2

Reviewing coordinate geometry

Objectives

- To find the **midpoint** of a line segment.
- To find the **distance** between two points.
- To calculate the **gradient** of a straight line.
- To calculate the **angle of slope** of a straight line given the gradient.
- To interpret and use different forms of an equation of a straight line.
- To establish and use necessary and sufficient conditions for two lines to be **perpendicular** or **parallel**.
- To use a parameter to describe families of straight lines.
- To apply knowledge of linear relations to solving problems.
- To solve and apply **simultaneous linear equations**.

The number plane (Cartesian plane) is divided into four quadrants by two perpendicular axes. These axes intersect at a point called the origin. The position of any point in the plane can be represented by an **ordered pair** of numbers (x, y), called the **coordinates** of the point. Given the coordinates of two points, we can find the equation of the straight line through the two points, the distance between the two points and the midpoint of the line segment joining the points. These are the beginning ideas of coordinate geometry. The topic of calculus, which is introduced later in this book, builds on these ideas.

A **relation** is defined as a set of ordered pairs in the form (x, y). Sometimes we can give a rule relating the x-value to the y-value of each ordered pair, such as $y = 2x + 1$, and this is a more convenient way of describing the relation. A relation may also be represented graphically on a set of axes. If the resulting graph is a straight line, then the relation is called a **linear relation**.

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2A Distance and midpoints

In this first section we look at the method to find the coordinates of the midpoint of a line segment and we apply Pythagoras' theorem to find the distance between two points.

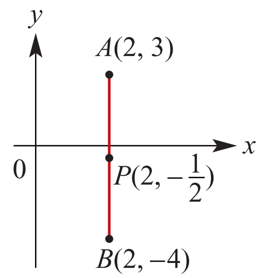
Midpoint of a line segment

Finding the midpoint of a line segment parallel to an axis is a simple special case, and it is useful in obtaining the more general result.

A line segment parallel to an axis

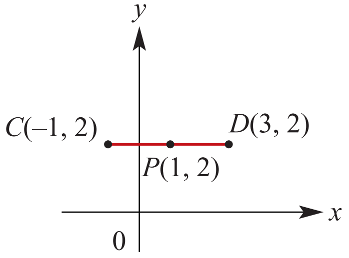
The midpoint of the line segment AB with endpoints $A(2, 3)$ and $B(2, -4)$ is the point P with coordinates $(2, -\frac{1}{2})$.

Note that $-\frac{1}{2}$ is the average of 3 and -4. The line through A and B is parallel to the y-axis.



Similarly for the line segment CD with endpoints $C(-1, 2)$ and $D(3, 2)$, the midpoint is the point P with coordinates $(1, 2)$.

Note that 1 is the average of -1 and 3. The line through C and D is parallel to the x-axis.

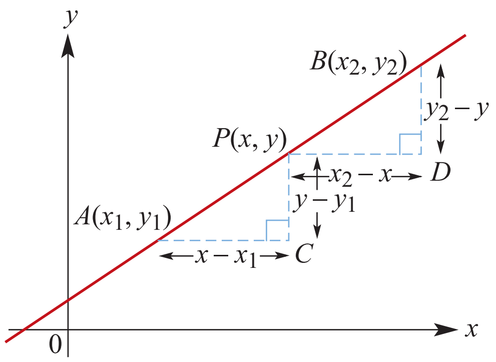


A line segment not parallel to one of the axes

Let $P(x, y)$ be the midpoint of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$, where the line through A and B is not parallel to either axis.

Let points C and D be chosen so that AC and PD are parallel to the x-axis, and PC and BD are parallel to the y-axis.

The triangles APC and PBD are congruent (AAS). Hence



$$AC = PD \quad \text{and} \quad PC = BD$$

$$\therefore \quad x - x_1 = x_2 - x \qquad y - y_1 = y_2 - y$$

$$2x = x_1 + x_2 \qquad 2y = y_1 + y_2$$

$$x = \frac{x_1 + x_2}{2} \qquad y = \frac{y_1 + y_2}{2}$$

We have proved the following result.

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The coordinates of the midpoint P of the line segment AB joining $A(x_1, y_1)$ and $B(x_2, y_2)$ are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

That is, we take the average of the x-coordinates and the average of the y-coordinates.

Example 1

Find the midpoint of the line segment joining $A(2, 6)$ with $B(-3, -4)$.

Solution

The midpoint of line segment AB has coordinates

$$\left(\frac{2 + (-3)}{2}, \frac{6 + (-4)}{2}\right) = \left(-\frac{1}{2}, 1\right)$$

Explanation

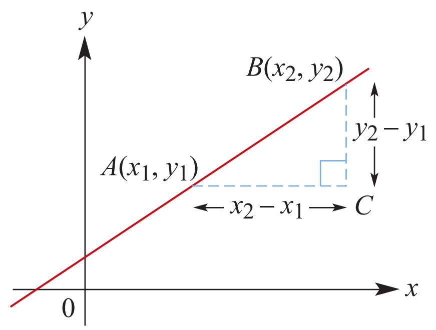
The coordinates of the midpoint of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

The distance between two points

The distance between given points $A(x_1, y_1)$ and $B(x_2, y_2)$ can be found by applying Pythagoras' theorem to the triangle ABC:

$$AB^2 = AC^2 + BC^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$



Therefore, the distance between the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

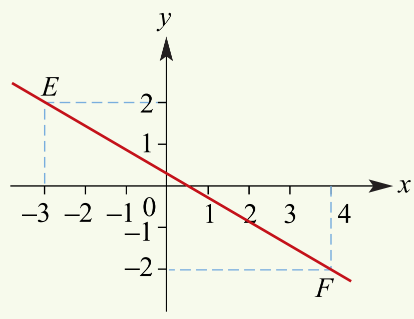
Example 2

Calculate the distance EF if E is $(-3, 2)$ and F is $(4, -2)$.

Solution

$$EF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - (-3))^2 + (-2 - 2)^2}$$
$$= \sqrt{7^2 + (-4)^2}$$
$$= \sqrt{65}$$
$$= 8.06 \quad \text{(to two decimal places)}$$

Explanation



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Summary 2A

- The coordinates of the midpoint of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- The distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Skill-sheet

Exercise 2A

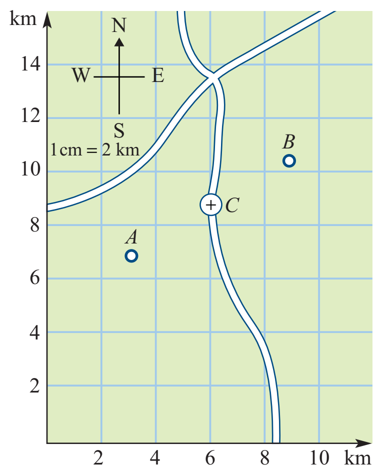
Example 1

1 Find the coordinates of M, the midpoint of AB, where A and B have the following coordinates:

a $A(2, 12)$, $B(8, 4)$ **b** $A(-3, 5)$, $B(4, -4)$
c $A(-1.6, 3.4)$, $B(4.8, -2)$ **d** $A(3.6, -2.8)$, $B(-5, 4.5)$

2 Find the midpoints of each of the sides of a triangle ABC, where A is $(1, 1)$, B is $(5, 5)$ and C is $(11, 2)$.

3 The secretary of a motocross club wants to organise two meetings on the same weekend. One is a hill climb starting from point $A(3.1, 7.1)$ and the other is a circuit event with the start at $B(8.9, 10.5)$, as shown on the map. Only one ambulance can be provided. The ambulance can be called up by radio, so it is decided to keep it at C, halfway between A and B. What are the coordinates of C?



4 If M is the midpoint of XY, find the coordinates of Y when X and M have the following coordinates:

a $X(-4, 2)$, $M(0, 3)$ **b** $X(-1, -3)$, $M(0.5, -1.6)$
c $X(6, -3)$, $M(2, 1)$ **d** $X(4, -3)$, $M(0, -3)$

5 Find the coordinates of the midpoint of the line segment joining $(1, 4)$ and (a, b), in terms of a and b. If $(5, -1)$ is the midpoint, find the values of a and b.

Example 2

6 Find the distance between each of the following (correct to two decimal places):

a $(3, 6)$ and $(-4, 5)$ **b** $(4, 1)$ and $(5, -3)$
c $(-2, -3)$ and $(-5, -8)$ **d** $(6, 4)$ and $(-7, 4)$

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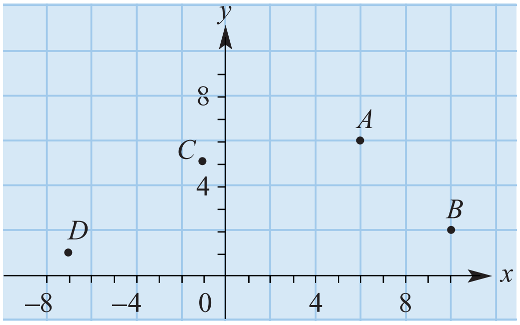
7 Calculate the perimeter of a triangle with vertices $(-3, -4)$, $(1, 5)$ and $(7, -2)$.

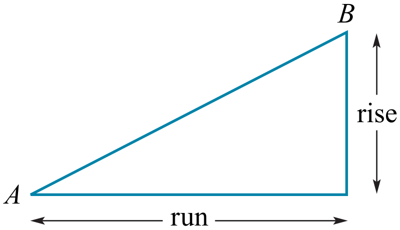
8 The diagram shows the four points $A(6, 6)$, $B(10, 2)$, $C(-1, 5)$ and $D(-7, 1)$. If the midpoint of AB is P and the midpoint of CD is M, calculate the distance PM.



9 There is an off-shore oil drilling platform in Bass Strait situated at $D(0, 6)$, where 1 unit = 5 km. Pipes for this oil drill come ashore at $M(-6, 1)$ and $N(3, -1)$. Assuming the pipelines are straight, which is the shorter DM or DN?

2B The gradient of a straight line

Through any two points it is only possible to draw a single straight line. Therefore a straight line is defined by any two points on the line.



In coordinate geometry the standard way to define the **gradient of a line segment** AB is $\frac{\text{rise}}{\text{run}}$ where:

- rise is the change in the y-values as you move from A to B
- run is the change in the x-values as you move from A to B.

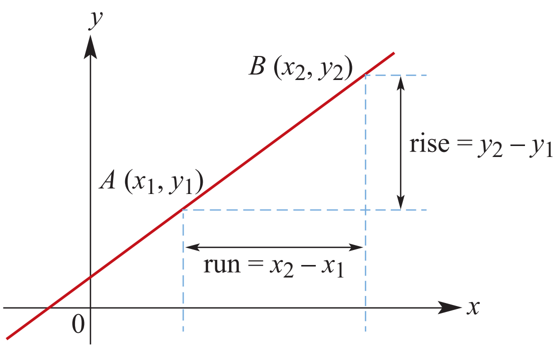
The **gradient of a line** is defined to be the gradient of any segment within the line. This definition depends on the fact that any two segments of a line have the same gradient. Hence given any two points on the line, $A(x_1, y_1)$ and $B(x_2, y_2)$, the gradient of the line can be found. The symbol used for gradient is m.

$$\text{Gradient } m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note that since

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

it does not matter which point we take as the first and which point we take as the second.



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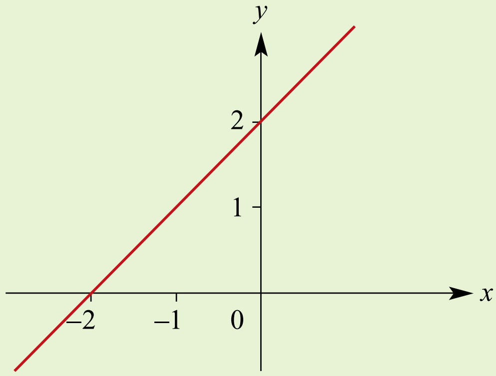
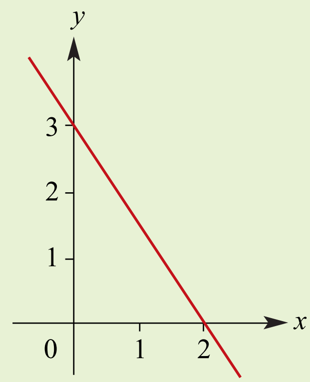
Example 3

Find the gradient of each line:

a



b



Solution

a Gradient $m = \dfrac{2-0}{0-(-2)}$

$= \dfrac{2}{2}$

$= 1$

b Gradient $m = \dfrac{0-3}{2-0}$

$= -\dfrac{3}{2}$

Explanation

Let $(x_1, y_1) = (-2, 0)$ and $(x_2, y_2) = (0, 2)$.

Gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Let $(x_1, y_1) = (0, 3)$ and $(x_2, y_2) = (2, 0)$.

Gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Notes:

- The gradient of a line that slopes upwards from left to right is **positive**, as illustrated in Example 3 **a**.
- The gradient of a line that slopes downwards from left to right is **negative**, as illustrated in Example 3 **b**.
- The gradient of a **horizontal line** (parallel to the x-axis) is zero, since $y_2 - y_1 = 0$.
- The gradient of a **vertical line** (parallel to the y-axis) is undefined, since $x_2 - x_1 = 0$.

Example 4

Find the gradient of the line that passes through the points $(1, 6)$ and $(-3, 7)$.

Solution

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-6}{-3-1} = -\frac{1}{4}$$

Explanation

The gradient can also be found using

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{6-7}{1-(-3)} = -\frac{1}{4}$$

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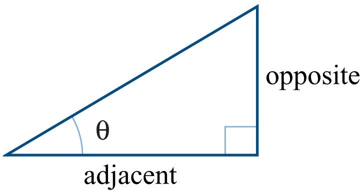
The tangent of the angle of slope

We will look first at the case when the gradient is positive and then when the gradient is negative.

Positive gradient

From Year 10 you will be familiar with the trigonometric ratio

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



Consider a straight line with positive gradient. The line forms an acute angle, θ, with the positive direction of the x-axis.

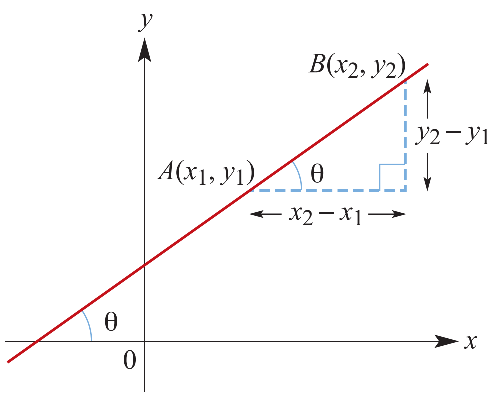
The gradient, m, of the line is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (x_1 \neq x_2)$$

From the diagram, it follows that

$$m = \tan\theta$$

where θ is the angle that the line makes with the positive direction of the x-axis.



Example 5

Determine the gradient of the line passing through the points (3, 2) and (5, 7) and the angle θ that the line makes with the positive direction of the x-axis.

Solution

$$m = \frac{7-2}{5-3} = \frac{5}{2}$$

$$\tan\theta = \frac{5}{2}$$

$$\therefore \quad \theta = 68.1986\ldots^\circ = 68.20^\circ$$

correct to two decimal places.

Explanation

The gradient is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The gradient is positive and so the angle θ is acute.

The angle can be found with a calculator using inverse tan.

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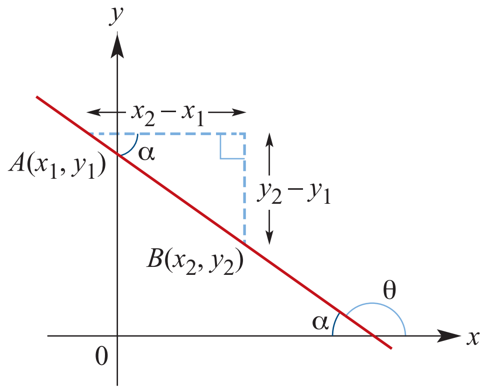
Negative gradient



Now consider a line with negative gradient. The line forms an acute angle α with the negative direction of the x-axis, and an obtuse angle θ with the positive direction of the x-axis. We have $\theta = 180° - \alpha$.

From the diagram, we see that the gradient satisfies

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-(y_1 - y_2)}{x_2 - x_1} = -\tan\alpha$$

From your work on circular functions in Year 10 you may recall that

$$\tan\theta = \tan(180° - \alpha) = -\tan\alpha$$

Thus the gradient satisfies

$$m = -\tan\alpha = \tan\theta$$

Example 6

Determine the gradient of the line passing through the points $(5, -3)$ and $(-1, 5)$ and the angle θ that the line makes with the positive direction of the x-axis.

Solution

$$m = \frac{5 - (-3)}{-1 - 5}$$

$$= -\frac{4}{3}$$

$$\tan\theta = -\frac{4}{3}$$

$$\therefore \quad \theta = 180° - (53.130\ldots°)$$

$$= 126.87°$$

correct to two decimal places.

Explanation

The gradient is negative and so the angle θ between the line and the positive direction of the x-axis is obtuse.

You first use your calculator to find the adjacent supplementary angle α, which is acute. You do this by finding the inverse tangent of $\frac{4}{3}$. The magnitude of this angle is $53.130\ldots°$.

You subtract this from 180° to obtain θ.

Summary 2B

- The gradient of a line segment AB joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\frac{y_2 - y_1}{x_2 - x_1} \quad \text{provided } x_1 \neq x_2$$

- The gradient of a line is defined as the gradient of any line segment of that line.
- A horizontal line has gradient zero.
- A vertical line does not have a gradient.

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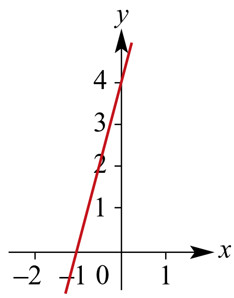
- Let θ be the angle that a line makes with the positive direction of the x-axis.
  - The gradient m is positive if and only if θ is acute.
  - The gradient m is negative if and only if θ is obtuse.
  - If θ is acute or obtuse, then $m = \tan\theta$.
  - If $\theta = 0°$, then $m = \tan 0° = 0$.
  - If $\theta = 90°$, then the gradient is not defined.

Exercise 2B

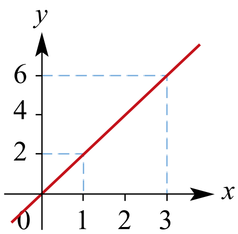
Example 3

1 Calculate the gradient of each of the following lines:

a



b



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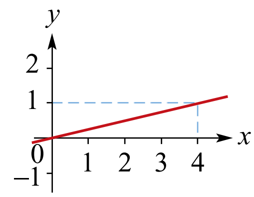

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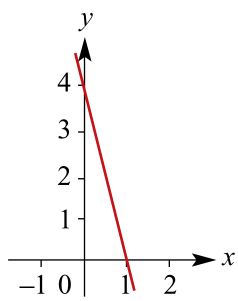

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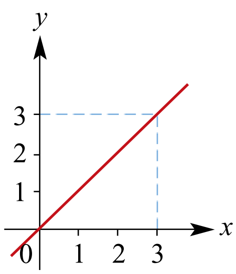

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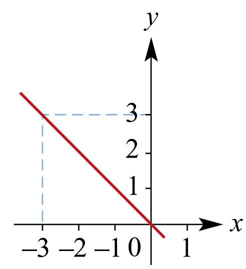

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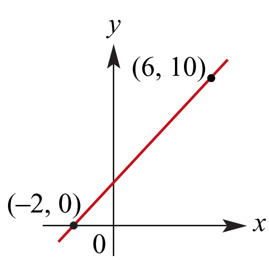

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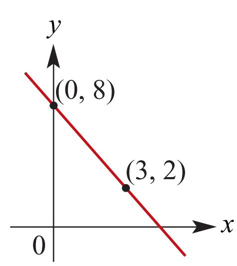

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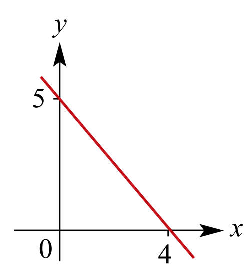

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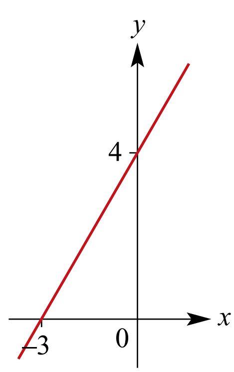

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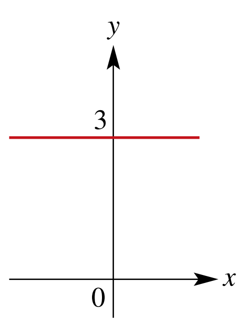

2 Sketch a graph of a line with gradient 1.

3 Sketch a graph of a line with gradient 0 which passes through the point $(1, 6)$.

Example 4

4 For each of the following, find the gradient of the line that passes through the two points with the given coordinates:

a $(6, 3)$, $(2, 4)$
b $(-3, 4)$, $(1, -6)$
c $(6, 7)$, $(11, -3)$
d $(5, 8)$, $(6, 0)$
e $(6, 0)$, $(-6, 0)$
f $(0, -6)$, $(-6, 0)$
g $(3, 9)$, $(4, 16)$
h $(5, 25)$, $(6, 36)$
i $(-5, 25)$, $(-8, 64)$
j $(1, 1)$, $(10, 100)$
k $(1, 1)$, $(10, 1000)$
l $(5, 125)$, $(4, 64)$

5 **a** Find the gradient of the straight line that passes through the points with coordinates $(5a, 2a)$ and $(3a, 6a)$.

b Find the gradient of the straight line that passes through the points with coordinates $(5a, 2a)$ and $(5b, 2b)$.

6 **a** A line has gradient 6 and passes through the points with coordinates $(-1, 6)$ and $(7, a)$. Find the value of a.

b A line has gradient -6 and passes through the points with coordinates $(1, 6)$ and $(b, 7)$. Find the value of b.

7 Find the angle, correct to two decimal places, that the lines joining the given points make with the positive direction of the x-axis:

a $(0, 3)$, $(-3, 0)$
b $(0, -4)$, $(4, 0)$
c $(0, 2)$, $(-4, 0)$
d $(0, -5)$, $(-5, 0)$

Example 5
Example 6

8 Find the angle, correct to two decimal places, that the lines joining the given points make with the positive direction of the x-axis:

a $(-4, -2)$, $(6, 8)$
b $(2, 6)$, $(-2, 4)$
c $(-3, 4)$, $(6, 1)$
d $(-4, -3)$, $(2, 4)$
e $(3b, a)$, $(3a, b)$
f (c, b), (b, c)

9 Find the gradient of a straight line which is:

a inclined at an angle of 45° to the positive direction of the x-axis
b inclined at an angle of 135° to the positive direction of the x-axis
c inclined at an angle of 60° to the positive direction of the x-axis
d inclined at an angle of 120° to the positive direction of the x-axis.

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2C The equation of a straight line

In this section we discuss different ways of determining the equation of a straight line. In general two 'independent pieces of information' are required. The following given information is considered:

- gradient and y-axis intercept
- gradient and a point
- two points.

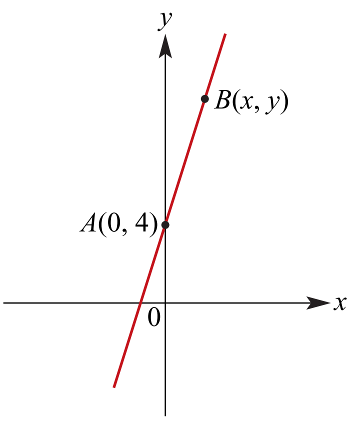
Sketching straight lines given the equation is discussed in Section 2D.

Gradient–intercept form of the equation of a straight line

We first consider an example before stating the general result. The argument in the general case is exactly the same.

The line $y = 2x + 4$

Consider the line with gradient 2 and y-axis intercept 4. This line passes through the point $A(0, 4)$. Let $B(x, y)$ be any other point on the line.



Gradient of line segment $AB = \dfrac{y-4}{x-0} = \dfrac{y-4}{x}$

We know that the gradient of the line is 2. Therefore

$$\frac{y-4}{x} = 2$$

$$y - 4 = 2x$$

$$y = 2x + 4$$

So the coordinates (x, y) satisfy the equation $y = 2x + 4$.

Conversely, if a point $B(x, y)$ in the plane satisfies $y = 2x + 4$, then

$$\frac{y-4}{x} = 2$$

Thus we know that the gradient of the line segment joining point B to the point $A(0, 4)$ is 2. Therefore the line through $A(0, 4)$ and $B(x, y)$ has gradient 2 and y-axis intercept 4.

The line $y = mx + c$

In the same way as for the line $y = 2x + 4$, we can show that:

- The line with gradient m and y-axis intercept c has equation $y = mx + c$.
- Conversely, the line with equation $y = mx + c$ has gradient m and y-axis intercept c.

This is called the **gradient–intercept form**.

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Example 7

Find the gradient and y-axis intercept of the line $y = 3x - 4$.

Solution	Explanation
The gradient is 3 and the y-axis intercept is -4.	$m = 3$ and $c = -4$

Example 8

Find the equation of the line with gradient -3 and y-axis intercept 5.

Solution	Explanation
$y = -3x + 5$	$y = mx + c$

Example 9

State the gradient and y-axis intercept of the line $3y + 6x = 9$.

Solution

$$3y + 6x = 9$$
$$3y = 9 - 6x$$
$$y = \frac{9 - 6x}{3}$$
$$y = 3 - 2x$$
i.e. $y = -2x + 3$

Therefore $m = -2$ and $c = 3$.

Explanation

Rearrange the equation $3y + 6x = 9$ into gradient–intercept form.

Now the gradient and y-axis intercept can be read directly from the equation.

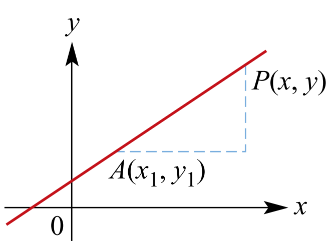
Point–gradient form of the equation of a straight line

If $A(x_1, y_1)$ is a point on a line with gradient m and $P(x, y)$ is any other point on the line, then

$$\frac{y - y_1}{x - x_1} = m$$

and so we have

$$y - y_1 = m(x - x_1)$$



The **point–gradient form** of the equation of a straight line is

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is a point on the line and m is the gradient.

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Example 10

Find the equation of the line which passes through the point $(-1, 3)$ and has gradient 4.

Solution

Method 1

$(x_1, y_1) = (-1, 3)$ and $m = 4$.

The equation is

$$y - 3 = 4(x - (-1))$$
$$y = 4(x + 1) + 3$$
$$= 4x + 4 + 3$$
$$= 4x + 7$$

Explanation

We use the equation $y - y_1 = m(x - x_1)$. (It is not necessary to work from first principles every time.)

Rearrange to make y the subject and write the equation in the form $y = mx + c$.

Method 2

Since $m = 4$, the equation is of the form $y = 4x + c$.

When $x = -1$, $y = 3$.

Therefore

$$3 = 4 \times (-1) + c$$
$$7 = c$$

The equation is $y = 4x + 7$.

Explanation

We can also use the equation $y = mx + c$ and find the value of c. The gradient is 4.

The point $(-1, 3)$ lies on the line.

Solve for c.

Example 11

Find the equation of the line that passes through the point $(3, 2)$ and has a gradient of -2.

Solution

$$y - 2 = -2(x - 3)$$
$$y - 2 = -2x + 6$$
$$y = -2x + 8$$

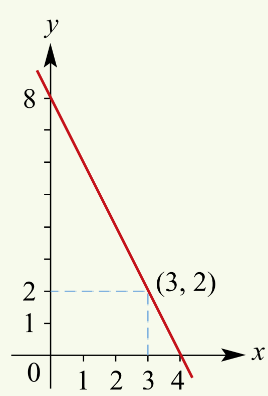
The equation is

$$y = -2x + 8$$

which could also be expressed as

$$2x + y - 8 = 0$$

Explanation



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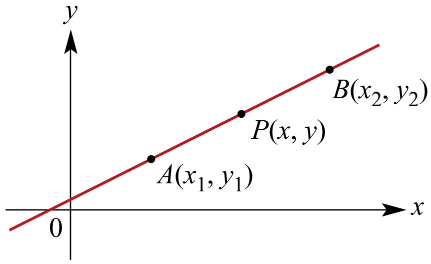
A line through two points

To find the equation of the line through two given points (x_1, y_1) and (x_2, y_2), first find the gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

and then use the point–gradient form

$$y - y_1 = m(x - x_1)$$



We can also find the equation directly by taking the point $P(x, y)$ and noting that

$$\frac{y - y_1}{x - x_1} = m$$

Example 12

Find the equation of the straight line passing through the points $(1, -2)$ and $(3, 2)$.

Solution

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-2)}{3 - 1} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$

$$\therefore \quad 2 = \frac{y - (-2)}{x - 1}$$

$$2x - 2 = y + 2$$

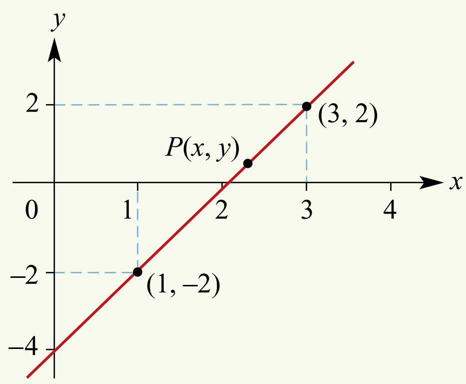
$$\therefore \quad y = 2x - 4$$

Explanation

First find the gradient m and then use

$$\frac{y - y_1}{x - x_1} = m$$

Choose $(x_1, y_1) = (1, -2)$.



Example 13

Find the equation of the straight line with y-axis intercept -3 which passes through the point with coordinates $(1, 10)$.

Solution

The gradient is

$$m = \frac{10 - (-3)}{1 - 0} = 13$$

Therefore the equation is $y = 13x - 3$.

Explanation

Find the gradient using $(x_1, y_1) = (0, -3)$ and $(x_2, y_2) = (1, 10)$.

The general equation of a line with y-axis intercept -3 is $y = mx - 3$.

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Two intercepts

A special case of finding the equation of a line given the coordinates of two points is when the intercept with each axis is known, that is, finding the line through $(a, 0)$ and $(0, b)$, where $a, b \neq 0$.

In this case, the gradient is

$$m = \frac{b-0}{0-a} = -\frac{b}{a}$$

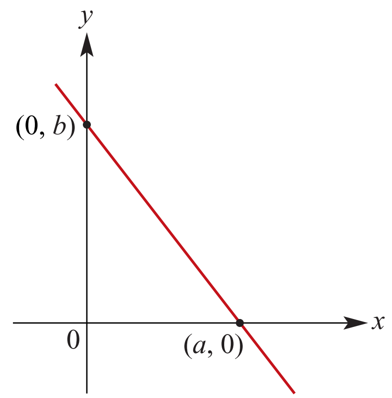
Thus the equation of the line is

$$y - 0 = -\frac{b}{a}(x-a)$$

Multiplying both sides of the equation by a gives

$$ay + bx = ab$$

Dividing both sides of the equation by ab gives the following:



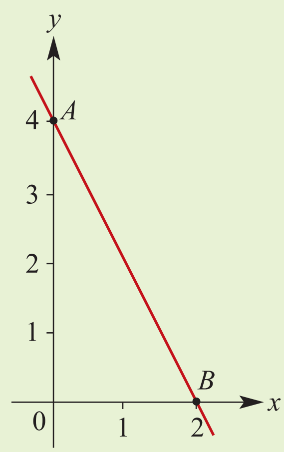
The **intercept form** of the equation of a straight line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

where a and b are the x-axis intercept and y-axis intercept respectively.

Example 14

Find the equation of the line shown in the graph.



Solution

The intercept form of the equation is

$$\frac{x}{2} + \frac{y}{4} = 1$$

Multiply both sides by 4:

$$2x + y = 4$$

The equation of the line is $y = -2x + 4$.

Explanation

The coordinates of A and B are $(0, 4)$ and $(2, 0)$.

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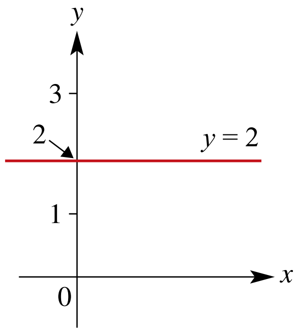
Vertical and horizontal lines

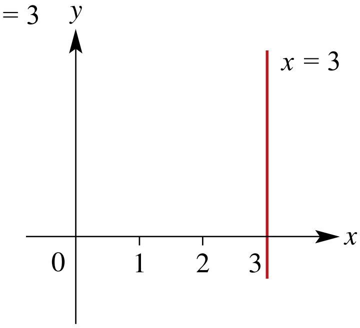
If a line is **horizontal**, then its gradient $m = 0$ and its equation is simply $\boldsymbol{y = c}$, where c is the y-axis intercept.

If a line is **vertical**, then its gradient is undefined and its equation is $\boldsymbol{x = a}$, where a is the x-axis intercept.

Equation $y = 2$



Equation $x = 3$



Note that the equation of a vertical line is not of the form $y = mx + c$.

General form of the equation of a straight line

We have seen that all points on the line through two given points satisfy an equation of the form $mx + ny + p = 0$, with m and n not both 0. Conversely, any 'linear equation' $mx + ny + p = 0$ is the equation of a (straight) line. This is called the **general form** of the equation of a line.

Summary 2C

- **Gradient–intercept form**
  - The line with gradient m and y-axis intercept c has equation $y = mx + c$.
  - Conversely, the line with equation $y = mx + c$ has gradient m and y-axis intercept c.
- **Point–gradient form** If (x_1, y_1) is a point on a line with gradient m and (x, y) is any other point on the line, then

  $$\frac{y - y_1}{x - x_1} = m$$

  which can be written as

  $$y - y_1 = m(x - x_1)$$

- **Two points** To find the equation of the line through two given points (x_1, y_1) and (x_2, y_2), first find the gradient

  $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

  and then use the point–gradient form

  $$y - y_1 = m(x - x_1)$$

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- **Intercept form** If a line has x-axis intercept a and y-axis intercept b, the equation of the line is
$$\frac{x}{a} + \frac{y}{b} = 1$$
- **Horizontal line** A line parallel to the x-axis through the point (a, c) has equation $y = c$.
- **Vertical line** A line parallel to the y-axis through the point (a, c) has equation $x = a$.
- **General form** Every straight line satisfies an equation of the form $mx + ny + p = 0$, with m and n not both 0. Conversely, any 'linear equation' $mx + ny + p = 0$ is the equation of a straight line.

Skill-sheet

Exercise 2C

Example 7

1 State the gradient and y-axis intercept of the graph of each equation:

a $y = 3x + 6$ **b** $y = -6x + 7$ **c** $y = 3x - 6$ **d** $y = -x - 4$

Example 8

2 **a** Find the equation of the straight line with gradient 3 and y-axis intercept 5.

b Find the equation of the straight line with gradient −4 and y-axis intercept 6.

c Find the equation of the straight line with gradient 3 and y-axis intercept −4.

Example 9

3 State the gradient and y-axis intercept of the graph of each equation:

a $3x - y = 6$ **b** $4x - 2y = 8$ **c** $5x - 10y = 20$ **d** $2x - 6y = 10$

4 Express in gradient–intercept form and hence state the gradient and y-axis intercept of each of the following linear relations:

a $2x - y = 9$ **b** $3x + 4y = 10$ **c** $-x - 3y = 6$ **d** $5x - 2y = 4$

Example 10

Example 11

5 **a** Find the equation of the straight line that has gradient 3 and passes through the point with coordinates $(6, 7)$.

b Find the equation of the straight line that has gradient −2 and passes through the point with coordinates $(1, 7)$.

Example 12

6 Find the equations of the straight lines passing through the following pairs of points. (Express your answer in gradient–intercept form.)

a $(-1, 4)$, $(2, 3)$ **b** $(0, 4)$, $(5, -3)$ **c** $(3, -2)$, $(4, -4)$ **d** $(5, -2)$, $(8, 9)$

Example 13

7 For the straight line that has y-axis intercept 6 and passes through the point with coordinates $(1, 8)$, find:

a the gradient **b** the equation

8 Find the equation of the straight line that passes through the point $(1, 6)$ and has gradient:

a 2 **b** −2

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9 Write, in the form $y = mx + c$, the equations of the lines which have the given gradient and pass through the given point:

a $m = 2$; $(-1, 4)$ **b** $m = -2$; $(0, 4)$ **c** $m = -5$; $(3, 0)$

Example 14

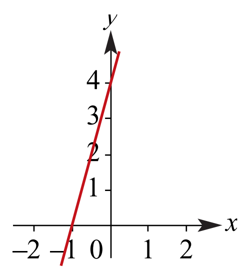
10 Find equations defining the lines which pass through the following pairs of points:

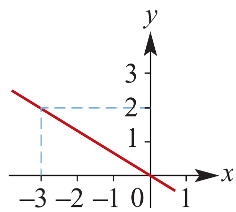
a $(0, 4)$, $(6, 0)$ **b** $(-3, 0)$, $(0, -6)$
c $(0, 4)$, $(4, 0)$ **d** $(2, 0)$, $(0, 3)$

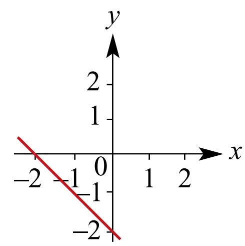
11 Find the equations, in the form $y = mx + c$, of the lines which pass through the following pairs of points:

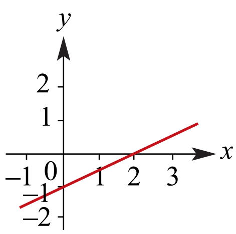
a $(0, 4)$, $(3, 6)$ **b** $(1, 0)$, $(4, 2)$
c $(-3, 0)$, $(3, 3)$ **d** $(-2, 3)$, $(4, 0)$
e $(-1.5, 2)$, $(4.5, 8)$ **f** $(-3, 1.75)$, $(4.5, -2)$

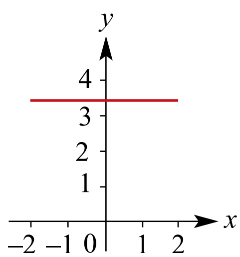
12 Find the equation of each of the following lines:

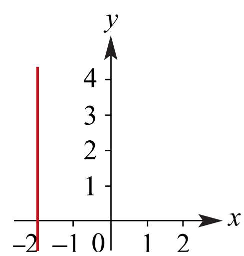
a


b


c


d


e


f


13 Do the points $P(1, -3)$, $Q(2, 1)$ and $R(2\frac{1}{2}, 3)$ lie on the same straight line?

14 For which of the following does the line pass through the origin?

a $y + x = 1$ **b** $y + 2x = 2(x + 1)$ **c** $x + y = 0$ **d** $x - y = 1$

15 **a** Find the equation of the line that is parallel to the y-axis and passes through the point with coordinates $(4, 7)$.

b Find the equation of the line that is parallel to the x-axis and passes through the point with coordinates $(-4, 11)$.

c Find the equation of the line that is parallel to the y-axis and passes through the point with coordinates $(11, -7)$.

d Find the equation of the line that is parallel to the x-axis and passes through the point with coordinates $(5, -1)$.

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2D Graphing straight lines

In the previous section we discussed methods of finding the equation of a straight line given suitable information. In this section we look at sketching a straight line from an equation. To sketch the graph we need to derive the coordinates of two points on the line. A convenient way to sketch graphs of straight lines is to plot the two axis intercepts.

Example 15

Sketch the graph of $2x + 4y = 10$.

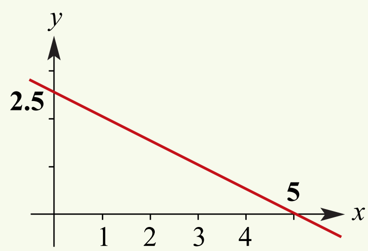
Solution

x-axis intercept ($y = 0$): $2x + 4(0) = 10$

$x = 5$

y-axis intercept ($x = 0$): $2(0) + 4y = 10$

$y = 2.5$



Example 16

Sketch the graph of $y = 2x - 6$ by first finding the intercepts.

Solution

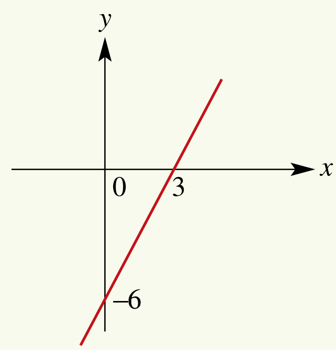
x-axis intercept ($y = 0$): $0 = 2x - 6$

$x = 3$

y-axis intercept ($x = 0$): $y = 2(0) - 6$

$y = -6$

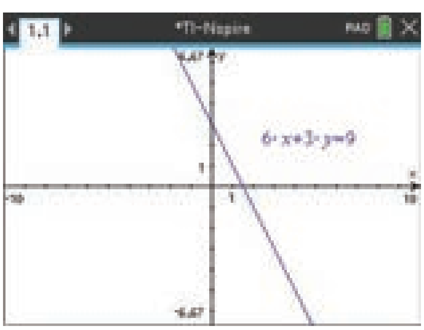
Note: You can also obtain the y-axis intercept directly from the equation.



Using the TI-Nspire

To plot the graph of $6x + 3y = 9$:

- Open a **Graphs** application: press [on] and select the **Graphs** icon, or use [ctrl] [I] and select **Add Graphs**.
- Equations of the form $a \cdot x + b \cdot y = c$ can be entered directly using [menu] > **Graph Entry/Edit** > **Relation**. Enter as $6x + 3y = 9$.



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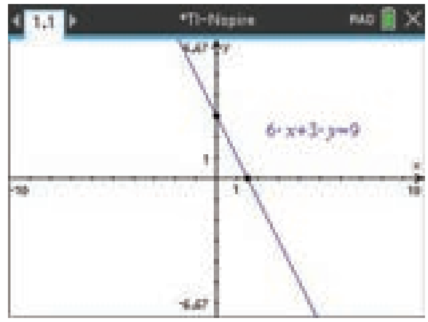
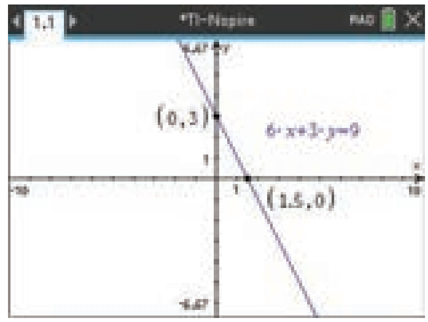
Note: The window settings (menu > **Window/Zoom** > **Window Settings**) will have to be changed if the axis intercepts do not appear on the screen.

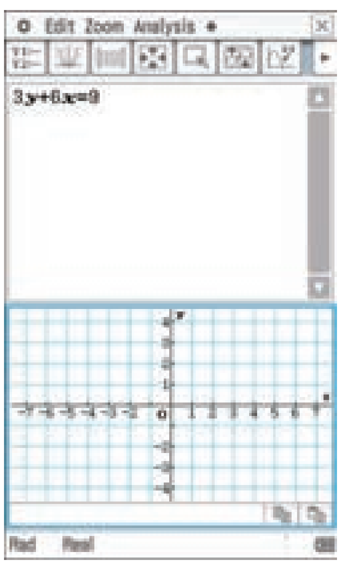
- The axis intercepts can be found using menu > **Geometry** > **Points & Lines** > **Intersection Point(s)**. Select the x-axis and the graph to display the x-axis intercept. Select the y-axis and the graph to display the y-axis intercept.
- To show the coordinates of these points, use menu > **Actions** > **Coordinates and Equations** and double click on each of the points.
- Press esc to exit the **Coordinates and Equations** tool.

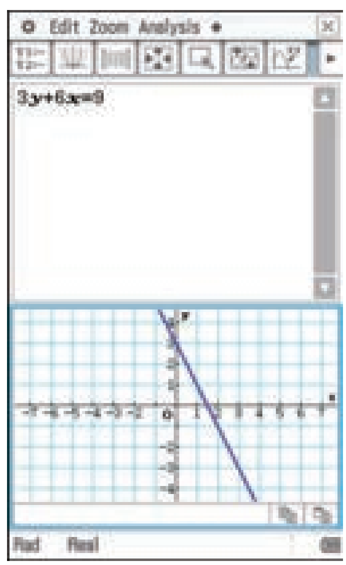




Using the Casio ClassPad

- Type the equation $3y + 6x = 9$ in the main screen Main $\sqrt{\alpha}$. Tap on the graph icon to display the graph window.
- Using your stylus, highlight the equation and drag it down into the graph window. Lift the stylus off the screen for the graph to appear.





- The graph window setting can be adjusted using the window setting icon.
- Ensure the graph window is selected and the intercepts are visible on the graph.

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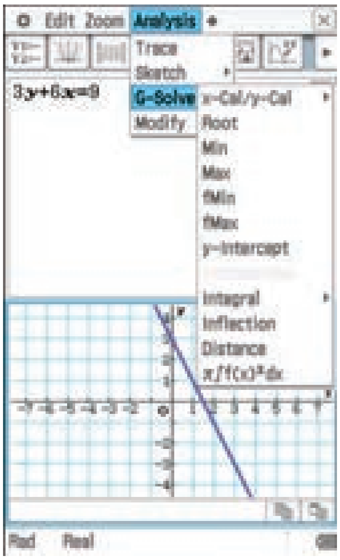
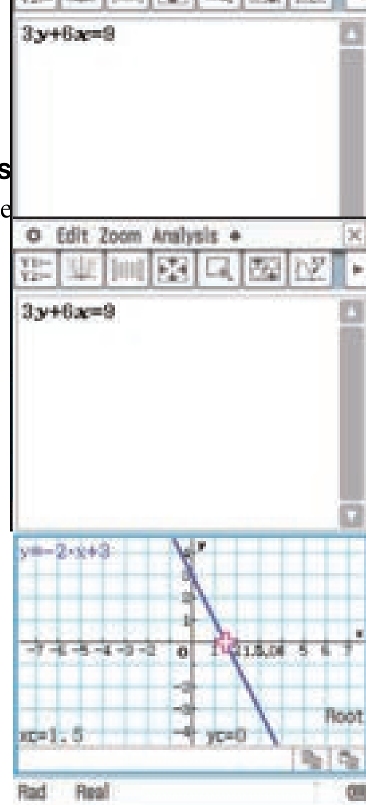
- To find the intercepts, go to **Analysis** > **G-S** ... y-axis intercept and select **Root** for the x-axis inte...





Note: The equation is displayed in gradient–intercept form in the top-left corner of the graph window.

From Section 2B we know that the gradient of a line is the tangent of the angle of slope (that is, the angle formed by the line with the positive direction of the x-axis).

Example 17

For each of the following lines, find the magnitude of the angle θ (correct to two decimal places) that the line makes with the positive direction of the x-axis:

a $y = 2x + 3$ **b** $3y = 3x - 6$

c $y = -0.3x + 1.5$

Solution

a
$$y = 2x + 3$$
Gradient $= 2$

Hence $\tan\theta = 2$

Therefore $\theta = 63.43^\circ$

correct to two decimal places

b
$$3y = 3x - 6$$
$$y = x - 2$$
Gradient $= 1$

Hence $\tan\theta = 1$

Therefore $\theta = 45^\circ$

c
$$y = -0.3x + 1.5$$
Gradient $= -0.3$

Hence $\tan\theta = -0.3$

Therefore $\theta = (180 - 16.699\ldots)^\circ$

$= 163.30^\circ$ correct to two decimal places

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Summary 2D

- The most practical way to sketch a straight line is to plot two points known to be on the required line and draw the line through them.
  - Two important points are the intercept with the x-axis and the intercept with the y-axis. These are the best two points to use in order to sketch a line that does not pass through the origin and is not parallel to one of the axes.
  - To sketch a line with equation of the form $y = mx$, plot one other point on the line besides the origin.
- The gradient m of a line $y = mx + c$ is equal to $\tan\theta$, where θ is the angle of slope measured between the line and the positive direction of the x-axis.

Exercise 2D

1 For each of the following, give the coordinates of the axis intercepts:

a $x + y = 4$ **b** $x - y = 4$ **c** $-x - y = 6$ **d** $y - x = 8$

Example 15

2 Sketch the graphs of each of the following linear relations:

a $2x - 3y = 12$ **b** $x - 4y = 8$ **c** $-3x + 4y = 24$
d $-5x + 2y = 20$ **e** $4x - 3y = 15$ **f** $7x - 2y = 15$

Example 16

3 For each of the following, sketch the graph by first finding the axis intercepts:

a $y = x - 1$ **b** $y = x + 2$ **c** $y = 2x - 4$

4 Sketch the graphs of each of the following by first determining the axis intercepts:

a $y = 2x - 10$ **b** $y = 3x - 9$ **c** $y = 5x + 10$ **d** $y = -2x + 10$

5 Sketch the graphs of each of the following:

a $y = x + 2$ **b** $y = -x + 2$ **c** $y = 2x + 1$ **d** $y = -2x + 1$

6 Sketch the graphs of each of the following:

a $x + y = 1$ **b** $x - y = 1$ **c** $y - x = 1$ **d** $-x - y = 1$

7 Sketch the graphs of each of the following:

a $y = x + 3$ **b** $y = 3x + 1$ **c** $y = 4 - \frac{1}{2}x$ **d** $y = 3x - 2$
e $4y + 2x = 12$ **f** $3x + 6y = 12$ **g** $4y - 6x = 24$ **h** $8x - 3y = 24$

8 Sketch the graphs of each of the following:

a $y = 3$ **b** $x = -2$ **c** $y = -2$ **d** $x = 5$

Example 17

9 Find the magnitude of the angle, correct to two decimal places, made by each of the following with the positive direction of the x-axis:

a $y = x$ **b** $y = -x$ **c** $y = x + 1$
d $x + y = 1$ **e** $y = 2x$ **f** $y = -2x$

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10 Find the magnitude of the angle, correct to two decimal places, made by each of the following with the positive direction of the x-axis:

a $y = 3x + 2$ **b** $2y = -2x + 1$ **c** $2y - 2x = 6$ **d** $3y + x = 7$

11 A straight line has equation $y = 3x - 4$. The points with coordinates $(0, a)$, $(b, 0)$, $(1, d)$ and $(e, 10)$ lie on the line. Find the values of a, b, d and e.

12 Sketch the graphs of each of the following using your calculator. Label the axis intercepts. CAS

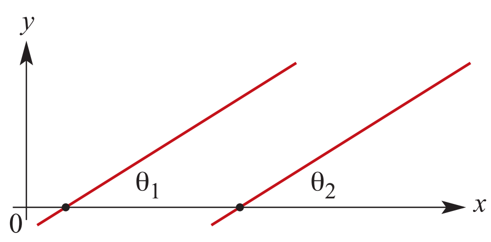
a $y = 2x - 3$ **b** $2x - 5y = 12$ **c** $5y + 2x = 10$ **d** $5x - 6y = 14$

2E Parallel and perpendicular lines

Parallel lines

- Two non-vertical lines are **parallel** if they have the same gradient.
- Conversely, if two non-vertical lines are parallel, then they have the same gradient.

This is easily proved through considering the angles of inclination of such lines to the positive direction of the x-axis and using the following two results:



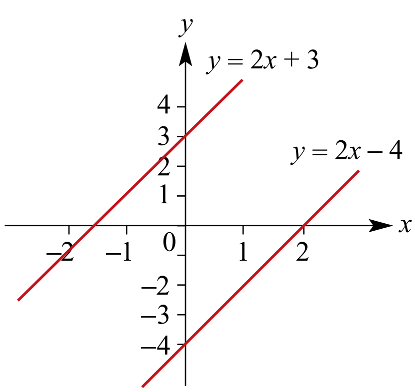
1 Two non-vertical lines are parallel if and only if the corresponding angles θ_1 and θ_2 formed by the lines and the positive direction of the x-axis are equal.

2 If two angles θ_1 and θ_2 are acute, obtuse or zero, then $\tan\theta_1 = \tan\theta_2$ implies $\theta_1 = \theta_2$.

For example, consider the lines

$$y = 2x + 3$$
$$y = 2x - 4$$

Both lines have gradient 2, and so they are parallel.



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Perpendicular lines

We prove that two lines are **perpendicular** if and only if the product of their gradients is -1 (or if one is horizontal and the other vertical).

Two lines with gradients m_1 and m_2 (both non-zero) are perpendicular if and only if $m_1 m_2 = -1$.

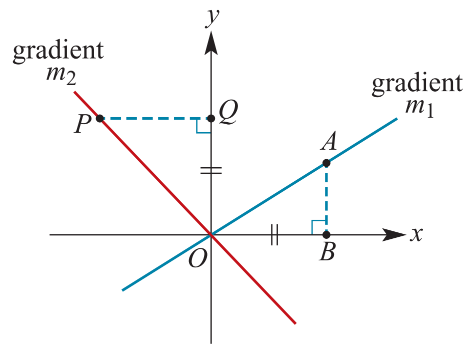
Proof Initially we consider the case where the two lines intersect at the origin.

Step 1 Draw two lines passing through the origin with one of the lines having positive gradient, m_1, and the other negative gradient, m_2. Form right-angled triangles OPQ and OAB with $OQ = OB$.

$$\text{Gradient } m_1 = \frac{AB}{BO}$$

$$\text{Gradient } m_2 = -\frac{OQ}{PQ}$$

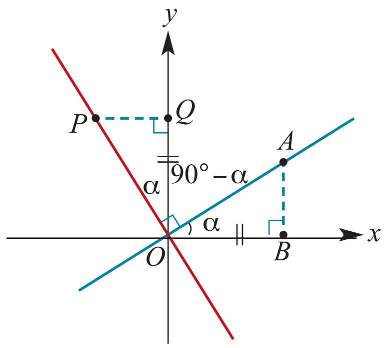
$$\begin{aligned}\text{Product } m_1 m_2 &= -\frac{OQ}{PQ} \times \frac{AB}{BO}\\ &= -\frac{OQ}{PQ} \times \frac{AB}{OQ}\\ &= -\frac{AB}{PQ}\end{aligned}$$



Step 2 We now prove: *If two lines passing through the origin are perpendicular, then the product of their gradients is* -1.

If the lines are perpendicular, then $\angle POQ = \angle AOB$. Therefore triangles OPQ and OAB are congruent. So $PQ = AB$ and therefore the product of the gradients is

$$m_1 m_2 = -\frac{AB}{PQ} = -\frac{AB}{AB} = -1$$



Step 3 We next prove the converse: *If for two lines passing through the origin the product of their gradients is* -1*, then the lines are perpendicular.*

If the product $m_1 m_2 = -1$, then $AB = PQ$, which implies that the triangles OAB and OPQ are congruent. Therefore $\angle POQ = \angle AOB$ and so $\angle AOP = 90°$.

Step 4 If we are given two lines anywhere in the plane, we can draw lines through the origin parallel to the original two lines. The slopes of the new lines are the same. So the result holds for lines that do not necessarily pass through the origin.

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Example 18

Find the equation of the straight line which passes through $(1, 2)$ and is:

a parallel to the line with equation $2x - y = 4$

b perpendicular to the line with equation $2x - y = 4$.

Solution

The equation $2x - y = 4$ is equivalent to $y = 2x - 4$. The line $y = 2x - 4$ has gradient 2.

a The required line passes through $(1, 2)$ and has gradient 2. Hence

$$y - 2 = 2(x - 1)$$

Therefore the line has equation $y = 2x$.

b The required line passes through $(1, 2)$ and has gradient $-\frac{1}{2}$. Hence

$$y - 2 = -\frac{1}{2}(x - 1)$$

Therefore $2y - 4 = -x + 1$ and equivalently $2y + x = 5$.

The line has equation $2y + x = 5$.

Explanation

A line parallel to $y = 2x - 4$ has gradient 2.

We use $y - y_1 = m(x - x_1)$ where $(x_1, y_1) = (1, 2)$ and $m = 2$.

A line perpendicular to a line with gradient m has gradient $-\frac{1}{m}$ (provided $m \neq 0$).

We use $y - y_1 = m(x - x_1)$ where $(x_1, y_1) = (1, 2)$ and $m = -\frac{1}{2}$.

Example 19

The coordinates of the vertices of a triangle ABC are $A(0, -1)$, $B(2, 3)$ and $C(3, -2\frac{1}{2})$. Show that the side AB is perpendicular to the side AC.

Solution

Let m_1 be the gradient of the line AB and let m_2 be the gradient of the line AC.

$$m_1 = \frac{3 - (-1)}{2 - 0}$$
$$= 2$$

$$m_2 = \frac{-2\frac{1}{2} - (-1)}{3 - 0}$$
$$= \frac{-1\frac{1}{2}}{3}$$
$$= -\frac{1}{2}$$

Since $m_1 \times m_2 = 2 \times (-\frac{1}{2}) = -1$, the lines AB and AC are perpendicular to each other.

Explanation

We use the fact that two lines with gradients m_1 and m_2 (both non-zero) are perpendicular if and only if $m_1 m_2 = -1$.

We show the product of the gradients is -1. Hence the lines are perpendicular.

Summary 2E

- Two non-vertical lines are **parallel** if they have the same gradient. Conversely, if two non-vertical lines are parallel, then they have the same gradient.
- Two lines are **perpendicular** if the product of their gradients is -1 (or if one is horizontal and the other vertical). Conversely, if two lines are perpendicular, then the product of their gradients is -1 (or one is horizontal and the other vertical).

Skill-sheet

Exercise 2E

Example 18

1 Find the equation of the straight line which passes through $(4, -2)$ and is:

a parallel to the line with equation $y = 2x + 1$
b perpendicular to the line with equation $y = 2x + 1$
c parallel to the line with equation $y = -2x + 1$
d perpendicular to the line with equation $y = -2x + 1$
e parallel to the line with equation $2x - 3y = 4$
f perpendicular to the line with equation $2x - 3y = 4$
g parallel to the line with equation $x + 3y = 5$
h perpendicular to the line with equation $x + 3y = -4$.

2 For which of the following pairs of equations are the corresponding lines parallel to each other? Sketch graphs to show the pairs of non-parallel lines.

a $2y = 6x + 4; \quad y = 3x + 4$
b $x = 4 - y; \quad 2x + 2y = 6$
c $3y - 2x = 12; \quad y + \frac{1}{3} = \frac{2}{3}x$
d $4y - 3x = 4; \quad 3y = 4x - 3$

3 Find the equation of the line:

a perpendicular to the line $x = 3$ and which passes through the point $(3, 4)$
b perpendicular to the line $y = 3$ and which passes through the point $(2, 3)$
c perpendicular to the line $x = -2$ and which passes through the point $(-2, 4)$
d perpendicular to the line $y = -4$ and which passes through the point $(3, -4)$.

4 Find the equation of the straight line which passes through the point $(1, 4)$ and is perpendicular to the line with equation $y = -\frac{1}{2}x + 6$.

5 Points A and B have coordinates $(1, 5)$ and $(-3, 7)$ respectively. Find the coordinates of the midpoint M of the line segment AB and find the equation of the line which passes through the point M and is perpendicular to the line AB.

Example 19

6 If the points A, B and C have the coordinates $A(5, 2)$, $B(2, -3)$ and $C(-8, 3)$, show that the triangle ABC is a right-angled triangle.

7 Given the points $A(3, 7)$, $B(6, 1)$ and $C(20, 8)$, prove that AB is perpendicular to BC.

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8 Show that $RSTU$ is a rectangle if the coordinates of the vertices are respectively $R(2, 6)$, $S(6, 4)$, $T(2, -4)$ and $U(-2, -2)$.

9 Given that the lines $4x - 3y = 10$ and $4x - \ell y = m$ are perpendicular and intersect at the point $(4, 2)$, find the values of ℓ and m.

10 The line $y = 2x + 3$ intersects the y-axis at A. The points B and C on this line are such that $AB = BC$. The line through B perpendicular to AC passes through the point $D(-1, 6)$. Find:

a the equation of BD

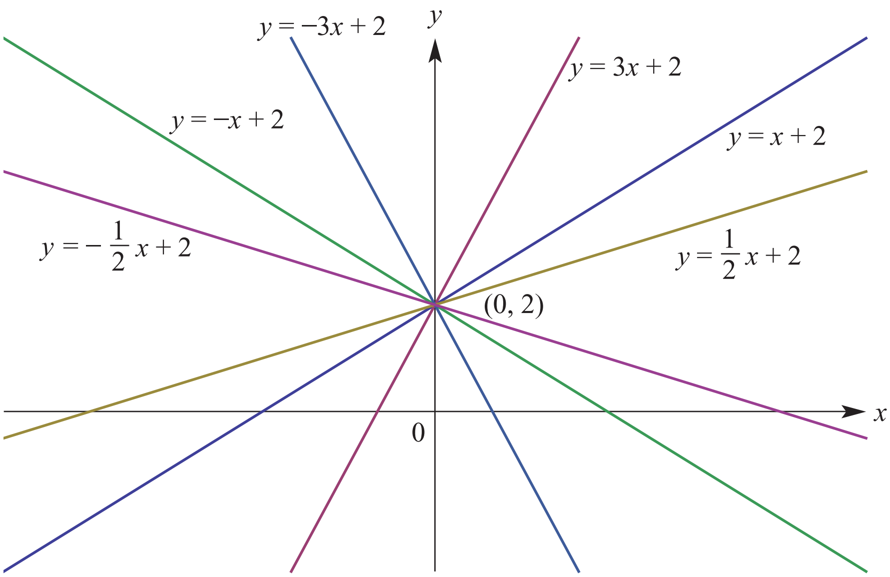
b the coordinates of B

c the coordinates of C.

2F Families of straight lines

Here are three families of straight lines:

- $y = mx$, where the gradient m of the lines varies – the graphs are the straight lines through the origin.
- $y = 3x + c$, where the y-axis intercept c of the lines varies – the graphs are the straight lines with gradient 3.
- $y = mx + 2$, where the gradient m of the lines varies – the graphs are the straight lines with y-axis intercept 2. Some graphs in this family are illustrated below.



The variable m is called a **parameter**. We will consider other families of graphs in later chapters of this book.

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Example 20

Find the value of m if the line $y = mx + 2$ passes through the point $(3, 11)$.

Solution

We can write

$$11 = 3m + 2$$

Therefore $3m = 9$ and hence $m = 3$. It is the line $y = 3x + 2$.

Example 21

A family of lines have equations of the form $y = mx + 2$, where m is a negative number.

a Find the x-axis intercept of a line in this family in terms of m.

b For which values of m is the x-axis intercept greater than 3?

c Find the equation of the line perpendicular to the line $y = mx + 2$ at the point $(0, 2)$.

Solution

a When $y = 0$:

$$\begin{aligned} mx + 2 &= 0 \\ mx &= -2 \\ x &= -\frac{2}{m} \end{aligned}$$

The x-axis intercept is $-\frac{2}{m}$.

b

$$\begin{aligned} -\frac{2}{m} &> 3 \\ -2 &< 3m \\ -\frac{2}{3} &< m \end{aligned}$$

Therefore the x-axis intercept is greater than 3 for $-\frac{2}{3} < m < 0$.

c The equation is $y - 2 = -\frac{1}{m}x$ and the gradient–intercept form is $y = -\frac{1}{m}x + 2$.

Explanation

a To find the x-axis intercept put $y = 0$.

b Multiply both sides of the inequality by m. Since m is negative, the inequality sign is reversed. Then divide both sides by 3.

c The perpendicular line has gradient $-\frac{1}{m}$ and passes through $(0, 2)$.

Summary 2F

Families of straight lines can be described through the use of a parameter. For example:

- All the non-vertical lines passing through $(0, 2)$ have equation of the form $y = mx + 2$, where m is a real number. (If $m = 0$, the line is horizontal.)
- All the lines with gradient 3 have equation of the form $y = 3x + c$, where c is a real number.
- All the lines with x-axis intercept 4 have equation of the form $\frac{x}{4} + \frac{y}{b} = 1$, where b is a non-zero real number.

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Skill-sheet

Exercise 2F

Example 20

1 The line with equation $y = mx - 3$, where m is a positive number, passes through the point $(2, 7)$. Find the value of m.

2 The line with equation $y = 2x + c$, where c is a number, passes through the point $(3, 11)$. Find the value of c.

3 A line has equation $y = mx + 3$, where $m \neq 0$. A second line is perpendicular to this line and passes through the point $(0, 3)$.

a Find the equation of the second line in terms of m.

b Find the value of m if the second line passes through the point $(1, -4)$.

4 The line with equation $y = mx + 2$, where m is a positive number, passes through the point $(3, 8)$. Find the value of m.

Example 21

5 A family of lines have equations of the form $y = mx - 3$, where m is a positive number.

a Find the x-axis intercept of a line in this family in terms of m.

b Find the value of m if the line passes through the point $(5, 6)$.

c For which values of m is the x-axis intercept less than or equal to 1?

d Find the equation of the line perpendicular to the line $y = mx - 3$ at the point $(0, -3)$.

6 A family of lines have equations of the form $y = 2x + c$, where c is a number.

a Find the x-axis intercept of a line in this family in terms of c.

b Find the value of c if the line passes through the point $(5, 6)$.

c For which values of c is the x-axis intercept less than or equal to 1?

d Find the equation of the line perpendicular to the line $y = 2x + c$ at the point $(0, c)$.

7 A family of lines have equations of the form $\frac{x}{a} - \frac{y}{12} = 4$, where a is a non-zero number.

a Find the x-axis intercept of a line in this family in terms of a.

b Find the gradient of the line in terms of a.

c Find the value of a if the gradient is

i 2 **ii** -2

8 A family of lines have equations of the form $3x + by = 12$, where b is a non-zero number.

a Find the y-axis intercept of a line in this family in terms of b.

b Find the gradient of the line in terms of b.

c Find the value of b if the gradient is

i 1 **ii** -2

d Find the equation of the line that is perpendicular to the line $3x + by = 12$ at the point $(4, 0)$.

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CAS

9 A family of lines have equations of the form $y = -bx + c$, where b and c are positive numbers.

a Find the x-axis intercept of a line in this family in terms of b and c.

b For a line in this family that passes through the point $(1, 7)$:

i find b in terms of c

ii find the values of c for which the x-axis intercept is less than or equal to 2.

c Let O denote the origin, and let A and B be the x-axis and y-axis intercepts of the line $y = -bx + c$.

i If the midpoint M of the line segment AB is $(3, 6)$, find the values of b and c.

ii If the triangle AOB has area 4, find c in terms of b.

iii If the midpoint M of AB is such that $OM = 5$, find c in terms of b. Then use your calculator to find the value of c when $b = 5, 10, 100$. Comment.

2G Linear models

There are many practical situations where a linear relation can be used.

Example 22

A historical site charges a tour company for priority entrance to the site. The charge consists of a monthly fee of \$200 plus \$3.50 for each tourist brought to the site. Construct a cost function that describes the monthly charge and sketch the linear graph for this.

Solution

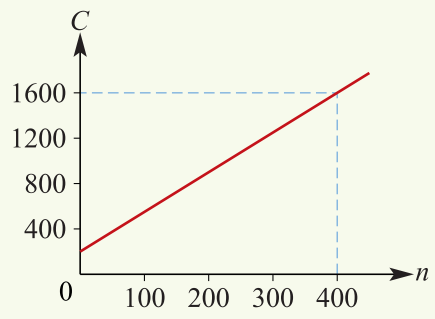
Let C = monthly charge (\$)

n = number of tourists

Then

$$C = 3.5n + 200$$

The number of tourists is counted in whole numbers including zero.



Note: The graph should be a series of discrete points rather than a continuous line, because n is a whole number. With the scale used it is not practical to show it correctly.

An important linear relation is the relation between distance travelled and time taken when an object is travelling with constant speed. If a car travels at 40 km/h, the relationship between distance travelled (s kilometres) and time taken (t hours) is

$$s = 40t \qquad (\text{for } t \geq 0)$$

The graph of s against t is a straight-line graph through the origin. The gradient of this graph is 40.

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Example 23

A car starts from point A on a highway 10 kilometres past the Wangaratta post office. The car travels at a constant speed of 90 km/h towards picnic stop B, which is 120 kilometres further on from A. Let t hours be the time after the car leaves point A.

a Find an expression for the distance d_1 of the car from the post office at time t hours.

b Find an expression for the distance d_2 of the car from point B at time t hours.

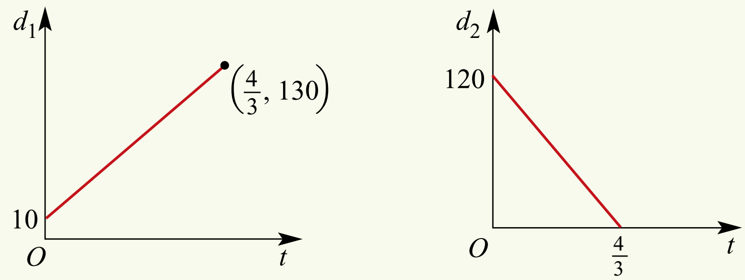
c On separate sets of axes, sketch the graphs of d_1 against t and d_2 against t and state the gradient of each graph.

Solution

a At time t the distance of the car from the post office is $10 + 90t$ kilometres.

b At time t the distance of the car from B is $120 - 90t$ kilometres.

c



Gradient = 90 Gradient = −90

Exercise 2G

Skill-sheet

Example 22

1 The weekly wage, $\$w$, of a vacuum cleaner salesperson consists of a fixed sum of \$350 plus \$20 for each cleaner sold. If n cleaners are sold per week, construct a rule that describes the weekly wage of the salesperson.

Example 23

2 **a** A train moves at 50 km/h in a straight line away from a town. Give a rule for the distance, d_1 km, from the town at time t hours after leaving the town.

b A train has stopped at a siding 80 km from the town and then moves at 40 km/h in a straight line away from the siding towards the town. Give a rule for the distance, d_2 km, from the town at time t hours after leaving the siding.

c On separate sets of axes, sketch the graphs of d_1 against t $(0 \le t \le 4)$ and d_2 against t and state the gradient of each graph.

3 **a** An initially empty container is being filled with water at a rate of 5 litres per minute. Give a rule for the volume, V litres, of water in the container at time t minutes after the filling of the container starts.

b A container contains 10 litres of water. Water is then poured in at a rate of 5 litres per minute. Give a rule for the volume, V litres, of water in the container at time t minutes after the pouring starts.

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4 The reservoir feeding an intravenous drip contains 500 mL of a saline solution. The drip releases the solution into a patient at the rate of 2.5 mL/minute.

a Construct a rule which relates the amount of solution left in the reservoir, v mL, to time, t minutes.

b State the possible values of t and v.

c Sketch the graph of the relation.

5 The cost (\$$C$) of hiring a taxi consists of two elements, a fixed flagfall and an amount that varies with the number (n) of kilometres travelled. If the flagfall is \$2.60 and the cost per kilometre is \$1.50, determine a rule which gives C in terms of n.

6 A car rental company charges \$85, plus an additional amount of 24c per kilometre.

a Write a rule to determine the total charge \$$C$ for hiring a car and travelling x kilometres.

b What would be the cost to travel 250 kilometres?

7 Two towns A and B are 200 km apart. A man leaves town A and walks at a speed of 5 km/h towards town B. Find the distance of the man from town B at time t hours after leaving town A.

8 The following table shows the extension of a spring when weights are attached to it.

x, extension (cm)	0	1	2	3	4	5	6
w, weight (g)	50	50.2	50.4	50.6	50.8	51.0	51.2

a Sketch a graph to show the relationship between x and w.

b Write a rule that describes the graph.

c What will be the extension if $w = 52.5$ g?

9 A printing firm charges \$35 for printing 600 sheets of headed notepaper and \$47 for printing 800 sheets.

a Find a formula, assuming the relationship is linear, for the charge, \$$C$, in terms of number of sheets printed, n.

b How much would they charge for printing 1000 sheets?

10 An electronic bank teller registered \$775 after it had counted 120 notes and \$975 after it had counted 160 notes.

a Find a formula for the sum registered (\$$C$) in terms of the number of notes (n) counted.

b Was there a sum already on the register when counting began?

c If so, how much?

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2H Simultaneous linear equations

In this section we revisit the geometry of simultaneous equations, first introduced in Section 1C, and also make use of parameters to explore these properties. Finally we consider some of the many applications of simultaneous equations.

The geometry of simultaneous equations

There are three possible outcomes when considering a system of two simultaneous linear equations in two unknowns:

- There is a unique solution. (Lines intersect at a point.)
- There are infinitely many solutions. (Lines coincide.)
- There is no solution. (Lines are parallel.)

Example 24

Explain why the simultaneous equations $2x + 3y = 6$ and $4x + 6y = 24$ have no solution.

Solution

First write the equations in the form $y = mx + c$. They become

$$y = -\frac{2}{3}x + 2 \qquad \text{and} \qquad y = -\frac{2}{3}x + 4$$

Each of the lines has gradient $-\frac{2}{3}$. The y-axis intercepts are 2 and 4 respectively. The equations have no solution as they correspond to parallel lines and they are different lines.

Example 25

The simultaneous equations $2x + 3y = 6$ and $4x + 6y = 12$ have infinitely many solutions. Describe these solutions through the use of a parameter.

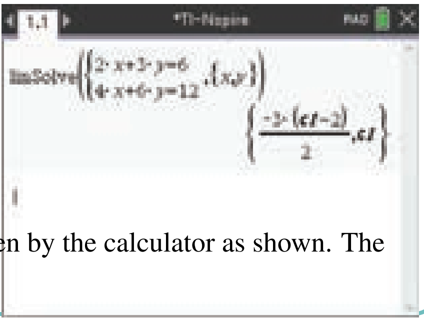
Solution

The two lines coincide, and so the solutions are all points on this line. We make use of a third variable λ as the parameter. If $y = \lambda$, then $x = \dfrac{6 - 3\lambda}{2}$. The points on the line are all points of the form $\left(\dfrac{6 - 3\lambda}{2}, \lambda\right)$.

Using the TI-Nspire

Simultaneous linear equations can be solved in a **Calculator** application.

- Use (menu) > **Algebra** > **Solve System of Equations** > **Solve System of Linear Equations**.
- Complete the pop-up screen.



The solution to this system of linear equations is given by the calculator as shown. The parameter $c1$ takes the place of λ.

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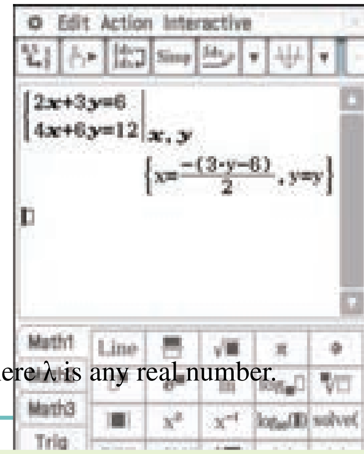
Using the Casio ClassPad

To solve the simultaneous equations $2x + 3y = 6$ and $4x + 6y = 12$:



- Open the Math1 keyboard.
- Select the simultaneous equations icon.
- Enter the two equations into the two lines and type x, y in the bottom-right square to indicate the variables.
- Select EXE.

Choose $y = \lambda$ to obtain the solution $x = \dfrac{6 - 3\lambda}{2}$, $y = \lambda$ where λ is any real number.

Example 26

The family of lines $y = mx + 2$ with varying gradient m all pass through the point $(0, 2)$.

a For what values of m does the line $y = mx + 2$ not intersect the line $y = 5x - 3$?

b For what values of m does the line $y = mx + 2$ intersect the line $y = 5x - 3$?

c If the line $y = mx + 2$ intersects the line $y = 5x - 3$ at the point $(5, 22)$, find the value of m.

Solution

a The y-axis intercept of $y = mx + 2$ is 2 and the y-axis intercept of $y = 5x - 3$ is -3. The lines will not intersect if they are parallel, that is, if they have the same gradient. So $m = 5$.

b The lines intersect when $m \neq 5$.

c If $(5, 22)$ lies on the line $y = mx + 2$, then

$$22 = 5m + 2$$
$$20 = 5m$$
$$m = 4$$

Thus the lines intersect at $(5, 22)$ when $m = 4$.

Example 27

The lines $y = x + k$ and $y = mx + 4$ intersect at $(1, 3)$. Find the values of m and k.

Solution

When $x = 1$, $y = 3$.

So $3 = 1 + k$ and $3 = m + 4$.

Hence $k = 2$ and $m = -1$.

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Example 28

The lines $(m - 2)x + y = 2$ and $mx + 2y = k$ intersect at $(2, 8)$. Find the values of m and k.

Solution

$$(m - 2)x + y = 2 \qquad (1)$$
$$mx + 2y = k \qquad (2)$$

When $x = 2$, $y = 8$. Substituting these values into equations (1) and (2), we have the equations

$$2(m - 2) + 8 = 2 \qquad (3)$$
$$2m + 16 = k \qquad (4)$$

From (3), we have $2m - 4 + 8 = 2$. Therefore $m = -1$.

From (4), we have $2(-1) + 16 = k$. Therefore $k = 14$.

Example 29

Consider the simultaneous linear equations $(m - 2)x + y = 2$ and $mx + 2y = k$. Find the values of m and k such that the system of equations has:

a no solution **b** infinitely many solutions **c** a unique solution.

Solution

$$(m - 2)x + y = 2 \qquad (1)$$
$$mx + 2y = k \qquad (2)$$

We know that for *no solution* or *infinitely many solutions*, the corresponding lines are parallel or coincide. If the corresponding lines are parallel or coincide, the gradients are the same.

Gradient of line (1) $= 2 - m$

Gradient of line (2) $= -\dfrac{m}{2}$

Hence
$$2 - m = -\frac{m}{2}$$
$$4 - 2m = -m$$
$$m = 4$$

Substitute $m = 4$ in equations (1) and (2). We have

$$2x + y = 2$$
$$4x + 2y = k$$

a There is no solution if $m = 4$ and $k \neq 4$.

b If $m = 4$ and $k = 4$, there are infinitely many solutions as the equations are equivalent.

c The solution is unique if $m \neq 4$ and k is any real number.

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Applications of simultaneous equations

Example 30

There are two possible methods for paying gas bills:

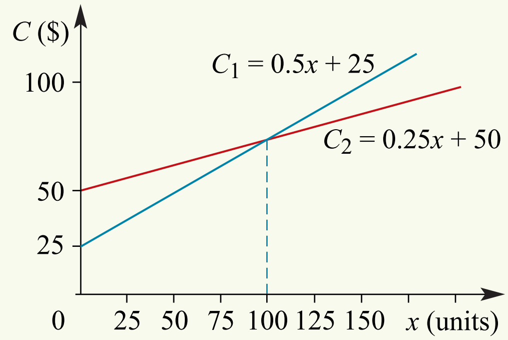
Method A A fixed charge of \$25 per quarter + 50c per unit of gas used
Method B A fixed charge of \$50 per quarter + 25c per unit of gas used

Determine the number of units which must be used before method B becomes cheaper than method A.

Solution

Let C_1 = charge (\$) using method A
C_2 = charge (\$) using method B
x = number of units of gas used

Then $C_1 = 25 + 0.5x$
$C_2 = 50 + 0.25x$



From the graph we see that method B is cheaper if the number of units exceeds 100.

The solution can be obtained by solving simultaneous linear equations:

$$C_1 = C_2$$
$$25 + 0.5x = 50 + 0.25x$$
$$0.25x = 25$$
$$x = 100$$

Example 31

Robyn and Cheryl race over 100 metres. Robyn runs so that it takes a seconds to run 1 metre, and Cheryl runs so that it takes b seconds to run 1 metre. Cheryl wins the race by 1 second. The next day they again race over 100 metres but Cheryl gives Robyn a 5-metre start so that Robyn runs 95 metres. Cheryl wins this race by 0.4 seconds. Find the values of a and b and the speed at which Robyn runs.

Solution

For the first race: Time for Robyn − time for Cheryl = 1 s.

$$100a - 100b = 1 \qquad (1)$$

For the second race: Time for Robyn − time for Cheryl = 0.4 s.

$$95a - 100b = 0.4 \qquad (2)$$

Subtract (2) from (1). This gives $5a = 0.6$ and therefore $a = 0.12$.

Substitute in (1) to find $b = 0.11$.

Robyn's speed $= \dfrac{1}{0.12} = \dfrac{25}{3}$ m/s.

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Summary 2H

- There are three cases for a system of two linear equations with two variables:
  - unique solution (lines intersect at a point), e.g. $y = 2x + 3$ and $y = 3x + 3$
  - infinitely many solutions (lines coincide), e.g. $y = 2x + 3$ and $2y = 4x + 6$
  - no solution (lines are parallel), e.g. $y = 2x + 3$ and $y = 2x + 4$.
- There are many applications of simultaneous linear equations with two variables. The problems often arise by working with two quantities both changing at a constant but often different rate.

Skill-sheet

Exercise 2H

Example 24

1 Explain why the simultaneous equations $x + y = 6$ and $2x + 2y = 13$ have no solution.

Example 25

2 The simultaneous equations $x + y = 6$ and $2x + 2y = 12$ have infinitely many solutions. Describe these solutions through the use of a parameter.

Example 26

3 The family of lines $y = mx + 6$ with varying gradient m all pass through the point $(0, 6)$.

a For what values of m does the line $y = mx + 6$ not intersect the line $y = 4x - 5$?

b For what values of m does the line $y = mx + 6$ intersect the line $y = 4x - 5$?

c If the line $y = mx + 6$ intersects the line $y = 4x - 5$ at the point $(5, 15)$, find the value of m.

Example 27

4 The lines $y = 2x + k$ and $y = mx - 4$ intersect at $(2, 6)$. Find the values of m and k.

Example 28

5 The lines $(m - 2)x + y = 4$ and $mx + 3y = k$ intersect at $(2, 8)$. Find the values of m and k.

Example 29

6 Find the value of m for which the simultaneous equations $mx - y = 5$ and $3x + y = 6$ have no solution.

7 Find the value of m for which the pair of simultaneous equations $3x + my = 5$ and $(m + 2)x + 5y = m$ have:

a infinitely many solutions

b no solutions.

Example 30

8 Two bicycle hire companies have different charges. Company A charges \$$C$, according to the rule $C = 10t + 20$, where t is the time in hours for which a bicycle is hired. Company B charges \$$C$, according to the rule $C = 8t + 30$.

a Sketch each of the graphs on the same set of axes.

b Find the time, t, for which the charge of both companies is the same.

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Example 31

9 John and Michael race over 50 metres. John runs so that it takes a seconds to run 1 metre and Michael runs so that it takes b seconds to run 1 metre. Michael wins the race by 1 second. The next day they again race over 50 metres but Michael gives John a 3-metre start so that John runs 47 metres. Michael wins this race by 0.1 seconds. Find the values of a and b and the speed at which Michael runs.

10 The distances, d_A km and d_B km, of cyclists A and B travelling along a straight road from a town hall step are given respectively by $d_A = 10t + 15$ and $d_B = 20t + 5$, where t is the time in hours after 1 p.m.

a Sketch each of the graphs on the one set of axes.

b Find the time in hours at which the two cyclists are at the same distance from the town hall step.

11 A school wishes to take some of its students on an excursion. If they travel by tram it will cost the school \$2.80 per student. Alternatively, the school can hire a bus at a cost of \$54 for the day plus a charge of \$1 per student.

CAS

a For each mode of transport, write an expression for the cost (\$$C$) of transport in terms of the number of students (x).

b On one set of axes, draw the graph of cost, \$$C$, versus number of students, x, for each mode of transport.

c Determine for how many students it will be more economical to hire the bus.

12 Anne and Maureen live in towns that are 57 km apart. Anne sets out at 9 a.m. one day to ride her bike to Maureen's town at a constant speed of 20 km/h. At the same time Maureen sets out to ride to Anne's town at a constant speed of 18 km/h.

a Write down a rule for the distance, d km, that each of them is from Anne's place at a time t minutes after 9 a.m.

b On the same set of axes, draw graphs of the distance, d km, versus time, t minutes after 9 a.m., for each cyclist.

c Find the time at which they will meet.

d How far has each of them travelled when they meet?

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Chapter summary

Assign-ment

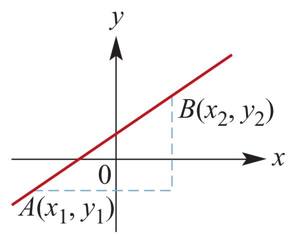
Nrich

- The **midpoint** of the line segment joining (x_1, y_1) and (x_2, y_2) is the point with coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

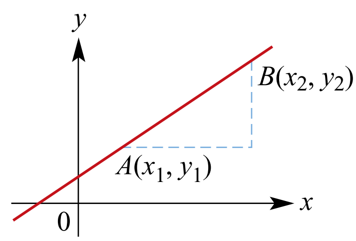
- The **distance** between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



- The **gradient** of a straight line joining two points:

$$\text{Gradient } m = \frac{y_2 - y_1}{x_2 - x_1}$$



- For a line with gradient m, the **angle of slope** (θ) can be found using

$$m = \tan\theta$$

where θ is the angle the line makes with the positive direction of the x-axis.

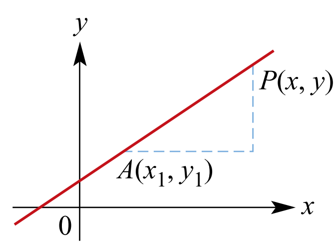
- The gradient–intercept form of the equation of a straight line is

$$y = mx + c$$

where m is the gradient and c is the y-axis intercept.

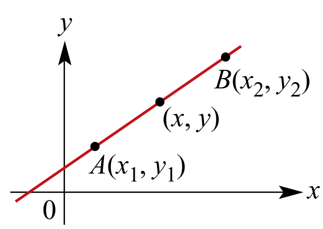
- The equation of a line passing through a given point (x_1, y_1) and having gradient m is

$$y - y_1 = m(x - x_1)$$



- The equation of a line passing through two given points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = m(x - x_1) \quad \text{where} \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$



- Two straight lines are **perpendicular** to each other if and only if the product of their gradients is −1 (or if one is horizontal and the other vertical):

$$m_1 m_2 = -1$$

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Technology-free questions

1 Find the length and the coordinates of the midpoint of the line segment joining each of the following pairs of points:

a $A(1, 2)$ and $B(5, 2)$ **b** $A(-4, -2)$ and $B(3, -7)$ **c** $A(3, 4)$ and $B(7, 1)$

2 Find the gradients of the lines joining each of the following pairs of points:

a $(4, 3)$ and $(8, 12)$ **b** $(-3, 4)$ and $(8, -6)$ **c** $(2, 1)$ and $(2, 9)$

d $(0, a)$ and $(a, 0)$ **e** $(0, 0)$ and (a, b) **f** $(0, b)$ and $(a, 0)$

3 Find the equation of the straight line of gradient 4 which passes through the point with coordinates:

a $(0, 0)$ **b** $(0, 5)$ **c** $(1, 6)$ **d** $(3, 7)$

4 **a** The point $(1, a)$ lies on the line with equation $y = 3x - 5$. Find the value of a.

b The point $(b, 15)$ lies on the line with equation $y = 3x - 5$. Find the value of b.

5 Find the equation of the straight line joining the points $(-5, 2)$ and $(3, -4)$.

6 Find the equation of the straight line of gradient $-\frac{2}{3}$ which passes through $(-4, 1)$.

7 Write down the equation of the straight line that:

a passes through $(5, 11)$ and is parallel to the x-axis

b passes through $(0, -10)$ and is parallel to the line with equation $y = 6x + 3$

c passes through the point $(0, -1)$ and is perpendicular to the line with equation $3x - 2y + 5 = 0$.

8 Find the equation of the straight line which passes through the point $(2, 3)$ and is inclined at $30°$ to the positive direction of the x-axis.

9 Find the equation of the straight line which passes through the point $(-2, 3)$ and makes an angle of $135°$ with the positive direction of the x-axis.

10 Find the equation of the straight line passing through the point $(4, 8)$ and which is perpendicular to the line with equation $y = -3x + 2$.

11 A straight line has equation $y = 2x + 1$. The points with coordinates $(0, a)$, $(b, 0)$, $(2, d)$ and $(e, 7)$ lie on this line. Find the values of a, b, d and e.

12 Sketch the graph of each of the following by first determining axis intercepts. Clearly label each axis intercept.

a $y = 2x - 8$ **b** $3x + y = 6$ **c** $3x + y + 6 = 0$

d $y - 2x - 8 = 0$ **e** $y = -6x + 6$ **f** $2x + 5y + 10 = 0$

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13 A family of straight lines satisfy the rule $y = ax + 2$.

a Find the equation of the straight line in this family for which $y = 6$ when $x = 2$.

b **i** Find the x-axis intercept of the line with equation $y = ax + 2$.

ii If $a < 0$, find the values of a for which the x-axis intercept is greater than 1.

c Find the coordinates of the point of intersection of the line with equation $y = x + 3$ and the line with equation $y = ax + 2$, given that $a \neq 1$.

Multiple-choice questions

1 The coordinates of the midpoint of AB, where A has coordinates $(4, 12)$ and B has coordinates $(6, 2)$, are

A $(4, 8)$ **B** $(4.5, 8)$ **C** $(5, 8)$ **D** $(5, 7)$ **E** $(1, 5)$

2 If $(6, 3)$ is the midpoint of the line segment joining the points $(-4, y)$ and $(x, -6)$, then the value of $x + y$ is

A 0 **B** 16 **C** 20 **D** -10 **E** 28

3 The gradient of the line passing through the points $(5, -8)$ and $(6, -10)$ is

A -2 **B** $-\frac{1}{2}$ **C** $\frac{1}{2}$ **D** $-\frac{1}{18}$ **E** $\frac{3}{2}$

4 The gradient of the line passing through points $(4a, 2a)$ and $(9a, -3a)$ is

A a **B** $-5a$ **C** 1 **D** -5 **E** -1

5 The equation of the straight line with gradient 3 that passes through the point $(1, 9)$ is

A $y = x + 9$ **B** $y = 3x + 9$ **C** $y = 3x + 6$
D $y = -\frac{1}{3}x + 1$ **E** $y = -\frac{1}{3}x + 6$

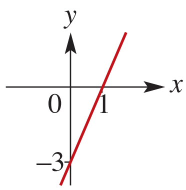
6 A straight line passes through the points $(2, -6)$ and $(-2, -14)$. The equation of the line is

A $y = x - 8$ **B** $y = \frac{1}{2}x - 7$ **C** $y = \frac{1}{2}x - 10$
D $y = 2x - 10$ **E** $y = -\frac{1}{2}x - 8$

7 The line with equation $y = 2x - 6$ passes through the point $(a, 2)$. The value of a is

A 2 **B** 4 **C** 5 **D** -4 **E** -2

8 The relation with graph as shown has rule



A $y = -3x - 3$ **B** $y = -\frac{1}{3}x - 3$
C $y = \frac{1}{3}x - 3$ **D** $y = 3x + 3$
E $y = 3x - 3$

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Review

9 If two lines $5x - y + 7 = 0$ and $ax + 2y - 11 = 0$ are parallel, then a equals

A -5 **B** 5 **C** -10 **D** 10 **E** $-\frac{1}{2}$

10 The cost ($\$C$) of hiring a car is given by the formula $C = 2.5x + 65$, where x is the number of kilometres travelled. A person is charged \$750 for the hire of the car. The number of kilometres travelled was

A 65 **B** 145 **C** 160 **D** 200 **E** 274

CAS

11 The solution of the two simultaneous equations $2ax + 2by = 3$ and $3ax - 2by = 7$ for x and y is

A $x = 2a,\ y = \dfrac{3 - 4a^2}{2b}$ **B** $x = 2,\ y = \dfrac{3 - 4a}{2b}$ **C** $x = \dfrac{2}{a},\ y = -\dfrac{1}{2b}$

D $x = 0,\ y = 0$ **E** $x = 3a,\ y = 7b$

Extended-response questions

1 The cost of hiring a motor cruiser consists of a down payment of \$500 and a running charge of \$100 per day, or part of a day. The cost of fuel is \$27.50 per day. There is also a charge of \$50 for filling the freshwater tanks and charging the batteries. Food for a cruise of n days costs \$62.50 per day.

a Give a formula for C, the total cost in dollars of hiring the cruiser for n days (all costs to be included).

b For how many days can a cruiser be hired if the cost of a cruise is to be no more than \$3000?

c A rival company has a fixed rate of \$300 per day. For how many days would it be cheaper to hire from this company?

2 The cost of fitting a new plug and cable for an electric drill is $\$C$, when the length of the cable is x metres and $C = 4.5 + 1.8x$.

a What meaning could be given for the constant term 4.5?

b What could be the meaning of the coefficient 1.8?

c What would be the gradient of the graph of C against x?

d What length of cable would give a total cost of \$24.50?

3 The profit made on a single journey of an Easyride bus tour is $\$P$, when there are x empty seats and $P = 1020 - 24x$.

a What do you think is the meaning of the constant term 1020?

b What is the least number of empty seats which would result in a loss on a single journey?

c Suggest a meaning for the coefficient 24.

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4 A quarterly electricity bill shows the following charges:

- For the first 50 kWh (kilowatt hours): 9.10c per kWh
- For the next 150 kWh: 5.80c per kWh
- Thereafter: 3.56c per kWh

a Write down a formula relating cost, $\$C$, to n, the number of kWh of electricity used:

i for the first 50 kWh

ii for the next 150 kWh

iii for more than 200 kWh.

b Draw a graph of C against n. Use the graph, or otherwise, to determine the charges for:

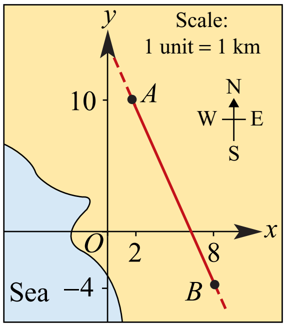
i 30 kWh

ii 90 kWh

iii 300 kWh

c How much electricity could be used for a cost of \$20?

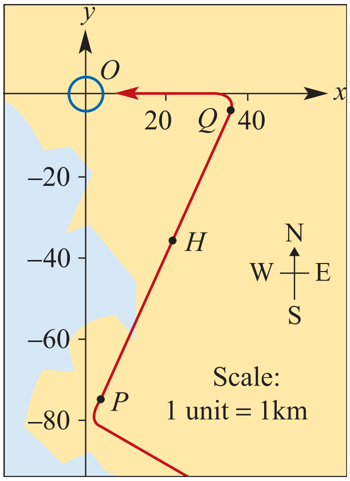
5 O is the position of the air traffic control tower at an airport. An aircraft travelling in a straight line is identified at $A(2, 10)$ and again at $B(8, -4)$.



a What is the equation that describes the flight path of the aircraft?

b How far south of O is the aircraft when $x = 15$ km?

6 A new light beacon is proposed at $P(4, -75)$ for air traffic flying into an airport located at $O(0, 0)$. It is intended that the aircraft should follow a course over beacons at P and $Q(36, -4)$, turning at Q towards the runway at O.



a Would a direct line from P to Q pass directly over a hospital located at $H(20, -36)$?

b If not, state how far east or west of H the aircraft would be when the y-coordinate of an aircraft's flight path is -36.

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7 Wheelrite, a small company that manufactures garden wheelbarrows, has overhead expenses of \$30 000 per year. In addition, it costs \$40 to manufacture each wheelbarrow.

- **a** Write a rule which determines the total cost, \$$C$, of manufacturing x wheelbarrows per year.
- **b** If the annual production is 6000 wheelbarrows, what is the overall cost per wheelbarrow?
- **c** How many wheelbarrows must be made so that the overall cost is \$46 per wheelbarrow?
- **d** Wheelrite sells wheelbarrows to retailers for \$80 each. Write a rule which determines the revenue, \$$R$, from the sale of x wheelbarrows to retailers.
- **e** Sketch the graphs for C and R against x on the same axes.
- **f** What is the minimum number of wheelbarrows that must be produced for Wheelrite to make a profit each year?
- **g** Write a rule which determines the profit, \$$P$, from the manufacture and sale of x number of wheelbarrows.

8 An electricity supply authority is offering customers a choice of two methods of paying electricity bills. Method 1 involves payment annually and method 2 involves payment each quarter (that is, every three months). The charges for each method are as follows:

Method 1 – per year		Method 2 – per quarter	
Fixed charge	\$100	Fixed charge	\$27.50
Price per unit	\$0.08125	Price per unit	\$0.075

- **a** Suppose a customer used 1560 units of electricity in a year. Calculate which is the cheaper method of payment.
- **b** Copy and then complete the following table:

	Number of units of electricity			
	0	1000	2000	3000
Cost (\$) calculated by method 1				
Cost (\$) calculated by method 2				

- **c** If C_1 is the cost by method 1, C_2 is the cost by method 2, and x is the number of units of electricity used in a year, write down the two formulas which show the cost of x units calculated by each method.
- **d** Use your calculator to find the number of units of electricity used in a year such that the cost is the same for both methods.

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