Exercise 16.7 Logarithms

1. **WE18** Write the following in logarithmic form.

   a. $4^2 = 16$

   b. $2^5 = 32$

   c. $3^4 = 81$

   d. $6^2 = 36$

   e. $1000 = 10^3$

   f. $25 = 5^2$

   g. $4^3 = x$

   h. $5^x = 125$

   i. $7^x = 49$

   j. $p^4 = 16$

   k. $9^{\frac{1}{2}} = 3$

   l. $0.1 = 10^{-1}$

   m. $2 = 8^{\frac{1}{3}}$

   n. $2^{-1} = \dfrac{1}{2}$

   o. $a^0 = 1$

   p. $4^{\frac{3}{2}} = 8$

2. **MC** The statement $w = h^t$ is equivalent to:

   A. $w = \log_t h$

   B. $h = \log_t w$

   C. $t = \log_w h$

   D. $t = \log_h w$

3. **WE19** Write the following in index form.

   a. $\log^2 16 = 4$

   b. $\log^3 27 = 3$

   c. $\log^{10} 1\ 000\ 000 = 6$

   d. $\log^5 125 = 3$

   e. $\log_{16} 4 = \dfrac{1}{2}$

   f. $\log^4 64 = x$

   g. $\dfrac{1}{2} = \log_{49} 7$

   h. $\log^3 x = 5$

   i. $\log_{81} 9 = \dfrac{1}{2}$

   j. $\log^{10} 0.01 = -2$

   k. $\log^8 8 = 1$

   l. $\log_{64} 4 = \dfrac{1}{3}$

4. **MC** The statement $q = \log_r p$ is equivalent to:

   A. $q = r^p$

   B. $p = r^q$

   C. $r = p^q$

   D. $r = q^p$

5. **WE20** Evaluate the following logarithms.

   a. $\log_2 16$

   b. $\log_4 16$

   c. $\log_{11} 121$

   d. $\log_{10} 100000$

   e. $\log_3 243$

   f. $\log_2 128$

   g. $\log_5 1$

   h. $\log_9 3$

   i. $\log_3 \left(\dfrac{1}{3}\right)$

   j. $\log_6 6$

6. Write the value of each of the following.

   a. $\log_{10} 1$

   b. $\log_{10} 10$

   c. $\log_{10} 100$

   d. $\log_{10} 1000$

   e. $\log_{10} 10\ 000$

   f. $\log_{10} 100\ 000$

Exercise 16.8 Logarithm laws

1. Use a calculator to evaluate the following, correct to 5 decimal places.

   1. $\log_{10} 50$
   2. $\log_{10} 25$
   3. $\log_{10} 5$
   4. $\log_{10} 2$

2. Use your answers to question 1 to show that each of the following statements is true.

   1. $\log_{10} 25 + \log_{10} 2 = \log_{10} 50$
   2. $\log_{10} 50 - \log_{10} 2 = \log_{10} 25$
   3. $\log_{10} 25 = 2\log_{10} 5$
   4. $\log_{10} 50 - \log_{10} 25 - \log_{10} 2 = \log_{10} 1$

3. WE21 Evaluate the following.

   1. $\log_6 3 + \log_6 2$
   2. $\log_4 8 + \log_4 8$
   3. $\log_{10} 25 + \log_{10} 4$
   4. $\log_8 32 + \log_8 16$
   5. $\log_6 108 + \log_6 12$
   6. $\log_{14} 2 + \log_{14} 7$

4. WE22 Evaluate the following.

   1. $\log_2 20 - \log_2 5$
   2. $\log_3 54 - \log_3 2$
   3. $\log_4 24 - \log_4 6$
   4. $\log_{10} 30\,000 - \log_{10} 3$
   5. $\log_6 648 - \log_6 3$
   6. $\log_2 224 - \log_2 7$

5. WE23 Evaluate the following.

   1. $\log_3 27 + \log_3 2 - \log_3 6$
   2. $\log_4 24 - \log_4 2 - \log_4 6$
   3. $\log_6 78 - \log_6 13 + \log_6 1$
   4. $\log_2 120 - \log_2 3 - \log_2 5$

6. Evaluate $2\ \log_4 8$.

7. WE24 Evaluate the following.

   1. $2\log_{10} 5 + \log_{10} 4$
   2. $\log_3 648 - 3\ \log_3 2$
   3. $4\log_5 10 - \log_5 80$
   4. $\log_2 50 + \frac{1}{2}\log_2 16 - 2\ \log_2 5$

8. Evaluate the following.

   1. $\log_8 8$
   2. $\log_5 1$
   3. $\log_2 \left(\frac{1}{2}\right)$
   4. $\log_4 4^5$
   5. $\log_6 6^{-2}$
   6. $\log_{20} 20$
   7. $\log_2 1$
   8. $\log_3 \left(\frac{1}{9}\right)$
   9. $\log_4 \left(\frac{1}{2}\right)$
   10. $\log_5 \sqrt{5}$
   11. $\log_3 \left(\frac{1}{\sqrt{3}}\right)$
   12. $\log_2 8\sqrt{2}$

9. Use the logarithm laws to simplify each of the following.

1. $\log_a 5 + \log_a 8$
2. $\log_a 12 + \log_a 3 - \log_a 2$
3. $4\log_x 2 + \log_x 3$
4. $\log_x 100 - 2\log_x 5$
5. $3\log_a x - \log_a x^2$
6. $5\log_a a - \log_a a^4$
7. $\log_x 6 - \log_x 6x$
8. $\log_a a^7 + \log_a 1$
9. $\log_p \sqrt{p}$
10. $\log_k k\sqrt{k}$
11. $6\log_a \left(\frac{1}{a}\right)$
12. $\log_a \left(\frac{1}{\sqrt[3]{a}}\right)$

10. MC *Note:* There may be more than one correct answer.

1. The equation $y = 10^x$ is equivalent to:
   1. $x = 10^y$
   2. $x = \log_{10} y$
   3. $x = \log_x 10$
   4. $x = \log_y 10$
2. The equation $y = 10^{4x}$ is equivalent to:
   1. $x = \log_{10} \sqrt{4y}$
   2. $x = \log_{10} \sqrt[4]{y}$
   3. $x = 10^{\frac{1}{4}y}$
   4. $x = \frac{1}{4} \log_{10} y$
3. The equation $y = 10^{3x}$ is equivalent to:
   1. $x = \frac{1}{3}\log_{10} y$
   2. $x = \log_{10} y^{\frac{1}{3}}$
   3. $x = \log_{10} y - 3$
   4. $x = 10^{y-3}$
4. The equation $y = ma^{nx}$ is equivalent to:
   1. $x = \frac{1}{n}a^{my}$
   2. $x = \log_a \left(\frac{m}{y}\right)^n$
   3. $x = \frac{1}{n}(\log_a y - \log_a m)$
   4. $x = \frac{1}{n}\log_a \left(\frac{y}{m}\right)$

11. Simplify, and evaluate where possible, each of the following without a calculator.

1. $\log_2 8 + \log_2 10$
2. $\log_3 7 + \log_3 15$
3. $\log_{10} 20 + \log_{10} 5$
4. $\log_6 8 + \log_6 7$
5. $\log_2 20 - \log_2 5$
6. $\log_3 36 - \log_3 12$
7. $\log_5 100 - \log_5 8$
8. $\log_2 \frac{1}{3} + \log_2 9$
9. $\log_4 25 + \log_4 \frac{1}{5}$
10. $\log_{10} 5 - \log_{10} 20$
11. $\log_3 \frac{4}{5} - \log_3 \frac{1}{5}$
12. $\log_2 9 + \log_2 4 - \log_2 12$
13. $\log_3 8 - \log_3 2 + \log_3 5$
14. $\log_4 24 - \log_4 2 - \log_4 6$

Exercise 16.9 Solving equations

1. WE25 Solve for x in the following.
   1. $\log_5 x = 2$
   2. $\log_3 x = 4$
   3. $\log_2 x = -3$
   4. $\log_4 x = -2$
   5. $\log_{10} x^2 = 4$
   6. $\log_2 x^3 = 12$
   7. $\log_3(x+1) = 3$
   8. $\log_5(x-2) = 3$
   9. $\log_4(2x-3) = 0$
   10. $\log_{10}(2x+1) = 0$
   11. $\log_2(-x) = -5$
   12. $\log_3(-x) = -2$
   13. $\log_5(1-x) = 4$
   14. $\log_{10}(5-2x) = 1$
2. WE26 Solve for x in the following, given that $x > 0$.
   1. $\log_x 9 = 2$
   2. $\log_x 16 = 4$
   3. $\log_x 25 = \frac{2}{3}$
   4. $\log_x 125 = \frac{3}{4}$
   5. $\log_x \left(\frac{1}{8}\right) = -3$
   6. $\log_x \left(\frac{1}{64}\right) = -2$
   7. $\log_x 6^2 = 2$
   8. $\log_x 4^3 = 3$
3. WE27 Solve for x in the following.
   1. $\log_2 8 = x$
   2. $\log_3 9 = x$
   3. $\log_5 \left(\frac{1}{5}\right) = x$
   4. $\log_4 \left(\frac{1}{16}\right) = x$
   5. $\log_4 2 = x$
   6. $\log_8 2 = x$
   7. $\log_6 1 = x$
   8. $\log_8 1 = x$
   9. $\log_{\frac{1}{2}} 2 = x$
   10. $\log_{\frac{1}{3}} 9 = x$
4. WE28 Solve for x in the following.
   1. $\log_2 x + \log_2 4 = \log_2 20$
   2. $\log_5 3 + \log_5 x = \log_5 18$
   3. $\log_3 x - \log_3 2 = \log_3 5$
   4. $\log_{10} x - \log_{10} 4 = \log_{10} 2$
   5. $\log_4 8 - \log_4 x = \log_4 2$
   6. $\log_3 10 - \log_3 x = \log_3 5$
   7. $\log_6 4 + \log_6 x = 2$
   8. $\log_2 x + \log_2 5 = 1$
   9. $3 - \log_{10} x = \log_{10} 2$
   10. $5 - \log_4 8 = \log_4 x$
   11. $\log_2 x + \log_2 6 - \log_2 3 = \log_2 10$
   12. $\log_2 x + \log_2 5 - \log_2 10 = \log_2 3$
   13. $\log_3 5 - \log_3 x + \log_3 2 = \log_3 10$
   14. $\log_5 4 - \log_5 x + \log_5 3 = \log_5 6$

5. MC

1. The solution to the equation $\log_7 343 = x$ is:
   1. $x = 2$
   2. $x = 3$
   3. $x = 1$
   4. $x = 0$
2. If $\log_8 x = 4$, then x is equal to:
   1. 4096
   2. 512
   3. 64
   4. 2
3. Given that $\log_x 3 = \frac{1}{2}$, x must be equal to:
   1. 3
   2. 6
   3. 81
   4. 9
4. If $\log_a x = 0.7$, then $\log_a x^2$ is equal to:
   1. 0.49
   2. 1.4
   3. 0.35
   4. 0.837

6. Solve for x in the following equations.

1. $2^x = 128$
2. $3^x = 9$
3. $7^x = \frac{1}{49}$
4. $9^x = 1$
5. $5^x = 625$
6. $64^x = 8$
7. $6^x = \sqrt{6}$
8. $2^x = 2\sqrt{2}$
9. $3^x = \frac{1}{\sqrt{3}}$
10. $4^x = 8$
11. $9^x = 3\sqrt{3}$
12. $2^x = \frac{1}{4\sqrt{2}}$
13. $3^{x+1} = 27\sqrt{3}$
14. $2^{x-1} = \frac{1}{32\sqrt{2}}$
15. $4^{x+1} = \frac{1}{8\sqrt{2}}$

7. WE29 Solve the following equations, correct to 3 $\mathrm{decimal}$ places.

1. $2^x = 11$
2. $2^x = 0.6$
3. $3^x = 20$
4. $3^x = 1.7$
5. $5^x = 8$
6. $0.7^x = 3$
7. $0.4^x = 5$
8. $3^{x+2} = 12$
9. $7^{-x} = 0.2$
10. $8^{-x} = 0.3$
11. $10^{-2x} = 7$
12. $8^{2-x} = 0.75$

8. The decibel (dB) scale for measuring loudness, d, is given by the formula $d = 10\log_{10}(I \times 10^{12})$, where I is the intensity of sound in watts per square metre.

1. Find the number of decibels of sound if the intensity is 1.
2. Find the number of decibels of sound produced by a jet engine at a distance of 50 metres if the intensity is 10 watts per square metre.
3. Find the intensity of sound if the sound level of a pneumatic drill 10 metres away is 90 decibels.
4. Find how the value of d changes if the intensity is doubled. Give your answer to the nearest decibel.
5. Find how the value of d changes if the intensity is 10 times as great.
6. By what factor does the intensity of sound have to be multiplied in order to add 20 decibels to the sound level?

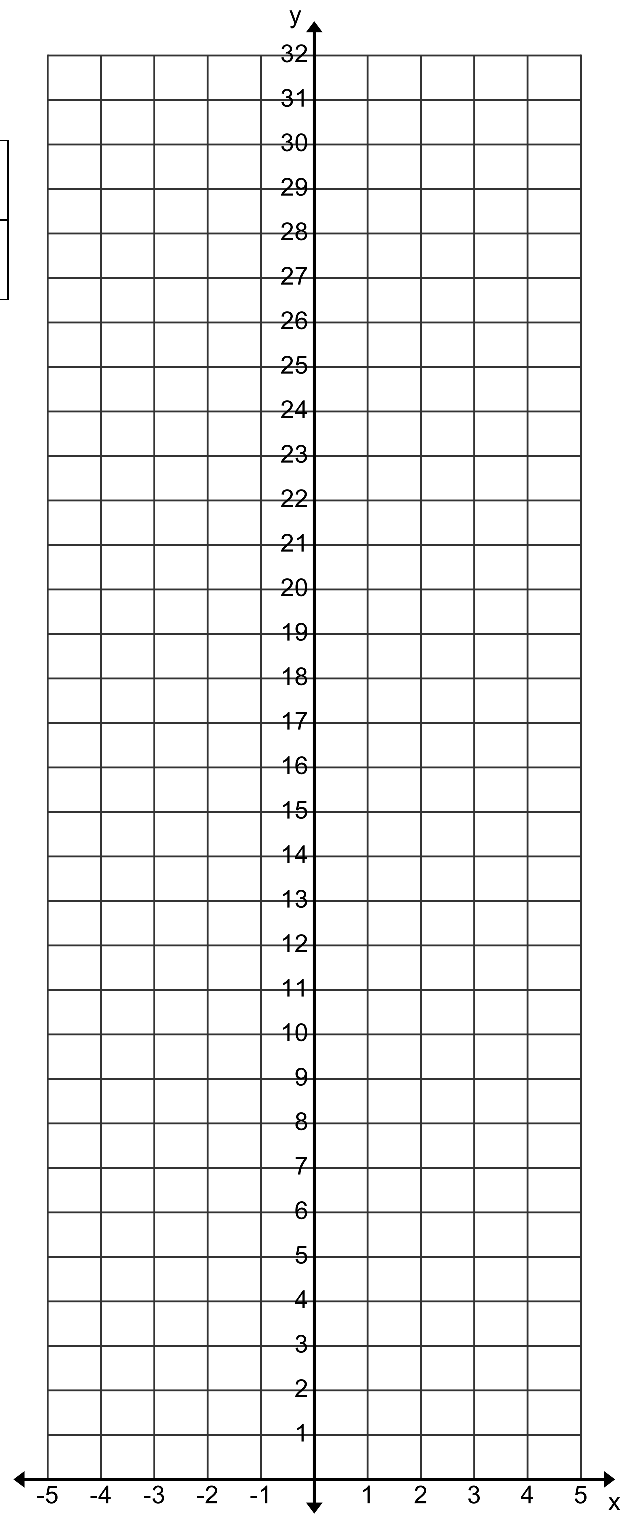
Name ______________________________________ Date ______________ Period ___________

Exponential and Logarithmic Graphs

1) Fill in the table below, make the graph, and identify the key features.

$$f(x) = 2^x$$

x	-3	-2	-1	0	1	2	3	4	5
f(x)									



a) What is the domain of this function?

____

b) What is the range of this function?

____

c) x-intercept = ___________ y-intercept = ___________

d) For what intervals is this function increasing?

____

e) For what intervals is this function decreasing?

____

f) When are the function values positive? When negative?

____

____

g) Describe the end behaviors of this function. What happens to the function values as x gets very small (i.e. large negative)? What happens to the function values as x gets very large?

____

____

2) Fill in the table below, make the graph, and identify the key features. $g(x) = \log_2 x$

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32
g(x)									

a) What is the domain of this function?

____

b) What is the range of this function?

____

c) x-intercept = ________ y-intercept = ___________

d) For what intervals is this function increasing?

____

e) For what intervals is this function decreasing?

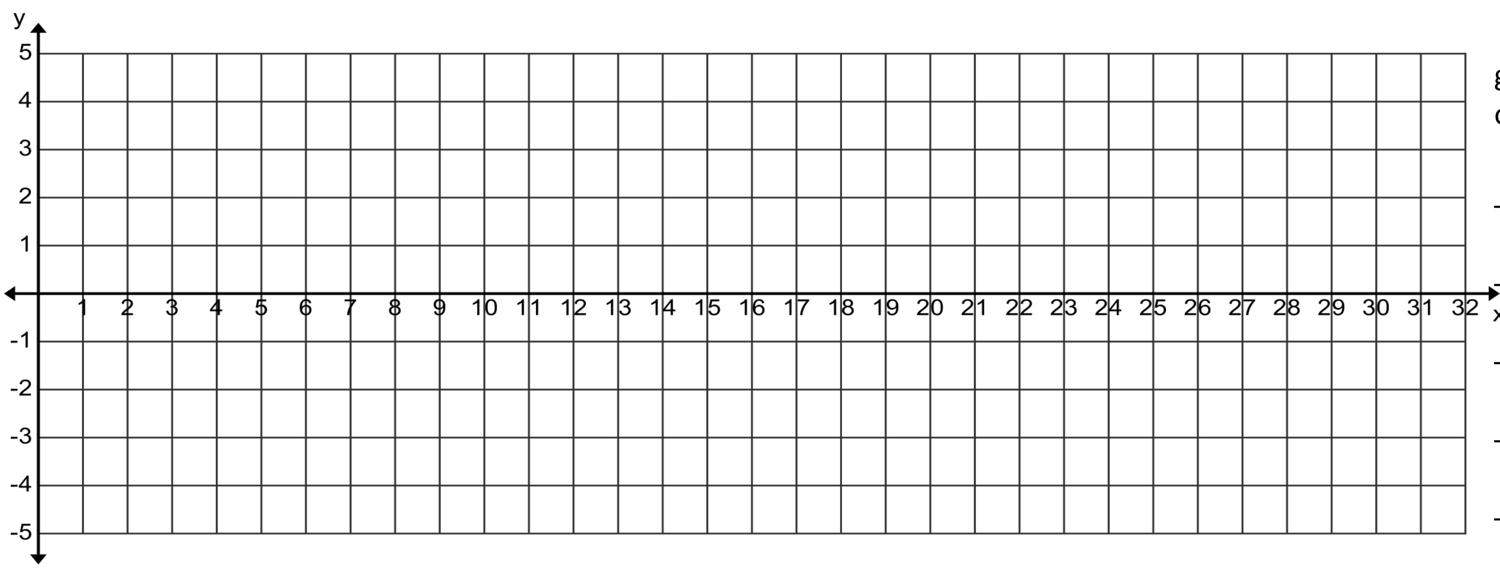
____

f) When are the function values positive? When negative?

____

____

____



g) Describe the end behaviors of the function.

____

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____

On the previous two pages you graphed and analyzed two functions,

$$f(x) = 2^x \qquad \text{and} \qquad g(x) = \log_2 x$$

Now you will compare the two functions to each other. Answer each prompt with complete sentences.

3) Compare the domains and ranges of the two functions. What do you notice?

____

____

4) Compare the intercepts of the two functions. What do you notice?

____

____

5) Compare the intervals on which the two functions are increasing.

____

____

6) Compare the end behaviors of the two functions. What do you notice?

____

____

7) What is the relationship between *f(*x) and *g*(x)?

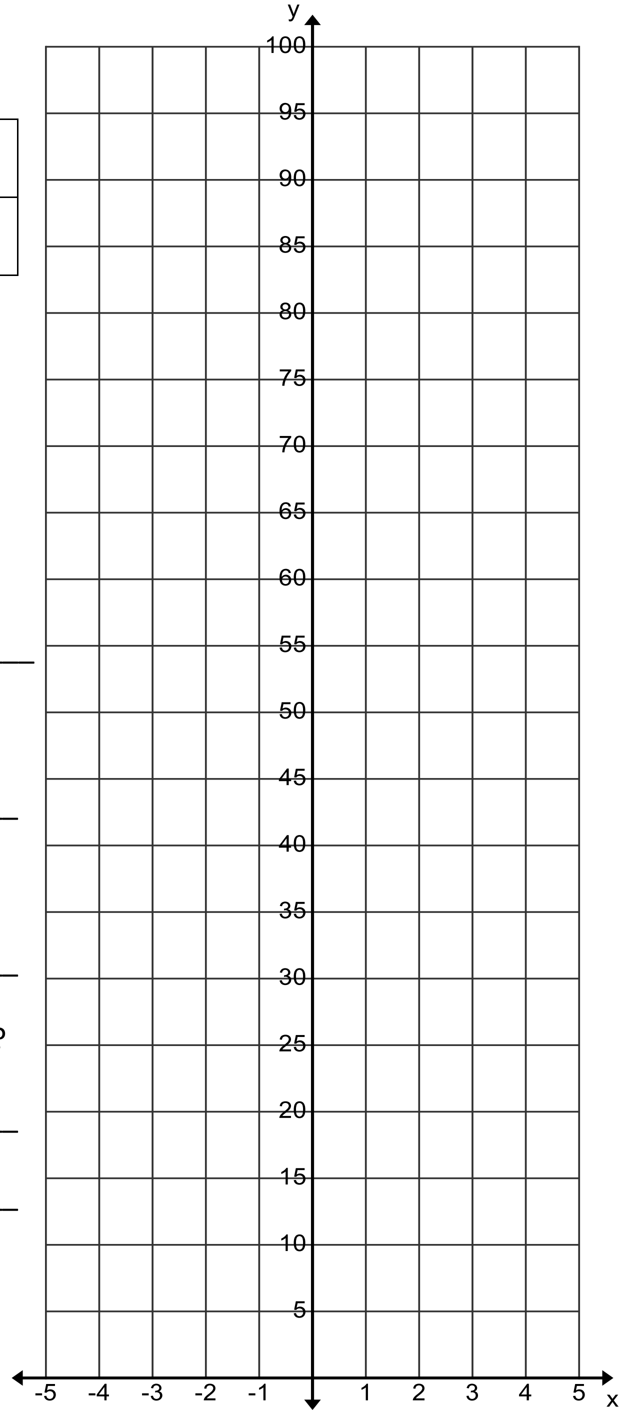
____

____

8) Fill in the table below, make the graph, and identify the key features.

$$j(x) = 10^x$$

x	-2	-1	0	1	2	3
j(x)						



a) What is the domain of this function?

____

b) What is the range of this function?

____

c) x-intercept = _____________ y-intercept = _____________

d) For what intervals is this function increasing?

____

e) For what intervals is this function decreasing?

____

f) When are the function values positive? When negative?

____

____

g) Describe the end behaviors of this function. What happens to the function values as x gets very small? What happens to the function values as x gets very large?

____

____

____

9) Fill in the table below, make the graph, and identify the key features. $k(x) = \log x$

x	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000
k(x)						

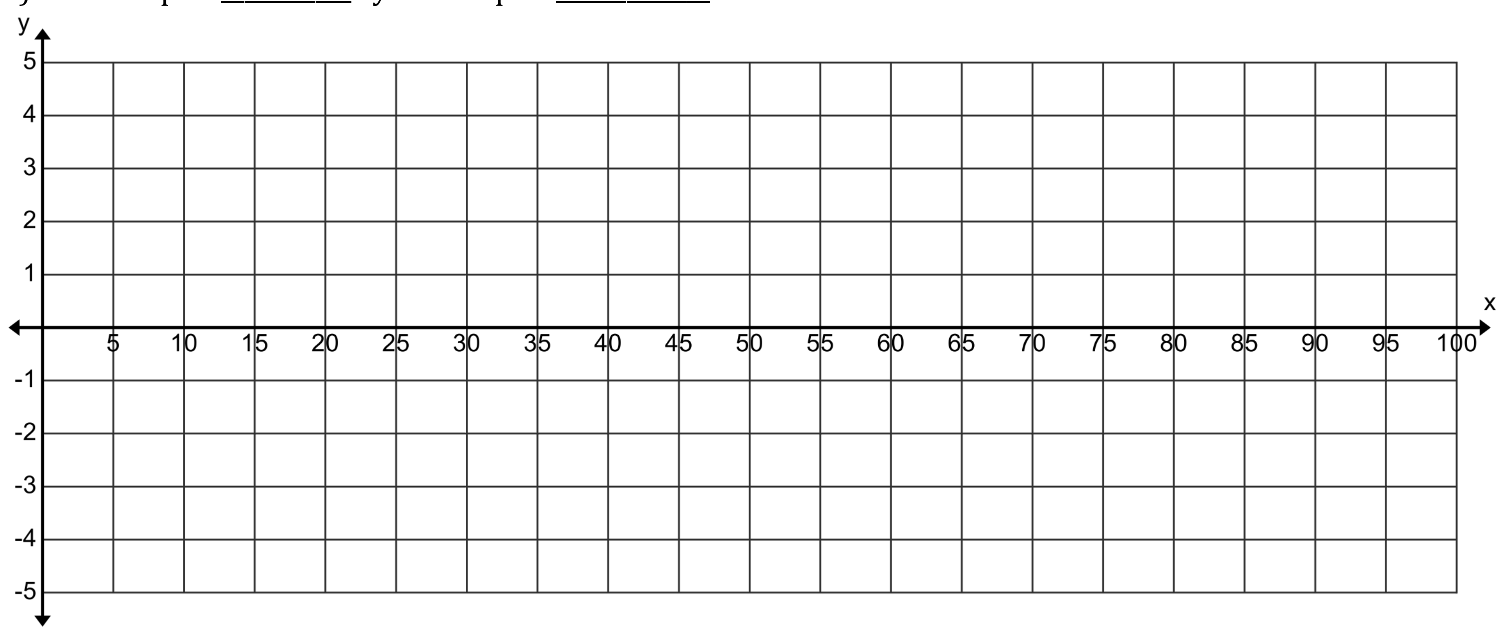
a) What is the domain of this function?

____

b) What is the range of this function?

____

c) x-intercept = _________ y-intercept = ___________



d) For what intervals is this function increasing?

____

e) For what intervals is this function decreasing?

____

f) When are the function values positive? When negative?

____

____

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g) Describe the end behaviors of the function.

____

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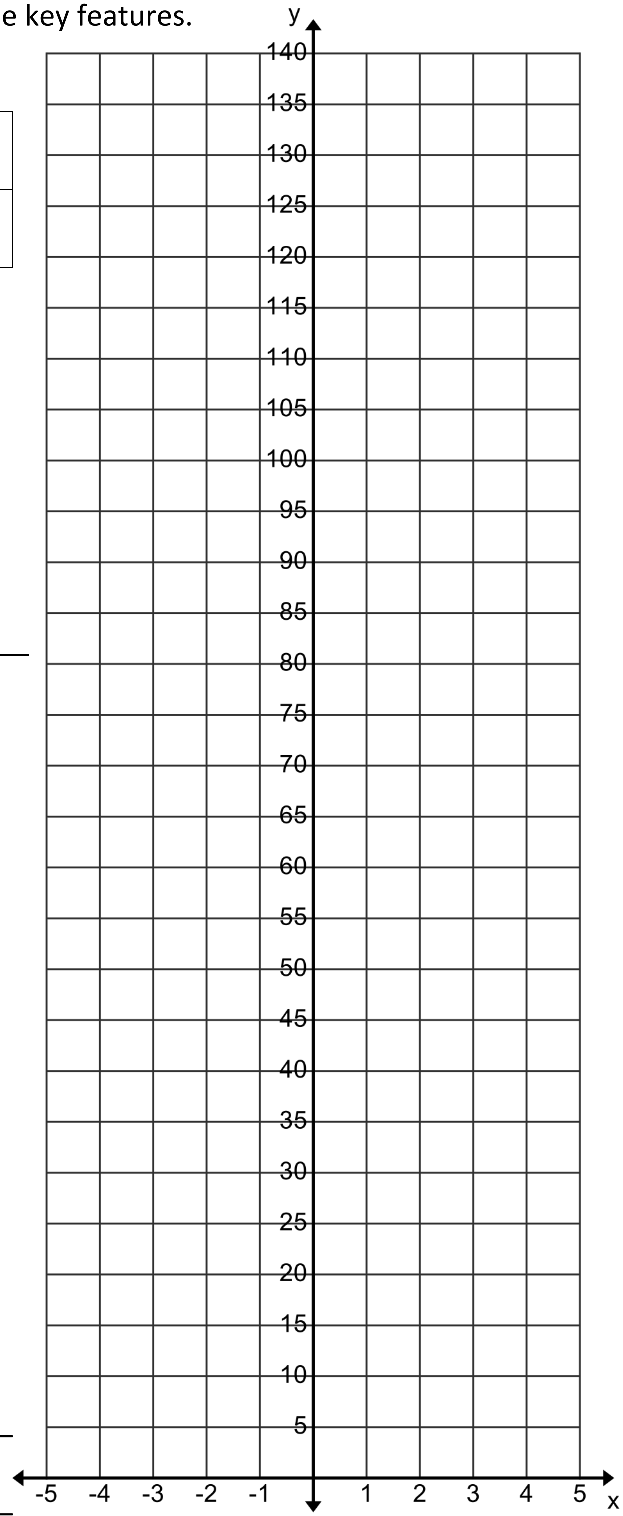
10) Fill in the table below, make the graph, and identify the key features.

$$m(x) = 5 \cdot 3^x$$

x	-2	-1	0	1	2	3
$m(x)$						



a) What is the domain of this function?

____

b) What is the range of this function?

____

c) x-intercept = ___________ y-intercept = _____________

d) For what intervals is this function increasing?

____

e) For what intervals is this function decreasing?

____

f) When are the function values positive? When negative?

____

____

g) Describe the end behaviors of this function. What happens to the function values as x gets very small? What happens to the function values as x gets very large?

____

____

____

____

11) Fill in the table below, make the graph, and identify the key features. $n(x) = \log_3 \frac{x}{5}$

x	$\frac{5}{9}$	$\frac{5}{3}$	5	15	45	135
n(x)						

a) What is the domain of this function?

____

b) What is the range of this function?

____

c) x-intercept = ________ y-intercept = __________

d) For what intervals is this function increasing?

____

e) For what intervals is this function decreasing?

____

f) When are the function values positive? When negative?

____

____

____

____

g) Describe the end behaviors of the function.

____

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____

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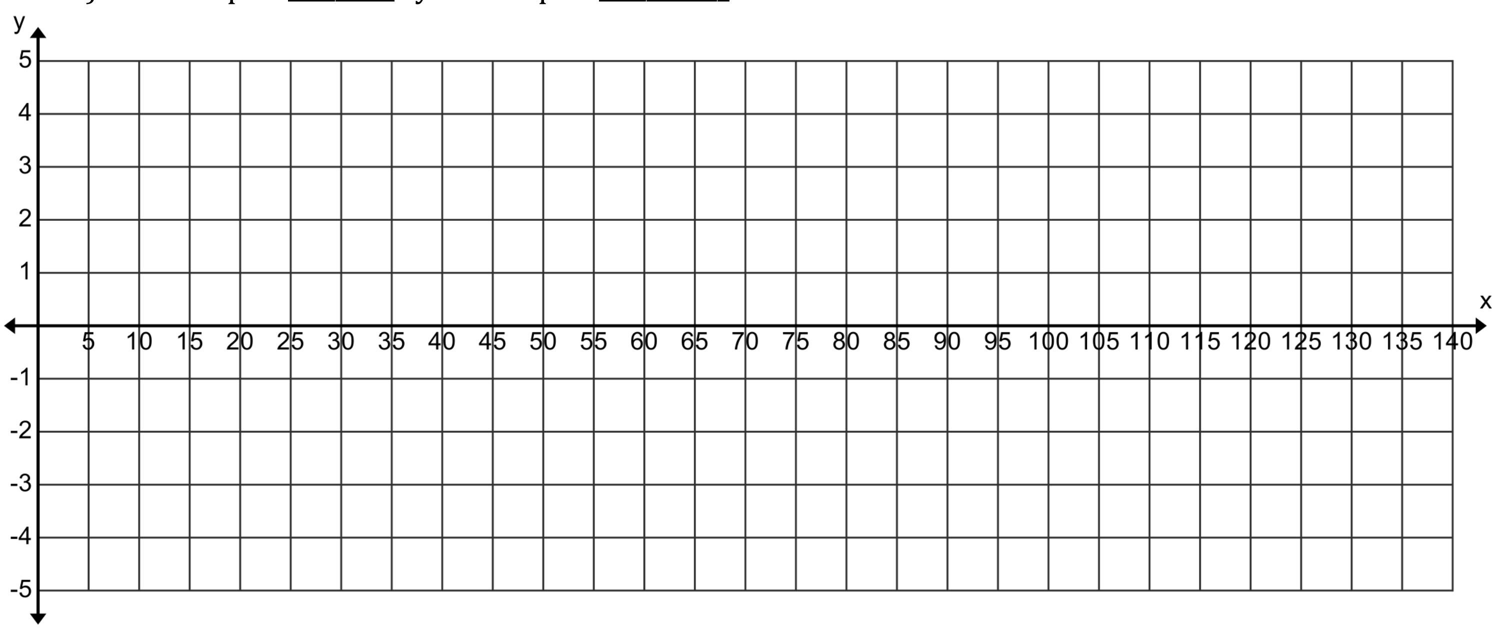

12) You have now graphed and analyzed three exponential functions and the three logarithmic functions that are their inverses. Look at the exponential functions and compare them. Look at the logarithmic functions and compare them.

What do the exponential functions have in common? Be sure to mention all of the key features of the graphs.

____

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13) What do the logarithmic functions have in common? Be sure to mention all of the key features of the graphs.

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Name________________________________ Date______________ Period_______

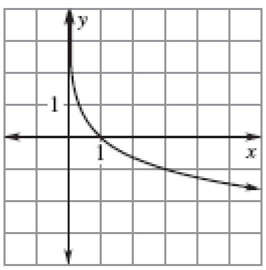
GRAPHING LOGARITHMIC FUNCTIONS WORKSHEET

Transformations of Logarithmic Functions: $y = a\log_b(x-h)+k$, where *a* is the vertical stretch or shrink, *h* is the horizontal translation and *k* is the vertical translation. The parent graph $y=\log_b x$ passes through the points (1, 0) and (b, 1) and has a vertical asymptote at $x=0$.

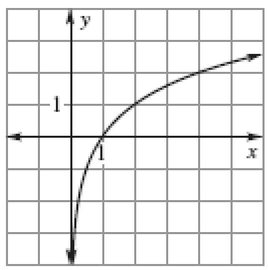
Match the function with its graph.

1. $f(x)=\log_2 x$ **2.** $f(x)=\log_5 x$ **3.** $f(x)=\log_{1/3} x$

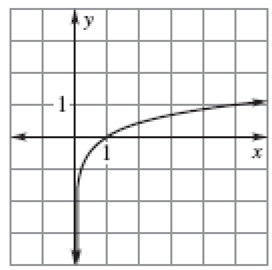
A.



B.

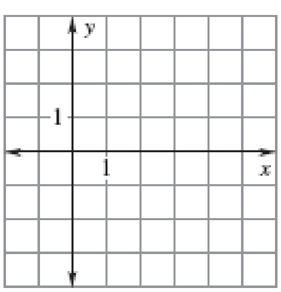


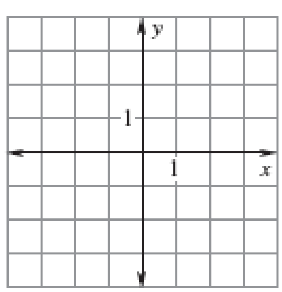
C.

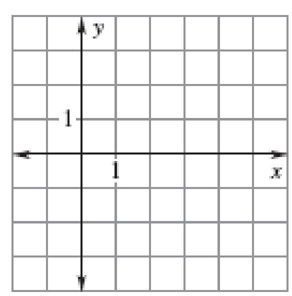


Graph the function. State the domain and range. Identify the parent function and describe the transformations.

4. $f(x) = \log_3 x$ **5.** $f(x) = \log_3(x + 2)$ **6.** $f(x)=-\log_3 x-1$



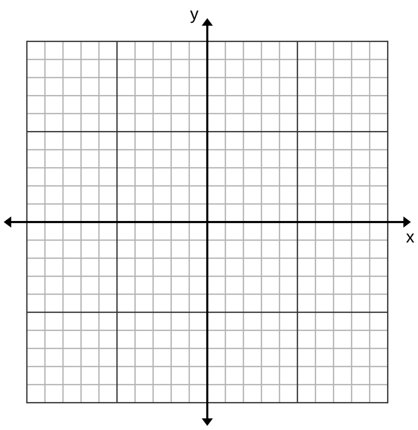


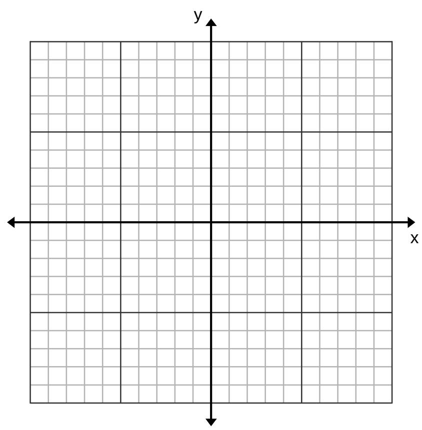


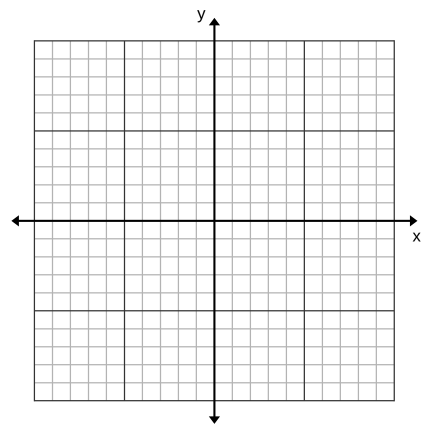
7. $f(x) = \log_2(x-3)+1$

8. $-\log_3(x+1)$

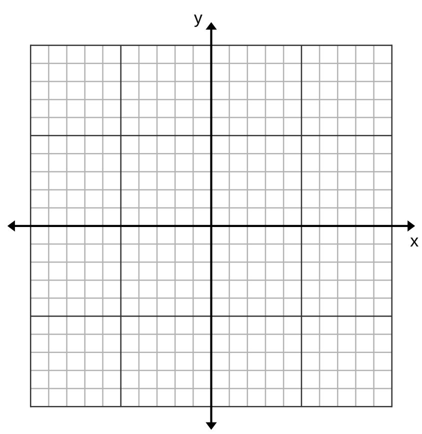
9. $f(x) = 3\log_2 x - 4$

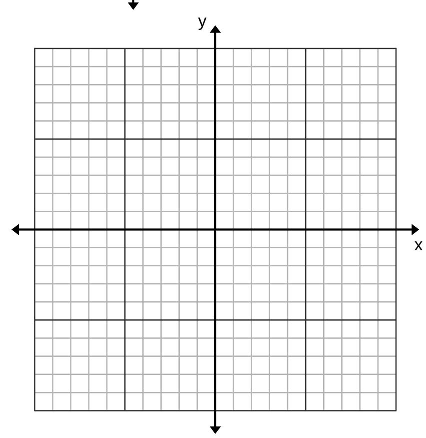






10. $f(x) = 4\log_{1/3}(x+2)$





11. $-\log_{1/2} x + 3$

12. The Palermo scale value of any object can be found using the equation $PS = \log_{10} R$, where R is the relative risk posed by the object. Write an equation in exponential form for the inverse of the function.