1

Reviewing linear equations

Objectives

- ► To solve **linear equations** in one unknown.
- ► To construct linear equations.
- ► To use linear equations to **solve problems**.
- ► To solve **simultaneous linear equations** by substitution and elimination methods.
- ► To solve **linear inequalities**.
- ► To use and **transpose** formulas.

Many problems may be solved by first translating them into mathematical equations and then solving the equations using algebraic techniques. An equation is solved by finding the value or values of the variables that would make the statement true.

Consider the equation $2x + 11 = 3 - 2x$. If $x = -2$, then

$$\text{LHS} = 2(-2) + 11 = 7 \quad \text{and} \quad \text{RHS} = 3 - 2(-2) = 7$$

The statement is true when $x = -2$. The solution to the equation is therefore $x = -2$. In this case there is no other value of x that would give a true statement.

The equations that we deal with in this chapter are called linear equations since they are related to the equation of a straight line.

Linear equations have either one solution (as shown in the example above), no solutions or infinitely many solutions. The equation $2x + 3 = 2x + 4$ has no solutions, since no value of x makes the statement true. The equation $2(x + 3) = 2x + 6$ has infinitely many solutions, since it is true for all values of x.

We note that the equation $x^2 = 16$ has exactly two solutions, $x = 4$ or $x = -4$, but this equation is not linear.

ISBN 978-1-009-11045-7

Cambridge University Press

1A Linear equations

A **linear equation** (in one unknown) is a particular type of polynomial equation in which the variable is to the first power. The following are examples of linear equations:

$$3x - 5 = 11, \qquad 7 - 2t = 8t - 11, \qquad \frac{z-3}{4} + \frac{2z-5}{3} = 11$$

In each of these equations, the variable is to the first power.

The following are examples of non-linear polynomial equations:

$$x^2 - x - 12 = 0 \quad \text{(quadratic)}, \qquad 2x^3 - x = 0 \quad \text{(cubic)}, \qquad x^4 = 16 \quad \text{(quartic)}$$

In each of these equations, the highest power of the variable is not the first power. You have met some of these in your previous study, and they are also in later chapters of this book.

Solving linear equations

It is important when setting out the solution to an equation that each step is written under the previous one with the equals signs aligned. This careful setting out makes the algebra easy to check. Unsystematic methods, such as guess and check, will generally be of limited use for more complicated equations.

It is often helpful to look at how the equation has been constructed so that the steps necessary to 'undo' the equation can be identified. It is most important that the steps taken to solve the equation are done in the correct order.

Linear equations of the form $ax + b = c$

Many linear equations that arise in applications are of the form $ax + b = c$.

Example 1

Solve the equation $3x + 4 = 16$ for x.

Solution	Explanation
$3x + 4 = 16$	
$3x = 12$	Subtract 4 from both sides.
$x = 4$	Divide both sides by 3.
Check: LHS $= 3(4) + 4 = 16$; RHS $= 16$; $\therefore$ the solution is correct.	Once a solution has been found it may be checked by substituting the value back into both sides of the original equation to ensure that the left-hand side (LHS) equals the right-hand side (RHS).

The first three equations in the above example are equivalent equations. Obtaining the second two equations enables us to solve the first equation.

ISBN 978-1-009-11045-7

Cambridge University Press

Given an equation, an equivalent equation can be formed by:

- adding or subtracting the same number on both sides of the equation
- multiplying or dividing both sides of the equation by the same non-zero number.

Importantly, two equivalent equations have the same solution. By forming suitable equivalent equations, we solve linear equations.

Equations with the unknown on both sides

Group all the terms containing the variable on one side of the equation and the remaining terms on the other side.

Example 2

Solve $4x + 3 = 3x - 5$.

Solution	Explanation
$4x + 3 = 3x - 5$	
$x + 3 = -5$	Subtract $3x$ from both sides and then subtract 3 from both sides.
$x = -8$	
Check:	The solution can be checked as previously shown.
LHS $= 4(-8) + 3 = -29$	
RHS $= 3(-8) - 5 = -29$	
$\therefore$ the solution is correct.	

Equations containing brackets

A frequently used first step is to remove brackets and then to follow the procedure for solving an equation without brackets.

Example 3

Solve $3(2x + 5) = 27$.

Solution	Explanation
$3(2x + 5) = 27$	We note that since 27 is divisible by 3, the following method is also possible:
$6x + 15 = 27$	$3(2x + 5) = 27$
$6x = 12$	$2x + 5 = 9$
$x = 2$	$2x = 4$
Check:	$x = 2$
LHS $= 3(2 \times 2 + 5) = 27$	
RHS $= 27$	
$\therefore$ the solution is correct.	

ISBN 978-1-009-11045-7

Equations containing fractions

A frequently used first step is to multiply both sides of the equation by the lowest common multiple of the denominators of the fractions.

Example 4

Solve $\frac{x}{5} - 2 = \frac{x}{3}$.

Solution

$$\frac{x}{5} - 2 = \frac{x}{3}$$

$$\frac{x}{5} \times 15 - 2 \times 15 = \frac{x}{3} \times 15$$

$$3x - 30 = 5x$$

$$-2x = 30$$

$$x = -15$$

Check: $\text{LHS} = \frac{-15}{5} - 2 = -3 - 2 = -5$

$\text{RHS} = \frac{-15}{3} = -5$

$\therefore$ the solution is correct.

Explanation

The denominators of the fractions are 3 and 5. The lowest common multiple of 3 and 5 is 15.

Multiply both sides of the equation by 15. This means that each term of the LHS and the RHS of the equation is multiplied by 15.

Example 5

Solve $\frac{x-3}{2} - \frac{2x-4}{3} = 5$.

Solution

$$\frac{x-3}{2} \times 6 - \frac{2x-4}{3} \times 6 = 5 \times 6$$

$$3(x-3) - 2(2x-4) = 30$$

$$3x - 9 - 4x + 8 = 30$$

$$-x = 31$$

$$x = -31$$

Check:

$$\text{LHS} = \frac{-31-3}{2} - \frac{2 \times (-31) - 4}{3}$$

$$= \frac{-34}{2} - \frac{-66}{3} = -17 + 22 = 5$$

$\text{RHS} = 5$

$\therefore$ the solution is correct.

Explanation

Remember that the line separating the numerator and the denominator (the vinculum) acts as brackets.

Multiply both sides of the equation by 6, the lowest common multiple of 2 and 3.

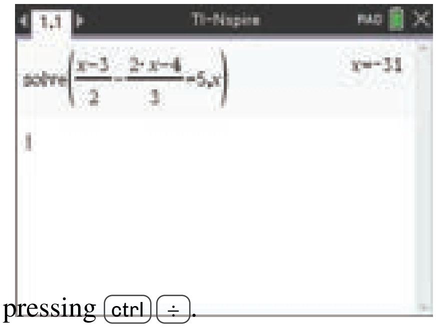
ISBN 978-1-009-11045-7

Using the TI-Nspire

- To find the solution to the linear equation, use a **Calculator** application.
- Select (menu) > **Algebra** > **Solve**.
- Enter the equation

$$\frac{x-3}{2} - \frac{2x-4}{3} = 5$$

- Press (enter) to obtain the solution.



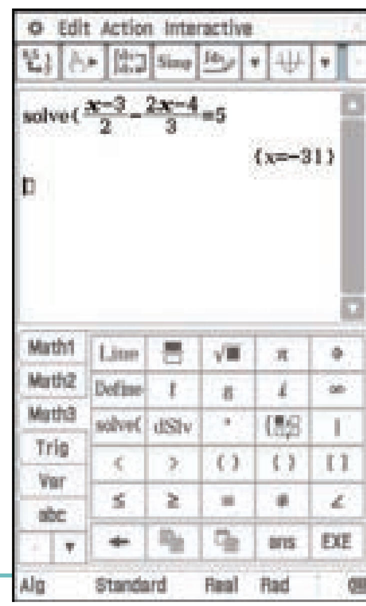
Note: A template for fractions may be obtained by pressing (ctrl) (÷).
For more details on the use of the calculator, refer to the TI-Nspire appendix in the Interactive Textbook.

Using the Casio ClassPad

- Go to the Main $\sqrt{\alpha}$ screen and turn on the keyboard.
- From either the (Math1) or the (Math3) keyboard, select (solve() and then select the fraction icon.
- Enter the equation

$$\frac{x-3}{2} - \frac{2x-4}{3} = 5$$

- Tap (EXE).



Note: The default setting is to solve for the variable x. To solve for a different variable, follow the equation by a comma and then the variable.
For more details on the use of the calculator, refer to the Casio ClassPad appendix in the Interactive Textbook.

Literal equations

An equation for the variable x in which all the coefficients of x, including the constants, are pronumerals is known as a **literal equation**.

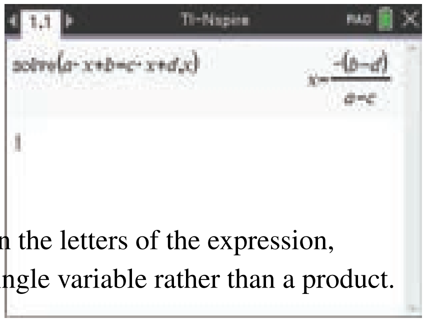
Example 6

Solve $ax + b = cx + d$ for x.

Solution	Explanation
$ax + b = cx + d$	
$ax - cx = d - b$	Collect terms in x on the left-hand side and constants on the right-hand side.
$(a - c)x = d - b$	Factorise the left-hand side.
$x = \dfrac{d-b}{a-c}$	

ISBN 978-1-009-11045-7

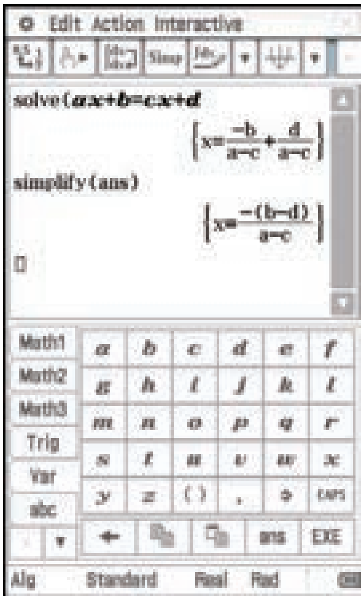
Using the TI-Nspire



- To solve the literal equation $ax + b = cx + d$, use a **Calculator** application.
- Select (menu) > **Algebra** > **Solve**.
- Enter $ax + b = cx + d$ as shown.
- Press (enter) to obtain the solution.

Note: Ensure a multiplication sign is placed between the letters of the expression, otherwise the calculator will read them as a single variable rather than a product. That is, enter $a \times x$ and not ax.

Using the Casio ClassPad



- To solve the literal equation $ax + b = cx + d$, first go to the Main $\sqrt{\alpha}$ screen and turn on the keyboard.
- Select [solve(] from the (Math1) or (Math3) keyboard.
- Now select the (Var) keyboard. This will bring up the variables. Enter $ax + b = cx + d$.
- Tap (EXE). (Recall that the default setting is to solve for the variable x.)
- To simplify the answer, tap [Simp] in the toolbar at the top of the screen.

Note: The variables x, y and z are found on the hard keyboard. Other variables should be entered using the (Var) keyboard.

The (abc) keyboard is used for typing text. If you use the (abc) keyboard for variables, then you must type $a \times x$, for example, because ax will be treated as a single variable.

Summary 1A

- An equation is solved by finding the value or values of the variables that would make the statement true.
- A linear equation is one in which the variable is to the first power.
- There are often several different ways to solve a linear equation. The following steps provide some suggestions:
  1. Expand brackets and, if the equation involves fractions, multiply through by the lowest common denominator of the terms.
  2. Group all of the terms containing a variable on one side of the equation and the terms without the variable on the other side.

ISBN 978-1-009-11045-7 Cambridge University Press

Exercise 1A

1 Solve each of the following equations for x:

a $x + 3 = 6$ **b** $x - 3 = 6$ **c** $3 - x = 2$ **d** $x + 6 = -2$

e $2 - x = -3$ **f** $2x = 4$ **g** $3x = 5$ **h** $-2x = 7$

i $-3x = -7$ **j** $\frac{3x}{4} = 5$ **k** $\frac{-3x}{5} = 2$ **l** $\frac{-5x}{7} = -2$

2 Solve each of the following literal equations for x:

a $x - b = a$ **b** $x + b = a$ **c** $ax = b$ **d** $\frac{x}{a} = b$ **e** $\frac{ax}{b} = c$

Example 1

3 Solve the following linear equations:

a $2y - 4 = 6$ **b** $3t + 2 = 17$ **c** $2y + 5 = 2$ **d** $7x - 9 = 5$

e $2a - 4 = 7$ **f** $3a + 6 = 14$ **g** $\frac{y}{8} - 11 = 6$ **h** $\frac{t}{3} + \frac{1}{6} = \frac{1}{2}$

i $\frac{x}{3} + 5 = 9$ **j** $3 - 5y = 12$ **k** $-3x - 7 = 14$ **l** $14 - 3y = 8$

Example 2

4 Solve the following linear equations:

a $6x - 4 = 3x$ **b** $x - 5 = 4x + 10$ **c** $3x - 2 = 8 - 2x$

5 Solve the following linear equations:

Example 3

a $2(y + 6) = 10$ **b** $2y + 6 = 3(y - 4)$ **c** $2(x + 4) = 7x + 2$

d $5(y - 3) = 2(2y + 4)$ **e** $x - 6 = 2(x - 3)$ **f** $\frac{y + 2}{3} = 4$

Example 4

g $\frac{x}{2} + \frac{x}{3} = 10$ **h** $x + 4 = \frac{3}{2}x$ **i** $\frac{7x + 3}{2} = \frac{9x - 8}{4}$

Example 5

j $\frac{2(1 - 2x)}{3} - 2x = -\frac{2}{5} + \frac{4(2 - 3x)}{3}$ **k** $\frac{4y - 5}{2} - \frac{2y - 1}{6} = y$

Example 6

6 Solve the following literal equations for x:

a $ax + b = 0$ **b** $cx + d = e$ **c** $a(x + b) = c$ **d** $ax + b = cx$

e $\frac{x}{a} + \frac{x}{b} = 1$ **f** $\frac{a}{x} + \frac{b}{x} = 1$ **g** $ax - b = cx - d$ **h** $\frac{ax + c}{b} = d$

7 Solve for x:

a $\frac{b - cx}{a} + \frac{a - cx}{b} + 2 = 0$ **b** $\frac{2}{x - a} + \frac{1}{x - b} = \frac{3}{x}$

CAS

8 Solve each of the following for x:

a $0.2x + 6 = 2.4$ **b** $0.6(2.8 - x) = 48.6$ **c** $\frac{2x + 12}{7} = 6.5$

d $0.5x - 4 = 10$ **e** $\frac{1}{4}(x - 10) = 6$ **f** $6.4x + 2 = 3.2 - 4x$

9 Solve for x:

a $\frac{a}{x + a} + \frac{b}{x - b} = \frac{a + b}{x + c}$ **b** $\frac{bx}{1 + bx} + \frac{x}{1 + x} = 2$

ISBN 978-1-009-11045-7 Cambridge University Press

1B Constructing linear equations

As stated earlier, many problems can be solved by translating them into mathematical language and using an appropriate mathematical technique to find the solution. By representing the unknown quantity in a problem with a symbol and constructing an equation from the information, the value of the unknown can be found by solving the equation.

Before constructing the equation, each symbol and what it stands for (including the units) should be stated. It is essential to remember that all the elements of the equation must be in units of the same system.

Example 7

A chef uses the following rule for cooking a turkey:
'Allow 30 minutes for each kilogram weight of turkey and then add an extra 15 minutes.'

If the chef forgot to weigh a turkey before cooking it, but knew that it had taken 3 hours to cook, calculate how much it weighed.

Solution

Let the weight of the turkey be x kilograms.
Then the time taken is $(30x + 15)$ minutes.

$$\begin{aligned} \therefore \quad 30x + 15 &= 180 \\ 30x &= 165 \\ x &= 5.5 \end{aligned}$$

The turkey weighed 5.5 kilograms.

Explanation

Assign a variable to the quantity that is to be found. In this example, the weight of the turkey is x kilograms.

Find, in terms of x, the time to cook the turkey. Then form the equation. Note that 3 hours is 180 minutes.

State the solution to the problem in words.

Example 8

Find the area of a rectangle whose perimeter is 1.08 m, if it is 8 cm longer than it is wide.

Solution

Let length = ℓ cm.
Then width = $(\ell - 8)$ cm.

$$\begin{aligned} \text{Perimeter} &= 2 \times \text{length} + 2 \times \text{width} \\ &= 2\ell + 2(\ell - 8) \\ &= 4\ell - 16 \text{ cm} \end{aligned}$$

$$\text{Perimeter} = 108 \text{ cm}$$

$$\begin{aligned} \therefore \quad 4\ell - 16 &= 108 \\ 4\ell &= 124 \\ \ell &= 31 \text{ cm} \end{aligned}$$

The length is 31 cm and the width is 23 cm.

Therefore the area is $31 \times 23 = 713$ cm^2.

Explanation

We know that

$$\text{Perimeter} = 2 \times \text{length} + 2 \times \text{width}$$

and that the width is 8 cm less than the length. Let ℓ cm be the length. Then the width is $(\ell - 8)$ cm.

Find the perimeter in terms of ℓ. Find the length and width, and hence find the area.

ISBN 978-1-009-11045-7 Cambridge University Press

Example 9

Adam normally takes 5 hours to travel between Higett and Logett. One day he increases his speed by 4 km/h and finds the journey from Higett to Logett takes half an hour less than the normal time. Find his normal speed.

Solution

Let x km/h be his normal speed.

The distance from Higett to Logett is $x \times 5 = 5x$ kilometres.

Adam's new speed is $(x + 4)$ km/h.

Hence
$$\begin{aligned}(x + 4) \times \frac{9}{2} &= 5x \\ 9(x + 4) &= 10x \\ 9x + 36 &= 10x \\ 36 &= x\end{aligned}$$

His normal speed is 36 km/h.

Explanation

In problems such as this, the speed is the average speed.

We note that

distance = speed × time

Summary 1B

Steps for solving a word problem with a linear equation

- Read the question carefully and write down the known information clearly.
- Identify the unknown quantity that is to be found.
- Assign a variable to this quantity.
- Form an expression in terms of x (or the variable being used) and use the other relevant information to form the equation.
- Solve the equation.
- Write a sentence answering the initial question.

Exercise 1B

1 For each of the following, write an equation using the variable x, then solve the equation for x:

a A number plus two is equal to six.

b A number multiplied by three is equal to ten.

c Six is added to a number multiplied by three and the result is twenty-two.

d Five is subtracted from a number multiplied by three and the result is fifteen.

e Three is added to a number. If the result of this is multiplied by six, then fifty-six is obtained.

f Five is added to a number and the result divided by four gives twenty-three.

ISBN 978-1-009-11045-7 Cambridge University Press

2 \$48 is divided among three students, A, B and C. If B receives three times as much as A, and C receives twice as much as A, how much does each receive?

3 The sum of two numbers is 42, and one number is twice the other. Find the two numbers.

Example 7 **4** A chef uses the following rule for cooking food on a spit: ‘Allow 20 minutes for each kilogram weight and then add an extra 20 minutes.’ If the chef forgot to weigh the food before cooking it but knew that it had taken 3 hours to cook, calculate how much it weighed.

Example 8 **5** Find the area of a rectangle whose perimeter is 4.8 m, if it is 0.5 m longer than it is wide.

6 Find three consecutive whole numbers with a sum of 150.

7 Find four consecutive odd numbers with a sum of 80.

8 Two tanks contain equal amounts of water. They are connected by a pipe and 3000 litres of water is pumped from one tank to the other. One tank then contains 6 times as much water as the other. How many litres of water did each tank contain originally?

9 A 120-page book has p lines to a page. If the number of lines were reduced by three on each page, the number of pages would need to be increased by 20 to give the same amount of writing space. How many lines were there on each page originally?

Example 9 **10** A rower travels upstream at 6 km/h and back to the starting place at 10 km/h. The total journey takes 48 minutes. How far upstream did the rower go?

11 A shopkeeper buys a crate of eggs at \$1.50 per dozen. He buys another crate, containing 3 dozen more than the first crate, at \$2.00 per dozen. He sells them all for \$2.50 a dozen and makes \$15 profit. How many dozens were there in each of the crates?

Example 9 **12** Jess walked for 45 minutes at 3 km/h and then ran for half an hour at x km/h. At the end of that time she was 6 km from the starting point. Find the value of x.

13 A man travels from A to B at 4 km/h and from B to A at 6 km/h. The total journey takes 45 minutes. Find the distance travelled. CAS

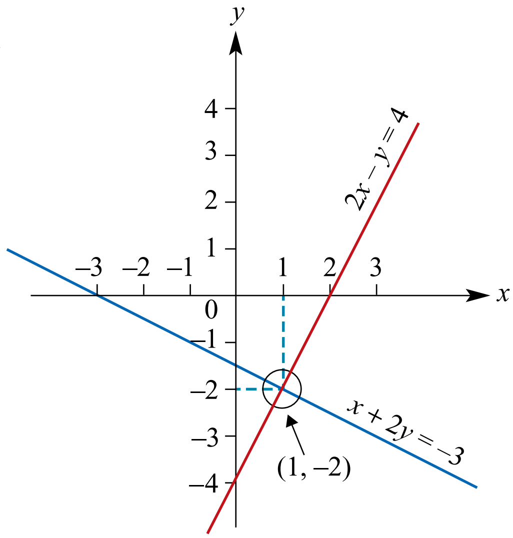
14 A boy is 24 years younger than his father. In two years’ time the sum of their ages will be 40. Find the present ages of father and son.

ISBN 978-1-009-11045-7 Cambridge University Press

1C Simultaneous equations

A linear equation that contains two unknowns, e.g. $2y + 3x = 10$, does not have a single solution. Such an equation actually expresses a relationship between pairs of numbers, x and y, that satisfy the equation. If all possible pairs of numbers (x, y) that satisfy the equation are represented graphically, the result is a straight line; hence the name **linear relation**.

If the graphs of two such equations are drawn on the same set of axes, and they are non-parallel, the lines will intersect at one point only. Hence there is one pair of numbers that will satisfy both equations simultaneously.



The intersection point of two straight lines can be found graphically; however, the accuracy of the solution will depend on the accuracy of the graphs.

Alternatively, the intersection point may be found algebraically by solving the pair of simultaneous equations. We shall consider two techniques for solving simultaneous equations.

Example 10

Solve the equations $2x - y = 4$ and $x + 2y = -3$.

Solution	Explanation
Method 1: Substitution	
$2x - y = 4$ (1) $x + 2y = -3$ (2)	Using one of the two equations, express one variable in terms of the other variable.
From equation (2), we get $x = -3 - 2y$.	
Substitute in equation (1): $2(-3 - 2y) - y = 4$ $-6 - 4y - y = 4$ $-5y = 10$ $y = -2$	Then substitute this expression into the other equation (reducing it to an equation in one variable, y). Solve the equation for y.
Substitute the value of y into (2): $x + 2(-2) = -3$ $x = 1$	Substitute this value for y in one of the equations to find the other variable, x.
Check in (1): LHS $= 2(1) - (-2) = 4$ RHS $= 4$	A check can be carried out with the other equation.

ISBN 978-1-009-11045-7 Cambridge University Press

Method 2: Elimination

$$2x - y = 4 \qquad (1)$$
$$x + 2y = -3 \qquad (2)$$

To eliminate x, multiply equation (2) by 2 and subtract the result from equation (1).

When we multiply equation (2) by 2, the pair of equations becomes:

$$2x - y = 4 \qquad (1)$$
$$2x + 4y = -6 \qquad (2')$$

Subtract (2′) from (1):

$$-5y = 10$$
$$y = -2$$

Now substitute for y in equation (2) to find x, and check as in the substitution method.

If one of the variables has the same coefficient in the two equations, we can eliminate that variable by subtracting one equation from the other.

It may be necessary to multiply one of the equations by a constant to make the coefficients of x or y the same in the two equations.

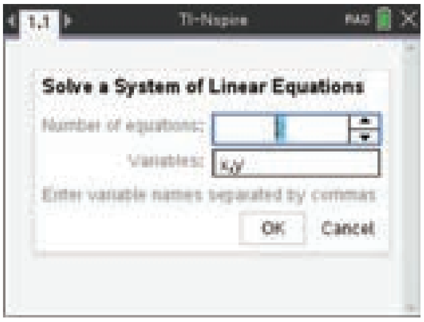
Note: This example shows that the point $(1, -2)$ is the point of intersection of the graphs of the two linear relations.

Using the TI-Nspire

Method 1: Using a Calculator application

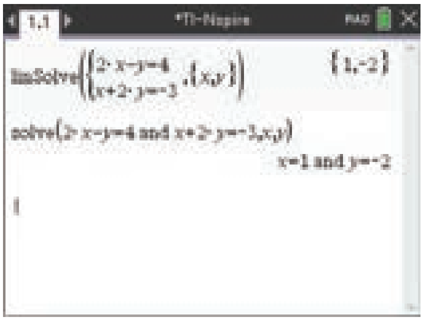
Simultaneous linear equations can be solved in a **Calculator** application.

- Use (menu) > **Algebra** > **Solve System of Equations** > **Solve System of Linear Equations**.
- Complete the pop-up screen.



- Enter the equations as shown to give the solution to the simultaneous equations $2x - y = 4$ and $x + 2y = -3$.
- Hence the solution is $x = 1$ and $y = -2$.

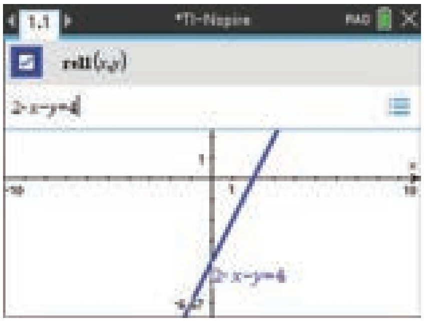
Note: The solution can also be found with solve($2x - y = 4$ and $x + 2y = -3, x, y$).



ISBN 978-1-009-11045-7 Cambridge University Press

Method 2: Using a Graphs application

Simultaneous linear equations can also be solved graphically in a **Graphs** application.



Entering the equations:

- The equations can be entered directly in the form $a \cdot x + b \cdot y = c$ using (menu) > **Graph Entry/Edit** > **Relation**.
- Enter the equations as shown.

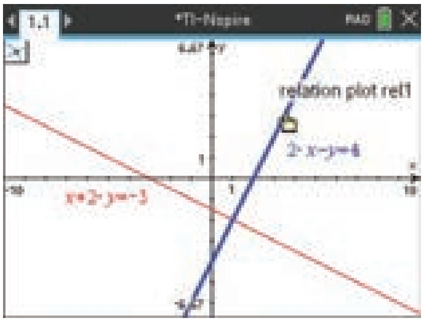
Note: Use ▼ to enter the second equation.

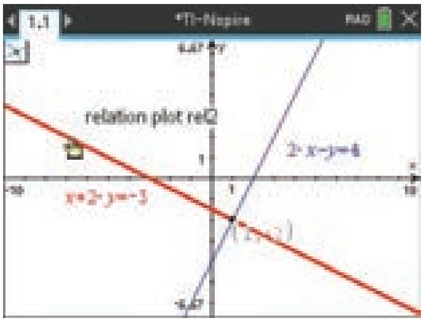
If the entry line is not visible, press (tab) or double click in an open area. Pressing (enter) will hide the entry line.

Equations of the form $a \cdot x + b \cdot y = c$ can also be entered using (menu) > **Graph Entry/Edit** > **Equation Templates** > **Line** > **Line Standard** $\boldsymbol{a \cdot x + b \cdot y = c}$.

Finding the intersection point:

- Use (menu) > **Geometry** > **Points & Lines** > **Intersection Point(s)**.
- Use the touchpad to move the cursor to select each of the two graphs.



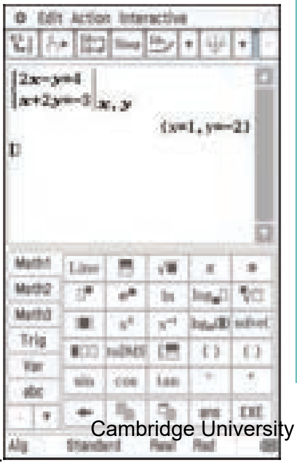


- The intersection point's coordinates will appear on the screen. Press (esc) to exit the **Intersection Point(s)** tool.

Note: Alternatively, you can find the intersection point using (menu) > **Analyze Graph** > **Intersection**.

Using the Casio ClassPad

To solve the simultaneous equations algebraically:

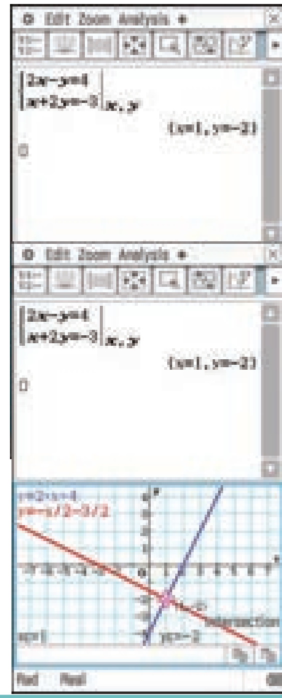


- Go to the Main $\sqrt{\alpha}$ screen and turn on the keyboard.
- Open the (Math1) keyboard and select the simultaneous equations icon.
- Enter the two equations $2x - y = 4$ and $x + 2y = -3$ into the two lines.
- Type x, y in the bottom-right square to indicate the variables.
- Tap (EXE).

ISBN 978-1-009-11045-7

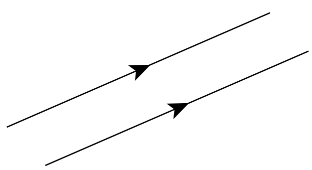
The simultaneous equations can also be solved graphically:

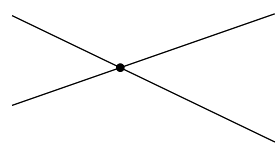
- Tap on the graph icon [icon] to display the graph window.
- Using your stylus, highlight the first equation $2x - y = 4$ and drag it down into the graph window. Lift the stylus off the screen for the graph to appear.
- Repeat by highlighting the second equation $x + 2y = -3$ and dragging it down into the graph window. Lift the stylus off the screen for the second graph to appear.
- To find the solution, tap into the graph window to select it, and then select **Analysis** > **G-Solve** > **Intersection**.



The geometry of simultaneous equations

Two distinct straight lines are either parallel or meet at a point.





There are three cases for a system of two linear equations with two variables.

	Example	Solutions	Geometry
Case 1	$2x + y = 5$ $x - y = 4$	Unique solution: $x = 3, \; y = -1$	Two lines meeting at a point
Case 2	$2x + y = 5$ $2x + y = 7$	No solutions	Distinct parallel lines
Case 3	$2x + y = 5$ $4x + 2y = 10$	Infinitely many solutions	Two copies of the same line

This is further discussed in Chapter 2.

ISBN 978-1-009-11045-7 Cambridge University Press

Summary 1C

We have two methods for solving simultaneous linear equations in two variables by hand.

1 Substitution

- Make one of the variables the subject in one of the equations.
- Substitute for that variable in the other equation.

2 Elimination

- Choose one of the two variables to eliminate.
- Obtain the same or opposite coefficients for this variable in the two equations. To do this, multiply both sides of one or both equations by a number.
- Add or subtract the two equations to eliminate the chosen variable.

Exercise 1C

1 Solve each of the following pairs of simultaneous equations by the substitution method:

a $y = 2x + 1$
$y = 3x + 2$

b $y = 5x - 4$
$y = 3x + 6$

c $y = 2 - 3x$
$y = 5x + 10$

d $y - 4 = 3x$
$y - 5x + 6 = 0$

e $y - 4x = 3$
$2y - 5x + 6 = 0$

f $y - 4x = 6$
$2y - 3x = 4$

2 Solve each of the following pairs of simultaneous equations by the elimination method:

a $x + y = 6$
$x - y = 10$

b $y - x = 5$
$x + y = 3$

c $x - 2y = 6$
$x + 6y = 10$

Example 10

3 Solve each of the following pairs of simultaneous linear equations by either the substitution or the elimination method:

a $2x - 3y = 7$
$y = 5 - 3x$

b $2x - 5y = 10$
$4x + 3y = 7$

c $2m - 1 = n$
$2n + m = 8$

d $7x - 6y = 20$
$3x + 4y = 2$

e $3s - 1 = t$
$5s + 2t = 20$

f $4x - 3y = 1$
$4y - 5x = 2$

4 For each of the following pairs of simultaneous linear equations, state whether there is one, none or infinitely many solutions:

a $3x + y = 6$
$6x + 2y = 7$

b $3x + y = 6$
$6x + 2y = 12$

c $3x + y = 6$
$6x - 2y = 7$

d $3x - y = 6$
$6x + 2y = 7$

5 Solve each of the following pairs of simultaneous linear equations:

CAS

a $15x - 4y = 6$
$9x - 2y = 5$

b $2p + 5q = -3$
$7p - 2q = 9$

c $2x - 4y = -12$
$2y + 3x - 2 = 0$

ISBN 978-1-009-11045-7

Cambridge University Press

1D Constructing simultaneous linear equations

Problems involving two unknowns can often be solved by using simultaneous equations with two variables. The following examples show how this may be done.

Example 11

The sum of two numbers is 24 and their difference is 96. Find the two numbers.

Solution

Let x and y be the two numbers. Then

$x + y = 24$ (1)

$x - y = 96$ (2)

Add equations (1) and (2):

$2x = 120$

$x = 60$

Substitute in equation (1):

$60 + y = 24$

$y = -36$

The two numbers are 60 and −36.

Check in (2): $60 - (-36) = 96$

Explanation

The problem can also be solved by eliminating x. Subtracting (2) from (1) gives $2y = -72$ and hence $y = -36$.

The problem can also be solved by substitution. From (1), we have $y = 24 - x$. Substitute in (2).

The values found for x and y have to make each of the equations true. The equation which has not been used in the final substitution is the one to use for the check.

Example 12

3 kg of jam and 2 kg of butter cost \$29, and 6 kg of jam and 3 kg of butter cost \$54.

Find the cost per kilogram of jam and butter.

Solution

Let the cost of 1 kg of jam be x dollars and the cost of 1 kg of butter be y dollars.

Then $3x + 2y = 29$ (1)

and $6x + 3y = 54$ (2)

Multiply (1) by 2: $6x + 4y = 58$ (1′)

Subtract (1′) from (2): $-y = -4$

$y = 4$

Substitute in (2): $6x + 3(4) = 54$

$6x = 42$

$x = 7$

Jam costs \$7 per kg and butter \$4 per kg.

Explanation

The unknowns are the cost per kilogram of jam and the cost per kilogram of butter.

Three kilograms of jam and two kilograms of butter cost \$29.

Six kilograms of jam and three kilograms of butter cost \$54.

Check in the original problem:

3 kg of jam = \$21 and 2 kg of butter = \$8
Total = \$29

6 kg of jam = \$42 and 3 kg of butter = \$12
Total = \$54

ISBN 978-1-009-11045-7

Summary 1D

Steps for solving a word problem with simultaneous linear equations

- Read the question carefully and write down the known information clearly.
- Identify the two unknown quantities that are to be found.
- Assign variables to these two quantities.
- Form expressions in terms of x and y (or other suitable variables) and use the other relevant information to form the two equations.
- Solve the system of equations.
- Write a sentence answering the initial question.

Skill-sheet

Exercise 1D

1 Find two numbers whose sum is 138 and whose difference is 88.

2 Find two numbers whose sum is 36 and whose difference is 9.

3 Six stools and four chairs cost \$58, while five stools and two chairs cost \$35.
- **a** How much do ten stools and four chairs cost?
- **b** How much do four stools cost?
- **c** How much does one stool cost?

4 A belt and a wallet cost \$42, while seven belts and four wallets cost \$213.
- **a** How much do four belts and four wallets cost?
- **b** How much do three belts cost?
- **c** How much does one belt cost?

Use simultaneous equations to solve the following.

Example 11

5 Find a pair of numbers whose sum is 45 and whose difference is 11.

6 In four years' time a mother will be three times as old as her son. Four years ago she was five times as old as her son. Find their present ages.

7 A party was organised for thirty people at which they could have either a hamburger or a pizza. If there were five times as many hamburgers as pizzas, calculate the number of each.

8 Two children had 110 marbles between them. After one child had lost half her marbles and the other had lost 20 they had an equal number. How many marbles did each child start with and how many did they finish with?

ISBN 978-1-009-11045-7

Cambridge University Press

9 One hundred and fifty tickets were sold for a basketball match and \$560 was the total amount collected. Adult tickets were sold at \$4.00 each and child tickets were sold at \$1.50 each. How many adult tickets and how many child tickets were sold?

10 The sum of the numerator and denominator of a fraction expressed in simplest form is 17. If 3 is added to the numerator, the value of the fraction will be 1. What is the fraction?

11 Linda thinks of a two-digit number. The sum of the digits is 8. If she reverses the digits, the new number is 36 greater than her original number. What was Linda's original number?

12 Tickets to a musical cost \$30 for adults and \$12 for children. At one particular performance 960 people attended and \$19 080 was collected in ticket sales. Find the number of adults and the number of children who attended the performance.

13 An investor received \$1400 interest per annum from a sum of money, with part of it invested at 10% and the remainder at 7% simple interest. This investor found that if she interchanged the amounts she had invested she could increase her return by \$90 per annum. Calculate the total amount invested.

Example 12
14 A shopkeeper sold his entire stock of shirts and ties in a sale for \$10 000. The shirts were priced at 3 for \$100 and the ties \$20 each. If he had sold only half the shirts and two-thirds of the ties he would have received \$6000. How many of each did he sell in the sale?

15 A tent manufacturer produces two models, the Outback and the Bush Walker. From earlier sales records, it is known that 20% more of the Outback model is sold than the Bush Walker. A profit of \$200 is made on each Outback sold, but \$350 is made on each Bush Walker. If during the next year a profit of \$177 000 is planned, how many of each model must be sold?

CAS

16 Oz Jeans has factories in Mydney and Selbourne. At the Mydney factory, fixed costs are \$28 000 per month and the cost of producing each pair of jeans is \$30. At the Selbourne factory, fixed costs are \$35 200 per month and the cost of producing each pair of jeans is \$24. During the next month Oz Jeans must manufacture 6000 pairs of jeans. Calculate the production order for each factory, if the total manufacturing costs for each factory are to be the same.

17 A tea wholesaler blends together three types of tea that normally sell for \$10, \$11 and \$12 per kilogram so as to obtain 100 kilograms of tea worth \$11.20 per kilogram. If the same amounts of the two higher priced teas are used, calculate how much of each type must be used in the blend.

ISBN 978-1-009-11045-7

1E Solving linear inequalities

An **inequality** is a mathematical statement that contains an inequality symbol rather than an equals sign: for example, $2x + 1 < 4$. When you solve the inequality $2x + 1 < 4$, you answer the question:

'Which numbers x satisfy the property that $2x + 1$ is less than 4?'

You will find that your answers can be described using a number line. This is a good way to represent the solution, as there are infinitely many numbers that satisfy an inequality such as $2x + 1 < 4$. For example:

$$2(1) + 1 = 3 < 4, \quad 2(0) + 1 = 1 < 4, \quad 2\left(\frac{1}{2}\right) + 1 = 2 < 4, \quad 2(-1) + 1 = -1 < 4$$

To solve linear inequalities, proceed exactly as for equations with the following exception:

- When multiplying or dividing both sides by a negative number, the 'direction' of the inequality symbol is reversed.

Example 13

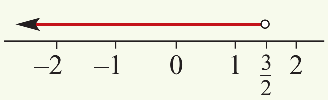
Solve the inequality $2x + 1 < 4$.

Solution

$$2x + 1 < 4$$

$$2x < 3$$

$$x < \frac{3}{2}$$



Explanation

Subtract 1 from both sides.

Divide both sides by 2.

The solution can be represented on a real number line.

Note: In a number-line diagram, the 'endpoint' of an interval is indicated with a closed circle if the point is included and with an open circle if it is not.

Example 14

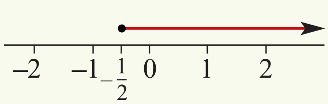
Solve the inequality $3 - 2x \le 4$.

Solution

$$3 - 2x \le 4$$

$$-2x \le 1$$

$$x \ge -\frac{1}{2}$$



Explanation

Subtract 3 from both sides.

Divide both sides by -2. Note that the inequality symbol is reversed.

ISBN 978-1-009-11045-7

Example 15

Solve the inequality $\dfrac{2x+3}{5} > \dfrac{3-4x}{3} + 2$.

Solution

$$\frac{2x+3}{5} > \frac{3-4x}{3} + 2$$

$$3(2x+3) > 5(3-4x) + 30$$

$$3(2x+3) - 5(3-4x) > 30$$

$$6x + 9 - 15 + 20x > 30$$

$$26x - 6 > 30$$

$$x > \frac{36}{26}$$

$$\therefore \quad x > \frac{18}{13}$$

Explanation

Multiply both sides by 15, the lowest common denominator of 5 and 3.

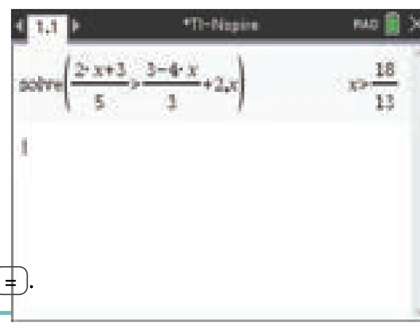
Collect the terms containing x on the left-hand side of the inequality.

Using the TI-Nspire

The inequality can be solved in a **Calculator** application.

- Choose **solve( )** from the **Algebra** menu to give the solution to

$$\frac{2x+3}{5} > \frac{3-4x}{3} + 2$$



Note: For the inequality signs template, press ctrl =.

Using the Casio ClassPad

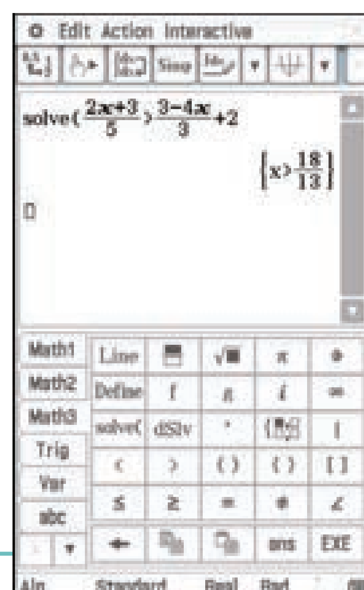
To solve the inequality:

- Go to the Main $\sqrt{\alpha}$ screen and turn on the keyboard.
- Select solve(from the Math3 keyboard.
- Enter the inequality

$$\frac{2x+3}{5} > \frac{3-4x}{3} + 2$$

Note: The fraction icon and the inequality signs (<, >, ≤, ≥) are also found in the Math3 keyboard.

- Tap EXE. (Recall that the default setting is to solve for the variable x.)



ISBN 978-1-009-11045-7

Summary 1E

- We can add or subtract the same number on both sides of an inequality, and the resulting inequality is equivalent to the original.
- We can multiply or divide both sides of an inequality by a positive number, and the resulting inequality is equivalent to the original.
- If we multiply or divide both sides of an inequality by a negative number, then we must reverse the inequality sign so that the resulting inequality is equivalent.

Skill-sheet

Exercise 1E

Example 13
Example 14

1 Solve each of the following inequalities for x:

a $x + 3 < 4$ **b** $x - 5 > 8$ **c** $2x \geq 6$

d $\frac{x}{3} \leq 4$ **e** $-x \geq 6$ **f** $-2x < -6$

g $6 - 2x > 10$ **h** $\frac{-3x}{4} \leq 6$ **i** $4x - 4 \leq 2$

Example 14
Example 15

2 Solve for x in each of the following and show the solutions on a real number line:

a $4x + 3 < 11$ **b** $3x + 5 < x + 3$ **c** $\frac{1}{2}(x + 1) - x > 1$

d $\frac{1}{6}(x + 3) \geq 1$ **e** $\frac{2}{3}(2x - 5) < 2$ **f** $\frac{3x - 1}{4} - \frac{2x + 3}{2} < -2$

g $\frac{4x - 3}{2} - \frac{3x - 3}{3} < 3$ **h** $\frac{1 - 7x}{-2} \geq 10$ **i** $\frac{5x - 2}{3} - \frac{2 - x}{3} > -1$

3 **a** For which real numbers x is $2x + 1$ a positive number?
b For which real numbers x is $100 - 50x$ a positive number?
c For which real numbers x is $100 + 20x$ a positive number?

4 In a certain country it costs \$1 to send a letter weighing less than 20 g. A sheet of paper weighs 3 g. Write a suitable inequality and hence state the maximum number of pages that can be sent for \$1. (Ignore the weight of the envelope in this question.)

5 A student receives marks of 66 and 72 on two tests. What is the lowest mark she can obtain on a third test to have an average for the three tests greater than or equal to 75?

6 Solve each of the following inequalities for x:

CAS

a $\frac{3x + 2}{3} + \frac{5x}{2} \geq 8$ **b** $\frac{3x + 2}{3} - \frac{5x}{2} \geq -8$

c $\frac{3ax + 2}{3} - \frac{5x}{2} \geq -8$ given that $a > \frac{5}{2}$ **d** $\frac{3ax + 2}{3} - \frac{5ax}{2} \geq -8$ given that $a > 0$

ISBN 978-1-009-11045-7 Cambridge University Press

1F Using and transposing formulas

An equation containing symbols that states a relationship between two or more quantities is called a **formula**. An example of a formula is $A = \ell w$ (area = length × width). The value of A, called the subject of the formula, can be found by substituting in given values of ℓ and w.

Example 16

Find the area of a rectangle with length (ℓ) 10 cm and width (w) 4 cm.

Solution	Explanation
$A = \ell w$	
$A = 10 \times 4$	Substitute $\ell = 10$ and $w = 4$.
$A = 40\ \text{cm}^2$	

Sometimes we wish to rewrite a formula to make a different symbol the subject of the formula. This process is called **transposing** the formula. The techniques for transposing formulas include those used for solving linear equations detailed in Section 1A.

Example 17

Transpose the formula $v = u + at$ to make a the subject.

Solution	Explanation
$v = u + at$	
$v - u = at$	Subtract u from both sides.
$\dfrac{v-u}{t} = a$	Divide both sides by t.

If we wish to evaluate an unknown that is not the subject of the formula, we can either substitute the given values for the other variables and then solve the resulting equation, or we can first transpose the formula and then substitute the given values.

Example 18

Evaluate p if $2(p + q) - r = z$, and $q = 2$, $r = -3$ and $z = 11$.

Solution	Explanation
Method 1: Substitute then solve	
$2(p + 2) - (-3) = 11$	First substitute $q = 2$, $r = -3$ and $z = 11$.
$2p + 4 + 3 = 11$	Then solve for p.
$2p = 4$	
$p = 2$	

ISBN 978-1-009-11045-7 Cambridge University Press

Method 2: Transpose then substitute

$$2(p+q) - r = z$$
First solve for p.
$$2(p+q) = z + r$$
$$p + q = \frac{z+r}{2}$$
$$p = \frac{z+r}{2} - q$$
Substitute $q = 2$, $r = -3$ and $z = 11$.
$$\therefore \quad p = \frac{11 + (-3)}{2} - 2$$
$$p = 2$$

Example 19

A path x metres wide surrounds a rectangular lawn. The lawn is ℓ metres long and b metres wide. The total area of the path is A m^2.

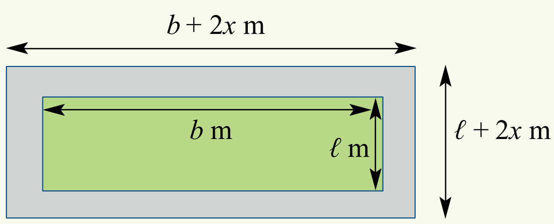
a Find A in terms of ℓ, b and x.

b Find b in terms of ℓ, A and x.

c Find the value of b if $\ell = 6$, $A = 72$ and $x = 1.5$.

Solution

a



The area of the path is

$$A = (b + 2x)(\ell + 2x) - b\ell$$
$$= b\ell + 2x\ell + 2xb + 4x^2 - b\ell$$
$$\therefore \quad A = 2x\ell + 2xb + 4x^2$$

b $A - (2x\ell + 4x^2) = 2xb$

Therefore

$$b = \frac{A - (2x\ell + 4x^2)}{2x}$$

c Substitute the given values into the expression for b:

$$b = \frac{A - (2x\ell + 4x^2)}{2x}$$
$$= \frac{72 - (2(1.5)(6) + 4(1.5)^2)}{2(1.5)}$$
$$= \frac{72 - (18 + 9)}{3}$$
$$\therefore \quad b = 15$$

ISBN 978-1-009-11045-7

Example 20

For each of the following, make c the subject of the formula:

a $e = \sqrt{3c - 7a}$

b $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{c - 2}$

Solution

a $e = \sqrt{3c - 7a}$

Square both sides of the equation:

$$e^2 = 3c - 7a$$

Therefore

$$3c = e^2 + 7a$$

$$c = \frac{e^2 + 7a}{3}$$

b $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{c - 2}$

Establish common denominator on the left-hand side of the equation:

$$\frac{b - a}{ab} = \frac{1}{c - 2}$$

Take the reciprocal of both sides:

$$\frac{ab}{b - a} = c - 2$$

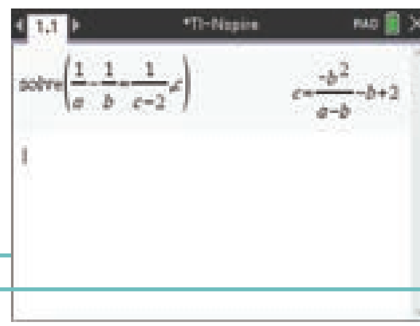
Therefore $c = \dfrac{ab}{b - a} + 2$

Using the TI-Nspire

Literal equations can be solved for a given variable in a **Calculator** application.

- Use **solve( )** from the **Algebra** menu to make c the subject of the formula

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{c - 2}$$



Using the Casio ClassPad

To solve a literal equation for a given variable:

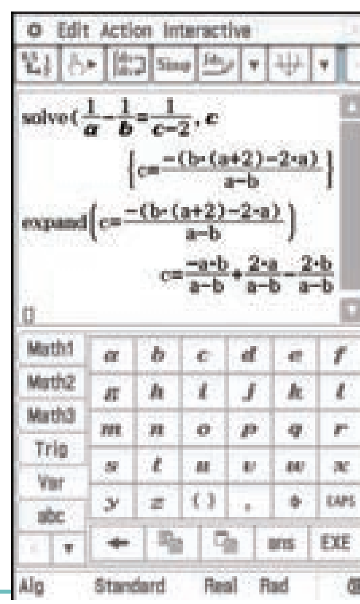
- Go to the Main $\sqrt{\alpha}$ screen and turn on the keyboard.
- Select solve(from the Math1 or Math3 keyboard.
- Enter the equation

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{c - 2}$$

- To solve for c, follow the equation by ', c'.

Note: Select the variables from the Var keyboard.
The comma is also found in the Var keyboard.

- Tap EXE.
- To produce a neater answer, copy the solution, paste it into the next line, highlight it and then use **Interactive** > **Transformation** > **expand**.



ISBN 978-1-009-11045-7

Cambridge University Press

Summary 1F

- A formula relates different quantities: for example, the formula $A = \pi r^2$ relates the radius r with the area A of the circle.
- The variable on the left is called the subject of the formula: for example, in the formula $A = \pi r^2$, the subject is A.
- To calculate the value of a variable which is not the subject of a formula:
  Method 1 Substitute the values for the known variables, then solve the resulting equation for the unknown variable.
  Method 2 Rearrange to make the required variable the subject, then substitute values.

Skill-sheet

Exercise 1F

1 For each of the following, find the value of the letter in parentheses:

a $c = ab,\ a = 6,\ b = 3 \quad (c)$
b $r = p + q,\ p = 12,\ q = -3 \quad (r)$
c $c = ab,\ a = 6,\ c = 18 \quad (b)$
d $r = p + q,\ p = 15,\ r = -3 \quad (q)$
e $c = \sqrt{a},\ a = 9 \quad (c)$
f $c = \sqrt{a},\ c = 9 \quad (a)$
g $p = \frac{u}{v},\ u = 10,\ v = 2 \quad (p)$
h $p = \frac{u}{v},\ p = 10,\ v = 2 \quad (u)$

2 For each of the following, construct a formula using the given symbols:

a S, the sum of three numbers a, b and c
b P, the product of two numbers x and y
c the cost, \$$C$, of five books which each cost \$$p$
d the total cost, \$$T$, of d chairs which cost \$$p$ each and c tables which cost \$$q$ each
e the time, T, in minutes, of a train journey that takes a hours and b minutes

Example 16

3 Find the values of the following:

a $E = IR$, when $I = 5$ and $R = 3$
b $C = pd$, when $p = 3.14$ and $d = 10$
c $P = \frac{RT}{V}$, when $R = 60$, $T = 150$ and $V = 9$
d $I = \frac{E}{R}$, when $E = 240$ and $R = 20$
e $A = \pi r\ell$, when $\pi = 3.14$, $r = 5$ and $\ell = 20$
f $S = 90(2n - 4)$, when $n = 6$

Example 17

4 For each of the following, make the symbol indicated the subject of the formula:

a $PV = c;\ V$
b $F = ma;\ a$
c $I = Prt;\ P$
d $w = H + Cr;\ r$
e $S = P(1 + rt);\ t$
f $V = \frac{2R}{R - r};\ r$

Example 18

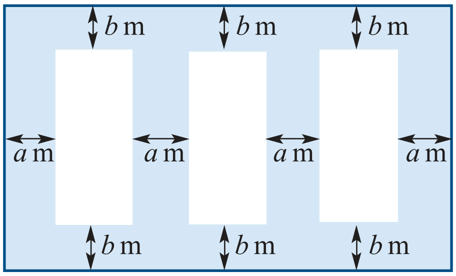
5 Find the value of the unknown symbol in each of the following:

a $D = \frac{T + 2}{P}$, when $D = 10$, $P = 5$
b $A = \frac{1}{2}bh$, when $A = 40$, $h = 10$
c $V = \frac{1}{3}\pi r^2 h$, when $\pi = 3.14$, $V = 100$, $r = 5$
d $A = \frac{1}{2}h(a + b)$, when $A = 50$, $h = 5$, $a = 10$

ISBN 978-1-009-11045-7

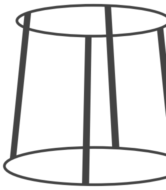
Cambridge University Press

6 The diagram represents the brick wall of a dwelling with three windows. Each of the windows is h m high and w m wide. Other dimensions are as shown.



a Find the length of the wall.
b Find the height of the wall.
c Find the total area of the three windows.
d Find the total area of brickwork.

7 A lampshade has a metal frame consisting of two circular hoops of radii p cm and q cm joined by four straight struts of length h cm. The total length of metal is T cm.



a **i** Find an expression for T in terms of p, q and h.
ii Find T when $p = 20$, $q = 24$ and $h = 28$.
b The area of the material covering the frame is A cm^2, where $A = \pi h(p + q)$. Find an expression for p in terms of A, h, q and π.

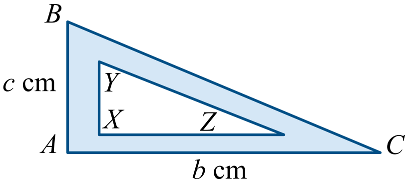
8 Find the value of the unknown symbol in each of the following:

a $P = \dfrac{T - M}{D}$, $P = 6$, $T = 8$, $M = 4$
b $H = \dfrac{a}{3} + \dfrac{a}{b}$, $H = 5$ and $a = 6$
c $a = \dfrac{90(2n - 4)}{n}$, $a = 6$
d $R = \dfrac{r}{a} + \dfrac{r}{3}$, $a = 2$ and $R = 4$

Example 19

9 Right-angled triangles XYZ and ABC are similar.

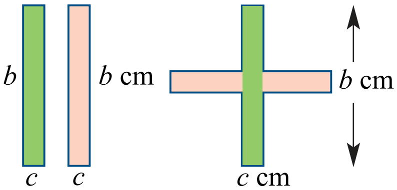
$$\frac{XY}{AB} = \frac{YZ}{BC} = \frac{ZX}{CA} = k$$



If $AB = c$ cm and $AC = b$ cm, find:

a the area, D cm^2, of the shaded region in terms of c, b and k
b k in terms of D, b and c
c the value of k if $D = 2$, $b = 3$ and $c = 4$.

10 Two rectangles each with dimensions c cm × b cm are used to form a cross as shown. The arms of the cross are all of equal length.



a Find the perimeter, P cm, of the cross in terms of b and c.
b Find the area, A cm^2, of the cross in terms of b and c.
c Find b in terms of A and c.

Example 20

11 For each of the following, make the symbol in brackets the subject of the formula:

a $a = \sqrt{a + 2b}$ (b)
b $\dfrac{a + x}{a - x} = \dfrac{b - y}{b + y}$ (x)
c $px = \sqrt{3q - r^2}$ (r)
d $\dfrac{x}{y} = \sqrt{1 - \dfrac{v^2}{u^2}}$ (v)

CAS

ISBN 978-1-009-11045-7 Cambridge University Press

Chapter summary

Assign-ment

Nrich

- A **linear equation** is one in which the variable is to the first power.
- It is often helpful to look at how the equation has been constructed so that the steps necessary to 'undo' the equation can be identified. It is most important that the steps taken to solve the equation are done in the correct order.
- An equation for the variable x in which all the coefficients of x, including the constants, are pronumerals is known as a **literal equation**: for example, $ax + b = c$.
- The two methods for solving simultaneous linear equations are **substitution** and **elimination**.
- An **inequality** is a mathematical statement that contains an inequality symbol rather than an equals sign: for example, $2x + 1 < 4$.
- To solve linear inequalities, proceed exactly as for equations except that, when multiplying or dividing both sides by a negative number, the 'direction' of the inequality symbol is reversed.
- An equation containing symbols that states a relationship between two or more quantities is called a **formula**. An example of a formula is $A = \ell w$ (area = length × width). The subject of this formula is A.
- If we wish to evaluate an unknown that is not the subject of the formula, we can either substitute the given values for the other variables and then solve the resulting equation, or we can first transpose the formula and then substitute the given values.

Technology-free questions

1 Solve each of the following equations for x:

a $2x + 6 = 8$ **b** $3 - 2x = 6$ **c** $2x + 5 = 3 - x$

d $\frac{3 - x}{5} = 6$ **e** $\frac{x}{3} = 4$ **f** $\frac{13x}{4} - 1 = 10$

g $3(2x + 1) = 5(1 - 2x)$ **h** $\frac{3x + 2}{5} + \frac{3 - x}{2} = 5$

2 Solve each of the following for t:

a $a - t = b$ **b** $\frac{at + b}{c} = d$ **c** $a(t - c) = d$

d $\frac{a - t}{b - t} = c$ **e** $\frac{at + b}{ct - b} = 1$ **f** $\frac{1}{at + c} = d$

3 Solve each of the following inequalities for x:

a $2 - 3x > 0$ **b** $\frac{3 - 2x}{5} \geq 60$

c $3(58x - 24) + 10 < 70$ **d** $\frac{3 - 2x}{5} - \frac{x - 7}{6} \leq 2$

4 Make x the subject of the formula $z = \frac{1}{2}x - 3t$. Find x when $z = 4$ and $t = -3$.

ISBN 978-1-009-11045-7 Cambridge University Press

5 A number d is equal to the square of a number e plus twice a number f.

a Find a formula for d in terms of e and f.

b Make f the subject of the formula.

c Find f when $d = 10$ and $e = 3$.

6 The surface area of a sphere of radius r is given by the formula $A = 4\pi r^2$. Calculate the surface area of a sphere of radius 10 cm. Give your answer in terms of π.

7 The volume of metal in a tube is given by the formula $V = \pi\ell[r^2 - (r - t)^2]$, where ℓ is the length of the tube, r is the radius of the outside surface and t is the thickness of the material. Find V when:

a $\ell = 100$, $r = 5$ and $t = 0.2$

b $\ell = 50$, $r = 10$ and $t = 0.5$

8 For each of the following, make the variable in brackets the subject of the formula:

a $A = \pi rs \quad (r)$

b $T = P(1 + rw) \quad (w)$

c $v = \sqrt{\dfrac{n - p}{r}} \quad (r)$

d $ac = b^2 + bx \quad (x)$

9 Let $s = \left(\dfrac{u + v}{2}\right)t$.

a Find the value of s if $u = 10$, $v = 20$ and $t = 5$.

b Find the value of t if $u = 10$, $v = 20$ and $s = 120$.

10 The volume, V cm^3, of a cylinder is given by $V = \pi r^2 h$, where r cm is the radius and h cm is the height. Find the radius of the cylinder if the volume of the cylinder is 500π cm^3 and the height is 10 cm.

11 A rope of length 205 m is cut into 10 pieces of one length and 5 pieces of another length. The total length of three of the first 10 lengths exceeds that of two of the second length by 2 m. Find the lengths of the pieces.

12 If I add one to the numerator of a fraction $\dfrac{m}{n}$ it simplifies to $\dfrac{1}{5}$. If I subtract one from the denominator it simplifies to $\dfrac{1}{7}$. Find the fraction $\dfrac{m}{n}$.

13 Mr Adonis earns \$7200 more than Mr Apollo, and Ms Aphrodite earns \$4000 less than Mr Apollo. If the total of the three incomes is \$303 200, find the income of each person.

14 Solve each of the following pairs of simultaneous equations for a and b:

a $4a - b = 11$
$3a + 2b = 6$

b $a = 2b + 11$
$4a - 3b = 11$

15 A motorist travelled a total distance of 424 km, and had an average speed of 80 km/h on highways and 24 km/h while passing through towns. If the journey took six hours, find how long the motorist spent travelling on highways.

ISBN 978-1-009-11045-7 Cambridge University Press

Review

Multiple-choice questions

1 The solution of the linear equation $3x - 7 = 11$ is

A $\frac{4}{3}$ **B** $\frac{11}{3}$ **C** $-\frac{3}{4}$ **D** 6 **E** -6

2 If $\frac{x}{3} + \frac{1}{3} = 2$ then $x =$

A $\frac{1}{3}$ **B** $\frac{2}{3}$ **C** $\frac{7}{3}$ **D** 5 **E** 7

3 The solution of the equation $x - 8 = 3x - 16$ is

A $x = -\frac{8}{3}$ **B** $x = \frac{11}{3}$ **C** $x = 4$ **D** $x = 2$ **E** $x = -2$

4 The statement that 7 is 11 times the result of subtracting 2 from x can be written as

A $7 = 11(x - 2)$ **B** $11x - 2 = 7$ **C** $7 = 11(x + 2)$

D $\frac{11}{x - 2} = 7$ **E** $\frac{7}{11} - 2 = x$

5 The solution of the simultaneous equations $2x - y = 10$ and $x + 2y = 0$ is

A $x = -2$ and $y = 3$ **B** $x = 2$ and $y = -3$ **C** $x = 4$ and $y = -2$

D $x = 6$ and $y = 2$ **E** $x = 1$ and $y = -8$

6 I bought x books for \$$a$ each and y magazines for \$$b$ each. The average price paid per item, in dollars, is

A $\frac{x + y}{a + b}$ **B** $\frac{a + b}{y + x}$ **C** $\frac{xa + yb}{y + x}$ **D** $\frac{y + x}{xa + yb}$ **E** $\frac{a}{x} + \frac{b}{y}$

7 The solution of the equation $\frac{x + 1}{4} - \frac{2x - 1}{6} = x$ is

A $x = \frac{8}{5}$ **B** $x = \frac{5}{13}$ **C** $x = 1$ **D** $x = -\frac{1}{5}$ **E** $x = -1$

CAS

8 The values of z that satisfy the inequality $\frac{72 + 15z}{3} > 4$ are

A $z > 4$ **B** $z > -4$ **C** $z = -4$ **D** $z < 4$ **E** $z < -4$

9 If $A = \frac{hw + k}{w}$ then

A $w = \frac{k}{A - h}$ **B** $w = \frac{ht + k}{A}$ **C** $w = \frac{A - 2k}{2h}$

D $w = \frac{3Ah}{2} - k$ **E** $w = \frac{2}{3}h(A + k)$

10 Bronwyn walks one lap of an oval at 2.5 km/h and then jogs another eight laps of the oval at 5 km/h. If it takes her 30 minutes in total, how long in metres is each lap?

A 200 m **B** 250 m **C** 300 m **D** 350 m **E** 400 m

ISBN 978-1-009-11045-7 Cambridge University Press

11 Which of the following equations has no solutions?

A $3x + 6 = 2x + 4$ **B** $2(x + 3) = 2x + 6$ **C** $3x - 4 = 5 - x$
D $3x + 6 = 2x + 6$ **E** $2x + 4 = 2x + 6$

12 Which of the following equations is true for all values of x?

A $4x - 6 = 2x + 4$ **B** $5(x + 3) = 5x + 15$ **C** $3x - 4 = 10 - x$
D $7x + 6 = -2x + 6$ **E** $2x + 4 = 2x + 6$

Extended-response questions

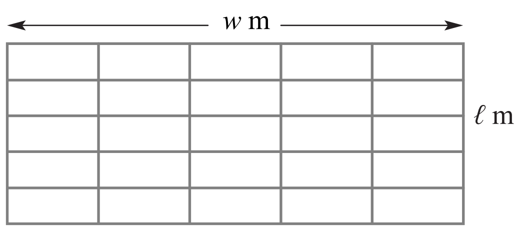
1 The formula for converting degrees Celsius, C, to degrees Fahrenheit, F, is $F = \frac{9}{5}C + 32$.

a Convert 30°F to Celsius.
b Convert 30°C to Fahrenheit.
c If x°C = x°F find x.
d If $(x + 10)$°C = x°F find x.
e If $2x$°C = x°F find the value of x.
f If k°F = $(-3k)$°C find k.

2 For a spherical mirror of radius r cm, $\frac{1}{v} + \frac{1}{u} = \frac{2}{r}$, where u cm is the distance from the mirror to the object and v cm is the distance from the mirror to the image. The magnification is given by $m = \frac{v - r}{r - u}$.

a Find r in terms of v and u from the first formula.
b Find m in terms of v and u only.

3 The diagram shows a section of wire mesh w metres in width and ℓ metres in length.



a Find an expression in terms of w and ℓ for the total length of wire required for the mesh.

b **i** If $w = 3\ell$, find an expression in terms of w for the total length of wire required.
ii If the length of wire used is 100 m, find the value of w and the value of ℓ.

c The total length of wire, L m, required for another type of rectangular mesh of dimensions x m by y m, is given by the formula $L = 6x + 8y$.
i Find y in terms of x and L.
ii Find y if $L = 200$ and $x = 4$.

d A third type of mesh can also be used to cover a rectangular region of dimensions x m by y m. In this case, the type of mesh introduced in part **c** requires 100 m of wire and so $6x + 8y = 100$. This third type of mesh requires 80 m and this gives the equation $3x + 2y = 40$. Find the values of x and y.

ISBN 978-1-009-11045-7 Cambridge University Press

4 Tom leaves town A and travels towards town B at a constant speed of u km/h. At the same time, Julie leaves town B and travels towards town A at a constant speed of v km/h. Town B is d km from town A.

a How far has each travelled after t hours?

b By considering that the sum of their distances travelled must be d km when they meet, find:

i the time it takes for them to meet

ii their distance from town A when they meet.

c If $u = 30$, $v = 50$ and $d = 100$, find the time it takes for them to meet and their distance from town A.

5 Xiu travels from town A to town B at u km/h and then returns at v km/h. Town A is d km from town B.

a Find the average speed at which Xiu travels for the complete journey, in terms of u and v. Remember that

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

b If it takes T hours to travel from A to B, find the time taken:

i for the return trip from B to A, in terms of T, u and v

ii for the entire trip, in terms of T, u and v.

6 A man on a bicycle rides one-third of the way from town A to town B at a speed a km/h and the remainder of the way at $2b$ km/h.

a If the distance between the two towns is 9 km, find the time taken to ride from A to B.

If the man had travelled at a uniform rate of $3c$ km/h, he could have ridden from A to B and back again in the same time.

b Show that $\frac{2}{c} = \frac{1}{a} + \frac{1}{b}$.

c **i** Make c the subject of this formula.

ii Find c, when $a = 10$ and $b = 20$.

7 A man walks 70 km. He walks x km at 8 km/h and y km at 10 km/h.

a Find the length of time he was walking at 8 km/h in terms of x, and the length of time he was walking at 10 km/h in terms of y.

b Find his average speed in terms of x and y.

c If the man walks at 10 km/h for the time he was walking at 8 km/h and at 8 km/h for the time he was walking at 10 km/h, he walks 72 km. Find x and y.

8 Prove that the lines with equations $2y - x = 2$, $y + x = 7$ and $y - 2x = -5$ meet at the one point.

ISBN 978-1-009-11045-7 Cambridge University Press