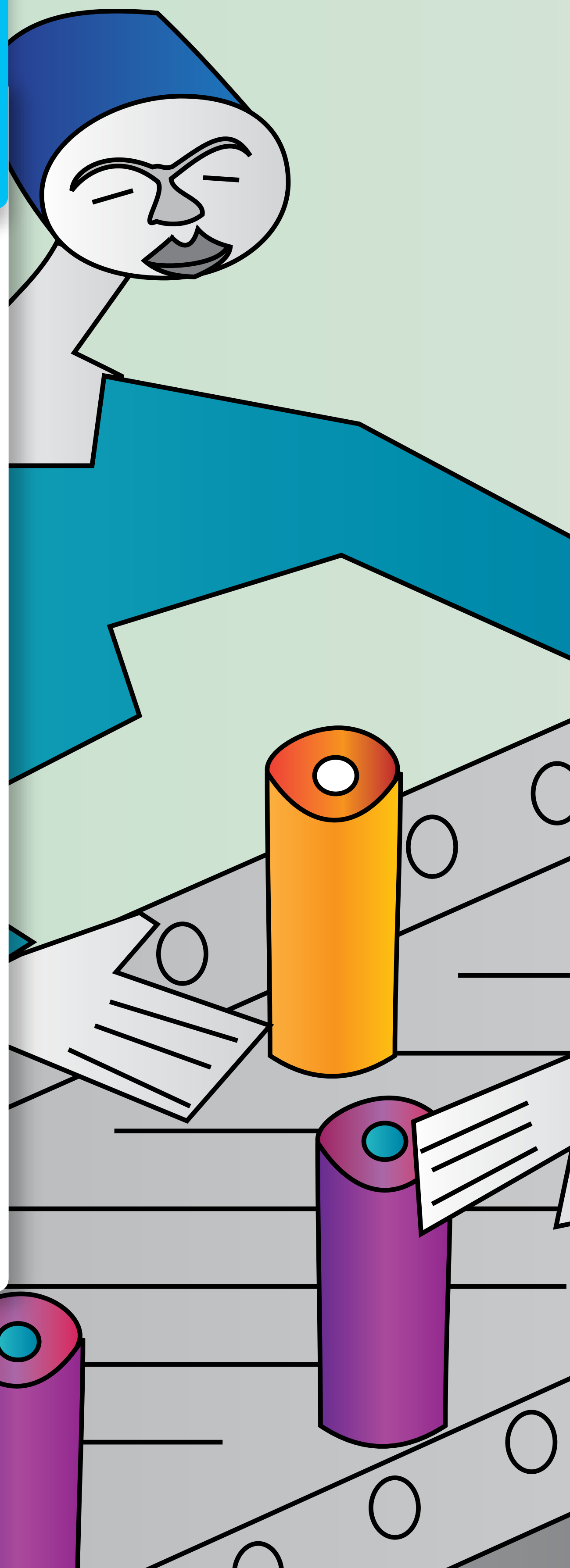
TOPIC 4

# Simultaneous linear equations and inequalities



## 4.1 Overview

### Why learn this?

Picture this — you own a factory that produces two different products, and you are planning to buy some new machines. The big machines are more expensive than the small ones, take up more floor space and need more staff to operate, but they can produce more. Which machines should you buy?

Solving simultaneous equations will help you determine feasible solutions to questions like this.

### What do you know?

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1 **THINK** List what you know about linear equations and linear inequations. Use a thinking tool such as a concept map to show your list.

2 **PAIR** Share what you know with a partner and then with a small group.

3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of linear equations and linear inequations.

### Learning sequence

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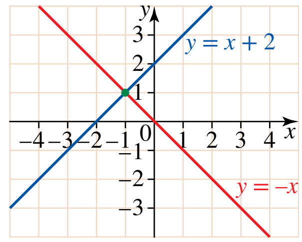
# 4.2 Graphical solution of simultaneous linear equations

## Simultaneous linear equations




- **Simultaneous** means occurring at the same time.
- When a point belongs to more than one line, the coordinates of the point satisfy all equations. The equations of the lines are called **simultaneous equations**. An example is shown at right.
- A **system of equations** is a set of two or more equations with the same variables.
- To solve simultaneous equations is to calculate the values of the variables that satisfy all equations in the system.
- Any two linear graphs will meet at a point, unless they are parallel.
- At this point, the two equations simultaneously share the same $x$- and $y$-coordinates, which are referred to as the solution.
- Simultaneous equations can be solved graphically or algebraically.

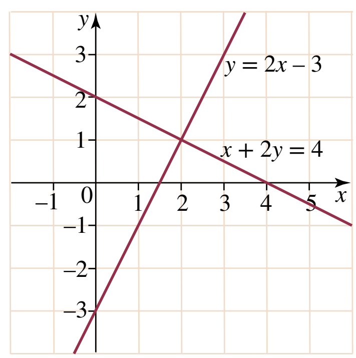
## Graphical solution

- The solution to a pair of simultaneous equations can be found by graphing the two equations and identifying the coordinates of the point of intersection.
- An accurate solution depends on drawing an accurate graph.
- Graph paper or graphing software can be used.

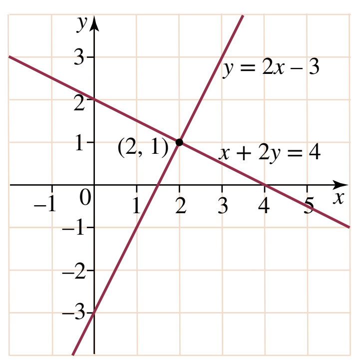
**WORKED EXAMPLE 1** TI CASIO

**Use the graphs of the given simultaneous equations to determine the point of intersection and, hence, the solution of the simultaneous equations.**

$$x + 2y = 4$$
$$y = 2x - 3$$




| THINK | WRITE/DRAW |
|---|---|
| 1 Write the equations and number them. | $x + 2y = 4$ [1]<br>$y = 2x - 3$ [2] |
| 2 Locate the point of intersection of the two lines. This gives the solution. | Point of intersection (2, 1)<br>Solution: $x = 2$ and $y = 1$ |



| | |
|---|---|
| **3** Check the solution by substituting $x = 2$ and $y = 1$ into the given equations. Comment on the results obtained. | Check equation [1]:<br>$\text{LHS} = x + 2y$ $\quad$ $\text{RHS} = 4$<br>$= 2 + 2(1)$<br>$= 4$<br>LHS = RHS<br>Check equation [2]:<br>$\text{LHS} = y$ $\quad$ $\text{RHS} = 2x - 3$<br>$= 1$ $\quad$ $= 2(2) - 3$<br>$= 4 - 3$<br>$= 1$<br>LHS = RHS<br>In both cases LHS = RHS, therefore the solution set (2, 1) is correct. |

**WORKED EXAMPLE 2** TI CASIO

**Check whether the given pair of coordinates, (5, −2), is the solution to the following pair of simultaneous equations.**

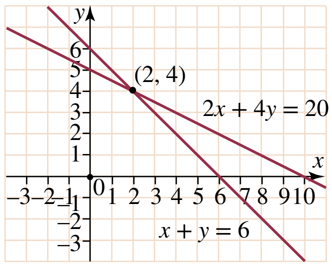
$$3x - 2y = 19$$
$$4y + x = -3$$

| THINK | WRITE |
|---|---|
| **1** Write the equations and number them. | $3x - 2y = 19$ $\quad$ [1]<br>$4y + x = -3$ $\quad$ [2] |
| **2** Substitute $x = 5$ and $y = -2$ into equation [1]. | Check equation [1]:<br>$\text{LHS} = 3x - 2y$ $\quad$ $\text{RHS} = 19$<br>$= 3(5) - 2(-2)$<br>$= 15 + 4$<br>$= 19$<br>LHS = RHS |
| **3** Substitute $x = 5$ and $y = -2$ into equation [2]. | Check equation [2]:<br>$\text{LHS} = 4y + x$ $\quad$ $\text{RHS} = -3$<br>$= 4(-2) + 5$<br>$= -8 + 5$<br>$= -3$<br>LHS = RHS<br>Therefore, the solution set (5, −2) is a solution to both equations. |

**WORKED EXAMPLE 3**

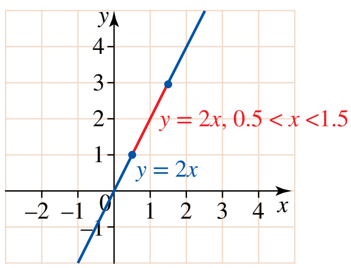
**Solve the following pair of simultaneous equations using a graphical method.**

$$x + y = 6$$
$$2x + 4y = 20$$

| THINK | WRITE/DRAW |
|---|---|
| 1 Write the equations, one under the other and number them. | $x + y = 6$ [1]<br>$2x + 4y = 20$ [2] |
| 2 Calculate the $x$- and $y$-intercepts for equation [1]. For the $x$-intercept, substitute $y = 0$ into equation [1]. | Equation [1]<br>$x$-intercept: when $y = 0$,<br>$x + 0 = 6$<br>$x = 6$<br>The $x$-intercept is at (6, 0). |
| For the $y$-intercept, substitute $x = 0$ into equation [1]. | $y$-intercept: when $x = 0$,<br>$0 + y = 6$<br>$y = 6$<br>The $y$-intercept is at (0, 6). |
| 3 Calculate the $x$- and $y$-intercepts for equation [2]. For the $x$-intercept, substitute $y = 0$ into equation [2]. | Equation [2]<br>$x$-intercept: when $y = 0$,<br>$2x + 0 = 20$<br>$2x = 20$ |
| Divide both sides by 2. | $x = 10$<br>The $x$-intercept is at (10, 0). |
| For the $y$-intercept, substitute $x = 0$ into equation [2]. | $y$-intercept: when $x = 0$,<br>$0 + 4y = 20$<br>$4y = 20$ |
| Divide both sides by 4. | $y = 5$<br>The $y$-intercept is at (0, 5). |
| 4 Use graph paper to rule up a set of axes and label the $x$-axis from 0 to 10 and the $y$-axis from 0 to 6. |  |
| 5 Plot the $x$- and $y$-intercepts for each equation. | |
| 6 Produce a graph of each equation by ruling a straight line through its intercepts. | |
| 7 Label each graph. | |
| 8 Locate the point of intersection of the lines. | The point of intersection is (2, 4). |

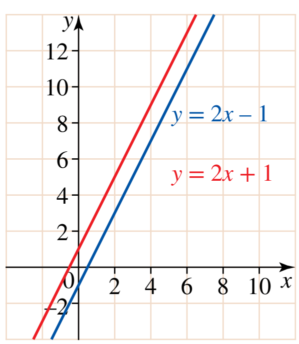
| | | |
|---|---|---|
| 9 | Check the solution by substituting $x = 2$ and $y = 4$ into each equation. | Check [1]: $\text{LHS} = x + y = 2 + 4 = 6$; $\text{RHS} = 6$; LHS = RHS<br>Check [2]: $\text{LHS} = 2x + 4y = 2(2) + 4(4) = 4 + 16 = 20$; $\text{RHS} = 20$; LHS = RHS |
| 10 | State the solution. | In both cases, LHS = RHS. Therefore, the solution set (2, 4) is correct.<br>The solution is $x = 2$, $y = 4$. |

## Equations with multiple solutions




- Two lines are **coincident** if they lie one on top of the other. For example, the line in blue and line segment in red at right are coincident.
- There are an infinite number of solutions to coincident equations. Every point where the lines coincide satisfies both equations and hence is a solution to the simultaneous equations.
- Coincident equations have the same equation, although the equations may have been transposed so they look different. For example, $y = 2x + 3$ and $2y - 4x = 6$ are coincident equations.

## Equations with no solutions



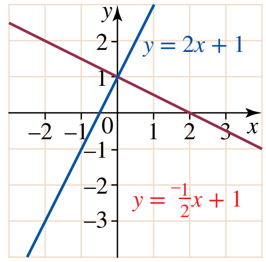

- If two lines do not intersect, there is no simultaneous solution to the equations. For example, the lines at right do not intersect, so there is no point that belongs to both lines.
- Parallel lines have the same gradient but a different $y$-intercept.
- For straight lines, the only situation in which the lines do not cross is if the lines are parallel *and* not coincident.
- Writing both equations in the form $y = mx + c$ confirms that the lines are parallel since the gradients are equal.

$$\begin{aligned} 2x - y &= 1 \quad [1] \\ -y &= 1 - 2x \\ -y &= -2x + 1 \\ y &= 2x - 1 \\ \text{Gradient } m &= 2 \end{aligned}$$

$$\begin{aligned} 4x - 2y &= -2 \quad [2] \\ -2y &= -2 - 4x \\ -2y &= -4x - 2 \\ y &= 2x + 1 \\ \text{Gradient } m &= 2 \end{aligned}$$

## Perpendicular lines

- Perpendicular lines meet at right angles (90°).
- Perpendicular lines have negative reciprocal gradients: $m_1 = \frac{-1}{m_2}$ or $m_1 m_2 = -1$, where $m_1$ is the gradient of the first line and $m_2$ is the gradient of the second line. For example, for the two lines at right, $m_1 = 2$ and $m_2 = \frac{-1}{2}$.

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# Exercise 4.2 Graphical solution of simultaneous linear equations

### INDIVIDUAL PATHWAYS

**REFLECTION**
What do you think is the major error made when solving simultaneous equations graphically?

| ■ PRACTISE | ■ CONSOLIDATE | ■ MASTER |
|---|---|---|
| Questions: 1, 2a–d, 3a–d, 4a–d, 7 | Questions: 1, 2c–g, 3a–d, 4a–f, 5, 7, 9 | Questions: 1, 2e–j, 3–10 |

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SkillSHEET
Graphing linear equations using the *x*- and *y*-intercept method
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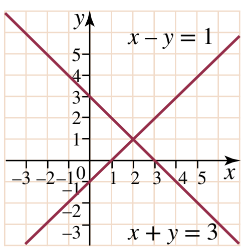
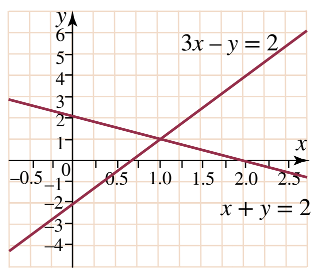
### FLUENCY

**1** **WE1** Use the graphs to find the solution of the simultaneous equations.

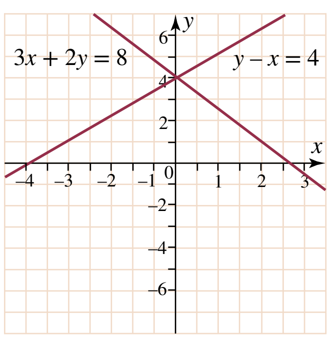
**a** $x + y = 3$
$x - y = 1$



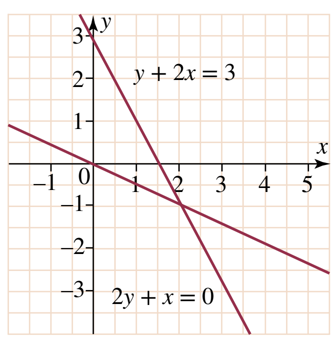

**b** $x + y = 2$
$3x - y = 2$



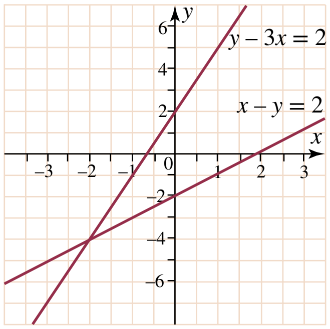

**c** $y - x = 4$
$3x + 2y = 8$



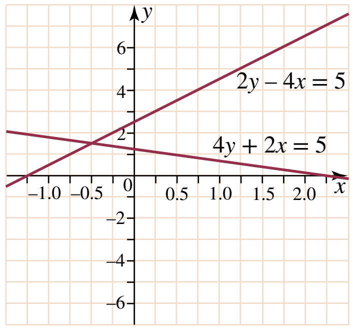

**d** $y + 2x = 3$
$2y + x = 0$



**e** $y - 3x = 2$
$x - y = 2$




**f** $2y - 4x = 5$
$4y + 2x = 5$




**2** **WE2** For the following simultaneous equations, use substitution to check if the given pair of coordinates is a solution.

**a** (7, 5) $3x + 2y = 31$, $2x + 3y = 28$
**b** (3, 7) $y - x = 4$, $2y + x = 17$
**c** (9, 1) $x + 3y = 12$, $5x - 2y = 43$
**d** (2, 5) $x - y = 7$, $2x + 3y = 18$
**e** (4, −3) $y = 3x - 15$, $4x + 7y = -5$
**f** (6, −2) $x - 2y = 2$, $3x + y = 16$
**g** (4, −2) $2x + y = 6$, $x - 3y = 8$
**h** (5, 1) $y - 5x = -24$, $3y + 4x = 23$
**i** (−2, −5) $3x - 2y = -4$, $2x - 3y = 11$
**j** (−3, −1) $y - x = 2$, $2y - 3x = 7$

**3** **WE3** Solve each of the following pairs of simultaneous equations using a graphical method.

**a** $x + y = 5$, $2x + y = 8$
**b** $x + 2y = 10$, $3x + y = 15$
**c** $2x + 3y = 6$, $2x - y = -10$
**d** $x - 3y = -8$, $2x + y = -2$
**e** $6x + 5y = 12$, $5x + 3y = 10$
**f** $y + 2x = 6$, $2y + 3x = 9$
**g** $y = 3x + 10$, $y = 2x + 8$
**h** $y = 8$, $3x + y = 17$
**i** $4x - 2y = -5$, $x + 3y = 4$
**j** $3x + y = 11$, $4x - y = 3$
**k** $3x + 4y = 27$, $x + 2y = 11$
**l** $3y + 3x = 8$, $3y + 2x = 6$

## UNDERSTANDING

**4** Using technology, determine which of the following pairs of simultaneous equations have no solutions. Confirm by finding the gradient of each line.

**a** $y = 2x - 4$, $3y - 6x = 10$
**b** $5x - 3y = 13$, $4x - 2y = 10$
**c** $x + 2y = 8$, $5x + 10y = 45$
**d** $y = 4x + 5$, $2y - 10x = 8$
**e** $3y + 2x = 9$, $6x + 4y = 22$
**f** $y = 5 - 3x$, $3y = -9x + 18$
**g** $4y + 3x = 7$, $12y + 9x = 22$
**h** $2y - x = 0$, $14y - 6x = 2$

**5** Two straight lines intersect at the point (3, −4). One of the lines has a $y$-intercept of 8. The second line is a mirror image of the first in the line $x = 3$. Determine the equation of the second line. (*Hint:* Draw a graph of both lines.)

### REASONING

6 At a well-known beach resort it is possible to hire a jet-ski by the hour in two different locations. On the northern beach the cost is $20 plus $12 per hour, while on the southern beach the cost is $8 plus $18 per hour. The jet-skis can be rented for up to 5 hours.

- a Write the rules relating cost to the length of rental.
- b On the same set of axes sketch a graph of cost ($y$-axis) against length of rental ($x$-axis) for 0–5 hours.
- c For what rental times, if any, is the northern beach rental cheaper than the southern beach rental? Use your graph to justify your answer.
- d For what length of rental time are the two rental schemes identical? Use the graph and your rules to justify your answer.

7 For each of the pairs of simultaneous equations below, determine whether they are the same line, parallel lines, perpendicular lines or intersecting lines. Show your working.

- a $2x - y = -9$ <br> $-4x - 18 = -2y$
- b $x - y = 7$ <br> $x + y = 7$
- c $x + 6 = y$ <br> $2x + y = 6$
- d $x + y = -2$ <br> $x + y = 7$

8 Which of the following problems has one solution, an infinite number of solutions or no solution? Explain your answers.

- a $x - y = 1$ <br> $2x - 3y = 2$
- b $2x - y = 5$ <br> $4x - 2y = -6$
- c $x - 2y = -8$ <br> $4x - 8y = -16$

### PROBLEM SOLVING

9 Line A is parallel to the line with equation $y - 3x - 3 = 0$ and passes through the point (1, 9). Line B is perpendicular to the line with equation $2y - x + 6 = 0$ and passes through the point (2, –3).

- a Find the equation of line A.
- b Find the equation of line B.
- c Sketch both lines on the one set of axes to find where they intersect.

10 Solve the system of three simultaneous equations graphically.

$$3x - y = 2$$
$$y + 3x = 4$$
$$2y - x = 1$$

# 4.3 Solving simultaneous linear equations using substitution

- There are two algebraic methods that are commonly used to solve simultaneous equations.
- They are the **substitution method** and the **elimination method**.

## Substitution method

- The substitution method is particularly useful when one (or both) of the equations is in a form where one of the two variables is the subject.
- This variable is then substituted into the other equation, producing a third equation with only one variable.
- This third equation can then be used to determine the value of the variable.

**WORKED EXAMPLE 4** TI CASIO

**Solve the simultaneous equations $y = 2x - 1$ and $3x + 4y = 29$ using the substitution method.**

| THINK | WRITE |
|---|---|
| 1 Write the equations, one under the other and number them. | $y = 2x - 1$ [1]<br>$3x + 4y = 29$ [2] |
| 2 $y$ and $2x - 1$ are equal so substitute expression $(2x - 1)$ for $y$ into equation [2]. | Substituting $(2x - 1)$ into [2]:<br>$3x + 4(2x - 1) = 29$ |
| 3 Solve for $x$.<br>(i) Expand the brackets on the LHS of the equation.<br>(ii) Collect like terms.<br>(iii) Add 4 to both sides of the equation.<br>(iv) Divide both sides by 11. | $3x + 8x - 4 = 29$<br>$11x - 4 = 29$<br>$11x = 33$<br>$x = 3$ |
| 4 Substitute $x = 3$ into either of the equations, say [1], to find the value of $y$. | Substituting $x = 3$ into [1]:<br>$y = 2(3) - 1$<br>$= 6 - 1$<br>$= 5$ |
| 5 Write your answer. | Solution: $x = 3$, $y = 5$ or $(3, 5)$ |
| 6 Check the solution by substituting $(3, 5)$ into equation [2]. | Check: Substitute $(3, 5)$ into $3x + 4y = 29$.<br>LHS $= 3(3) + 4(5)$ RHS $= 29$<br>$= 9 + 20$<br>$= 29$<br>As LHS = RHS, the solution is correct. |

**WORKED EXAMPLE 5**

**Solve the pair of simultaneous equations $y = 5x - 8$ and $y = -3x + 16$ using the substitution method.**

| THINK | WRITE |
|---|---|
| 1 Write the equations, one under the other and number them. | $y = 5x - 8$ [1]<br>$y = -3x + 16$ [2] |
| 2 Both equations are written with $y$ as the subject, so equate them. | $5x - 8 = -3x + 16$ |
| 3 Solve for $x$.<br>(i) Add $3x$ to both sides of the equation.<br>(ii) Add 8 to both sides of the equation.<br>(iii) Divide both sides of the equation by 8. | <br>$8x - 8 = 16$<br>$8x = 24$<br>$x = 3$ |
| 4 Substitute the value of $x$ into either of the original equations, say [1], and solve for $y$. | Substituting $x = 3$ into [1]:<br>$y = 5(3) - 8$<br>$= 15 - 8$<br>$= 7$ |

| | |
|---|---|
| 5 Write your answer. | Solution: $x = 3, y = 7$ or $(3, 7)$ |
| 6 Check the answer by substituting the point of intersection into equation [2]. | Check: Substitute into $y = -3x + 16$.<br>$\text{LHS} = y$<br>$= 7$<br>$\text{RHS} = -3x + 16$<br>$= -3(3) + 16$<br>$= -9 + 16$<br>$= 7$<br>As LHS = RHS, the solution is correct. |

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## Exercise 4.3 Solving simultaneous linear equations using substitution

**REFLECTION**
When would you choose the substitution method in solving simultaneous equations?

### INDIVIDUAL PATHWAYS

| PRACTICE | CONSOLIDATE | MASTER |
|---|---|---|
| Questions:<br>1a–d, 2a–d, 4, 6, 8 | Questions:<br>1a–d, 2c–f, 5–9 | Questions:<br>1–11 |

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### FLUENCY

1 **WE4** Solve the following simultaneous equations using the substitution method. Check your solutions using technology.

a $x = -10 + 4y$
$3x + 5y = 21$

b $3x + 4y = 2$
$x = 7 + 5y$

c $3x + y = 7$
$x = -3 - 3y$

d $3x + 2y = 33$
$y = 41 - 5x$

e $y = 3x - 3$
$-5x + 3y = 3$

f $4x + y = 9$
$y = 11 - 5x$

g $x = -5 - 2y$
$5y + x = -11$

h $x = -4 - 3y$
$-3x - 4y = 12$

i $x = 7 + 4y$
$2x + y = -4$

j $x = 14 + 4y$
$-2x + 3y = -18$

k $3x + 2y = 12$
$x = 9 - 4y$

l $y = 2x + 1$
$-5x - 4y = 35$

2 **WE5** Solve the following pairs of simultaneous equations using the substitution method. Check your solutions using technology.

a $y = 2x - 11$ and $y = 4x + 1$

b $y = 3x + 8$ and $y = 7x - 12$

c $y = 2x - 10$ and $y = -3x$

d $y = x - 9$ and $y = -5x$

e $y = -4x - 3$ and $y = x - 8$

f $y = -2x - 5$ and $y = 10x + 1$

g $y = -x - 2$ and $y = x + 1$

h $y = 6x + 2$ and $y = -4x$

i $y = 0.5x$ and $y = 0.8x + 0.9$

j $y = 0.3x$ and $y = 0.2x + 0.1$

k $y = -x$ and $y = -\frac{2}{7}x + \frac{4}{7}$

l $y = -x$ and $y = -\frac{3}{4}x - \frac{1}{4}$

### UNDERSTANDING

3 A small farm has sheep and chickens. There are twice as many chicken as sheep, and there are 104 legs between the sheep and the chickens. How many chickens are there?

**4** Use substitution to solve each of the following pairs of simultaneous equations.

**a** $5x + 2y = 17$
$y = \dfrac{3x - 7}{2}$

**b** $2x + 7y = 17$
$x = \dfrac{1 - 3y}{4}$

**c** $2x + 3y = 13$
$y = \dfrac{4x - 15}{5}$

**d** $-2x - 3y = -14$
$x = \dfrac{2 + 5y}{3}$

**e** $3x + 2y = 6$
$y = 3 - \dfrac{5x}{3}$

**f** $-3x - 2y = -12$
$y = \dfrac{5x - 20}{3}$

**5** Use substitution to solve each of the following pairs of simultaneous equations for $x$ and $y$ in terms of $m$ and $n$.

**a** $mx + y = n$
$y = mx$

**b** $x + ny = m$
$y = nx$

**c** $mx - y = n$
$y = nx$

**d** $mx - ny = n$
$y = x$

**e** $mx - ny = -m$
$x = y - n$

**f** $mx + y = m$
$x = \dfrac{y + m}{n}$

**6** Determine the values of $a$ and $b$ so that the pair of equations $ax + by = 17$ and $2ax - by = -11$ has a unique solution of $(-2, 3)$.

**7** The earliest record of magic squares is from China in about 2200 BC. In magic squares the sums of the numbers of each row, column and diagonal are all equal to a magic number. Let $z$ be the magic number. By creating a set of equations, solve to find the magic number and the missing values in the magic square.

| | | |
|---|---|---|
| $m$ | 11 | 7 |
| 9 | | |
| $n$ | 5 | 10 |

## REASONING

**8 a** For the pair of simultaneous equations:
$8x - 7y = 9$
$x + 2y = 4$,
which of the equations is the logical choice to make $x$ the subject of the equation?

**b** Use the substitution method to solve the system of equations. Show all your working.

**9** A particular chemistry book costs $6 less than a particular physics book, while two such chemistry books and three such physics books cost a total of $123. Construct two simultaneous equations and solve them using the substitution method. Show your working.

## PROBLEM SOLVING

**10** Use the substitution method to solve the following.
$2x + y - 9 = 0$
$4x + 5y + 3 = 0$

**11** Use the substitution method to solve the following.
$\dfrac{y - x}{2} - \dfrac{x + y}{3} = \dfrac{1}{6}$
$\dfrac{x}{5} + \dfrac{y}{2} = \dfrac{1}{2}$

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**Interactivity**
Simultaneous linear equations
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# 4.4 Solving simultaneous linear equations using elimination

- The elimination method is an algebraic method to solve simultaneous equations without graphing.
- If two balanced equations contain the same variables, the equations can be added or subtracted to eliminate one of the variables. For example, the equations $2x + y = 5$ and $x + y = 3$ are shown at right on balance scales.

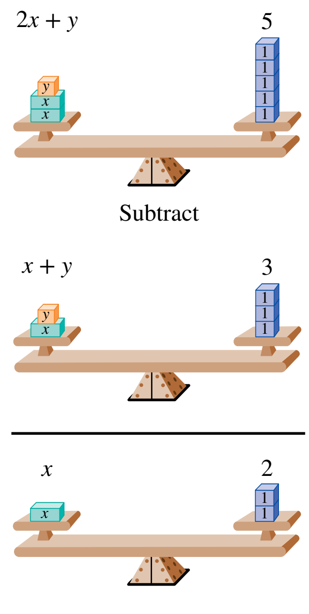
  If the left-hand side of the second equation is subtracted from the left-hand side of the first equation, and the right-hand side of the second equation is subtracted from the right-hand side of the first equation, the variable $y$ is eliminated, leaving $x = 2$.

  Another way to represent this situation is:

$$\begin{array}{r} 2x + y \ = 5 \\ -\ (x + y = 3) \\ \hline x \qquad = 2 \\ \hline \end{array}$$

  In this example, the variable is eliminated by subtraction to reveal the value of $x$. The value of $y$ can then be calculated by substituting $x = 2$ into either equation.

$$2(2) + y = 5 \Rightarrow y = 1$$



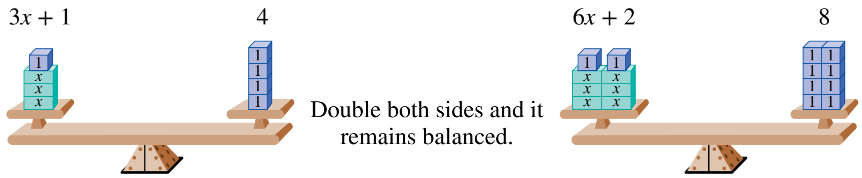

## WORKED EXAMPLE 6

**Solve the following pair of simultaneous equations using the elimination method.**

$$-2x - 3y = -9 \qquad 2x + y = 7$$

| THINK | WRITE |
|---|---|
| **1** Write the equations, one under the other and number them. | $-2x - 3y = -9$ [1]<br>$2x + y = 7$ [2] |
| **2** Look for an addition or subtraction that will eliminate either $x$ or $y$.<br>*Note:* Adding equations [1] and [2] in order will eliminate $x$. | [1] + [2]:<br>$-2x - 3y + (2x + y) = -9 + 7$<br>$-2x - 3y + 2x + y = -2$<br>$-2y = -2$ |
| **3** Solve for $y$ by dividing both sides of the equation by $-2$. | $y = 1$ |
| **4** Substitute the value of $y$ into equation [2].<br>*Note:* $y = 1$ may be substituted into either equation. | Substituting $y = 1$ into [2]:<br>$2x + 1 = 7$ |
| **5** Solve for $x$.<br>(i) Subtract 1 from both sides of the equation.<br>(ii) Divide both sides of the equation by 2. | $2x = 6$<br>$x = 3$ |
| **6** Write the solution. | Solution: $x = 3$, $y = 1$ or $(3, 1)$ |
| **7** Check the solution by substituting $(3, 1)$ into equation [1] since equation [2] was used to find the value of $x$. | Check: Substitute into $-2x - 3y = -9$.<br>LHS $= -2(3) - 3(1)$<br>$= -6 - 3$<br>$= -9$<br>RHS $= -9$<br>LHS = RHS, so the solution is correct. |

- If a variable is not eliminated when the equations are simply added or subtracted, it may be necessary to multiply one or both equations by some number or numbers so that when the equations are added, one of the variables is then eliminated.
- If two equal quantities are multiplied by the same number, the results remain equal.




**WORKED EXAMPLE 7**

**Solve the following pair of simultaneous equations using the elimination method.**

$$x - 5y = -17 \qquad 2x + 3y = 5$$

| THINK | WRITE |
|---|---|
| 1 Write the equations, one under the other and number them. | $x - 5y = -17$ [1]<br>$2x + 3y = 5$ [2] |
| 2 Look for a single multiplication that will create the same coefficient of either $x$ or $y$. Multiply equation [1] by 2 and call the new equation [3]. | [1] × 2: $2x - 10y = -34$ [3] |
| 3 Subtract equation [2] from [3] in order to eliminate $x$. | [3] − [2]:<br>$2x - 10y - (2x + 3y) = -34 - 5$<br>$2x - 10y - 2x - 3y = -39$<br>$-13y = -39$ |
| 4 Solve for $y$ by dividing both sides of the equation by −13. | $y = 3$ |
| 5 Substitute the value of $y$ into equation [2]. | Substituting $y = 3$ into [2]:<br>$2x + 3(3) = 5$<br>$2x + 9 = 5$ |
| 6 Solve for $x$.<br>(i) Subtract 9 from both sides of the equation.<br>(ii) Divide both sides of the equation by 2. | $2x = -4$<br>$x = -2$ |
| 7 Write the solution. | Solution: $x = -2$, $y = 3$ or $(-2, 3)$ |
| 8 Check the solution by substituting into equation [1]. | Check: Substitute into $x - 5y = -17$.<br>LHS $= (-2) - 5(3)$<br>$= -2 - 15$<br>$= -17$<br>RHS $= -17$<br>LHS = RHS, so the solution is correct. |

*Note:* In this example, equation [1] could have been multiplied by −2 (instead of by 2), then the two equations added (instead of subtracted) to eliminate $x$.

**WORKED EXAMPLE 8**

**Solve the following pair of simultaneous equations using the elimination method.**

$$6x + 5y = 3 \qquad 5x + 4y = 2$$

| THINK | WRITE |
|---|---|
| 1 Write the equations, one under the other and number them. | $6x + 5y = 3$ [1]<br>$5x + 4y = 2$ [2] |
| 2 Decide which variable to eliminate, say $y$.<br>Multiply equation [1] by 4 and call the new equation [3].<br>Multiply equation [2] by 5 and call the new equation [4]. | Eliminate $y$.<br>[1] × 4: $24x + 20y = 12$ [3]<br>[2] × 5: $25x + 20y = 10$ [4] |
| 3 Subtract equation [3] from [4] in order to eliminate $y$. | [4] − [3]:<br>$25x + 20y - (24x + 20y) = 10 - 12$<br>$25x + 20y - 24x - 20y = -2$<br>$x = -2$ |
| 4 Substitute the value of $x$ into equation [1]. | Substituting $x = -2$ into [1]:<br>$6(-2) + 5y = 3$<br>$-12 + 5y = 3$ |
| 5 Solve for $y$.<br>(i) Add 12 to both sides of the equation.<br>(ii) Divide both sides of the equation by 5. | $5y = 15$<br>$y = 3$ |
| 6 Write your answer. | Solution $x = -2$, $y = 3$ or $(-2, 3)$ |
| 7 Check the answer by substituting the solution into equation [2]. | Check: Substitute into $5x + 4y = 2$.<br>LHS $= 5(-2) + 4(3)$<br>$= -10 + 12$<br>$= 2$<br>RHS $= 2$<br>LHS = RHS, so the solution is correct. |

*Note:* Equation [1] could have been multiplied by −4 (instead of by 4), then the two equations added (instead of subtracted) to eliminate $y$.

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## Exercise 4.4 Solving simultaneous linear equations using elimination

**INDIVIDUAL PATHWAYS**

**REFLECTION**
How does eliminating one variable help to solve simultaneous equations?

| PRACTISE | CONSOLIDATE | MASTER |
|---|---|---|
| Questions:<br>1, 2, 3a–c, 4a–c, 5a–c, 6, 7 | Questions:<br>1, 2, 3a–d, 4a–e, 5a–d, 7, 8, 9 | Questions:<br>1, 2, 3d–f, 4e–i, 5c–f, 6–10 |

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### FLUENCY

**1** **WE6** Solve the following pairs of simultaneous equations by adding equations to eliminate either $x$ or $y$.

**a** $x + 2y = 5$
$-x + 4y = 1$

**b** $5x + 4y = 2$
$5x - 4y = -22$

**c** $-2x + y = 10$
$2x + 3y = 14$

**2** Solve the following pairs of equations by subtracting equations to eliminate either $x$ or $y$.

**a** $3x + 2y = 13$
$5x + 2y = 23$

**b** $2x - 5y = -11$
$2x + y = 7$

**c** $-3x - y = 8$
$-3x + 4y = 13$

**3** Solve each of the following equations using the elimination method.

**a** $6x - 5y = -43$
$6x - y = -23$

**b** $x - 4y = 27$
$3x - 4y = 17$

**c** $-4x + y = -10$
$4x - 3y = 14$

**d** $-5x + 3y = 3$
$-5x + y = -4$

**e** $5x - 5y = 1$
$2x - 5y = -5$

**f** $4x - 3y - 1 = 0$
$4x + 7y - 11 = 0$

**4** **WE7** Solve the following pairs of simultaneous equations.

**a** $6x + y = 9$
$-3x + 2y = 3$

**b** $x + 3y = 14$
$3x + y = 10$

**c** $5x + y = 27$
$4x + 3y = 26$

**d** $-6x + 5y = -14$
$-2x + y = -6$

**e** $2x + 5y = 14$
$3x + y = -5$

**f** $-3x + 2y = 6$
$x + 4y = -9$

**g** $3x - 5y = 7$
$x + y = -11$

**h** $2x + 3y = 9$
$4x + y = -7$

**i** $-x + 5y = 7$
$5x + 5y = 19$

**5** **WE8** Solve the following pairs of simultaneous equations.

**a** $-4x + 5y = -9$
$2x + 3y = 21$

**b** $2x + 5y = -6$
$3x + 2y = 2$

**c** $2x - 2y = -4$
$5x + 4y = 17$

**d** $2x - 3y = 6$
$4x - 5y = 9$

**e** $\frac{x}{2} + \frac{y}{3} = 2$
$\frac{x}{4} + \frac{y}{3} = 4$

**f** $\frac{x}{3} + \frac{y}{2} = \frac{3}{2}$
$\frac{x}{2} + \frac{y}{5} = -\frac{1}{2}$

### UNDERSTANDING

**6** Solve the following simultaneous equations using an appropriate method. Check your answer using technology.

**a** $7x + 3y = 16$
$y = 4x - 1$

**b** $2x + y = 8$
$4x + 3y = 16$

**c** $-3x + 2y = 19$
$4x + 5y = 13$

**d** $-3x + 7y = 9$
$4x - 3y = 7$

**e** $-4x + 5y = -7$
$x = 23 - 3y$

**f** $y = -x$
$y = -\frac{2}{5}x - \frac{1}{5}$

### REASONING

**7** Ann, Beth and Celine wanted to weigh themselves, but the scales they had were broken and would only give readings over 100 kg. They decided to weigh themselves in pairs and calculate their weights from the results.

- Ann and Beth weighed 119 kg
- Beth and Celine weighed 112 kg
- Celine and Ann weighed 115 kg

How much did each of the girls weigh? Show your working.

**8 a** For the general case $ax + by = e$ [1]
$cx + dy = f$ [2]

$y$ can be found by eliminating $x$.

**i** Multiply equation [1] by $c$ to create equation 3.

**ii** Multiply equation [2] by $a$ to create equation 4.

**iii** Use the elimination method to find a general solution for $y$.

**b** Use a similar process to that outlined above to find a general solution for $x$.

**c** Use the general solution for $x$ and $y$ to solve each of the following.

**i** $2x + 5y = 7$
$7x + 2y = 24$

**ii** $3x - 5y = 4$
$x + 3y = 5$

Choose another method to check that your solutions are correct in each part.

**d** For $y$ to exist, it is necessary to state that $bc - ad \neq 0$. Why?

**e** Is there a necessary condition for $x$ to exist? Explain.

## PROBLEM SOLVING

**9** Use the method of elimination to solve

$$\frac{x-4}{3} + y = -2$$

$$\frac{2y-1}{7} + x = 6.$$

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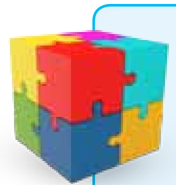
**10** Use an appropriate method to solve

$$2x + 3y + 3z = -1$$
$$3x - 2y + z = 0$$
$$z + 2y = 0.$$



### CHALLENGE 4.1

If $x + y = 17$, $y + z = 15$ and $x + z = 14$, what is the value of $z$?

# 4.5 Applications of simultaneous linear equations

- There are many practical applications of simultaneous equations, some examples of which are shown below.
- When solving practical problems, the following steps can be useful.
  - Define the unknown quantities using appropriate pronumerals.
  - Use the information given in the problem to form two equations in terms of these pronumerals.
  - Solve these equations using an appropriate method.
  - Write the solution in words.
  - Check the solution.

**WORKED EXAMPLE 9**

**Ashley received better results for his Mathematics test than for his English test. If the sum of the two marks is 164 and the difference is 22, calculate the mark he received for each subject.**

| THINK | WRITE |
|---|---|
| 1 Define the two variables. | Let $x$ = the Mathematics mark.<br>Let $y$ = the English mark. |
| 2 Formulate two equations from the information given and number them. The sum of the two marks is $x + y$. The difference of the two marks is $x - y$. | $x + y = 164$ [1]<br>$x - y = 22$ [2] |
| 3 Use the elimination method by adding equations [1] and [2] to eliminate $y$. | [1] + [2]: $2x = 186$ |
| 4 Solve for $x$ by dividing both sides of the equation by 2. | $x = 93$ |
| 5 Substitute the value of $x$ into equation [1]. | Substituting $x = 93$ into [1]:<br>$x + y = 164$<br>$93 + y = 164$ |
| 6 Solve for $y$ by subtracting 93 from both sides of the equation. | $y = 71$ |
| 7 Write the solution. | Solution:<br>Mathematics mark ($x$) = 93<br>English mark ($y$) = 71 |
| 8 Check the solution by substituting $x = 93$ and $y = 71$ into equation [1]. | Check: Substitute into $x + y = 164$.<br>LHS = 93 + 71 = 164 RHS = 164<br>As LHS = RHS, the solution is correct. |

## WORKED EXAMPLE 10

**To finish a project, Genevieve buys a total of 25 nuts and bolts from a hardware store. If each nut costs 12 cents, each bolt costs 25 cents and the total purchase price is \$4.30, how many nuts and how many bolts does Genevieve buy?**

| THINK | WRITE |
|---|---|
| 1 Define the two variables. | Let $x$ = the number of nuts.<br>Let $y$ = the number of bolts. |
| 2 Formulate two equations from the information given and number them.<br>*Note:* The total number of nuts and bolts is 25. Each nut cost 12 cents, each bolt cost 25 cents and the total cost is 430 cents (\$4.30). | $x + y = 25$ [1]<br>$12x + 25y = 430$ [2] |
| 3 Solve simultaneously using the substitution method, since equation [1] is easy to rearrange. | |
| 4 Rearrange equation [1] to make $x$ the subject by subtracting $y$ from both sides of equation [1]. | Rearrange equation [1]:<br>$x + y = 25$<br>$x = 25 - y$ |
| 5 Substitute the expression $(25 - y)$ for $x$ into equation [2]. | Substituting $(25 - y)$ into [2]:<br>$12(25 - y) + 25y = 430$ |
| 6 Solve for $y$. | $300 - 12y + 25y = 430$<br>$300 + 13y = 430$<br>$13y + 300 = 430$<br>$13y = 130$<br>$y = 10$ |
| 7 Substitute the value of $y$ into the rearranged equation $x = 25 - y$ from step **4**. | Substituting $y = 10$ into $x = 25 - y$:<br>$x = 25 - 10$<br>$x = 15$ |
| 8 Write the solution. | Solution:<br>The number of nuts $(x) = 15$.<br>The number of bolts $(y) = 10$. |
| 9 Check the solution by substituting $x = 15$ and $y = 10$ into equation [1]. | Check: Substitute into $x + y = 25$.<br>LHS $= 15 + 10$ $= 25$ RHS $= 25$<br>As LHS = RHS, the solution is correct. |

# Exercise 4.5 Applications of simultaneous linear equations

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## INDIVIDUAL PATHWAYS

| PRACTISE | CONSOLIDATE | MASTER |
|---|---|---|
| Questions: 1–3, 6, 8, 12, 14, 16, 19 | Questions: 1, 2, 4, 7, 9, 11, 14, 16, 17, 19 | Questions: 1, 2, 5, 7, 9, 10, 13, 15–20 |

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**REFLECTION**
How do you decide which method to use when solving problems using simultaneous linear equations?

### FLUENCY

1 WE9 Rick received better results for his Maths test than for his English test. If the sum of his two marks is 163 and the difference is 31, find the mark for each subject.

2 WE10 Rachael buys 30 nuts and bolts to finish a project. If each nut costs 10 cents, each bolt costs 20 cents and the total purchase price is $4.20, how many nuts and how many bolts does she buy?

### UNDERSTANDING

3 Find two numbers whose difference is 5 and whose sum is 11.

4 The difference between two numbers is 2. If three times the larger number minus twice the smaller number is 13, find the two numbers.

5 One number is 9 less than three times a second number. If the first number plus twice the second number is 16, find the two numbers.

6 A rectangular house has a perimeter of 40 metres and the length is 4 metres more than the width. What are the dimensions of the house?

7 Mike has 5 lemons and 3 oranges in his shopping basket. The cost of the fruit is $3.50. Voula, with 2 lemons and 4 oranges, pays $2.10 for her fruit. How much does each type of fruit cost?

8 A surveyor measuring the dimensions of a block of land finds that the length of the block is three times the width. If the perimeter is 160 metres, what are the dimensions of the block?

9 Julie has $3.10 in change in her pocket. If she has only 50 cent and 20 cent pieces and the total number of coins is 11, how many coins of each type does she have?

10 Mr Yang's son has a total of twenty-one $1 and $2 coins in his moneybox. When he counts his money, he finds that its total value is $30. How many coins of each type does he have?

11 If three Magnums and two Paddlepops cost $8.70 and the difference in price between a Magnum and a Paddlepop is 90 cents, how much does each type of ice-cream cost?

**12** If one Redskin and 4 Golden Roughs cost \$1.65, whereas 2 Redskins and 3 Golden Roughs cost \$1.55, how much does each type of sweet cost?

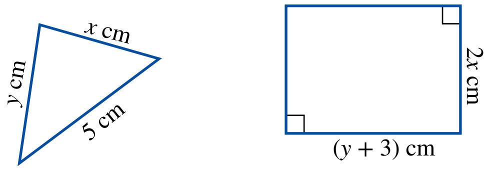
**13** A catering firm charges a fixed cost for overheads and a price per person. It is known that a party for 20 people costs \$557, whereas a party for 35 people costs \$909.50. What is the fixed cost and the cost per person charged by the company?

**14** The difference between Sally's PE mark and Science mark is 12, and the sum of the marks is 154. If the PE mark is the higher mark, what did Sally get for each subject?

**15** Mozza's Cheese Supplies sells six Mozzarella cheeses and eight Swiss cheeses to Munga's deli for \$83.60, and four Mozzarella cheeses and four Swiss cheeses to Mina's deli for \$48. How much does each type of cheese cost?

### REASONING

**16** If the perimeter of the triangle in the diagram is 12 cm and the length of the rectangle is 1 cm more than the width, find the value of $x$ and $y$. Show your working.




**17** Mr and Mrs Waugh want to use a caterer for a birthday party for their twin sons. The manager says the cost for a family of four would be \$160. However, the sons want to invite 8 friends, making 12 people in all. The cost for this would be \$360. If the total cost in each case is made up of the same cost per person and the same fixed cost, find the cost per person and the fixed cost. Show your working.

**18** Joel needs to buy some blank DVDs and zip disks to back up a large amount of data that has been generated by an accounting firm. He buys 6 DVDs and 3 zip disks for \$96. He later realises these are not sufficient and so buys another 5 DVDs and 4 zip disks for \$116. How much did each DVD and each zip disk cost? (Assume the same rate per item was charged for each visit.) Show your working.

### PROBLEM SOLVING

**19** At the football hot chips are twice as popular as meat pies and three times as popular as hot dogs. Over the period of half an hour during half time, a fast-food outlet serves 121 people who each bought one item. How many serves of each of the foods were sold during this half-hour period?

**20** Three jet-skis in a 300 kilometre handicap race leave at two hour intervals. Jet-ski 1 leaves first and has an average speed of 25 kilometres per hour for the entire race. Jet-ski 2 leaves two hours later and has an average speed of 30 kilometres per hour for the entire race. Jet-ski 3 leaves last, two hours after jet-ski 2 and has an average speed of 40 kilometres per hour for the entire race.

**a** Sketch a graph to show each jet-ski's journey on the one set of axes.

**b** Determine who wins the race.

**c** Check your findings algebraically and describe what happened to each jet-ski during the course of the race.

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