

# Glossary

## A

**Acceleration** [p. 642] the rate of change of a particle's velocity with respect to time

**Acceleration, average** [p. 642] The average acceleration of a particle for the time interval  $[t_1, t_2]$  is given by  $\frac{v_2 - v_1}{t_2 - t_1}$ , where  $v_2$  is the velocity at time  $t_2$  and  $v_1$  is the velocity at time  $t_1$ .

**Acceleration, instantaneous** [p. 642]

$$a = \frac{dv}{dt}$$

**Addition rule for choices** [p. 364] To determine the total number of choices from disjoint alternatives, simply add up the number of choices available for each alternative.

**Addition rule for probability** [p. 324] The probability of  $A$  or  $B$  or both occurring is given by  $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

**Algorithm** [p. 735] a finite, unambiguous sequence of instructions for performing a specific task

**Amplitude of circular functions** [p. 487] The distance between the mean position and the maximum position is called the amplitude. The graph of  $y = \sin x$  has an amplitude of 1.

**Antiderivative** [p. 595] To find the general antiderivative of  $f(x)$ : If  $F'(x) = f(x)$ , then  $\int f(x) dx = F(x) + c$  where  $c$  is an arbitrary real number.

**Antidifferentiation rules** [pp. 596, 687]

- $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \in \mathbb{Q} \setminus \{-1\}$
- $\int f(x) + g(x) dx = \int f(x) dx + \int g(x) dx$
- $\int kf(x) dx = k \int f(x) dx$

**Arrangements** [p. 367] counted when order is important. The number of ways of selecting and arranging  $r$  objects from a total of  $n$  objects is

$$\frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1)$$

**Asymptote** [pp. 138, 690] A straight line is an asymptote of the graph of a function  $y = f(x)$  if the graph of  $y = f(x)$  gets arbitrarily close to the straight line. An asymptote can be horizontal, vertical or oblique.

## B

**Binomial distribution** [p. 397] The probability of observing  $x$  successes in  $n$  independent trials, each with probability of success  $p$ , is given by

$$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \dots, n$$

where  $\binom{n}{x} = \frac{n!}{x!(n-x)!}$

**Binomial experiment** [p. 397]

- The experiment consists of a number,  $n$ , of identical trials.
- Each trial results in one of two outcomes, which are usually designated either a success,  $S$ , or a failure,  $F$ .
- The probability of success on a single trial,  $p$ , is constant for all trials.
- The trials are independent (so that the outcome of any trial is not affected by the outcome of any previous trial).

**Bisection method** [p. 253] A numerical method for solving polynomial equations. If the values of  $f(a)$  and  $f(b)$  have opposite signs, where  $a < b$ , then the equation  $f(x) = 0$  has a solution in the interval  $[a, b]$ . The method is to bisect the interval and replace it with one half or the other.

## C

**Chain rule** [p. 679] The chain rule can be used to differentiate a complicated function  $y = f(x)$  by transforming it into two simpler functions, which are ‘chained’ together:

$$x \xrightarrow{h} u \xrightarrow{g} y$$

Using Leibniz notation, the chain rule is stated as

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

**Circle, general equation** [p. 148] The general equation for a circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

where the centre of the circle is the point  $(h, k)$  and the radius is  $r$ .

**Circular functions** [pp. 478, 480] the sine, cosine and tangent functions

**Circular functions, exact values** [p. 484]

|               |   |                      |                      |                      |                 |
|---------------|---|----------------------|----------------------|----------------------|-----------------|
| $\theta$      | 0 | $\frac{\pi}{6}$      | $\frac{\pi}{4}$      | $\frac{\pi}{3}$      | $\frac{\pi}{2}$ |
| $\sin \theta$ | 0 | $\frac{1}{2}$        | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1               |
| $\cos \theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$        | 0               |
| $\tan \theta$ | 0 | $\frac{1}{\sqrt{3}}$ | 1                    | $\sqrt{3}$           | undef           |

**Coefficient** [p. 206] the number that multiplies a power of  $x$  in a polynomial. E.g. for  $2x^5 - 7x^2 + 4$ , the coefficient of  $x^2$  is  $-7$ .

**Combinations** [p. 373] *see* selections

**Complement,  $A'$**  [pp. 309, 322] the set of outcomes that are in the sample space,  $\epsilon$ , but not in  $A$ . The probability of the event  $A'$  is

$$\Pr(A') = 1 - \Pr(A)$$

**Complementary relationships** [p. 503]

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \quad \sin\left(\frac{\pi}{2} + \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta \quad \cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta$$

**Conditional probability** [p. 331] the probability of an event  $A$  occurring when it is known that some event  $B$  has occurred, given by

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

**Constant function** [p. 177] a function  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = a$

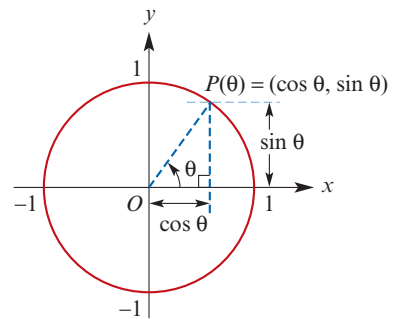
**Continuous function** [p. 604] A function  $f$  is continuous at the point  $x = a$  if the following conditions are met:

- $f(x)$  is defined at  $x = a$
- $\lim_{x \rightarrow a} f(x) = f(a)$

We say that a function is continuous everywhere if it is continuous for all real numbers.

**Coordinates** [p. 32] an ordered pair of numbers that identifies a point in the Cartesian plane; the first number identifies the position with respect to the  $x$ -axis, and the second number identifies the position with respect to the  $y$ -axis

**Cosine function** [p. 478] cosine  $\theta$  is defined as the  $x$ -coordinate of the point  $P$  on the unit circle where  $OP$  forms an angle of  $\theta$  radians with the positive direction of the  $x$ -axis.



**Cubic function** [p. 205] A polynomial of degree 3 is called a cubic, and is a function  $f$  with rule  $f(x) = ax^3 + bx^2 + cx + d$ , where  $a \neq 0$ .

## D

**Definite integral** [pp. 703, 708]  $\int_a^b f(x) dx$  denotes the signed area enclosed by the graph of  $y = f(x)$  between  $x = a$  and  $x = b$ .

*See also* fundamental theorem of calculus

**Degree of a polynomial** [pp. 76, 206] given by the highest power of  $x$  with a non-zero coefficient. For example, the polynomial  $2x^5 - 7x^2 + 4$  has degree 5.

**Dependent trials or events** [p. 393] The probability of one event is influenced by the outcome of another event. *See* sampling without replacement

**Derivative function** [p. 571] also called the gradient function. The derivative  $f'$  of a function  $f$  is given by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Desk check** [p. 739] To carry out a desk check of an algorithm, you carefully follow the algorithm step by step, and construct a table of the values of all the variables after each step.

**Difference of two cubes** [p. 222]

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

**Difference of two squares** [pp. 79, 83]

$$x^2 - y^2 = (x - y)(x + y)$$

**Differentiable** [p. 607] A function  $f$  is said to be differentiable at the point  $x = a$  if

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \text{ exists.}$$

**Differentiation rules** [pp. 576, 684]

- Power:  $f(x) = x^n$ ,  $f'(x) = nx^{n-1}$ , for  $n \in \mathbb{Q} \setminus \{0\}$
- Constant:  $f(x) = c$ ,  $f'(x) = 0$
- Sum:  $f(x) = g(x) + h(x)$ ,  $f'(x) = g'(x) + h'(x)$
- Multiple:  $f(x) = k g(x)$ ,  $f'(x) = k g'(x)$

**Dilation from the x-axis** [p. 270] A dilation of factor  $b$  from the  $x$ -axis is described by the rule  $(x, y) \rightarrow (x, by)$ . The curve with equation  $y = f(x)$  is mapped to the curve with equation  $y = bf(x)$ .

**Dilation from the y-axis** [p. 270] A dilation of factor  $a$  from the  $y$ -axis is described by the rule  $(x, y) \rightarrow (ax, y)$ . The curve with equation  $y = f(x)$  is mapped to the curve with equation  $y = f\left(\frac{x}{a}\right)$ .

**Discontinuity** [p. 604] A function is said to be discontinuous at a point if it is not continuous at that point.

**Discrete random variable** [p. 387] a random variable  $X$  which can take only a countable number of values, usually whole numbers

**Discriminant,  $\Delta$ , of a quadratic** [p. 110] the expression  $b^2 - 4ac$ , which is part of the quadratic formula. For the quadratic equation  $ax^2 + bx + c = 0$ :

- If  $b^2 - 4ac > 0$ , there are two solutions.
- If  $b^2 - 4ac = 0$ , there is one solution.
- If  $b^2 - 4ac < 0$ , there are no real solutions.

**Disjoint sets** [pp. 165, 323] If sets  $A$  and  $B$  have no elements in common, we say  $A$  and  $B$  are disjoint and write  $A \cap B = \emptyset$ .

**Displacement** [p. 640] The displacement of a particle moving in a straight line is defined as the change in position of the particle.

**Distance between two points** [p. 34] The distance between points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Division of polynomials** [p. 211] When we divide the polynomial  $P(x)$  by the polynomial  $D(x)$  we obtain two polynomials,  $Q(x)$  the quotient and  $R(x)$  the remainder, such that

$$P(x) = D(x)Q(x) + R(x)$$

and either  $R(x) = 0$  or  $R(x)$  has degree less than  $D(x)$ .

**Domain** [p. 169] the set of all the first coordinates of the ordered pairs in a relation

## E

**Element** [p. 165] a member of a set.

- If  $x$  is an element of a set  $A$ , we write  $x \in A$ .
- If  $x$  is *not* an element of a set  $A$ , we write  $x \notin A$ .

**Empty set,  $\emptyset$**  [pp. 165, 322] the set that has no elements

**Even function** [p. 245] A function  $f$  is even if  $f(-x) = f(x)$ . This means that the graph is symmetric about the  $y$ -axis.

**Event** [p. 305] a subset of the sample space. It may consist of a single outcome, or it may consist of several outcomes.

**Exponential function** [p. 424] a function  $f(x) = ka^x$ , where  $k$  is a non-zero constant and the base  $a$  is a positive real number other than 1

## F

**Factor** [p. 211] a number or expression that divides another number or expression without remainder

**Factor theorem** [p. 218] If  $\beta x + \alpha$  is a factor of a polynomial  $P(x)$ , then  $P\left(-\frac{\alpha}{\beta}\right) = 0$ . Conversely, if  $P\left(-\frac{\alpha}{\beta}\right) = 0$ , then  $\beta x + \alpha$  is a factor of  $P(x)$ .

**Factorise** [p. 82] express as a product of factors

**Formula** [p. 22] an equation containing symbols that states a relationship between two or more quantities; e.g.  $A = \ell w$  (area = length  $\times$  width). The value of  $A$ , the subject of the formula, can be found by substituting given values of  $\ell$  and  $w$ .

**Function** [p. 175] a relation such that for each  $x$ -value there is only one corresponding  $y$ -value. This means that, if  $(a, b)$  and  $(a, c)$  are ordered pairs of a function, then  $b = c$ .

**Function, one-to-one** [p. 183] different  $x$ -values map to different  $y$ -values. For example, the function  $y = x + 1$  is one-to-one. But  $y = x^2$  is not one-to-one, as both 2 and  $-2$  map to 4.

**Fundamental theorem of calculus** [p. 705]

If  $f$  is a continuous function on an interval  $[a, b]$ , then

$$\int_a^b f(x) dx = F(b) - F(a)$$

where  $F$  is any antiderivative of  $f$  and  $\int_a^b f(x) dx$  is the definite integral from  $a$  to  $b$ .

**G**

**Gradient function** *see* derivative function

**Gradient of a line** [pp. 36, 37] The gradient is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

where  $(x_1, y_1)$  and  $(x_2, y_2)$  are the coordinates of two points on the line. The gradient of a vertical line (parallel to the  $y$ -axis) is undefined.

**H**

**Horizontal-line test** [p. 183] If a horizontal line can be drawn anywhere on the graph of a function and it only ever intersects the graph a maximum of once, then the function is *one-to-one*.

**Hybrid function** *see* piecewise-defined function

**I**

**Implied domain** *see* maximal domain

**Indefinite integral** *see* antiderivative

**Independent events** [p. 339] Two events  $A$  and  $B$  are independent if  $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$  or  $\Pr(A | B) = \Pr(A)$  or  $\Pr(B | A) = \Pr(B)$

**Independent trials** *see* sampling with replacement

**Index laws** [p. 425]

- To multiply two powers with the same base, add the indices:  $a^m \times a^n = a^{m+n}$
- To divide two powers with the same base, subtract the indices:  $a^m \div a^n = a^{m-n}$
- To raise a power to another power, multiply the indices:  $(a^m)^n = a^{m \times n}$
- Rational indices:  $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$
- For base  $a \in \mathbb{R}^+ \setminus \{1\}$ , if  $a^x = a^y$ , then  $x = y$ .

**Inequality** [p. 19] a mathematical statement that contains an inequality symbol rather than an equals sign; e.g.  $2x + 1 < 4$

**Integers** [p. 166]  $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

**Integration (definite), properties** [p. 712]

- $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$
- $\int_a^a f(x) dx = 0$
- $\int_a^b k f(x) dx = k \int_a^b f(x) dx$
- $\int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$
- $\int_a^b f(x) dx = - \int_b^a f(x) dx$

**Intersection of sets** [pp. 165, 322] The intersection of two sets  $A$  and  $B$ , written  $A \cap B$ , is the set of all elements common to  $A$  and  $B$ .

**Interval** [p. 167] a subset of the real numbers of the form  $[a, b]$ ,  $[a, b)$ ,  $(a, \infty)$ , etc.

**Inverse function** [p. 191] For a one-to-one function  $f$ , the inverse function  $f^{-1}$  is defined by  $f^{-1}(x) = y$  if  $f(y) = x$ , for  $x \in \text{ran } f$ ,  $y \in \text{dom } f$ .

**Irrational number** [p. 166] a real number that is not rational; e.g.  $\pi$  and  $\sqrt{2}$

**L**

**Law of total probability** [p. 334] In the case of two events,  $A$  and  $B$ :

$$\Pr(A) = \Pr(A | B) \Pr(B) + \Pr(A | B') \Pr(B')$$

**Leading term** [p. 206] The leading term,  $a_n x^n$ , of a polynomial is the term of highest index among those terms with a non-zero coefficient. E.g. the leading term of  $2x^5 - 7x^2 + 4$  is  $2x^5$ .

**Limit** [p. 601] The notation  $\lim_{x \rightarrow a} f(x) = \ell$  says that the limit of  $f(x)$ , as  $x$  approaches  $a$ , is  $\ell$ . We can also say: 'As  $x$  approaches  $a$ ,  $f(x)$  approaches  $\ell$ .'

**Limits, properties** [p. 602]

- Sum:  $\lim_{x \rightarrow a} (f(x) + g(x)) = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
- Multiple:  $\lim_{x \rightarrow a} (k f(x)) = k \lim_{x \rightarrow a} f(x)$
- Product:  $\lim_{x \rightarrow a} (f(x) g(x)) = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$
- Quotient:  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ , if  $\lim_{x \rightarrow a} g(x) \neq 0$

**Linear equation** [p. 2] a polynomial equation of degree 1; e.g.  $2x + 1 = 0$

**Linear function** [p. 177] a function  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = mx + c$ ; e.g.  $f(x) = 3x + 1$

**Literal equation** [p. 5] an equation for the variable  $x$  in which the coefficients of  $x$ , including the constants, are pronumerals; e.g.  $ax + b = c$

**Logarithm** [p. 445] If  $a \in \mathbb{R}^+ \setminus \{1\}$  and  $x \in \mathbb{R}$ , then the statements  $a^x = y$  and  $\log_a y = x$  are equivalent.

**Logarithm laws** [p. 445]

- $\log_a(mn) = \log_a m + \log_a n$
- $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$
- $\log_a\left(\frac{1}{n}\right) = -\log_a n$
- $\log_a(m^p) = p \log_a m$
- $\log_a 1 = 0$  and  $\log_a a = 1$

**Logarithms, change of base** [p. 450]

$$\log_a c = \frac{\log_b c}{\log_b a}$$

## M

**Maximal domain** [pp. 173, 184] When the rule for a relation is given and no domain is specified, then the domain taken is the largest for which the rule has meaning.

**Maximum and minimum value** [p. 635] For a continuous function  $f$  defined on an interval  $[a, b]$ :

- if  $M$  is a value of the function such that  $f(x) \leq M$  for all  $x \in [a, b]$ , then  $M$  is the *absolute maximum* value of the function
- if  $N$  is a value of the function such that  $f(x) \geq N$  for all  $x \in [a, b]$ , then  $N$  is the *absolute minimum* value of the function.

**Midpoint of a line segment** [p. 33] If  $P(x, y)$  is the midpoint of the line segment joining  $A(x_1, y_1)$  and  $B(x_2, y_2)$ , then

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$

**Monic polynomial** [p. 206] a polynomial whose leading term has coefficient 1; e.g.  $x^3 + 4x^2 + 5$

**Multiplication rule for choices** [p. 365] When sequential choices are involved, the total number of possibilities is found by multiplying the number of choices at each successive stage.

**Multiplication rule for probability** [p. 333] the probability of events  $A$  and  $B$  both occurring is  $\Pr(A \cap B) = \Pr(A | B) \times \Pr(B)$

**Multi-stage experiment** [p. 317] a random experiment that could be considered to take place in more than one stage; e.g. tossing two coins

**Mutually exclusive** [p. 323] Two events  $A$  and  $B$  are said to be mutually exclusive if they have no outcomes in common, i.e. if  $A \cap B = \emptyset$ .

## N

$n!$  [p. 369] The notation  $n!$  (read as 'n factorial') is an abbreviation for the product of all the integers from  $n$  down to 1:

$$n! = n \times (n - 1) \times (n - 2) \times (n - 3) \times \cdots \times 2 \times 1$$

**Natural numbers** [p. 166]  $\mathbb{N} = \{1, 2, 3, 4, \dots\}$

${}^n C_r$  [p. 374] the number of combinations of  $n$  objects in groups of size  $r$ :

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

An alternative notation for  ${}^n C_r$  is  $\binom{n}{r}$ .

**Newton's method** [p. 649] A method for finding successive approximations to a solution of an equation  $f(x) = 0$  using the iterative formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

**Normal line, equation** [p. 617] Let  $(x_1, y_1)$  be a point on the curve  $y = f(x)$ . If  $f$  is differentiable at  $x = x_1$ , the equation of the normal at  $(x_1, y_1)$  is

$$y - y_1 = \frac{-1}{f'(x_1)}(x - x_1)$$

**Null factor theorem** [p. 87] If  $ab = 0$ , then  $a = 0$  or  $b = 0$ .

## O

**Odd function** [p. 245] A function  $f$  is odd if  $f(-x) = -f(x)$ . The graph of an odd function has rotational symmetry about the origin.

**Ordered pair** [p. 169] a pair of elements, denoted  $(x, y)$ , where  $x$  is the first coordinate and  $y$  is the second coordinate

## P

**Period of a function** [p. 487] A function  $f$  with domain  $\mathbb{R}$  is periodic if there is a positive constant  $a$  such that  $f(x + a) = f(x)$  for all  $x$ . The smallest such  $a$  is called the period of  $f$ . For example, the period of the sine function is  $2\pi$ , as  $\sin(x + 2\pi) = \sin x$ .

**Permutations** [p. 367] *see* arrangements

**Piecewise-defined function** [p. 187] a function which has different rules for different subsets of its domain

**Polynomial function** [p. 206] A polynomial has a rule of the type  $y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ ,  $n \in \mathbb{N} \cup \{0\}$  where  $a_0, a_1, \dots, a_n$  are numbers called coefficients.

**Position** [pp. 554, 639] For a particle moving in a straight line, the position of the particle relative to a point  $O$  on the line is determined by its distance from  $O$  and whether it is to the right or left of  $O$ . The direction to the right of  $O$  is positive.

**Power function** [p. 245] a function of the form  $f(x) = x^r$ , where  $r$  is a non-zero real number

**Probability** [p. 304] a numerical value assigned to the likelihood of an event occurring. If the event  $A$  is impossible, then  $\Pr(A) = 0$ ; if the event  $A$  is certain, then  $\Pr(A) = 1$ ; otherwise  $0 < \Pr(A) < 1$ .

**Probability distribution** [p. 388] a function, denoted  $p(x)$  or  $\Pr(X = x)$ , which assigns a probability to each value of a discrete random variable  $X$

**Probability table** [p. 327] a table used for illustrating a probability problem diagrammatically

**Pseudocode** [pp. 735, 747] a notation for describing algorithms that is less formal than a programming language

**Pythagorean identity** [p. 504]  
 $\cos^2 \theta + \sin^2 \theta = 1$

## Q

**Q** [p. 166] the set of all rational numbers

**Quadratic formula** [p. 106] The solutions of the quadratic equation  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Quadratic function** [p. 76] A quadratic has a rule of the form  $y = ax^2 + bx + c$ , where  $a$ ,  $b$  and  $c$  are constants and  $a \neq 0$ .

**Quadratic, turning point form** [p. 91] The turning point form of a quadratic function is  $y = a(x - h)^2 + k$ , where  $(h, k)$  is the turning point.

**Quartic function** [p. 205] A polynomial of degree 4 is called a quartic, and is a function  $f$  with rule  $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ , where  $a \neq 0$ .

## R

$\mathbb{R}^+$  [p. 167]  $\{x : x > 0\}$ , positive real numbers

$\mathbb{R}^-$  [p. 167]  $\{x : x < 0\}$ , negative real numbers

$\mathbb{R} \setminus \{0\}$  [p. 167] real numbers excluding 0

$\mathbb{R}^2$  [p. 266]  $\{(x, y) : x, y \in \mathbb{R}\}$ ; i.e.  $\mathbb{R}^2$  is the set of all ordered pairs of real numbers

**Radian** [p. 475] One radian (written  $1^\circ$ ) is the angle subtended at the centre of the unit circle by an arc of length 1 unit.

**Random experiment** [p. 305] an experiment, such as the rolling of a die, in which the outcome of a single trial is uncertain but observable

**Random variable** [p. 387] a variable that takes its value from the outcome of a random experiment; e.g. the number of heads observed when a coin is tossed three times

**Range** [p. 169] the set of all the second coordinates of the ordered pairs in a relation

**Rational number** [p. 166] a number that can be written as  $\frac{p}{q}$ , for some integers  $p$  and  $q$  with  $q \neq 0$

**Rational-root theorem** [p. 220]

Let  $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$  be a polynomial of degree  $n$  with all coefficients  $a_i$  integers. Let  $\alpha$  and  $\beta$  be integers such that the highest common factor of  $\alpha$  and  $\beta$  is 1. If  $\beta x + \alpha$  is a factor of  $P(x)$ , then  $\beta$  divides  $a_n$  and  $\alpha$  divides  $a_0$ .

**Rectangular hyperbola** [p. 138] The basic rectangular hyperbola has equation  $y = \frac{1}{x}$ .

**Reflection in the x-axis** [p. 271] A reflection in the  $x$ -axis is described by the rule  $(x, y) \rightarrow (x, -y)$ . The curve with equation  $y = f(x)$  is mapped to the curve with equation  $y = -f(x)$ .

**Reflection in the y-axis** [p. 271] A reflection in the  $y$ -axis is described by the rule  $(x, y) \rightarrow (-x, y)$ . The curve with equation  $y = f(x)$  is mapped to the curve with equation  $y = f(-x)$ .

**Relation** [p. 169] a set of ordered pairs; e.g.  $\{(x, y) : y = x^2\}$

**Remainder theorem** [p. 216]

When a polynomial  $P(x)$  is divided by  $\beta x + \alpha$ , the remainder is  $P\left(-\frac{\alpha}{\beta}\right)$ .

**Repeated factor of a polynomial** [p. 235]

a factor that occurs more than once in the factorised form of a polynomial. For example,  $x - a$  is a repeated factor of  $P(x) = (x - a)^3(x - b)$ .

## S

**Sample space,  $\varepsilon$**  [p. 305] the set of all possible outcomes for a random experiment

**Sampling with replacement** [p. 396] selecting individual objects sequentially from a group of objects, and replacing the selected object, so that the probability of obtaining a particular object does not change with each successive selection



**Sampling without replacement** [p. 393] selecting individual objects sequentially from a group of objects, and not replacing the selected object, so that the probability of obtaining a particular object changes with each successive selection

**Secant** [p. 544] a straight line that passes through two points  $(a, f(a))$  and  $(b, f(b))$  on the graph of a function  $y = f(x)$

**Selections** [p. 373] counted when order is not important. The number of ways of selecting  $r$  objects from a total of  $n$  objects is

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

An alternative notation for  ${}^n C_r$  is  $\binom{n}{r}$ .

**Set difference** [p. 166] The set  $A \setminus B$  contains all the elements of  $A$  that are not in  $B$ .

**Set notation** [p. 165]

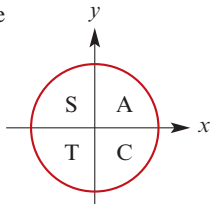
- $\in$  means 'is an element of'
- $\notin$  means 'is not an element of'
- $\subseteq$  means 'is a subset of'
- $\cap$  means 'intersection'
- $\cup$  means 'union'
- $\emptyset$  is the empty set, containing no elements

**Sets of numbers** [p. 166]

- $\mathbb{N}$  is the set of natural numbers
- $\mathbb{Z}$  is the set of integers
- $\mathbb{Q}$  is the set of rational numbers
- $\mathbb{R}$  is the set of real numbers

**Signs of circular functions** [p. 481]

- 1st quadrant all are positive
- 2nd quadrant sin is positive
- 3rd quadrant tan is positive
- 4th quadrant cos is positive

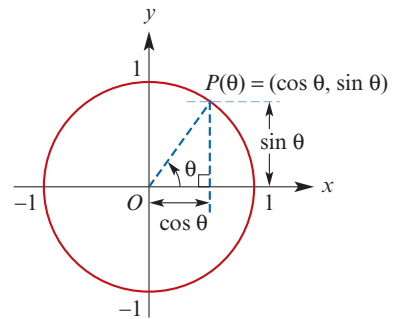


**Simulation** [p. 346] the process of finding an approximate solution to a probability problem by repeated trials using a simulation model

**Simulation model** [p. 346] a simple model which is analogous to a real-world situation. For example, the outcomes from a toss of a coin (head, tail) could be used as a simulation model for the sex of a child (male, female) under the assumption that in both situations the probabilities are 0.5 for each outcome.

**Simultaneous equations** [p. 11] equations of two or more lines or curves in the Cartesian plane, the solutions of which are the points of intersection of the lines or curves

**Sine function** [p. 478] sine  $\theta$  is defined as the  $y$ -coordinate of the point  $P$  on the unit circle where  $OP$  forms an angle of  $\theta$  radians with the positive direction of the  $x$ -axis.



**Speed** [p. 641] the magnitude of velocity

**Speed, average** [pp. 543, 641]

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

**Stationary point** [pp. 624, 627] A point with coordinates  $(a, f(a))$  on a curve  $y = f(x)$  is said to be a stationary point if  $f'(a) = 0$ .

**Straight line, equation given two points**

[p. 45]  $y - y_1 = m(x - x_1)$ , where  $m = \frac{y_2 - y_1}{x_2 - x_1}$

**Straight line, gradient–intercept form** [p. 42]

The gradient–intercept form of the equation of a straight line is  $y = mx + c$ , where  $m$  is the gradient and  $c$  is the  $y$ -axis intercept.

**Straight lines, parallel** [p. 54] Two straight lines are parallel to each other if and only if they have the same gradient (or if both are vertical).

**Straight lines, perpendicular** [p. 55]

Two straight lines are perpendicular to each other if and only if the product of their gradients is  $-1$  (or if one is horizontal and the other vertical).

**Strictly decreasing** [p. 589] A function  $f$  is strictly decreasing on an interval if  $a < b$  implies  $f(a) > f(b)$ .

**Strictly increasing** [p. 589] A function  $f$  is strictly increasing on an interval if  $a < b$  implies  $f(a) < f(b)$ .

**Subset** [p. 165] A set  $B$  is called a subset of a set  $A$  if every element of  $B$  is also an element of  $A$ . We write  $B \subseteq A$ .

**Sum of two cubes** [p. 222]

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

## T

**Tangent function** [p. 480] The tangent function is given by

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

**Tangent line, equation** [p. 616] Let  $(x_1, y_1)$  be a point on the curve  $y = f(x)$ . Then, if  $f$  is differentiable at  $x = x_1$ , the equation of the tangent at  $(x_1, y_1)$  is given by

$$y - y_1 = f'(x_1)(x - x_1)$$

**Translation** [p. 266] A translation of  $h$  units in the positive direction of the  $x$ -axis and  $k$  units in the positive direction of the  $y$ -axis is described by the rule  $(x, y) \rightarrow (x + h, y + k)$ , where  $h$  and  $k$  are positive numbers. The curve with equation  $y = f(x)$  is mapped to the curve with equation  $y - k = f(x - h)$ .

**Tree diagram** [pp. 319, 334] a diagram representing the outcomes of a multi-stage experiment

## U

**Union of sets** [pp. 165, 322] The union of two sets  $A$  and  $B$ , written  $A \cup B$ , is the set of all elements which are in  $A$  or  $B$  or both.

## V

**Velocity** [pp. 557, 640] the rate of change of a particle's position with respect to time

**Velocity, average** [pp. 555, 640]

$$\text{average velocity} = \frac{\text{change in position}}{\text{change in time}}$$

**Velocity, instantaneous** [pp. 557, 640]

$$v = \frac{dx}{dt}$$

**Venn diagram** [pp. 165, 322] a diagram showing sets and the relationships between sets

**Vertical-line test** [p. 175] used to identify whether a relation is a function or not. If a vertical line can be drawn anywhere on the graph and it only ever intersects the graph a maximum of once, then the relation is a *function*.

## Z

$\mathbb{Z}$  [p. 166] the set of all integers

**Zero polynomial** [p. 206] The number 0 is called the zero polynomial.