right)}{9-b^3}$; $\frac{4}{11}$

18 $m^{\frac{1}{5}} - n^{\frac{1}{5}} + p^{\frac{1}{5}}$

19 **a** $a^{-\frac{1}{4}} \times b^{\frac{13}{2}}$

b No, because you can't take the fourth root of a negative number.

c $a = 1$

Exercise 1.5 — Combining index laws

1 **a** $54a^{10}b^9$ **b** $48a^5b^{16}$ **c** $\frac{2n^{13}}{m^9}$ **d** $500p^8q^{18}$

e $36a^{20}b^{10}$ **f** $\frac{15b^2}{c^{26}}$ **g** $12x^{\frac{7}{8}}y^{\frac{11}{15}}$ **h** $8m^{\frac{15}{4}}n^{\frac{15}{4}}$

i $\frac{6}{p^{\frac{7}{12}}}$ **j** $8p^{\frac{7}{45}}q^{\frac{5}{18}}$

2 **a** $\frac{5}{8a^7}$ **b** $\frac{x}{4y^6}$ **c** $\frac{27}{128m^{29}n^{26}}$

d $\frac{64y^{36}}{x^{24}}$ **e** $24a^{24}b^7$ **f** $\frac{27h^{12}}{8g^6}$

g $p^{\frac{35}{3}}q^{\frac{1}{2}}$ **h** $\frac{625}{81b^{20}c^{28}}$ **i** $x^{\frac{5}{3}}y^{\frac{1}{8}}z^{\frac{3}{2}}$

3 **a** $\frac{3a^2}{2}$ **b** $8n^2$ **c** $\frac{m^2n^4}{3}$

d $\frac{4x^5}{3y^8}$ **e** $\frac{36x^6}{y}$ **f** $\frac{y^2}{x^4}$

g $\frac{b^7}{3a^4}$ **h** $\frac{75q^5}{2p^{11}}$ **i** $x^{\frac{17}{10}}y^{\frac{7}{10}}$

4 **a** $\frac{2}{5a^4b^7}$ **b** $\frac{4a^3b^3}{15}$ **c** $\frac{n^9}{4m^9}$

d $\frac{4m^5}{9n^{15}}$ **e** $\frac{4}{81x^2y^{14}}$ **f** $48x^{11}y^6$

g $\frac{3p^4}{5q^9}$ **h** $\frac{2b^{\frac{1}{12}}}{3a^{\frac{17}{24}}}$ **i** $\frac{4x^{\frac{1}{12}}}{3y^{\frac{21}{20}}}$

5 **a** $\frac{5}{2a^{13}}$ **b** $\frac{56a^{11}b^6}{81}$ **c** $\frac{1024b^2}{81a}$ **d** $\frac{25}{128x^{23}y^4}$

e $\frac{4y^{36}}{27x^{16}}$ **f** $6m^{19}n^{19}$ **g** $\frac{16m^{\frac{11}{12}}n}{3}$ **h** $\frac{4b^{\frac{11}{2}}}{3^{\frac{1}{2}}c^{\frac{7}{30}}}$

6 **a** $\frac{125}{8}$ **b** 1

7 1

8 **a** 5^{y-1} **b** $y = 4$

9 E

10 A

11 **a** $m^{\frac{1}{6}}n^{-\frac{7}{6}}$ or $\sqrt[6]{\frac{m}{n^7}}$ **b** $g^{-6}h^3n^{\frac{3}{2}}$ **c** $3^{-\frac{7}{3}} \times 5^{-\frac{7}{6}}$

d 2^{-2} or $\frac{1}{4}$ **e** a^6b^{-8} or $\frac{a^6}{b^8}$ **f** $d^{\frac{14}{15}}$ or $\sqrt[15]{d^{14}}$

12 **a** 79 koalas **b** During the 6th year

13 **a** 79%

b 56%

c 31%

14 $\frac{4}{21}$

15 $z^2 + z^{-2} + \sqrt{5}$

Challenge 1.2

$x = \frac{2y}{y-z}$

Investigation — Rich task

1

Unit	Symbol	Power of 2 and value in bytes	Power of 10 and value in bytes
Byte	B	$2^0 = 1$	$10^0 = 1$
Kilobyte	KB	$2^{10} = 1024$	$10^3 = 1000$
Megabyte	MB	$2^{20} = 1\,048\,576$	$10^6 = 1\,000\,000$
Gigabyte	GB	$2^{30} = 1\,073\,741\,824$	$10^9 = 1\,000\,000\,000$
Terabyte	TB	$2^{40} = 1\,099\,511\,627\,776$	$10^{12} = 1\,000\,000\,000\,000$

2 Approximately 1.1 GB
3 3993.6 MB
4 1248 photos
5 Discuss with your teacher.
6 Discuss with your teacher. The discussion will depend on the latest information from the internet.

Code puzzle

The first manned hot-air balloon flight in a balloon invented by the Montgolfier brothers