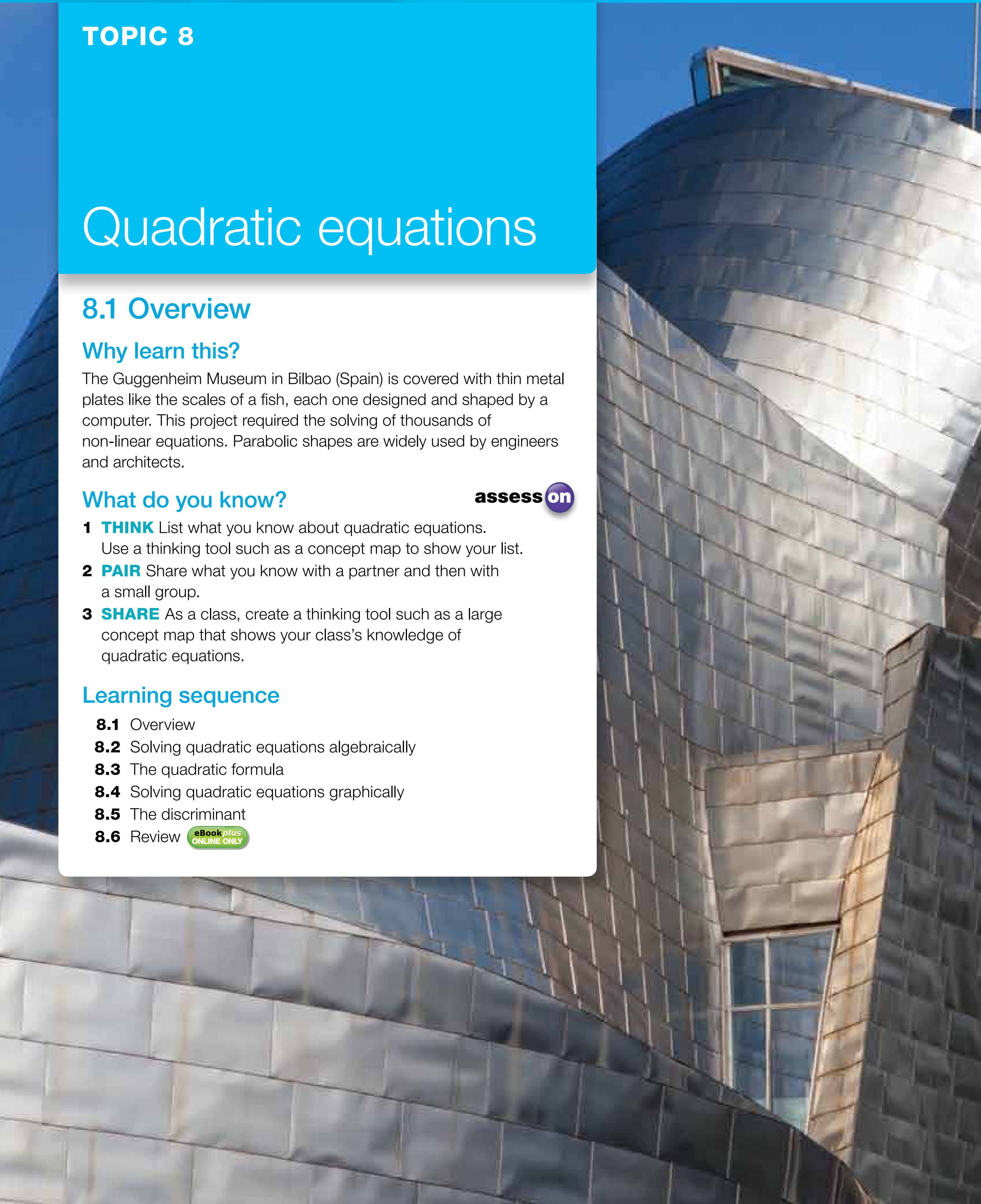


TOPIC 8

Quadratic equations

8.1 Overview

Why learn this?

The Guggenheim Museum in Bilbao (Spain) is covered with thin metal plates like the scales of a fish, each one designed and shaped by a computer. This project required the solving of thousands of non-linear equations. Parabolic shapes are widely used by engineers and architects.

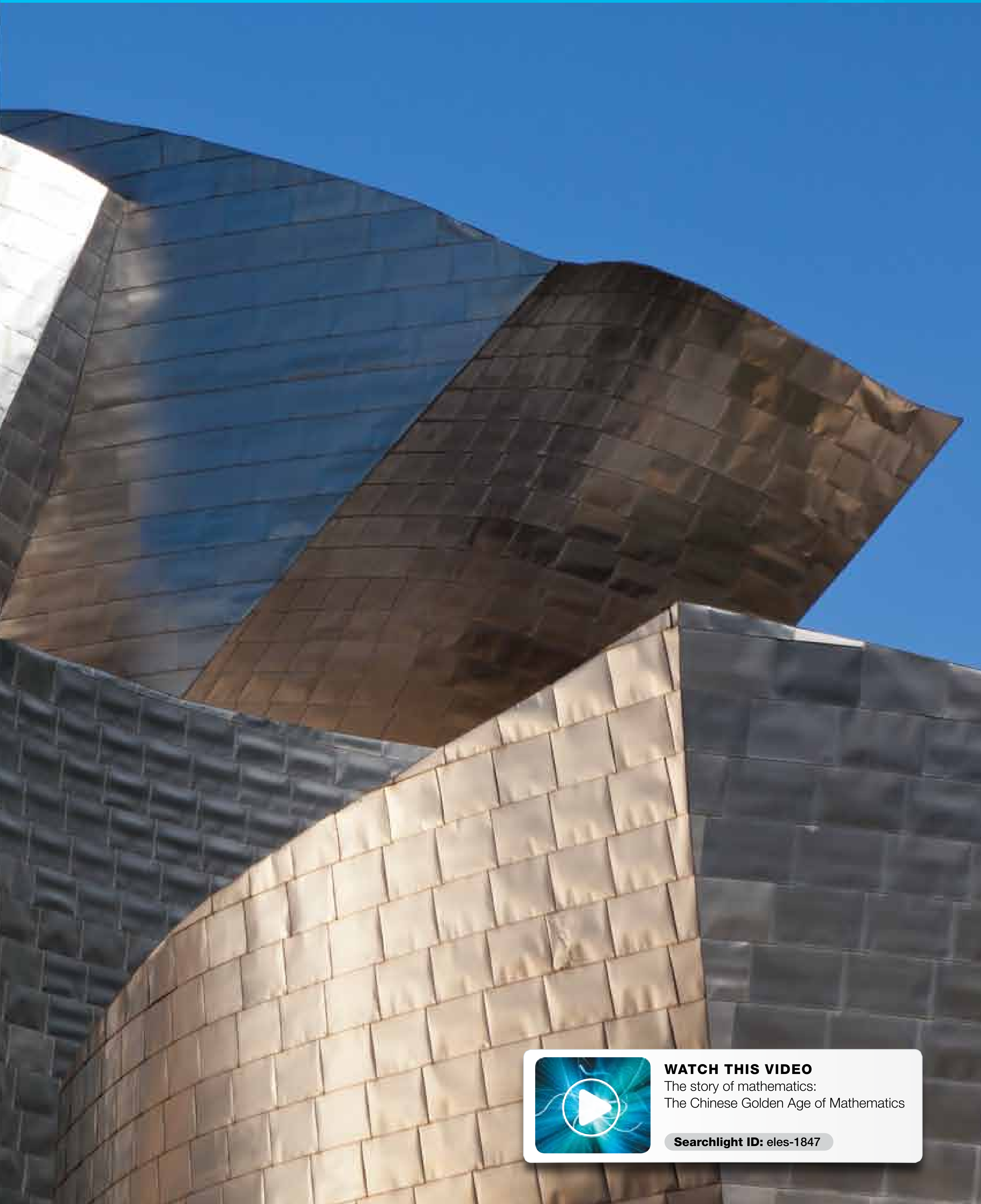
What do you know?

1 **THINK** List what you know about quadratic equations. Use a thinking tool such as a concept map to show your list.

2 **PAIR** Share what you know with a partner and then with a small group.

3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of quadratic equations.

Learning sequence



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8.2 Solving quadratic equations algebraically

Quadratic equations

- The general form of a quadratic equation is $ax^2 + bx + c = 0$.
- To solve an equation means to find the value of the pronumeral(s) or variables, which when substituted, will make the equation a true statement.

The Null Factor Law

- The **Null Factor Law** states that if the product of two numbers is zero then one or both of the numbers must equal zero.
- In other words, there are two solutions to the equation $pq = 0$; they are $p = 0$ and $q = 0$.
- When solving quadratic equations by applying the Null Factor Law, it is best to write the equations equal to zero.

WORKED EXAMPLE 1

Solve the equation $(x - 7)(x + 11) = 0$.

	THINK	WRITE
1	Write the equation and check that the right-hand side equals zero. (The product of the two numbers is zero.)	$(x - 7)(x + 11) = 0$
2	The left-hand side is factorised, so apply the Null Factor Law.	$x - 7 = 0$ or $x + 11 = 0$
3	Solve for x.	$x = 7 \qquad x = -11$

WORKED EXAMPLE 2

Solve each of the following equations.

a $x^2 - 3x = 0$ **b** $3x^2 - 27 = 0$

c $x^2 - 13x + 42 = 0$ **d** $36x^2 - 21x = 2$

	THINK	WRITE
a 1	Write the equation. Check that the right-hand side equals zero.	a $x^2 - 3x = 0$
2	Factorise by taking out the common factor of x^2 and $3x$, which is x.	$x(x - 3) = 0$
3	Apply the Null Factor Law.	$x = 0$ or $x - 3 = 0$
4	Solve for x.	$x = 0 \qquad x = 3$
b 1	Write the equation. Check that the right-hand side equals zero.	b $3x^2 - 27 = 0$
2	Factorise by taking out the common factor of $3x^2$ and 27, which is 3.	$3(x^2 - 9) = 0$

3 Factorise using the difference of two squares rule.

$$3(x^2 - 3^2) = 0$$
$$3(x + 3)(x - 3) = 0$$

4 Apply the Null Factor Law.

$x + 3 = 0$ or $x - 3 = 0$

5 Solve for x.

$x = -3 \quad x = 3$

(Alternatively, $x = \pm 3$.)

c **1** Write the equation. Check that the right-hand side equals zero.

c $x^2 - 13x + 42 = 0$

2 Factorise by finding a factor pair of 42 that adds to −13.

Factors of 42	Sum of factors
−6 and −7	−13

$$(x - 6)(x - 7) = 0$$

3 Use the Null Factor Law to write two linear equations.

$x - 6 = 0$ or $x - 7 = 0$

4 Solve for x.

$x = 6 \quad x = 7$

d **1** Write the equation. Check that the right-hand side equals zero. (It does not.)

d $36x^2 - 21x = 2$

2 Rearrange the equation so the right-hand side of the equation equals zero.

$$36x^2 - 21x - 2 = 0$$

3 Recognise that the expression to factorise is a quadratic trinomial.

Factors of −72	Sum of factors
3 and −24	−21

$$36x^2 - 24x + 3x - 2 = 0$$

4 Factorise the expression.

$$12x(3x - 2) + (3x - 2) = 0$$
$$(3x - 2)(12x + 1) = 0$$

5 Use the Null Factor Law to write two linear equations.

$3x - 2 = 0$ or $12x + 1 = 0$

$3x = 2 \qquad 12x = -1$

6 Solve for x.

$x = \frac{2}{3} \qquad x = -\frac{1}{12}$

Solving quadratic equations by completing the square

- Sometimes it is necessary to complete the square in order to factorise a quadratic trinomial.
- This is often necessary if the solutions are not rational numbers.

WORKED EXAMPLE 3

TI CASIO

Find the solutions to the equation $x^2 + 2x - 4 = 0$. Give exact answers.

THINK	WRITE
1 Write the equation.	$x^2 + 2x - 4 = 0$
2 Identify the coefficient of x, halve it and square the result.	$\left(\frac{1}{2} \times 2\right)^2$
3 Add the result of step 2 to the equation, placing it after the x-term. To balance the equation, we need to subtract the same amount as we have added.	$x^2 + 2x + \left(\frac{1}{2} \times 2\right)^2 - 4 - \left(\frac{1}{2} \times 2\right)^2 = 0$ $x^2 + 2x + (1)^2 - 4 - (1)^2 = 0$ $x^2 + 2x + 1 - 4 - 1 = 0$
4 Insert brackets around the first three terms to group them and then simplify the remaining terms.	$(x^2 + 2x + 1) - 5 = 0$
5 Factorise the first three terms to produce a perfect square.	$(x + 1)^2 - 5 = 0$
6 Express as the difference of two squares and then factorise.	$(x + 1)^2 - (\sqrt{5})^2 = 0$ $(x + 1 + \sqrt{5})(x + 1 - \sqrt{5}) = 0$
7 Apply the Null Factor Law to find linear equations.	$x + 1 + \sqrt{5} = 0$ or $x + 1 - \sqrt{5} = 0$
8 Solve for x. Keep the answer in surd form to provide an exact answer.	$x = -1 - \sqrt{5}$ or $x = -1 + \sqrt{5}$ (Alternatively, $x = -1 \pm \sqrt{5}$.)

Solving problems

- There are many problems that can be modelled by a quadratic equation. You should first form the quadratic equation that represents the situation before attempting to solve such problems.
- Recall that worded problems should always be answered with a sentence.

WORKED EXAMPLE 4

When two consecutive numbers are multiplied together, the result is 20. Determine the numbers.

THINK	WRITE
1 Define the unknowns. First number = x, second number = $x + 1$.	Let the two numbers be x and $(x + 1)$.
2 Write an equation using the information given in the question.	$x(x + 1) = 20$
3 Transpose the equation so that the right-hand side equals zero.	$x(x + 1) - 20 = 0$
4 Expand to remove the brackets.	$x^2 + x - 20 = 0$
5 Factorise.	$(x + 5)(x - 4) = 0$

6	Apply the Null Factor Law to solve for x.	$x + 5 = 0$ or $x - 4 = 0$ $x = -5$ $x = 4$
7	Use the answer to determine the second number.	If $x = -5, x + 1 = -4$. If $x = 4, x + 1 = 5$.
8	Check the solutions.	Check: $4 \times 5 = 20$ $-5 \times -4 = 20$
9	Answer the question in a sentence.	The numbers are 4 and 5 or −5 and −4.

WORKED EXAMPLE 5

The height of a football after being kicked is determined by the formula $h = -0.1d^2 + 3d$, where d is the horizontal distance from the player.

a How far away is the ball from the player when it hits the ground?

b What horizontal distance has the ball travelled when it first reaches a height of 20 m?

		THINK		WRITE
a	1	Write the formula.	a	$h = -0.1\ d^2 + 3d$
	2	The ball hits the ground when $h = 0$. Substitute $h = 0$ into the formula.		$-0.1\ d^2 + 3d = 0$
	3	Factorise.		$-0.1d^2 + 3d = 0$ $d(-0.1d + 3) = 0$
	4	Apply the Null Factor Law and simplify.		$d = 0$ or $-0.1d + 3 = 0$ $-0.1d = -3$ $d = \frac{-3}{-0.1}$ $= 30$
	5	Interpret the solutions.		$d = 0$ is the origin of the kick. $d = 30$ is the distance from the origin that the ball has travelled when it lands.
	6	Answer the question in a sentence.		The ball is 30 m from the player when it hits the ground.
b	1	The height of the ball is 20 m, so, substitute $h = 20$ into the formula.	b	$h = -0.1d^2 + 3d$ $20 = -0.1d^2 + 3d$
	2	Transpose the equation so that zero is on the right-hand side.		$0.1d^2 - 3d + 20 = 0$
	3	Multiply both sides of the equation by 10 to remove the decimal from the coefficient.		$d^2 - 30d + 200 = 0$

4	Factorise.	$(d-20)(d-10)=0$
5	Apply the Null Factor Law.	$d-20=0$ or $d-10=0$
6	Solve.	$d=20 \qquad d=10$
7	Interpret the solution. The ball reaches a height of 20 m on the way up and on the way down. The *first* time the ball reaches a height of 20 m is the smaller value of d. Answer in a sentence.	The ball first reaches a height of 20 m after it has travelled a distance of 10 m.

Exercise 8.2 Solving quadratic equations algebraically

REFLECTION
What does the Null Factor Law mean?

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1a–f, 2a–c, 3a–d, 4a–c, 5a–h, 6–8, 10, 14, 16, 19, 20, 22	Questions: 1b–g, 2a–d, 3a–f, 4a–f, 5d–l, 6, 7, 8a–d, 9, 11, 14–16, 18, 20–22, 26	Questions: 1, 2, 3f–i, 4f–l, 5g–l, 6, 7, 8g–l, 9f–i, 10d–i, 11d–i, 12–27

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SkillSHEET
Finding a factor pair that adds to a given number
doc-5257
SkillSHEET
Simplifying surds
doc-5258
SkillSHEET
Substituting into quadratic equations
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SkillSHEET
Equation of a vertical line
doc-5260

FLUENCY

1 **WE1** Solve each of the following equations.

a $(x+7)(x-9)=0$ **b** $(x-3)(x+2)=0$ **c** $(x-2)(x-3)=0$
d $x(x-3)=0$ **e** $x(x-1)=0$ **f** $x(x+5)=0$
g $2x(x-3)=0$ **h** $9x(x+2)=0$ **i** $(x-\frac{1}{2})(x+\frac{1}{2})=0$
j $-(x+1.2)(x+0.5)=0$ **k** $2(x-0.1)(2x-1.5)=0$ **l** $(x+\sqrt{2})(x-\sqrt{3})=0$

2 Solve each of the following equations.

a $(2x-1)(x-1)=0$ **b** $(3x+2)(x+2)=0$ **c** $(4x-1)(x-7)=0$
d $(7x+6)(2x-3)=0$ **e** $(5x-3)(3x-2)=0$ **f** $(8x+5)(3x-2)=0$
g $x(x-3)(2x-1)=0$ **h** $x(2x-1)(5x+2)=0$ **i** $x(x+3)(5x-2)=0$

3 **WE2a** Solve each of the following equations.

a $x^2-2x=0$ **b** $x^2+5x=0$ **c** $x^2=7x$
d $3x^2=-2x$ **e** $4x^2-6x=0$ **f** $6x^2-2x=0$
g $4x^2-2\sqrt{7}x=0$ **h** $3x^2+\sqrt{3}x=0$ **i** $15x-12x^2=0$

4 **WE2b** Solve each of the following equations.

a $x^2-4=0$ **b** $x^2-25=0$ **c** $3x^2-12=0$
d $4x^2-196=0$ **e** $9x^2-16=0$ **f** $4x^2-25=0$
g $9x^2=4$ **h** $36x^2=9$ **i** $x^2-\frac{1}{25}=0$
j $\frac{1}{36}x^2-\frac{4}{9}=0$ **k** $x^2-5=0$ **l** $9x^2-11=0$

5 **WE2c** Solve each of the following equations.

a $x^2-x-6=0$ **b** $x^2+6x+8=0$ **c** $x^2-6x-7=0$
d $x^2-8x+15=0$ **e** $x^2-2x+1=0$ **f** $x^2-3x-4=0$

g $x^2 - 10x + 25 = 0$ **h** $x^2 - 3x - 10 = 0$ **i** $x^2 - 8x + 12 = 0$
j $x^2 - 4x - 21 = 0$ **k** $x^2 - x - 30 = 0$ **l** $x^2 - 7x + 12 = 0$
m $x^2 - 8x + 16 = 0$ **n** $x^2 + 10x + 25 = 0$ **o** $x^2 - 20x + 100 = 0$

6 **MC** The solutions to the equation $x^2 + 9x - 10 = 0$ are:
A $x = 1$ and $x = 10$ **B** $x = 1$ and $x = -10$
C $x = -1$ and $x = 10$ **D** $x = -1$ and $x = -10$
E $x = 1$ and $x = 9$

7 **MC** The solutions to the equation $x^2 - 100 = 0$ are:
A $x = 0$ and $x = 10$ **B** $x = 0$ and $x = -10$
C $x = -10$ and $x = 10$ **D** $x = 0$ and $x = 100$
E $x = -100$ and $x = 100$

8 **WE2d** Solve each of the following equations.
a $2x^2 - 5x = 3$ **b** $3x^2 + x - 2 = 0$ **c** $5x^2 + 9x = 2$
d $6x^2 - 11x + 3 = 0$ **e** $14x^2 - 11x = 3$ **f** $12x^2 - 7x + 1 = 0$
g $6x^2 - 7x = 20$ **h** $12x^2 + 37x + 28 = 0$ **i** $10x^2 - x = 2$
j $6x^2 - 25x + 24 = 0$ **k** $30x^2 + 7x - 2 = 0$ **l** $3x^2 - 21x = -36$

9 **WE3** Find the solutions for each of the following equations. Give exact answers.
a $x^2 - 4x + 2 = 0$ **b** $x^2 + 2x - 2 = 0$ **c** $x^2 + 6x - 1 = 0$
d $x^2 - 8x + 4 = 0$ **e** $x^2 - 10x + 1 = 0$ **f** $x^2 - 2x - 2 = 0$
g $x^2 + 2x - 5 = 0$ **h** $x^2 + 4x - 6 = 0$ **i** $x^2 + 4x - 11 = 0$

10 Find the solutions for each of the following equations. Give exact answers.
a $x^2 - 3x + 1 = 0$ **b** $x^2 + 5x - 1 = 0$ **c** $x^2 - 7x + 4 = 0$
d $x^2 - 5 = x$ **e** $x^2 - 11x + 1 = 0$ **f** $x^2 + x = 1$
g $x^2 + 3x - 7 = 0$ **h** $x^2 - 3 = 5x$ **i** $x^2 - 9x + 4 = 0$

11 Solve each of the following equations, rounding answers to 2 decimal places.
a $2x^2 + 4x - 6 = 0$ **b** $3x^2 + 12x - 3 = 0$ **c** $5x^2 - 10x - 15 = 0$
d $4x^2 - 8x - 8 = 0$ **e** $2x^2 - 6x + 2 = 0$ **f** $3x^2 - 9x - 3 = 0$
g $5x^2 - 15x - 25 = 0$ **h** $7x^2 + 7x - 21 = 0$ **i** $4x^2 + 8x - 2 = 0$

UNDERSTANDING

12 **WE4** When two consecutive numbers are multiplied, the result is 72. Find the numbers.

13 When two consecutive even numbers are multiplied, the result is 48. Find the numbers.

14 When a number is added to its square the result is 90. Find the number.

15 Twice a number is added to three times its square. If the result is 16, find the number.

16 Five times a number is added to two times its square. If the result is 168, find the number.

17 **WE5** A soccer ball is kicked. The height, h, in metres, of the soccer ball t seconds after it is kicked can be represented by the equation $h = -t(t - 6)$. Find how long it takes for the soccer ball to hit the ground again.

18 The length of an Australian flag is twice its width and the diagonal length is 45 cm.

- **a** If x cm is the width of the flag, find the length in terms of x.
- **b** Draw a diagram of the flag marking in the diagonal. Mark the length and the width in terms of x.
- **c** Use Pythagoras' theorem to write an equation relating the lengths of the sides to the length of the diagonal.
- **d** Solve the equation to find the dimensions of the Australian flag. Round your answer to the nearest cm.

19 If the length of a paddock is 2 m more than its width and the area is 48 m^2, find the length and width of the paddock.

20 Solve for x.

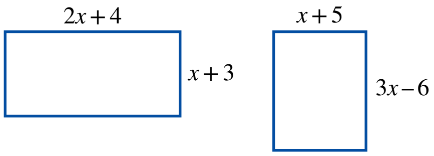
- **a** $x + 5 = \dfrac{6}{x}$
- **b** $x = \dfrac{24}{x - 5}$
- **c** $x = \dfrac{1}{x}$

REASONING

21 The sum of the first n numbers 1, 2, 3, 4 … n is given by the formula $S = \dfrac{n(n + 1)}{2}$.

- **a** Use the formula to find the sum of the first 6 counting numbers.
- **b** How many numbers are added to give a sum of 153?

22 If these two rectangles have the same area, what is the value of x?



23 Henrietta is a pet rabbit who lives in an enclosure that is 2 m wide and 4 m long. Her human family has decided to purchase some more rabbits to keep her company and so the size of the enclosure must be increased.

- **a** Draw a diagram of Henrietta's enclosure, clearly marking the lengths of the sides.
- **b** If the length and width of the enclosure are increased by x m, find the new dimensions.
- **c** If the new area is to be 24 m^2, write an equation relating the sides and the area of the enclosure (Area = length × width).
- **d** Use the equation to find the value of x and, hence, the length of the sides of the new enclosure. Justify your answer.

24 The cost per hour, C, in thousands of dollars of running two cruise ships, *Annabel* and *Betty*, travelling at a speed of s knots is given by the following relationships.

$C_{Annabel} = 0.3s^2 + 4.2s + 12$ and $C_{Betty} = 0.4s^2 + 3.6s + 8$

a Determine the cost per hour for each ship if they are both travelling at 28 knots.

b Find the speed in knots at which both ships must travel for them to have the same cost.

c Explain why only one of the solutions obtained in your working for part **b** is valid.

25 Explain why the equation $x^2 + 4x + 10 = 0$ has no real solutions.

PROBLEM SOLVING

26 Solve $(x^2 - x)^2 - 32(x^2 - x) + 240 = 0$ for x.

27 Solve $\dfrac{3z^2 - 35}{16} - z = 0$ for z.

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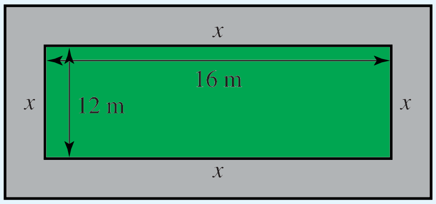
CHALLENGE 8.1

A garden measuring 12 metres by 16 metres is to have a pedestrian pathway installed all around it, increasing the total area to 285 square metres. What will be the width of the pathway?



8.3 The quadratic formula

- The method of solving quadratic equations by completing the square can be generalised to produce what is called the **quadratic formula**.
- The general equation $ax^2 + bx + c = 0$ can be solved by completing the square. We will first follow the steps involved in completing the square.

1. Divide both sides of the equation by a.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

2. Complete the square.

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$$

3. Factorise the first three terms as a perfect square.

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

4. Add the final two terms.

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2} = 0$$

5. Write as the difference of two squares.

$$\left(x + \frac{b}{2a}\right)^2 - \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)^2 = 0$$

6. Factorise using the difference of two squares rule.

$$\left(x + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}\right)\left(x + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}\right) = 0$$

7. Solve the two linear factors.

$$x + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} = 0 \text{ or } x + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} = 0$$

$$x + \ = \frac{-b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$

- The solution can be summarised as $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where a is the coefficient of x^2, b is the coefficient of x and c is the constant or the term without an x.
- The quadratic formula can be used to solve any quadratic equation.
- If the value inside the square root sign is negative, then there are no solutions to the equation.

WORKED EXAMPLE 6 TI CASIO

Use the quadratic formula to solve each of the following equations.

a $3x^2 + 4x + 1 = 0$ (exact answer)

b $-3x^2 - 6x - 1 = 0$ (round to 2 decimal places)

THINK	WRITE
a 1 Write the equation.	**a** $3x^2 + 4x + 1 = 0$
2 Write the quadratic formula.	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3 State the values for a, b and c.	where $a = 3, b = 4, c = 1$
4 Substitute the values into the formula.	$x = \dfrac{-4 \pm \sqrt{(4)^2 - (4 \times 3 \times 1)}}{2 \times 3}$

5 Simplify and solve for x.

$$= \frac{-4 \pm \sqrt{4}}{6}$$

$$= \frac{-4 \pm 2}{6}$$

$$x = \frac{-4 + 2}{6} \quad \text{or} \quad x = \frac{-4 - 2}{6}$$

6 Write the two solutions.

$$x = -\tfrac{1}{3} \qquad x = -1$$

b **1** Write the equation.

b $-3x^2 - 6x - 1 = 0$

2 Write the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3 State the values for a, b and c.

where $a = -3$, $b = -6$, $c = -1$

4 Substitute the values into the formula.

$$x = \frac{-(-6) \pm \sqrt{36 - 4 \times -3 \times -1}}{2 \times -3}$$

5 Simplify the fraction.

$$= \frac{6 \pm \sqrt{24}}{-6}$$

$$= \frac{6 \pm 2\sqrt{6}}{-6}$$

$$= \frac{3 \pm \sqrt{6}}{-3}$$

$$x = \frac{3 + \sqrt{6}}{-3} \text{ or } \frac{3 - \sqrt{6}}{-3}$$

6 Write the two solutions correct to two decimal places.

$x \approx -1.82$ or $x \approx -0.18$

Note: When asked to give an answer in exact form, you should simplify any surds as necessary.

Exercise 8.3 The quadratic formula

assess on

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1a–d, 2a–f, 3a–f, 4–7, 8a–g, 9	Questions: 1d–g, 2d–h, 3d–h, 4–7, 8d–i, 10, 12, 13	Questions: 1e–h, 2g–l, 3g–n, 4–7, 8i–o, 9–14

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REFLECTION
What kind of answer will you get if the value inside the square root sign in the quadratic formula is zero?

FLUENCY

1 State the values for a, b and c in each of the following equations of the form $ax^2 + bx + c = 0$.

a $3x^2 - 4x + 1 = 0$
b $7x^2 - 12x + 2 = 0$
c $8x^2 - x - 3 = 0$
d $x^2 - 5x + 7 = 0$
e $5x^2 - 5x - 1 = 0$
f $4x^2 - 9x - 3 = 0$
g $12x^2 - 29x + 103 = 0$
h $43x^2 - 81x - 24 = 0$

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SkillSHEET
Substituting into the quadratic formula
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2 **WE6a** Use the quadratic formula to solve each of the following equations. Give exact answers.

a $x^2 + 5x + 1 = 0$ **b** $x^2 + 3x - 1 = 0$ **c** $x^2 - 5x + 2 = 0$
d $x^2 - 4x - 9 = 0$ **e** $x^2 + 2x - 11 = 0$ **f** $x^2 - 7x + 1 = 0$
g $x^2 - 9x + 2 = 0$ **h** $x^2 - 6x - 3 = 0$ **i** $x^2 + 8x - 15 = 0$
j $-x^2 + x + 5 = 0$ **k** $-x^2 + 5x + 2 = 0$ **l** $-x^2 - 2x + 7 = 0$

3 **WE6b** Use the quadratic formula to solve each of the following equations. Give approximate answers rounded to 2 decimal places.

a $3x^2 - 4x - 3 = 0$ **b** $4x^2 - x - 7 = 0$ **c** $2x^2 + 7x - 5 = 0$
d $7x^2 + x - 2 = 0$ **e** $5x^2 - 8x + 1 = 0$ **f** $2x^2 - 13x + 2 = 0$
g $-3x^2 + 2x + 7 = 0$ **h** $-7x^2 + x + 8 = 0$ **i** $-12x^2 + x + 9 = 0$
j $-6x^2 + 4x + 5 = 0$ **k** $-11x^2 - x + 1 = 0$ **l** $-4x^2 - x + 7 = 0$
m $-2x^2 + 12x - 1 = 0$ **n** $-5x^2 + x + 3 = 0$

4 **MC** The solutions of the equation $3x^2 - 7x - 2 = 0$ are:

A 1, 2 **B** 1, −2 **C** −0.257, 2.59
D −0.772, 7.772 **E** −1.544, 15.544

5 **MC** In the expansion of $(6x - 5)(3x + 4)$, the coefficient of x is:

A 18 **B** −15 **C** 9 **D** 6 **E** −2

6 **MC** In the expanded form of $(x - 2)(x + 4)$, which of the following is incorrect?

A The value of the constant is −8. **B** The coefficient of the x term is −6.
C The coefficient of the x term is 2. **D** The coefficient of the x^2 term is 1.
E The expansion shows this to be a trinomial expression.

7 **MC** An exact solution to the equation $x^2 + 2x - 5 = 0$ is:

A −3.449 **B** $-1 + \sqrt{24}$ **C** $-1 + \sqrt{6}$
D $\frac{2 + \sqrt{-16}}{2}$ **E** $\frac{2 + \sqrt{24}}{2}$

UNDERSTANDING

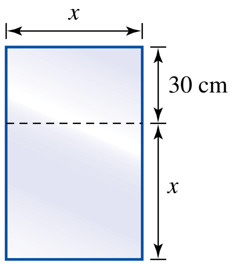
8 Solve each of the following equations using any suitable method. Round to 3 decimal places where appropriate.

a $2x^2 - 7x + 3 = 0$ **b** $x^2 - 5x = 0$ **c** $x^2 - 2x - 3 = 0$
d $x^2 - 3x + 1 = 0$ **e** $x^2 - 7x + 2 = 0$ **f** $x^2 - 6x + 8 = 0$
g $x^2 - 5x + 8 = 0$ **h** $x^2 - 7x - 8 = 0$ **i** $x^2 + 2x - 9 = 0$
j $3x^2 + 3x - 6 = 0$ **k** $2x^2 + 11x - 21 = 0$ **l** $7x^2 - 2x + 1 = 0$
m $-x^2 + 9x - 14 = 0$ **n** $-6x^2 - x + 1 = 0$ **o** $-6x^2 + x - 5 = 0$

9 The surface area of a closed cylinder is given by the formula $SA = 2\pi r(r + h)$, where r cm is the radius of the can and h cm is the height.

The height of a can of wood finish is 7 cm and its surface area is 231 cm^2.

a Substitute values into the formula to form a quadratic equation using the pronumeral, r.
b Use the quadratic formula to solve the equation and, hence, find the radius of the can correct to 1 decimal place.
c Calculate the area of the curved surface of the can, correct to the nearest square centimetre.



10 To satisfy lighting requirements, a window must have an area of 1500 cm^2.

a Find an expression for the area of the window in terms of x.

b Write an equation so that the window satisfies the lighting requirements.

c Use the quadratic formula to solve the equation and find x to the nearest mm.

REASONING

11 Two competitive neighbours build rectangular pools that cover the same area but are different shapes. Pool A has a width of $(x + 3)$ m and a length that is 3 m longer than its width. Pool B has a length that is double the width of Pool A. The width of Pool B is 4 m shorter than its length.

a Find the exact dimensions of each pool if their areas are the same.

b Verify that the areas are the same.

12 A block of land is in the shape of a right-angled triangle with a perimeter of 150 m and a hypotenuse of 65 m. Determine the lengths of the other two sides. Show your working.

PROBLEM SOLVING

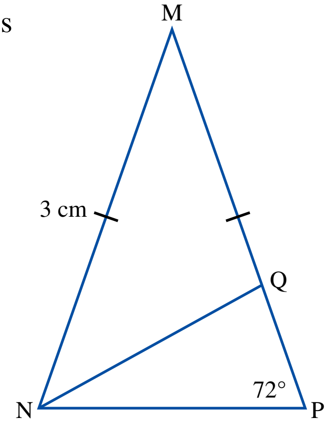
13 Solve $\left(x + \frac{1}{x}\right)^2 - 14\left(x + \frac{1}{x}\right) = 72$ for x.

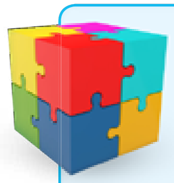
14 Triangle MNP is an isosceles triangle with sides MN = MP = 3 cm. Angle MPN is equal to 72°. The line NQ bisects the angle MNP.

a Prove that triangles MNP and NPQ are similar.

b If NP = m cm and $PQ = 3 - m$, show that $m^2 + 3m - 9 = 0$.

c Solve the equation $m^2 + 3m - 9 = 0$ and find the side length of NP, giving your answer correct to two decimal places.





CHALLENGE 8.2

The equation $ax^4 + bx^2 + c = 0$ can be solved by applying substitution and the rules used to solve quadratics. For example, $x^4 - 5x^2 + 4 = 0$ is solved for x as follows.

Notice that $x^4 - 5x^2 + 4 = (x^2)^2 - 5(x)^2 + 4$. Now let $x^2 = u$ and substitute.

$(x^2)^2 - 5(x)^2 + 4 = u^2 - 5u + 4$.

Solve for u. That is,

$$u^2 - 5u + 4 = 0$$
$$(u - 4)(u - 1) = 0$$
$$u - 4 = 0 \text{ or } u - 1 = 0$$
$$u = 4 \text{ or } u = 1$$

Since $x^2 = u$, that implies that

$x^2 = 4$ or $x^2 = 1$

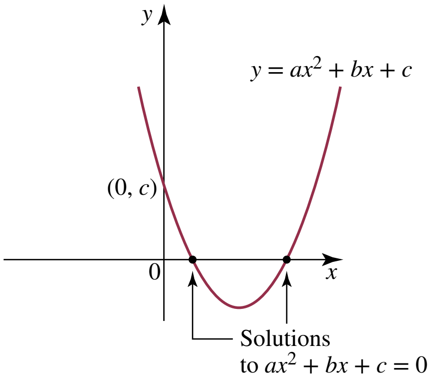
$x = \pm 2$ or $x = \pm 1$

Using this or another method, solve the following for x.

1 $x^4 - 13x^2 + 36 = 0$ **2** $4x^4 - 17x^2 = -4$

8.4 Solving quadratic equations graphically

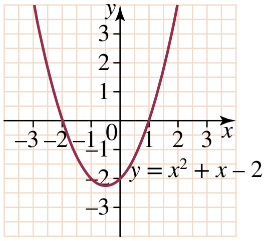
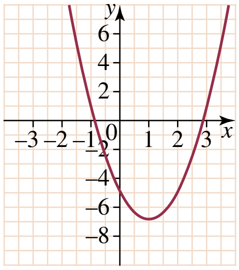
- The graph of $y = ax^2 + bx + c$ is in the shape of a parabola.
- The graph can be used to locate the solutions to quadratic equations such as $ax^2 + bx + c = 0$.



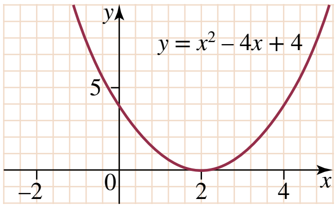
WORKED EXAMPLE 7 TI CASIO

Determine the solutions of each of the following quadratic equations by inspecting their corresponding graphs. Give answers to 1 decimal place where appropriate.

a $x^2 + x - 2 = 0$ **b** $2x^2 - 4x - 5 = 0$

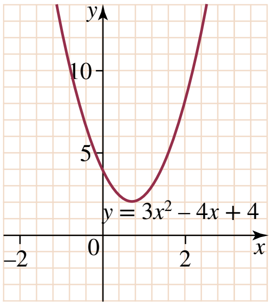
THINK	WRITE/DRAW
a 1 Examine the graph of $y = x^2 + x - 2$ and locate the points where $y = 0$, that is, where the graph intersects the x-axis.	**a** 
2 The graph cuts the x-axis ($y = 0$) at $x = 1$ and $x = -2$. Write the solutions.	$x^2 + x - 2 = 0$ From the graph, the solutions are $x = 1$ and $x = -2$.
b 1 The graph of $y = 2x^2 - 4x - 5$ is equal to zero when $y = 0$. Look at the graph to see where $y = 0$; that is, where it intersects the x-axis. By sight, we can only give estimates of the solutions.	**b** 
2 The graph cuts the x-axis at approximately $x = -0.9$ and approximately $x = 2.9$. Write the solutions.	$2x^2 - 4x - 5 = 0$ From the graph, the solutions are $x \approx -0.9$ and $x \approx 2.9$.

Quadratic equations with only one solution



- Some quadratic equations have only one solution. For example, the graph of $y = x^2 - 4x + 4$ has only one solution at $x = 2$. That is, the graph of $y = x^2 - 4x + 4$ touches the x-axis at one point only at $x = 2$.

Quadratic equations with no solutions



- There are also quadratic equations that have no real solutions. For example, the graph of $y = 3x^2 - 4x + 4$ does not intersect the x-axis and so $3x^2 - 4x + 4 = 0$ has no real solutions (that is, no solutions that are real numbers).

Confirming solutions

- It is possible to confirm the solutions obtained by sight. This is achieved by substituting the solution or solutions into the original quadratic equation, and determining whether they make a true statement.

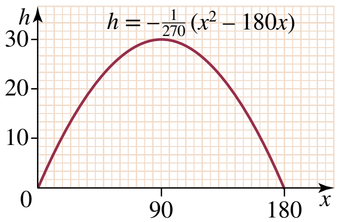
WORKED EXAMPLE 8

Confirm, by substitution, the solutions obtained in Worked example 7.
$x^2 + x - 2 = 0$; solutions: $x = 1$ and $x = -2$

	THINK	WRITE
1	Write the left-hand side of the equation and substitute $x = 1$ into the expression.	When $x = 1$, LHS: $x^2 + x - 2 = 1^2 + 1 - 2$ $= 0$
2	Write the right-hand side.	RHS: $= 0$
3	Confirm the solution.	LHS = RHS ⇒ Solution is confirmed.
4	Write the left-hand side and substitute $x = -2$.	When $x = -2$, LHS: $x^2 + x - 2 = (-2)^2 + -2 - 2$
5	Write the right-hand side.	$= 4 - 2 - 2$ $= 0$ RHS: $= 0$
6	Confirm.	LHS = RHS ⇒ Solution is confirmed.

WORKED EXAMPLE 9

A golf ball hit along a fairway follows the path shown in the following graph. The height, h metres after it has travelled x metres horizontally, follows the rule $h = -\frac{1}{270}(x^2 - 180x)$. Use the graph to find how far the ball landed from the golfer.



THINK	**WRITE**
On the graph, the ground is represented by the x-axis since this is where $h = 0$. The golf ball lands when the graph intersects the x-axis.	The golf ball lands 180 m from the golfer.

Exercise 8.4 Solving quadratic equations graphically

REFLECTION
What does 'the solution of a graph' mean?

INDIVIDUAL PATHWAYS

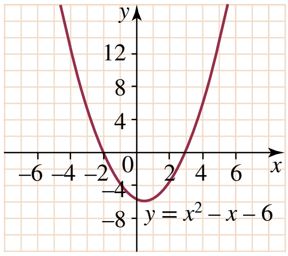
■ **PRACTISE**	■ **CONSOLIDATE**	■ **MASTER**
Questions: 1a–d, 2–4, 7	Questions: 1c–h, 2–5, 7	Questions: 1e–j, 2–8

■■■ **Individual pathway interactivity** int-4603

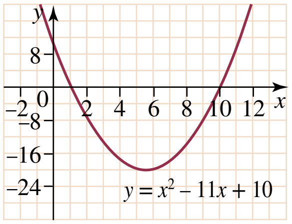
FLUENCY

1 **WE7** Determine the solutions of each of the following quadratic equations by inspecting the corresponding graphs. Give answers correct to 1 decimal place where appropriate.

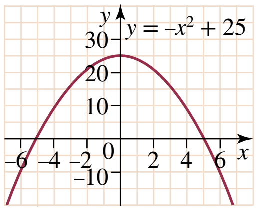
a $x^2 - x - 6 = 0$



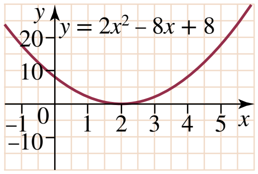
b $x^2 - 11x + 10 = 0$



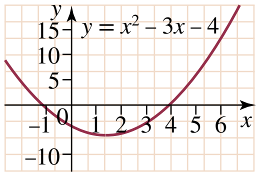
c $-x^2 + 25 = 0$



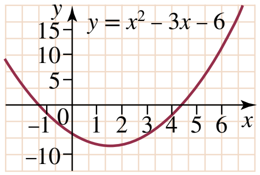
d $2x^2 - 8x + 8 = 0$



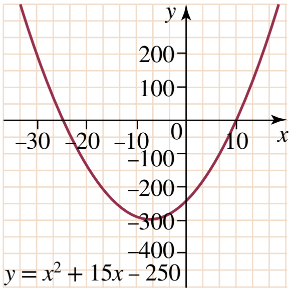
e $x^2 - 3x - 4 = 0$



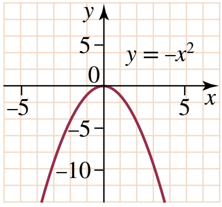
f $x^2 - 3x - 6 = 0$



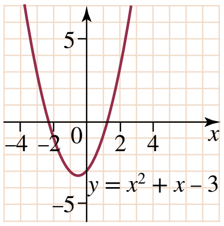
g $x^2 + 15x - 250 = 0$



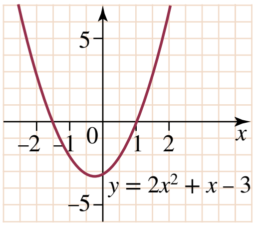
h $-x^2 = 0$



i $x^2 + x - 3 = 0$



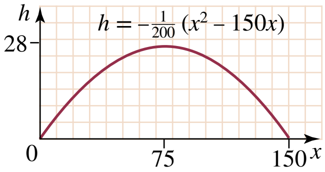
j $2x^2 + x - 3 = 0$



UNDERSTANDING

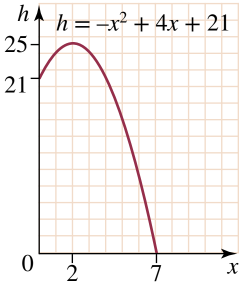
2 **WE8** Confirm, by substitution, the solutions obtained in question **1**.

3 **WE9** A golf ball hit along a fairway follows the path shown in the graph.



The height, h metres after it has travelled x metres horizontally, follows the rule $h = -\frac{1}{200}(x^2 - 150x)$. Use the graph to find how far the ball lands from the golfer.

4 A ball is thrown upwards from a building and follows the path shown in the graph until it lands on the ground.



The ball is h metres above the ground when it is a horizontal distance of x metres from the building. The path of the ball follows the rule $h = -x^2 + 4x + 21$. Use the graph to find how far from the building the ball lands.

REASONING

5 **a** The x-intercepts of a particular equation are $x = 2$ and $x = 5$. Suggest a possible equation.

b If the y-intercept in part **a** is (0, 4), give the exact equation.

6 **a** The x-intercepts of a particular equation are $x = p, q$. Suggest a possible equation.

b If the y-intercept in part **a** is $(0, r)$, give the exact equation.

PROBLEM SOLVING

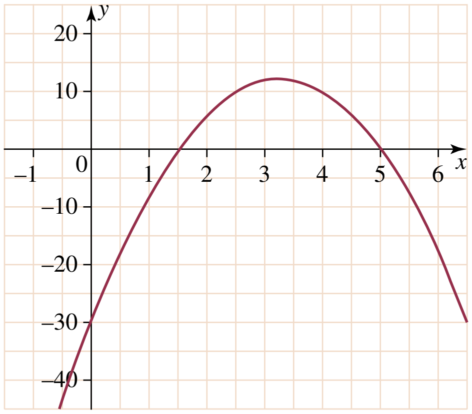
7 A platform diver follows a path determined by the equation $h = -0.5d^2 + 2d + 6$, where h represents the height of the diver above the water and d represents the distance from the diving board. Both pronumerals are measured in metres.

Use the graph to determine:

a how far the diver landed from the edge of the diving board

b how high the diving board is above the water.

8 Find the equation of the given parabola. Give your answer in the form $y = ax^2 + bx + c$.



8.5 The discriminant

eBook*plus*

Interactivity
Simultaneous quadratic equations
Int-2784

- Where $ax^2 + bx + c = 0$, the expression $\Delta = b^2 - 4ac$ is known as the **discriminant**.

Discriminant
$\Delta = b^2 - 4ac$

- The symbol used for the discriminant, Δ, is the Greek capital letter delta.
- The discriminant is found in the quadratic formula, as shown below.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

- The discriminant is the value that determines the number of solutions to the quadratic equation.
  - If $\Delta < 0$, there are *no* real solutions to the quadratic equation.
  - If $\Delta = 0$, there is *only one* solution to the quadratic equation.
  - If $\Delta > 0$, there are *two* solutions to the quadratic equation.

WORKED EXAMPLE 10

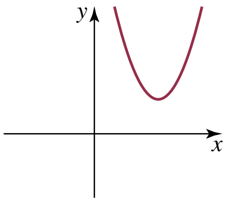
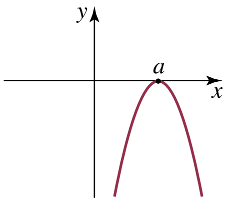
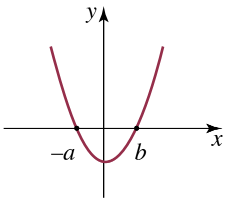
Calculate the value of the discriminant for each of the following and use it to determine how many solutions the equation will have.

a $2x^2 + 9x - 5 = 0$ **b** $x^2 + 10 = 0$

	THINK	WRITE
a	1 Write the expression and determine the values of *a*, *b* and *c* given $ax^2 + bx + c = 0$.	**a** $2x^2 + 9x - 5 = 0$ $2x^2 + 9x + -5 = 0$ $a = 2, b = 9, c = -5$
	2 Write the formula for the discriminant and substitute values of *a*, *b* and *c*.	$\Delta = b^2 - 4ac$ $= 9^2 - 4 \times 2 \times -5$
	3 Simplify the equation and solve.	$= 81 - -40$ $= 121$
	4 State the number of solutions. In this case $\Delta > 0$, which means there are two solutions.	$\Delta > 0$ There will be two solutions to the equation $2x^2 + 9x - 5 = 0$.
b	1 Write the expression and determine the values of *a*, *b* and *c* given $ax^2 + bx + c = 0$.	**b** $x^2 + 10 = 0$ $1x^2 + 0x + 10 = 0$ $a = 1, b = 0, c = 10$
	2 Write the formula for the discriminant and substitute the values of *a*, *b* and *c*.	$\Delta = b^2 - 4ac$ $= (0)^2 - 4 \times 1 \times 10$ $= 0 - 40$ $= -40$
	3 State the number of solutions. In this case $\Delta < 0$, which means there are no solutions.	$\Delta < 0$, so there will be no solutions to the equation $x^2 + 10 = 0$.

Types of solutions

- The table below summarises the types of solutions indicated by the discriminant.

			$\Delta > 0$ **(positive)**	
	$\Delta < 0$ **(negative)**	$\Delta = 0$ **(zero)**	**Perfect square**	**Not a perfect square**
Number of solutions	No real solutions	1 rational solution	2 rational solutions	2 irrational (surd) solutions
Description	Graph does not cross or touch the x-axis	Graph touches the x-axis	Graph intersects the x-axis twice	
Graph				

WORKED EXAMPLE 11 TI CASIO

By using the discriminant, determine whether the following equations have:

- **i** **two rational solutions**
- **ii** **two irrational solutions**
- **iii** **one rational solution (two equal solutions)**
- **iv** **no real solutions.**

a $x^2 - 9x - 10 = 0$ **b** $x^2 - 2x - 14 = 0$

c $x^2 - 2x + 14 = 0$ **d** $x^2 + 14x = -49$

THINK	**WRITE**
a 1 Write the equation.	**a** $x^2 - 9x - 10 = 0$
2 Identify the coefficients a, b and c.	$a = 1, b = -9, c = -10$
3 Find the discriminant.	$\Delta = b^2 - 4ac$ $= (-9)^2 - (4 \times 1 \times -10)$ $= 121$
4 Identify the number and type of solutions when $\Delta > 0$ and is a perfect square.	The equation has two rational solutions.
b 1 Write the equation.	**b** $x^2 - 2x - 14 = 0$
2 Identify the coefficients a, b and c.	$a = 1, b = -2, c = -14$
3 Find the discriminant.	$\Delta = b^2 - 4ac$ $= (-2)^2 - 4 \times 1 \times -14$ $= 60$

	4	Identify the number and type of solutions when $\Delta > 0$ but not a perfect square.		The equation has two irrational solutions.
c	1	Write the equation.	**c**	$x^2 - 2x + 14 = 0$
	2	Identify the coefficients a, b and c.		$a = 1, b = -2, c = 14$
	3	Find the discriminant.		$\Delta = b^2 - 4ac$ $= (-2)^2 - (4 \times 1 \times 14)$ $= -52$
	4	Identify the number and type of solutions when $\Delta < 0$.		The equation has no real solutions.
d	1	Write the equation, then rewrite it so the right side equals zero.	**d**	$x^2 + 14x = -49$ $x^2 + 14x + 49 = 0$
	2	Identify the coefficients a, b and c.		$a = 1, b = 14, c = 49$
	3	Find the discriminant.		$\Delta = b^2 - 4ac$ $= 14^2 - (4 \times 1 \times 49)$ $= 0$
	4	Identify the number and types of solutions when $\Delta = 0$.		The equation has 1 rational solution.

Using the discriminant to determine if graphs intersect

- The discriminant can be used to determine the number of points of intersection between graphs.

WORKED EXAMPLE 12

Determine whether the parabola $y = x^2 - 2$ and the line $y = x - 3$ intersect.

	THINK	WRITE
1	If the parabola and the line intersect, there will be at least one solution to the simultaneous equations: let $y_1 = y_2$.	$y_1 = x^2 - 2$ $y_2 = x - 3$ $y_1 = y_2$ $x^2 - 2 = x - 3$
2	Collect all terms on one side and simplify.	$x^2 - 2 - x + 3 = x - 3 - x + 3$ $x^2 - 2 - x + 3 = 0$ $x^2 - x + 1 = 0$
3	• Use the discriminant to check if any solutions exist. • If $\Delta < 0$, then no solutions exist.	$\Delta = b^2 - 4ac$ $a = 1, b = -1$ $\Delta = (-1)^2 - 4(1)(1)$ $= 1 - 4$ $= -3$ $\Delta < 0 \therefore$ no solutions exist
4	Answer the question.	The parabola and the line do not intersect.

Exercise 8.5 The discriminant

REFLECTION
What does the discriminant tell us?

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1a–f, 2a–f, 3, 4, 6–8, 10, 13, 15	Questions: 1e–j, 2d–i, 3, 5–9, 11, 12, 14, 15, 17	Questions: 1h–n, 2f–l, 3, 5–18

■■■ **Individual pathway interactivity** int-4604 eBook*plus*

FLUENCY

1 **WE10** Calculate the value of the discriminant for each of the following and use it to determine how many solutions the equation will have.

a $6x^2 + 13x - 5 = 0$
b $x^2 + 9x - 90 = 0$
c $x^2 + 4x - 2 = 0$
d $36x^2 - 1 = 0$
e $x^2 + 2x + 8 = 0$
f $x^2 - 5x - 14 = 0$
g $36x^2 + 24x + 4 = 0$
h $x^2 - 19x + 88 = 0$
i $x^2 - 10x + 17 = 0$
j $30x^2 + 17x - 21 = 0$
k $x^2 + 16x + 62 = 0$
l $9x^2 - 36x + 36 = 0$
m $2x^2 - 16x = 0$
n $x^2 - 64 = 0$

2 **WE11** By using the discriminant, determine whether the equations below have:

i two rational solutions
ii two irrational solutions
iii one rational solution (two equal solutions)
iv no real solutions.

a $x^2 - 3x + 5$
b $4x^2 - 20x + 25 = 0$
c $x^2 + 9x - 22 = 0$
d $9x^2 + 12x + 4$
e $x^2 + 3x - 7 = 0$
f $25x^2 - 10x + 1 = 0$
g $3x^2 - 2x - 4 = 0$
h $2x^2 - 5x + 4 = 0$
i $x^2 - 10x + 26 = 0$
j $3x^2 + 5x - 7 = 0$
k $2x^2 + 7x - 10 = 0$
l $x^2 - 11x + 30 = 0$

3 **WE12** Determine whether the following graphs intersect.

a $y = -x^2 + 3x + 4$ and $y = x - 4$
b $y = -x^2 + 3x + 4$ and $y = 2x + 5$
c $y = -(x + 1)^2 + 3$ and $y = -4x - 1$
d $y = (x - 1)^2 + 5$ and $y = -4x - 1$

4 Consider the equation $3x^2 + 2x + 7 = 0$.

a What are the values of a, b and c?
b What is the value of $b^2 - 4ac$?
c How many real solutions, and hence x-intercepts, are there for this equation?

5 Consider the equation $-6x^2 + x + 3 = 0$.

a What are the values of a, b and c?
b What is the value of $b^2 - 4ac$?
c How many real solutions, and hence x-intercepts, are there for this equation?
d With the information gained from the discriminant, use the most efficient method to solve the equation. Give an exact answer.

6 **MC** The discriminant of the equation $x^2 - 4x - 5 = 0$ is:

A 36
B 11
C 4
D 0
E -4

7 **MC** Which of the following quadratic equations has two irrational solutions?

A $x^2 - 8x + 16 = 0$
B $2x^2 - 7x = 0$
C $x^2 + 8x + 9 = 0$
D $x^2 - 4 = 0$
E $x^2 - 6x + 15 = 0$

8 **MC** The equation $x^2 = 2x - 3$ has:

A two rational solutions

B exactly one solution

C no solutions

D two irrational solutions

E one rational and one irrational solution

UNDERSTANDING

9 Find the value of k if $x^2 - 2x - k = 0$ has one solution.

10 Find the values of m for which $mx^2 - 6x + 5 = 0$ has one solution.

11 Find the values of n when $x^2 - 3x - n = 0$ has two solutions.

12 Show that $3x^2 + px - 2 = 0$ will have real solutions for all values of p.

13 The path of a dolphin as it leaps out of the water can be modelled by the equation $h = -0.4d^2 + d$, where h is the dolphin's height above water and d is the horizontal distance from its starting point. Both h and d are in metres.

a How high above the water is the dolphin when it has travelled 2 m horizontally from its starting point?

b What horizontal distance has the dolphin covered when it first reaches a height of 25 cm?

c What horizontal distance has the dolphin covered when it next reaches a height of 25 cm? Explain your answer.

d What horizontal distance does the dolphin cover in one leap? (*Hint:* What is the value of h when the dolphin has completed its leap?)

e During a leap, can this dolphin reach a height of:

i 0.5 m

ii 1 m?

How can you determine this without actually solving the equation?

f Find the greatest height the dolphin reaches during a leap.

14 The parabolas $y = x^2 - 4$ and $y = 4 - x^2$ intersect in two places. Find the coordinates of their points of intersection.

REASONING

15 **a** For what values of a will the straight line $y = ax + 1$ have one intersection with the parabola $y = -x^2 - x - 8$?

b For what values of b will the straight line $y = 2x + b$ *not* intersect the parabola $y = x^2 + 3x - 5$?

16 **a** Find how many points of intersection exist between the parabola $y = -2(x + 1)^2 - 5$, where $y = f(x), x \in R$, and the straight line $y = mx - 7$, where $y = f(x), x \in R$.

b Find $m\,(m < 0)$ such that $y = mx - 7$ has one intersection point with $y = -m(x + 1)^2 - 5$.

PROBLEM SOLVING

eBook*plus*

Digital doc
WorkSHEET 8.2
doc-13854

17 The parabola with the general equation $y = ax^2 + bx + 9$ where $0 < a < 10$ and $0 < b < 20$ touches the x-axis at one point only. The graph passes through the point (1, 25). Find the values of a and b.

18 The line with equation $kx + y = 3$ is a tangent to the curve with equation $y = kx^2 + kx - 1$. Find the value of k.

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8.6 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

eBook*plus*

Interactivities

Word search

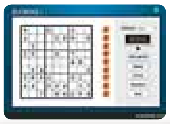
int-2847

Crossword

int-2848

Sudoku

int-3595



Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

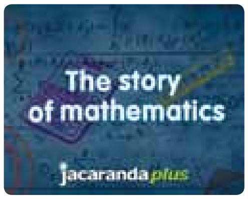
coefficient
completing the square
constant term
discriminant
factorise
intersection
irrational
linear equation
null factor law
parabola
perfect square
product
quadratic equation
quadratic formula
rational
real
solutions
substitution

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

The story of mathematics is an exclusive Jacaranda video series that explores the history of mathematics and how it helped shape the world we live in today.



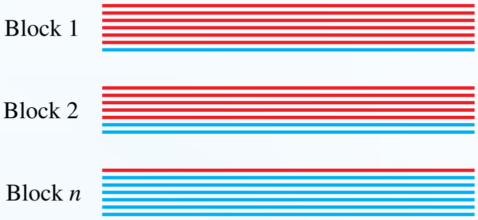
The Chinese Golden Age of Mathematics (eles-1847) explores the life of Qin Jiushao, whose mathematical work was centuries ahead of his Western peers, and the controversial figure of Huangdi, the Yellow Emperor.

INVESTIGATION

RICH TASK

Weaving

Many articles of clothing are sewn from materials that show designs and patterns made by weaving together threads of different colours. Intricate and complex designs can result. Let's investigate some very simple repetitive-type patterns. Knowledge of quadratic equations and the quadratic formula is helpful in creating these designs.



We need to understand the process of weaving. Weaving machines have parts called *warps*. Each warp is divided into a number of *blocks*. Consider a pattern that is made up of a series of blocks, where the first block is all one colour except for the last thread, which is a different colour. Let's say our pattern is red and blue. The first block contains all red threads, except for the last one, which is blue. The next block has all red threads, except for the last two threads, which are blue. The pattern continues in this manner. The last block has the first thread as red and the remainder as blue. The warp consists of a particular number of threads, let's say 42 threads. How many blocks and how many threads per block would be necessary to create a pattern of this type?

To produce this pattern, we need to divide the warp into equally sized blocks, if possible. What size block and how many threads per block would give us the 42-thread warp? We will need to look for a mathematical pattern. Look at the table opposite, where we consider the smallest block consisting of 2 threads through to a block consisting of 7 threads.

Pattern	Number of threads per block	Number of blocks	Total threads in warp
RB	2	1	2
RRB RBB	3	2	6
RRRB RRBB RBBB	4		
	5		
	6		
	7		

1 Complete the entries in the table.

2 Consider a block consisting of *n* threads.
- **a** How many blocks would be needed?
- **b** What would be the total number of threads in the warp?

The 42-thread warp was chosen as a simple example to show the procedure involved in determining the number of blocks required and the number of threads per block. In this particular case, 6 blocks of 7 threads per block would give us our design for a 42-thread warp. In practice, you would not approach the problem by drawing up a table to determine the number of blocks and the size of each block.

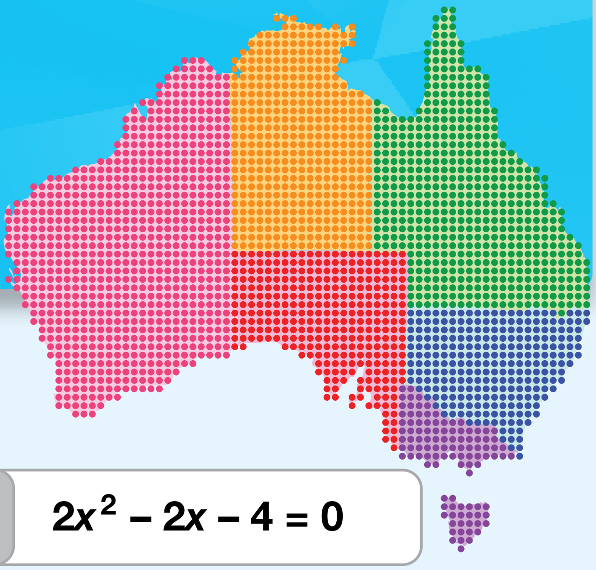
3 Take your expression in question **2b** and let it equal 42. This should form a quadratic equation. Solve this equation to verify that you would need 6 blocks with 7 threads per block to fulfil the size of a 42-thread warp.

4 In reality, the size of each block is not always clearly defined. Also, the thread warp sizes are generally much larger, say, about 250. Let's determine the number of threads per block and the number of blocks required for a 250-thread warp.
- **a** Form your quadratic equation with the thread warp size equal to 250.
- **b** A solution to this equation can be found using the quadratic formula. Use the quadratic formula to determine a solution.
- **c** The number of threads per block is represented by *n* and this obviously must be a whole number. Round your solution down to the nearest whole number.
- **d** How many whole blocks are needed?
- **e** Use your solutions to **c** and **d** to determine the total number of threads used for the pattern.
- **f** How many more threads do you need to make the warp size equal to 250 threads?
- **g** Distribute these threads by including them at the beginning of the first block and the end of the last block. Describe your overall pattern.

5 Investigate the number of blocks required and threads per block required for a 400-thread warp.

6 Investigate changing the pattern. Let the first block be all red. In the next block change the colour of the first and last threads to blue. With each progressive block, change the colour of an extra thread at the top and bottom to blue until the last block is all blue. On a separate sheet of paper, draw a table to determine the thread warp size for a block size of *n* threads. Draw the pattern and describe the result for a particular warp size.

CODE PUZZLE



Aussie fact

Factorise each of the following and then solve for *x* to discover the puzzle code.

A	$x^2 - 4x = 0$	**N**	$2x^2 - 2x - 4 = 0$
C	$x^2 + 7x = 0$	**O**	$3x^2 + 12x = 0$
D	$x^2 + 6x + 5 = 0$	**R**	$3x^2 - 12 = 0$
E	$x^2 + 2x + 1 = 0$	**S**	$x^2 + 7x + 10 = 0$
F	$x^2 - 8x + 12 = 0$	**T**	$3x^2 - 3x - 18 = 0$
H	$x^2 - 81 = 0$	**U**	$x^2 - 8x + 16 = 0$
I	$x^2 - 16x + 64 = 0$	**W**	$x^2 - 9x + 20 = 0$
L	$x^2 - 2x - 3 = 0$	**X**	$x^2 + 6x - 27 = 0$

3, −2	9, −9	−1	0, 4	4	−2, −5	3, −2	2, −2	0, 4	3, −1	8	0, 4	2, −1	6, 2	−1
−5, −1	−1	2, −2	0, 4	3, −2	8	0, −4	2, −1	0, −7	0, −4	2, −1	−2, −5	8	−2, −5	3, −2
−2, −5	0, −4	6, 2	−2, −5	8	−9, 3	−2, −5	3, −2	0, 4	3, −2	−1	−2, −5	0, 4	2, −1	−5, −1
3, −2	4, 5	0, −4	3, −2	−1	2, −2	2, −2	8	3, −2	0, −4	2, −2	8	−1	−2, −5	

eBook*plus* Activities

8.1 Overview

Video

- The story of mathematics (eles-1847)

8.2 Solving quadratic equations algebraically

Interactivity

- IP interactivity 8.2 (int-4601): Solving quadratic equations algebraically

Digital docs

- SkillSHEET (doc-5256): Factorising by taking out the highest common factor
- SkillSHEET (doc-5257): Finding a factor pair that adds to a given number
- SkillSHEET (doc-5258): Simplifying surds
- SkillSHEET (doc-5259): Substituting into quadratic equations
- SkillSHEET (doc-5260): Equation of a vertical line
- WorkSHEET 8.1 (doc-5261): Solving quadratic equations

8.3 The quadratic formula

Interactivity

- IP interactivity 8.3 (int-4602): The quadratic formula

Digital doc

- SkillSHEET (doc-5262): Substituting into the quadratic formula

8.4 Solving quadratic equations graphically

Interactivity

- IP interactivity 8.4 (int-4603): Solving quadratic equations graphically

8.5 The discriminant

Interactivities

- Simultaneous quadratic equations (int-2784)
- IP interactivity 8.5 (int-4604): The discriminant

Digital doc

- WorkSHEET 8.2 (doc-13854): Using the discriminant

8.6 Review

Interactivities

- Word search (int-2847)
- Crossword (int-2848)
- Sudoku (int-3595)

Digital docs

- Topic summary (doc-13808)
- Concept map (doc-13809)

To access eBookPLUS activities, log on to **www.jacplus.com.au**

Answers

TOPIC 8 Quadratic equations

Exercise 8.2 — Solving quadratic equations algebraically

1 a $-7, 9$ **b** $-2, 3$ **c** $2, 3$
d $0, 3$ **e** $0, 1$ **f** $-5, 0$
g $0, 3$ **h** $-2, 0$ **i** $-\frac{1}{2}, \frac{1}{2}$
j $-1.2, -0.5$ **k** $0.1, 0.75$ **l** $-\sqrt{2}, \sqrt{3}$

2 a $\frac{1}{2}, 1$ **b** $-2, -\frac{2}{3}$ **c** $\frac{1}{4}, 7$
d $-\frac{6}{7}, 1\frac{1}{2}$ **e** $\frac{3}{5}, \frac{2}{3}$ **f** $-\frac{5}{8}, \frac{2}{3}$
g $0, \frac{1}{2}, 3$ **h** $0, \frac{1}{2}, -\frac{2}{5}$ **i** $0, -3, \frac{2}{5}$

3 a $0, 2$ **b** $-5, 0$ **c** $0, 7$
d $-\frac{2}{3}, 0$ **e** $0, 1\frac{1}{2}$ **f** $0, \frac{1}{3}$
g $0, \frac{\sqrt{7}}{2}$ **h** $-\frac{\sqrt{3}}{3}, 0$ **i** $0, 1\frac{1}{4}$

4 a $-2, 2$ **b** $-5, 5$ **c** $-2, 2$
d $-7, 7$ **e** $-1\frac{1}{3}, 1\frac{1}{3}$ **f** $-2\frac{1}{2}, 2\frac{1}{2}$
g $-\frac{2}{3}, \frac{2}{3}$ **h** $-\frac{1}{2}, \frac{1}{2}$ **i** $-\frac{1}{5}, \frac{1}{5}$
j $-4, 4$ **k** $-\sqrt{5}, \sqrt{5}$ **l** $-\frac{\sqrt{11}}{3}, \frac{\sqrt{11}}{3}$

5 a $-2, 3$ **b** $-4, -2$ **c** $-1, 7$
d $3, 5$ **e** 1 **f** $-1, 4$
g 5 **h** $-2, 5$ **i** $2, 6$
j $-3, 7$ **k** $-5, 6$ **l** $3, 4$
m 4 **n** -5 **o** 10

6 B **7** C

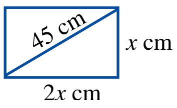
8 a $-\frac{1}{2}, 3$ **b** $\frac{2}{3}, -1$ **c** $-2, \frac{1}{5}$
d $\frac{1}{3}, 1\frac{1}{2}$ **e** $-\frac{3}{14}, 1$ **f** $\frac{1}{4}, \frac{1}{3}$
g $-1\frac{1}{3}, 2\frac{1}{2}$ **h** $-1\frac{3}{4}, -1\frac{1}{3}$ **i** $-\frac{2}{5}, \frac{1}{2}$
j $1\frac{1}{2}, 2\frac{2}{3}$ **k** $-\frac{2}{5}, \frac{1}{6}$ **l** $3, 4$

9 a $2+\sqrt{2}, 2-\sqrt{2}$ **b** $-1+\sqrt{3}, -1-\sqrt{3}$
c $-3+\sqrt{10}, -3-\sqrt{10}$ **d** $4+2\sqrt{3}, 4-2\sqrt{3}$
e $5+2\sqrt{6}, 5-2\sqrt{6}$ **f** $1+\sqrt{3}, 1-\sqrt{3}$
g $-1+\sqrt{6}, -1-\sqrt{6}$ **h** $-2+\sqrt{10}, -2-\sqrt{10}$
i $-2+\sqrt{15}, -2-\sqrt{15}$

10 a $\frac{3}{2}+\frac{\sqrt{5}}{2}, \frac{3}{2}-\frac{\sqrt{5}}{2}$ **b** $-\frac{5}{2}+\frac{\sqrt{29}}{2}, -\frac{5}{2}-\frac{\sqrt{29}}{2}$
c $\frac{7}{2}+\frac{\sqrt{33}}{2}, \frac{7}{2}-\frac{\sqrt{33}}{2}$ **d** $\frac{1}{2}+\frac{\sqrt{21}}{2}, \frac{1}{2}-\frac{\sqrt{21}}{2}$
e $\frac{11}{2}+\frac{\sqrt{117}}{2}, \frac{11}{2}-\frac{\sqrt{117}}{2}$ **f** $-\frac{1}{2}+\frac{\sqrt{5}}{2}, -\frac{1}{2}-\frac{\sqrt{5}}{2}$
g $-\frac{3}{2}+\frac{\sqrt{37}}{2}, -\frac{3}{2}-\frac{\sqrt{37}}{2}$ **h** $\frac{5}{2}+\frac{\sqrt{37}}{2}, \frac{5}{2}-\frac{\sqrt{37}}{2}$
i $\frac{9}{2}+\frac{\sqrt{65}}{2}, \frac{9}{2}-\frac{\sqrt{65}}{2}$

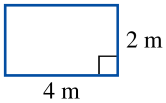
11 a $-3, 1$ **b** $-4.24, 0.24$ **c** $-1, 3$
d $-0.73, 2.73$ **e** $0.38, 2.62$ **f** $-0.30, 3.30$
g $-1.19, 4.19$ **h** $-2.30, 1.30$ **i** $-2.22, 0.22$

12 8 and 9 or −8 and −9
13 6 and 8, −6 and −8
14 9 or −10
15 2 or $-2\frac{2}{3}$
16 8 or $-10\frac{1}{2}$
17 6 seconds
18 a $l = 2x$
b



c $x^2 + (2x)^2 = 45^2, 5x^2 = 2025$
d Length 40 cm, width 20 cm

19 8 m, 6 m
20 a $-6, 1$ **b** $8, -3$ **c** $x = \pm 1$
21 a 21 **b** 17
22 7
23 a



b $(2+x)$ m, $(4+x)$ m
c $(2+x)(4+x) = 24$
d $x = 2$, 4 m wide, 6 m long

24 a $C_{Annabel}(28) = \$364\,800, C_{Betty}(28) = \$422\,400$
b 10 knots
c Speed can only be a positive quantity, so the negative solution is not valid.

25 No real solutions — when we complete the square we get the sum of two squares, not the difference of two squares and we cannot factorise the expression.

26 $x = 5, -4, 4, -3$

27 $z = -\frac{5}{3}, 7$

Challenge 8.1

The width of the pathway is 1.5 m.

Exercise 8.3 — The quadratic formula

1 a $a = 3, b = -4, c = 1$ **b** $a = 7, b = -12, c = 2$
c $a = 8, b = -1, c = -3$ **d** $a = 1, b = -5, c = 7$
e $a = 5, b = -5, c = -1$ **f** $a = 4, b = -9, c = -3$
g $a = 12, b = -29, c = 103$ **h** $a = 43, b = -81, c = -24$

2 a $\frac{-5 \pm \sqrt{21}}{2}$ **b** $\frac{-3 \pm \sqrt{13}}{2}$ **c** $\frac{5 \pm \sqrt{17}}{2}$
d $2 \pm \sqrt{13}$ **e** $-1 \pm 2\sqrt{3}$ **f** $\frac{7 \pm \sqrt{45}}{2}$
g $\frac{9 \pm \sqrt{73}}{2}$ **h** $3 \pm 2\sqrt{3}$ **i** $-4 \pm \sqrt{31}$
j $\frac{1 \pm \sqrt{21}}{2}$ **k** $\frac{5 \pm \sqrt{33}}{2}$ **l** $-1 \pm 2\sqrt{2}$

3 a $-0.54, 1.87$ **b** $-1.20, 1.45$ **c** $-4.11, 0.61$
d $-0.61, 0.47$ **e** $0.14, 1.46$ **f** $0.16, 6.34$
g $-1.23, 1.90$ **h** $-1.00, 1.14$ **i** $-0.83, 0.91$
j $-0.64, 1.31$ **k** $-0.35, 0.26$ **l** $-1.45, 1.20$
m $0.08, 5.92$ **n** $-0.68, 0.88$

4 C
5 C
6 B
7 C

8 a $0.5, 3$ **b** $0, 5$
c $-1, 3$ **d** $0.382, 2.618$
e $0.298, 6.702$ **f** $2, 4$
g No real solution **h** $-1, 8$
i $-4.162, 2.162$ **j** $-2, 1$
k $-7, 1.5$ **l** No real solution
m $2, 7$ **n** $-\frac{1}{2}, \frac{1}{3}$
o No real solution

9 a $2\pi r^2 + 14\pi r - 231 = 0$ **b** 3.5 cm
c 154 cm^2
10 a $x(x + 30)$ **b** $x(x + 30) = 1500$ **c** 265 mm
11 a Pool A: $3\frac{2}{3}$ m by $6\frac{2}{3}$ m; Pool B: $3\frac{1}{3}$ m by $7\frac{1}{3}$ m
b The area of each is $24\frac{4}{9}$ m^2.
12 25 m, 60 m
13 $-2 \pm \sqrt{3}, 9 \pm 4\sqrt{5}$
14 a Teacher to check
b Teacher to check
c $m = 1.85$ so NP is 1.85 cm.

Challenge 8.2

1 $x = \pm 2$ or $x = \pm 3$ **2** $x = \pm\frac{1}{2}$ or $x = \pm 2$

Exercise 8.4 — Solving quadratic equations graphically

1 a $x = -2, x = 3$ **b** $x = 1, x = 10$
c $x = -5, x = 5$ **d** $x = 2$
e $x = -1, x = 4$ **f** $x \approx\ -1.4, x \approx 4.4$
g $x = -25, x = 10$ **h** $x = 0$
i $x \approx -2.3, x \approx 1.3$ **j** $x \approx -1.5, x = 1$
2 a–j Confirm by substitution of above values into quadratic equations.
3 150 m
4 7 m
5 a $y = a(x - 2)(x - 5)$ **b** $y = \frac{2}{5}(x - 2)(x - 5)$
6 a $y = a(x - p)(x - q)$ **b** $y = \frac{r}{pq}(x - p)(x - q)$
7 a 6 m **b** 6 m
8 $y = -4x^2 + 26x - 30$

Exercise 8.5 — The discriminant

1 a $\Delta = 289$, 2 solutions **b** $\Delta = 441$, 2 solutions
c $\Delta = 24$, 2 solutions **d** $\Delta = 144$, 2 solutions
e $\Delta = -28$, 0 solutions **f** $\Delta = 81$, 2 solutions
g $\Delta = 0$, 1 solution **h** $\Delta = 9$, 2 solutions
i $\Delta = 32$, 2 solutions **j** $\Delta = 2809$, 2 solutions
k $\Delta = 8$, 2 solutions **l** $\Delta = 0$, 1 solution
m $\Delta = 256$, 2 solutions **n** $\Delta = 256$, 2 solutions
2 a No real solutions **b** 1 rational solution
c 2 rational solutions **d** 1 rational solution
e 2 irrational solutions **f** 1 rational solution
g 2 irrational solutions **h** No real solutions
i No real solutions **j** 2 irrational solutions
k 2 irrational solutions **l** 2 rational solutions
3 a Yes **b** No
c Yes **d** No
4 a $a = 3, b = 2, c = 7$
b -80
c No real solutions
5 a $a = -6, b = 1, c = 3$ **b** 73
c 2 real solutions **d** $\frac{1 \pm \sqrt{73}}{12}$
6 A
7 C
8 C
9 $k = -1$
10 $m = 1, 8$
11 $n > \frac{9}{4}$
12 p^2 can only give a positive number, which, when added to 24, is always a positive solution.

13 a 0.4 m
b 0.28 m
c 2.22 m
d 2.5 m
e i Yes
ii No
Find the halfway point between the beginning and the end of the leap, and substitute this value into the equation to find the maximum height.
f 0.625 m
14 $(-2, 0), (2, 0)$
15 a $a = -7$ or 5 will give one intersection point.
b For values of $< -\frac{21}{4}$, there will be no intersection points.
16 a The straight line crosses the parabola at $(0, -7)$, so no matter what value m takes, there will be at least one intersection point and a maximum of two.
b $m = -\frac{8}{5}$
17 $a = 4, b = 12$
18 $k = -4$

Investigation — Rich task

1

Pattern	Number of threads per block	Number of blocks	Total threads in warp
RB	2	1	2
RRB RRB	3	2	6
RRRB RRBB RBBB	4	3	12
RRRRB RRRBB RRBBB RBBBB	5	4	20
RRRRRB RRRRBB RRRBBB RRBBBB RBBBBB	6	5	30
RRRRRRB RRRRRBB RRRRBBB RRRBBBB RRBBBBB RBBBBBB	7	6	42

2 a $n - 1$ **b** $n^2 - n$
3 Teacher to check
4 a $n^2 - n = 250$ **b** $n = \frac{\sqrt{1001} + 1}{2}$ **c** $n = 16$
d 15 **e** 240 **f** 10
g Answers will vary.
5 Answers will vary.
6 Answers will vary.

Code puzzle

The Australian federation consists of six states and two territories.