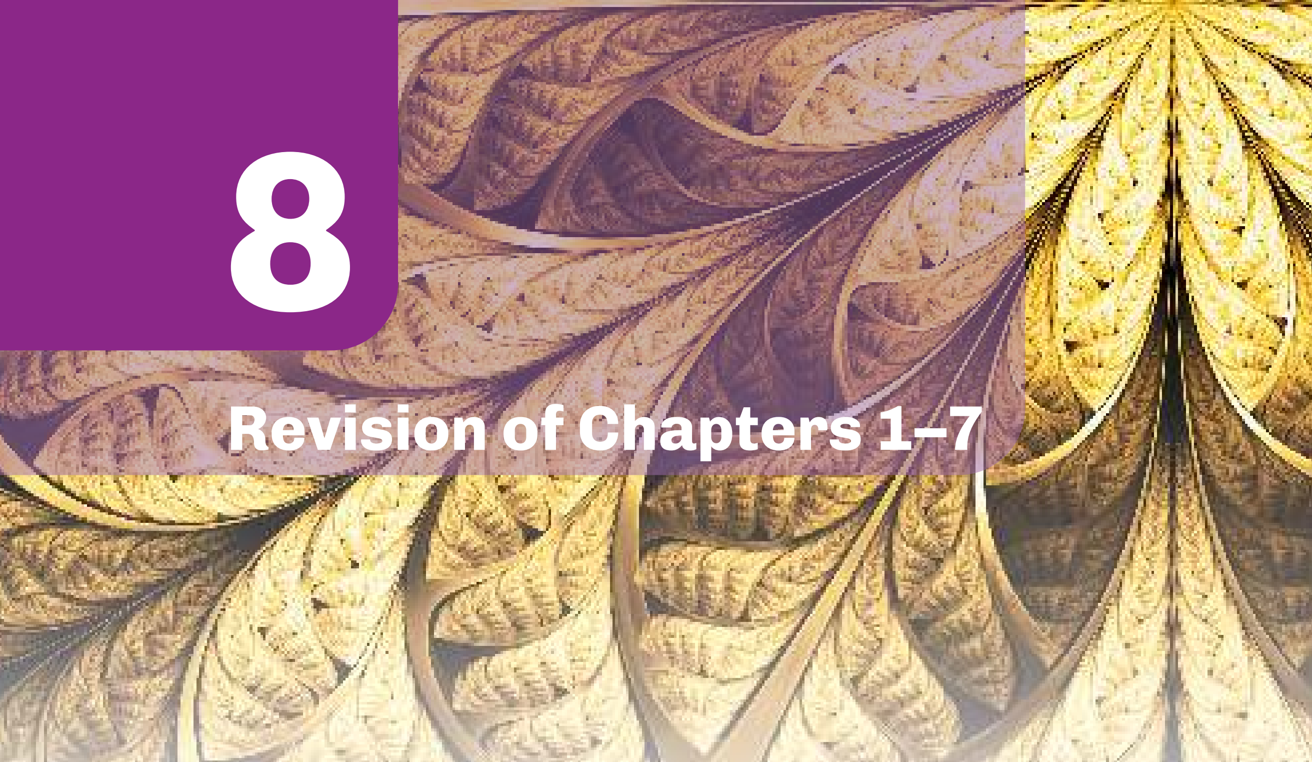

8 Revision of Chapters 1–7

8A Technology-free questions

1 State the maximal domain and range of each of the following:

a $f(x) = \dfrac{1}{x} + 2$ **b** $f(x) = 3 - 2\sqrt{3x - 2}$ **c** $f(x) = \dfrac{4}{(x-2)^2} + 3$

d $h(x) = 4 - \dfrac{3}{x - 2}$ **e** $f(x) = \sqrt{x - 2} - 5$ **f** $f(x) = \sqrt{(x - 2)(x + 4)}$.

2 Find the inverse of the function with the rule $f(x) = \sqrt{x - 2} + 4$ and sketch both functions on the one set of axes.

3 Find the inverse of the function with the rule $f(x) = \dfrac{x - 2}{x + 1}$.

4 Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 2e^{3x} - 1$.

a Find the rule and domain of f^{-1}.

b Sketch the graphs of f and f^{-1} on the one set of axes.

c Sketch the graph of $y = f(f^{-1}(x))$ for its maximal domain.

d Sketch the graph of $y = f^{-1}(f(x))$ for its maximal domain.

e Find $y = f(f^{-1}(2x))$.

5 Simplify $2\log_{10} 5 + 3\log_{10} 2 - \log_{10} 20$.

6 Find x in terms of a if $3\log_a x = 3 + \log_a 12$.

7 Solve $2 \times 2^{-x} = 1024$.

8 Solve the equation $\log_e(x + 12) = 1 + \log_e(2 - x)$.

9 Evaluate $\log_a 4 \times \log_{16} a$.

10 Solve the equation $4e^{2x} = 9$ for x.

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11 **a** The graph of the function f with rule $f(x) = 2\log_e(x+2)$ intersects the axes at the points $(a, 0)$ and $(0, b)$. Find the exact values of a and b.

b Hence sketch the graph of $y = f(x)$.

12 Solve the equation $2^{4x} - 5 \times 2^{2x} + 4 = 0$ for x.

13 Solve the equation $\sin\left(\frac{3x}{2}\right) = \frac{1}{2}$ for $x \in [-\pi, \pi]$.

14 **a** State the range and period of the function $h\colon \mathbb{R} \to \mathbb{R}$, $h(x) = 5 - 3\cos\left(\frac{\pi x}{3}\right)$.

b Solve the equation $\cos\left(2x + \frac{\pi}{6}\right) = \frac{1}{2}$ for $x \in [0, \pi]$.

15 Consider the simultaneous equations

$$mx + y = 2$$
$$2x + (m-1)y = -4$$

Find the values of m such that the system of equations has:

a a unique solution **b** no solution **c** infinitely many solutions.

16 If a graph has rule $y = \frac{a}{x^2} + b$ and passes through the points $(1, -1)$ and $(-2, \frac{1}{2})$, find the values of a and b.

17 Find the value(s) of m for which the equation $x^2 + mx + 2 = 0$ has:

a one solution **b** two solutions **c** no solution.

18 Find all non-zero values of a for which the equation $2ax^2 + 2(a^2 + a)x + 3(a+1) = 0$ has only one solution.

19 Two points A and B have coordinates $(a, -2)$ and $(3, 1)$.

a Find the value(s) of a if:

i the midpoint of AB is $(0, -\frac{1}{2})$

ii the length of AB is $\sqrt{13}$

iii the gradient of AB is $\frac{1}{2}$.

b Find the equation of the line passing through A and B if $a = -2$, and find the angle the line makes with the positive direction of the x-axis.

20 Let $f\colon \mathbb{R} \to \mathbb{R}$, $f(x) = 2x^3$.

a State whether the function f is even, odd or neither.

b Find the inverse function f^{-1}.

c Find:

i $f^{-1}(16)$ **ii** $f^{-1}(-2)$ **iii** $\{x : f(x) = f^{-1}(x)\}$

21 Let $f(x) = 2 - x$ and $g(x) = \sqrt{2x - 3}$. Find:

a $f(-2)$ **b** $g(4)$ **c** $f(2a)$ **d** $g(a-1)$

e $\{x : f(x) = 10\}$ **f** $\{x : g(x) = 10\}$ **g** $\{x : f(2x) > 0\}$

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22 Let $f(x) = 4x - 3$ and $g(x) = x^2 + 2x$.

a Find:

i $f \circ g$ **ii** $g \circ f$ **iii** $g \circ f^{-1}$

b Find a transformation that takes the graph of $y = g(x)$ to the graph of $y = g(f(x))$.

c Find a transformation that takes the graph of $y = x^2$ to the graph of $y = g(x)$.

23 Solve the equation $1 - \sin\left(\frac{x}{4}\right) = \sin\left(\frac{x}{4}\right)$ for $-2\pi \le x \le 2\pi$

24 Solve $\cos x = \frac{\sqrt{3}}{2}$, giving the general solution.

25 A function has rule $y = Ae^{kt}$. Given that $y = 4$ when $t = 1$ and that $y = 10$ when $t = 2$, find the values of A and k.

26 Define $f(x) = 8\sin(5x)$ for all $x \in \mathbb{R}$.

a State the period of f.

b State the amplitude of f.

c Describe a sequence of transformations that maps:

i the graph of $y = \sin x$ to the graph of $y = f(x)$

ii the graph of $y = \cos x$ to the graph of $y = f(x)$.

27 Let $P(x) = x^4 + ax^3 + bx^2 - 12x + 4$, where a and b are real constants.

a Find the values of a and b given that $P(x) = 0$ when $x = 1$ and when $x = 2$.

b Use the values of a and b obtained in part **a**. By factorising $P(x)$, show that the equation $P(x) = 0$ has no real solutions other than $x = 1$ and $x = 2$.

28 Suppose that functions f, g and h satisfy $g(x) = \frac{1}{2}f(x + 4)$ and $h(x) = 2g(5x - 11) + 3$ for all $x \in \mathbb{R}$. Write a rule for the function h in terms of the function f.

29 Let $h(x) = x^3 + ax + b$. Given that the equation $h(x) = x$ has solutions $x = 2$ and $x = 3$, find the values of a and b.

30 Find the value of n such that dividing $x^{2n} - 8x^n + 10$ by $x - 2$ gives remainder 10.

31 Let $f(x) = x^2 - 7x + 6$ and $g(x) = e^x$. Solve the equation $f(g(x)) = 0$ for x.

32 Describe the transformation that takes the graph of $y = 2x\cos x$ to the graph of $y = 2(x - 5\pi)\cos(x)$

33 Solve each of the following inequalities for x:

a $2x^3 - 3x^2 - 11x + 6 \ge 0$ **b** $-x^3 + 4x^2 - 4x > 0$

34 Let $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = e^{3x+2}$ and let $g : (0, \infty) \to \mathbb{R}$ where $g(x) = \log_e(x)$.

a Find $f(g(x))$

b Find the value of k such that $f(g(2)) = ke^2$

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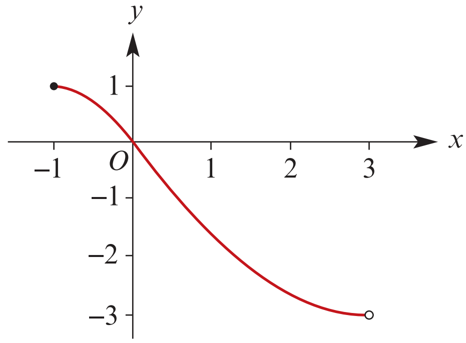
Revision

8B Multiple-choice questions

1 The domain of the function whose graph is shown is

A $(-3, 1]$
B $(-1, 3]$
C $[1, 3]$
D $[-1, 3)$
E $(-1, 3)$



2 Which of the following sets of ordered pairs does not represent a function where y is the value of the function?

A $\{(x, y) : x = 2y^2,\ x \geq 0\}$
B $\left\{(x, y) : y = \frac{1}{x},\ x \in \mathbb{R} \setminus \{0\}\right\}$
C $\{(x, y) : y = 2x^3 + 3,\ x \in \mathbb{R}\}$
D $\{(x, y) : y = 3x^2 + 7,\ x \in \mathbb{R}\}$
E $\{(x, y) : y = e^x - 1,\ x \in \mathbb{R}\}$

3 The implied (largest possible) domain for the function with the rule $y = \dfrac{1}{\sqrt{2 - x}}$ is

A $\mathbb{R} \setminus \{2\}$ **B** $(-\infty, 2)$ **C** $(2, \infty)$ **D** $(-\infty, 2]$ **E** $\mathbb{R}^+$

4 If $f(x) = \dfrac{x}{x - 1}$, then $f\left(-\dfrac{1}{a}\right)$ can be simplified as

A $\dfrac{1}{-1 - a}$ **B** -1 **C** 0 **D** $\dfrac{a^2}{1 - a}$ **E** $\dfrac{1}{a + 1}$

5 If $f : [0, 2\pi] \to \mathbb{R}$ where $f(x) = \sin(2x)$ and $g : [0, 2\pi] \to \mathbb{R}$ where $g(x) = 2\sin x$, then the value of $(f + g)\left(\dfrac{3\pi}{2}\right)$ is

A 2 **B** 0 **C** -1 **D** 1 **E** -2

6 If $f(x) = 3x + 2$ and $g(x) = 2x^2$, then $f(g(3))$ equals

A 36 **B** 20 **C** 56 **D** 144 **E** 29

7 If $f(x) = 3x^2$, $0 \leq x \leq 6$ and $g(x) = \sqrt{2 - x}$, $x \leq 2$, then the domain of $f + g$ is

A $[0, 2]$ **B** $[0, 6]$ **C** $(-\infty, 2]$ **D** $\mathbb{R}^+ \cup \{0\}$ **E** $[2, 6]$

8 If $g(x) = 2x^2 + 1$ and $f(x) = 3x + 2$, then the rule of the product function $(fg)(x)$ equals

A $2x^2 + 3x + 3$
B $6x^3 + 4x^2 + 3x + 2$
C $6x^3 + 3$
D $6x^3 + 2x^2 + 3$
E $6x^3 + 2$

9 The implied domain for the function with rule $y = \sqrt{4 - x^2}$ is

A $[2, \infty)$ **B** $\{x : -2 < x < 2\}$ **C** $[-2, 2]$ **D** $(-\infty, 2)$ **E** $\mathbb{R}^+$

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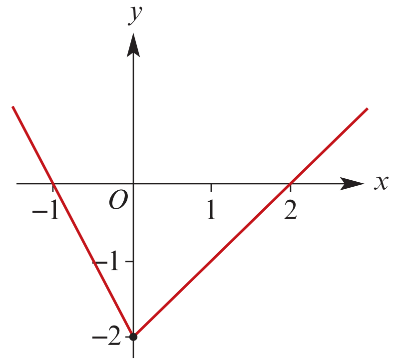
10 The graph shown has the equation



A $y = \begin{cases} x - 2, & x > 0 \\ -2x - 2, & x \le 0 \end{cases}$

B $y = \begin{cases} 2x - 2, & x \ge 0 \\ -2x - 2, & x < 0 \end{cases}$

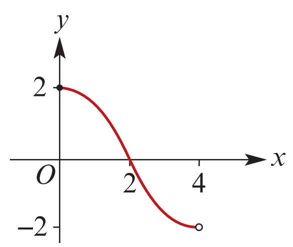
C $y = \begin{cases} x - 2, & x > 0 \\ -2x - 1, & x \le 0 \end{cases}$

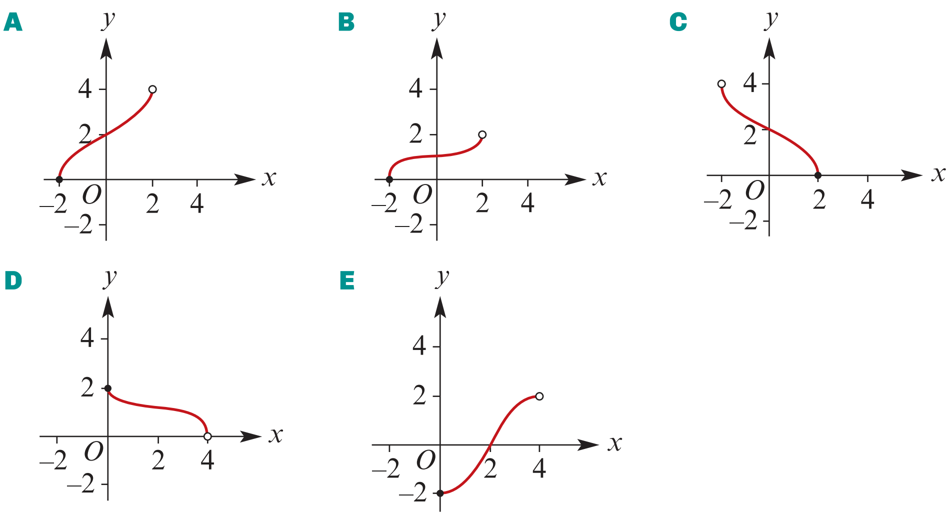
D $y = \begin{cases} x + 2, & x > 0 \\ -2x - 2, & x \le 0 \end{cases}$

E $y = \begin{cases} x - 2, & x > 0 \\ -x - 2, & x \le 0 \end{cases}$

11 The graph of the function with rule $y = f(x)$ is shown.

Which one of the following graphs is the graph of the inverse of f?





12 Let $h\colon [a, 2] \to \mathbb{R}$ where $h(x) = 2x - x^2$. If a is the smallest real value such that h has an inverse function, h^{-1}, then a equals

A -1 **B** 0 **C** 1 **D** -2 **E** $\frac{1}{2}$

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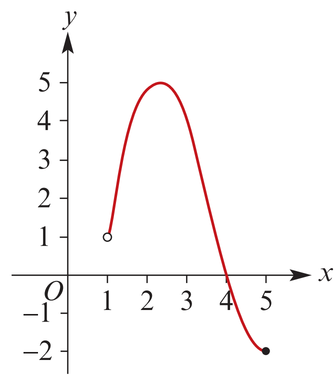
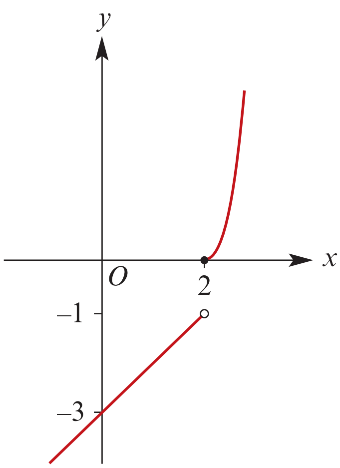
13 The domain of the function whose graph is shown is



A $[1, 5]$
B $(1, 5]$
C $(-2, 5]$
D $(1, 5)$
E $(-2, 5)$

14 The graph shown has the rule



A $y = \begin{cases} (x-2)^2, & x \geq 2 \\ x - 3, & x < 2 \end{cases}$

B $y = \begin{cases} x-2, & x \geq 2 \\ x - 3, & x < 2 \end{cases}$

C $y = \begin{cases} (2-x)^2, & x \geq 2 \\ 2x - 3, & x < 2 \end{cases}$

D $y = \begin{cases} (x-2)^2, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$

E $y = \begin{cases} (x-2)^2, & x \geq 2 \\ 2x - 3, & x < 2 \end{cases}$

15 The inverse, f^{-1}, of the function $f\colon [2, 3] \to \mathbb{R}$, $f(x) = 2x - 4$ is

A $f^{-1}\colon [0, 2] \to \mathbb{R}$, $f^{-1}(x) = \dfrac{x}{2} + 4$
B $f^{-1}\colon [3, 2] \to \mathbb{R}$, $f^{-1}(x) = \dfrac{x+4}{2}$
C $f^{-1}\colon [2, 3] \to \mathbb{R}$, $f^{-1}(x) = \dfrac{1}{2x-4}$
D $f^{-1}\colon [0, 2] \to \mathbb{R}$, $f^{-1}(x) = \dfrac{1}{2x-4}$
E $f^{-1}\colon [0, 2] \to \mathbb{R}$, $f^{-1}(x) = \dfrac{x+4}{2}$

16 Let f be the function defined by $f(x) = \dfrac{1}{x^2+2}$, $x \in \mathbb{R}$. A suitable restriction f^* of f such that $(f^*)^{-1}$ exists would be

A $f^*\colon [-1, 1] \to \mathbb{R}$, $f^*(x) = \dfrac{1}{x^2+2}$
B $f^*\colon \mathbb{R} \to \mathbb{R}$, $f^*(x) = \dfrac{1}{x^2+2}$
C $f^*\colon [-2, 2] \to \mathbb{R}$, $f^*(x) = \dfrac{1}{x^2+2}$
D $f^*\colon [0, \infty) \to \mathbb{R}$, $f^*(x) = \dfrac{1}{x^2+2}$
E $f^*\colon [-1, \infty) \to \mathbb{R}$, $f^*(x) = \dfrac{1}{x^2+2}$

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17 If $f(x) = 3x - 2$, $x \in \mathbb{R}$, then $f^{-1}(x)$ equals

A $\dfrac{1}{3x-2}$ **B** $3x + 2$ **C** $\dfrac{1}{3}(x-2)$ **D** $3x + 6$ **E** $\dfrac{1}{3}(x+2)$

18 The straight line with equation $y = \dfrac{4}{5}x - 4$ meets the x-axis at A and the y-axis at B. If O is the origin, the area of the triangle OAB is

A $3\frac{1}{5}$ square units **B** $9\frac{2}{5}$ square units **C** 10 square units
D 15 square units **E** 20 square units

19 If the equations $2x - 3y = 12$ and $3x - 2y = 13$ are simultaneously true, then $x + y$ equals

A -5 **B** -1 **C** 0 **D** 1 **E** 5

20 The graphs of the relations $7x - 6y = 20$ and $3x + 4y = 2$ are drawn on the same pair of axes. The x-coordinate of the point of intersection is

A -2 **B** -1 **C** 1 **D** 2 **E** 3

21 It is known that the graph of the function with rule $y = ax + \sin(x)$ has an x-axis intercept when $x = -\dfrac{\pi}{2}$. The value of a is

A 2 **B** $-\dfrac{2}{\pi}$ **C** 2π **D** -2π **E** $\dfrac{-1}{2\pi}$

22 Consider the polynomial $p(x) = (x - 2a)^2(x^2 - a^2)(x^2 + a^2)$ where $a > 0$. The equation $p(x) = 0$ has exactly

A 1 distinct real solution **B** 2 distinct real solutions
C 3 distinct real solutions **D** 4 distinct real solutions
E 5 distinct real solutions

23 The graph of $y = kx - 1$ will not intersect or touch the graph of $y = x^2 + 3x$ when

A $\{k : 1 < k < 5\}$ **B** $\{k : k < 1\} \cup \{k : k > 5\}$ **C** $\{k : k > 5\}$
D $\{k : 1 \le k \le 5\}$ **E** $\{1, 5\}$

24 Let f and g be functions such that $f(2) = 6$, $f(5) = 7$, $g(7) = 5$, $g(6) = 4$ and $g(5) = 11$. The value of $f(g(7))$ is

A 6 **B** 7 **C** 5 **D** 4 **E** 11

25 The sum of the solutions to the equation $3\sin(2x) + \sqrt{3}\cos(2x) = 0$ for $x \in \left[0, \dfrac{3\pi}{2}\right]$ is

A 2π **B** $\dfrac{5\pi}{6}$ **C** $\dfrac{4\pi}{3}$ **D** $\dfrac{14\pi}{3}$ **E** $\dfrac{11\pi}{4}$

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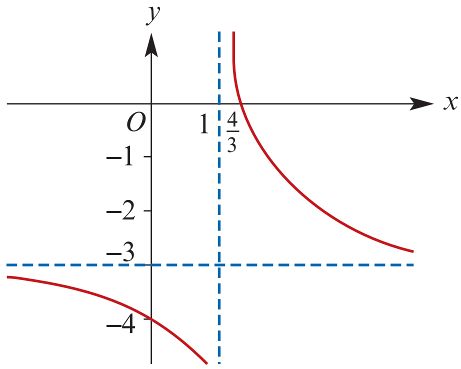
26 A possible equation for the graph shown is



A $y - 3 = \dfrac{1}{x-1}$

B $y + 3 = \dfrac{1}{x+1}$

C $y - 3 = \dfrac{1}{x+1}$

D $y - 4 = \dfrac{1}{x+1}$

E $y = \dfrac{1}{x-1} - 3$

27 The function given by $f(x) = \dfrac{1}{x+3} - 2$ has the range

A $\mathbb{R} \setminus \{-2\}$ **B** $\mathbb{R}$ **C** $\mathbb{R} \setminus \{3\}$ **D** $\mathbb{R} \setminus \{2\}$ **E** $\mathbb{R} \setminus \{-3\}$

28 A parabola has its vertex at $(2, 3)$. A possible equation for this parabola is

A $y = (x+2)^2 + 3$ **B** $y = (x-2)^2 - 3$ **C** $y = (x+2)^2 - 3$

D $y = (x-2)^2 + 3$ **E** $y = 3 - (x+2)^2$

29 Which one of the following is an even function of x?

A $f(x) = 3x + 1$ **B** $f(x) = x^3 - x$ **C** $f(x) = (1-x)^2$

D $f(x) = -x^2$ **E** $f(x) = x^3 + x^2$

30 The graph of $y = 3\sqrt{x+2}$ can be obtained from the graph of $y = \sqrt{x}$ by

A a translation $(x, y) \to (x-2, y)$ followed by a dilation of factor 3 from the x-axis

B a translation $(x, y) \to (x+2, y)$ followed by a dilation of factor $\frac{1}{3}$ from the x-axis

C a translation $(x, y) \to (x+3, y)$ followed by a dilation of factor 3 from the y-axis

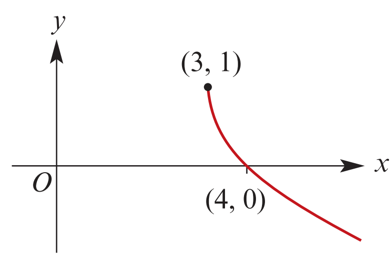
D a translation $(x, y) \to (x-2, y)$ followed by a dilation of factor 3 from the y-axis

E a translation $(x, y) \to (x+2, y)$ followed by a dilation of factor 3 from the y-axis

31 A function with rule $f(x) = 3\sqrt{x-2} + 1$ has maximal domain

A $(-\infty, 2)$ **B** $[1, \infty)$ **C** $(2, \infty)$ **D** $[-2, \infty)$ **E** $[2, \infty)$

32 A possible equation for the graph shown is



A $y = 2\sqrt{x-3} + 1$

B $y = -2\sqrt{x-3} + 1$

C $y = \sqrt{x-3} + 1$

D $y = -\sqrt{x-3} + 1$

E $y = -2\sqrt{x-3} + 2$

33 The range of the function $f : \mathbb{R} \setminus \{2\} \to \mathbb{R}$, $f(x) = \dfrac{3}{(x-2)^2} + 4$ is

A $(3, 4]$ **B** $(-\infty, 4)$ **C** $[3, 4)$ **D** $[4, \infty)$ **E** $(4, \infty)$

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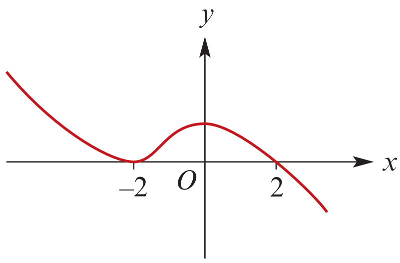
34 If $3x^2 + kx + 1 = 0$ when $x = 1$, then k equals

A -4 **B** -1 **C** 1 **D** 4 **E** 0

35 If $x^3 - 5x^2 + x + k$ is divisible by $x + 1$, then k equals

A -7 **B** -5 **C** -2 **D** 5 **E** 7

36 Which one of the following could be the equation of the graph shown?



A $y = x(x - 2)(x + 2)$

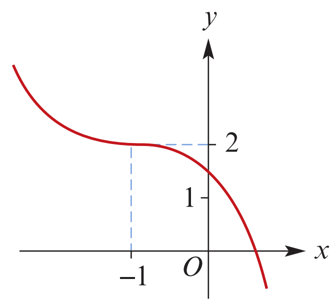
B $y = -x(x + 2)(x - 2)$

C $y = -(x + 2)^2(x - 2)$

D $y = (x - 2)^2(x + 2)$

E $y = x(x - 2)^2$

37 The graph shown is



A $y + 2 = -2(x + 1)^3$

B $y - 2 = 2(x - 1)^3$

C $y = x^3 + 2$

D $y = -\frac{1}{2}(x + 1)^3 + 2$

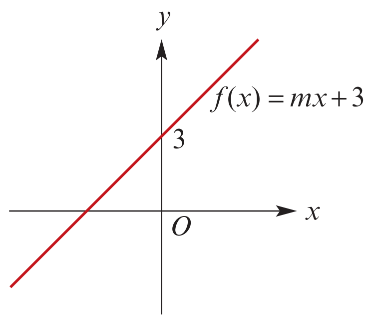
E $y = 2(x - 1)^3 - 2$

38 $P(x) = x^3 + 2x^2 - 5x - 6$ has the factorisation

A $(x - 1)(x - 2)(x + 3)$ **B** $(x + 1)(x + 2)(x + 3)$ **C** $(x + 1)(x - 2)(x + 3)$

D $(x + 1)(x - 2)(x - 3)$ **E** $(x - 1)(x - 2)(x - 3)$

39 The graph shown is that of the function $f(x) = mx + 3$, where m is a constant.
The inverse function is $f^{-1}: \mathbb{R} \to \mathbb{R}$, $f^{-1}(x) = ax + b$, where a and b are constants.
Which one of the following statements is true?



A $a = \dfrac{-3}{m}, \ b = \dfrac{1}{m}$

B $a < 0$ and $b < 0$

C $a = -m, \ b = 3$

D $a > 0$ and $b > 0$

E $a = \dfrac{1}{m}, \ b = \dfrac{-3}{m}$

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40 Let $P(x) = 2x^3 - 2x^2 + 3x + 1$. When $P(x)$ is divided by $x - 2$, the remainder is

A 31 **B** 15 **C** 1 **D** -2 **E** -29

41 If $x^3 + 2x^2 + ax - 4$ has remainder 1 when divided by $x + 1$, then a equals

A -8 **B** -4 **C** -2 **D** 0 **E** 2

42 Which of these equations is represented by the graph shown?

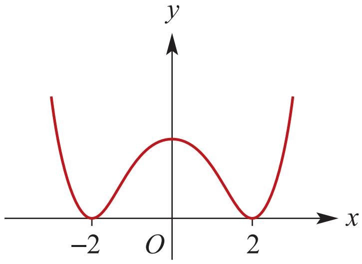
A $y = (x + 2)^2(x - 2)$

B $y = 16 - x^4$

C $y = (x^2 - 4)^2$

D $y = (x + 2)^2(2 - x)$

E $y = x^4 - 16$



43 The function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = e^{-x} + 1$ has an inverse function f^{-1}. The domain of f^{-1} is

A $(0, \infty)$ **B** $\mathbb{R}$ **C** $[1, \infty)$ **D** $(1, \infty)$ **E** $[0, \infty)$

44 The function $f: \mathbb{R}^+ \to \mathbb{R}$, $f(x) = 2\log_e x + 1$ has an inverse function f^{-1}. The rule for f^{-1} is given by

A $f^{-1}(x) = 2e^{x-1}$ **B** $f^{-1}(x) = e^{\frac{1}{2}(x-1)}$ **C** $f^{-1}(x) = e^{\frac{x}{2}-1}$

D $f^{-1}(x) = 2e^{x+1}$ **E** $f^{-1}(x) = \dfrac{1}{2}e^{x-1}$

45 Let $f: \mathbb{R} \to \mathbb{R}$ where $f(x) = e^{-x}$ and let $g: (-1, \infty) \to \mathbb{R}$ where $g(x) = \log_e(x + 2)$. The function with the rule $y = f(g(x))$ has the range

A $(1, \infty)$ **B** $(0, 1)$ **C** $(0, 1]$ **D** $[1, \infty)$ **E** $[0, 1]$

46 The function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = e^x - 1$ has an inverse whose rule is given by

A $f^{-1}(x) = \dfrac{1}{e^x - 1}$ **B** $f^{-1}(x) = -\log_e(x + 1)$ **C** $f^{-1}(x) = \log_e(x - 1)$

D $f^{-1}(x) = \log_e(1 - x)$ **E** $f^{-1}(x) = \log_e(x + 1)$

47 The function $f: [4, \infty) \to \mathbb{R}$, $f(x) = \log_e(x - 3)$ has an inverse. The domain of this inverse is

A $[0, \infty)$ **B** $(0, \infty)$ **C** $[4, \infty)$ **D** $(3, \infty)$ **E** $\mathbb{R}$

48 The function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = e^{x-1}$ has an inverse whose rule is given by $f^{-1}(x) =$

A $e^{-(x-1)}$ **B** $-\log_e x$ **C** $1 + \log_e x$ **D** $\log_e(x + 1)$ **E** $\log_e(x - 1)$

49 The function $f: \mathbb{R}^+ \to \mathbb{R}$, $f(x) = \log_e\left(\dfrac{x}{2}\right)$ has an inverse function f^{-1}. The rule for f^{-1} is given by $f^{-1}(x) =$

A $e^{\frac{1}{2}x}$ **B** $\log_e\left(\dfrac{2}{x}\right)$ **C** $\dfrac{1}{2}e^{\frac{x}{2}}$ **D** $2e^x$ **E** $\dfrac{1}{\log_e\left(\dfrac{2}{x}\right)}$

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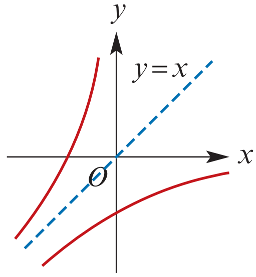
50 For which values of x is the function f with the rule $f(x) = -2 + \log_e(3x - 2)$ defined?

A $(-2, \infty)$ **B** $\left(\frac{2}{3}, \infty\right)$ **C** $[-2, \infty)$ **D** $\left[\frac{2}{3}, \infty\right)$ **E** $(2, \infty)$

51 The graphs of the function $f\colon (-2, \infty) \to \mathbb{R}$ where $f(x) = 2 + \log_e(x + 2)$ and its inverse f^{-1} are best shown by which one of the following?

A



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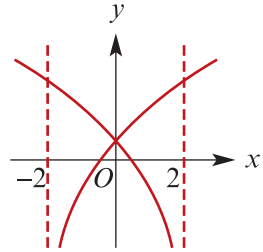

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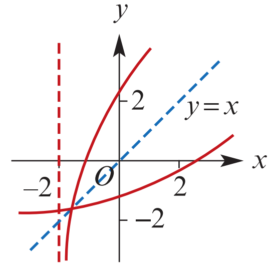

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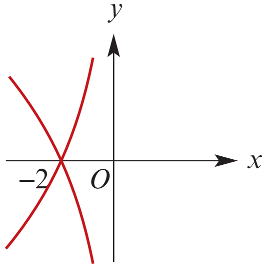
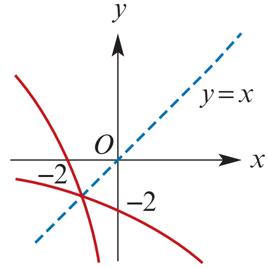


E



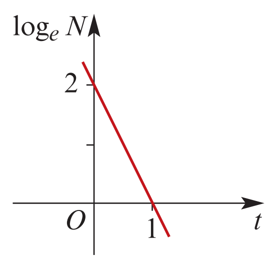
52 If $\log_2(8x) + \log_2(2x) = 6$, then $x =$

A 1.5 **B** ± 1.5 **C** 2 **D** ± 2 **E** 6.4

53 The equation $\log_{10} x = y(\log_{10} 3) + 1$ is equivalent to the equation

A $x = 10(3^y)$ **B** $x = 30^y$ **C** $x = 3^y + 10$
D $x = y^3 + 10$ **E** $x = 10y^3$

54 The graph indicates that the relationship between N and t is

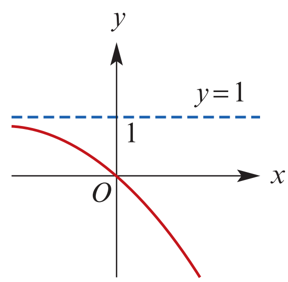


A $N = 2 - e^{-2t}$ **B** $N = e^{2-2t}$ **C** $N = e^{2t} + 2$

D $N = \dfrac{e^{-2t}}{100}$ **E** $N = -2e^{2t}$

55 A possible equation for the graph is

A $y = 1 - e^x$
B $y = 1 - e^{-x}$
C $y = 1 + e^x$
D $y = 1 + e^{-x}$
E $y = e^{-x} - 1$



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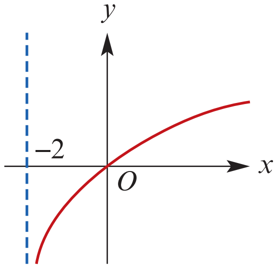
56 A possible equation for the graph is



A $y = \log_e(x - 2)$

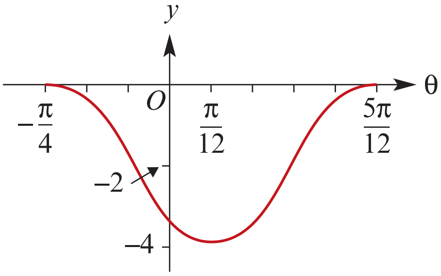
B $y = \log_e \frac{1}{2}(x + 2)$

C $y = \log_e 2(x + 1)$

D $y = 2\log_e(x + 1)$

E $y = \frac{1}{2}\log_e(x + 2)$

57 A possible equation for the graph shown is



A $y = 2\cos 3\left(\theta + \frac{\pi}{4}\right) - 4$

B $y = 2\cos 2\left(\theta + \frac{\pi}{4}\right) - 2$

C $y = 2\sin 3\left(\theta + \frac{\pi}{4}\right) - 2$

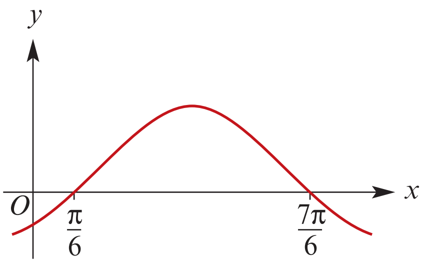
D $y = 2\cos 3\left(\theta + \frac{\pi}{4}\right) - 2$

E $y = 2\cos 3\left(\theta - \frac{\pi}{4}\right) - 2$

58 The function $f: \mathbb{R} \to \mathbb{R}$ where $f(x) = 2 - 3\cos 2\left(\theta + \frac{\pi}{2}\right)$ has range

A $[-3, 5]$ **B** $[2, 5]$ **C** $\mathbb{R}$ **D** $[-1, 5]$ **E** $[-3, 2]$

59 A possible equation for the graph shown is



A $y = \sin\left(x - \frac{\pi}{6}\right)$

B $y = \sin\left(x + \frac{\pi}{6}\right)$

C $y = -\sin\left(x - \frac{\pi}{6}\right)$

D $y = \cos\left(x - \frac{\pi}{6}\right)$

E $y = \cos\left(x + \frac{\pi}{6}\right)$

60 The function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 3\sin(2x)$ has

A amplitude 3 and period π

B amplitude 2 and period $\frac{\pi}{2}$

C amplitude 1 and period $\frac{\pi}{2}$

D amplitude $\frac{3}{2}$ and period 2π

E amplitude $1\frac{1}{2}$ and period 2π

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61 The function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 3\sin(2x)$ has range

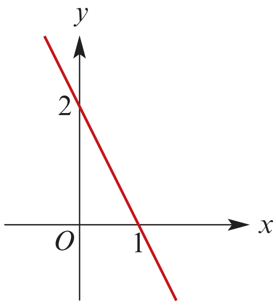
A $[0, 3]$ **B** $[-2, 2]$ **C** $[2, 3]$ **D** $[-3, 3]$ **E** $[-1, 5]$

62 Consider the polynomial $p(x) = (x - 2a)^2(x + a)(x^2 + a)$ where $a > 0$. The equation $p(x) = 0$ has exactly

A 1 distinct real solution **B** 2 distinct real solutions **C** 3 distinct real solutions
D 4 distinct real solutions **E** 5 distinct real solutions

63 The gradient of a straight line perpendicular to the line shown is

A 2 **B** -2 **C** $\dfrac{-1}{2}$ **D** $\dfrac{1}{2}$ **E** 3



64 The graph of a function f whose rule is $y = f(x)$ has exactly one asymptote, for which the equation is $y = 6$. The inverse function f^{-1} exists. The inverse function will have

A a horizontal asymptote with equation $y = 6$
B a vertical asymptote with equation $x = 6$
C a vertical asymptote with equation $x = -\frac{1}{6}$
D a horizontal asymptote with equation $y = -6$
E no asymptote

65 The function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = a\sin(bx) + c$, where a, b and c are positive constants, has period

A a **B** b **C** c **D** $\dfrac{2\pi}{a}$ **E** $\dfrac{2\pi}{b}$

66 The functions $f: [18, 34] \to \mathbb{R}$, $f(x) = 2x - 4$ and $g: \mathbb{R}^+ \to \mathbb{R}$, $g(x) = \log_2 x$ are used to define the composite function $g \circ f$. The range of $g \circ f$ is

A $[2, \infty)$ **B** $\left[\dfrac{3}{2}, \infty\right)$ **C** $[5, 6]$ **D** $\mathbb{R}^+$ **E** $\mathbb{R}$

67 The rule for the inverse function of the function $f: (-\infty, 2) \to \mathbb{R}$, $f(x) = x^2 - 4x + 5$ is

A $y = 2 + \sqrt{x + 1}$ **B** $y^2 = 2x + 5$ **C** $y = 2 - \sqrt{x - 1}$
D $y = \sqrt{4x - 5}$ **E** $y = 2x - 1$

68 The function $f: B \to \mathbb{R}$, $f(x) = x^2 - 4x + 3$ will have an inverse function for

A $B = \mathbb{R}$ **B** $B = (2, \infty)$ **C** $B = [-1, \infty)$ **D** $B = (-\infty, 4]$ **E** $B = \mathbb{R}^+$

69 Let $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 - 6x$ and let $g: [-5, -3) \to \mathbb{R}$, $g(x) = x + 6$. Then the domain of the inverse function of $h = f + g$ is

A $[-5, 3)$ **B** $[-5, 3) \cup \mathbb{R}$ **C** $(30, 56]$ **D** $[-5, 3) \cap \mathbb{R}$ **E** $[30, 56)$

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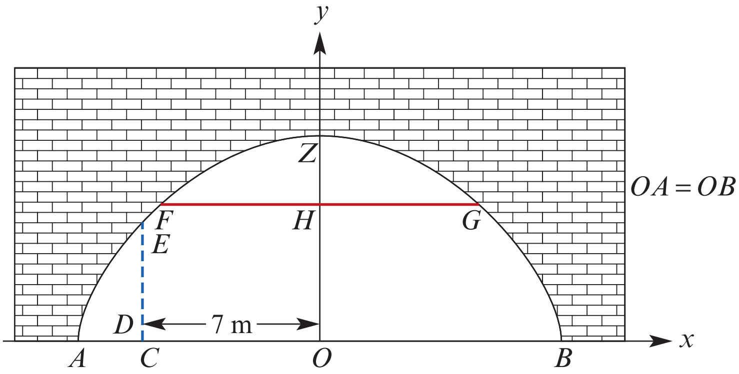
Revision

8C Extended-response questions

1 An arch is constructed as shown.



The height of the arch is 9 metres ($OZ = 9$ m). The width of the arch is 20 metres ($AB = 20$ m). The equation of the curve is of the form $y = ax^2 + b$, taking axes as shown.

a Find the values of a and b.

b A man of height 1.8 m stands at C ($OC = 7$ m). How far above his head is the point E on the arch? (That is, find the distance DE.)

c A horizontal bar FG is placed across the arch as shown. The height, OH, of the bar above the ground is 6.3 m. Find the length of the bar.

2 **a** The expression $2x^3 + ax^2 - 72x - 18$ leaves a remainder of 17 when divided by $x + 5$. Determine the value of a.

b Solve the equation $2x^3 = x^2 + 5x + 2$.

c **i** Given that the expression $x^2 - 5x + 7$ leaves the same remainder whether divided by $x - b$ or $x - c$, where $b \neq c$, show that $b + c = 5$.

ii Given further that $4bc = 21$ and $b > c$, find the values of b and c.

3 **a** Find the minimum integer value of a such that $ax^2 + 7x + 3$ is positive for all x.

b Find the minimum integer value of b such that $-3x^2 + bx - 4$ is negative for all x.

c Consider the quadratic equation $ax^2 + bx + c = 0$, where a, b and c are integers.

i Show that if $a + b + c = 0$, then $b^2 - 4ac$ is a perfect square.

ii Show that if $b - a - c = 0$, then $b^2 - 4ac$ is a perfect square.

iii Explain how these two results can help you to choose the coefficients of a quadratic equation so that it has rational solutions.

iv Give several examples of quadratic equations with rational solutions.

v Give an example to show that not all quadratic equations with rational solutions satisfy one of the two properties $a + b + c = 0$ or $b - a - c = 0$.

4 As a pendulum swings, its horizontal position, x cm, measured from the central position, varies from -4 cm (at A) to 4 cm (at B).

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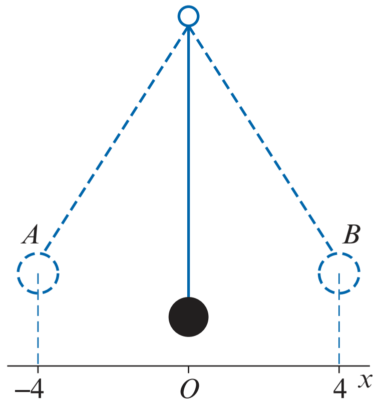
The position x is given by the rule

$$x = -4\sin(\pi t)$$



a Sketch the graph of x against t for $t \in [0, 2]$.

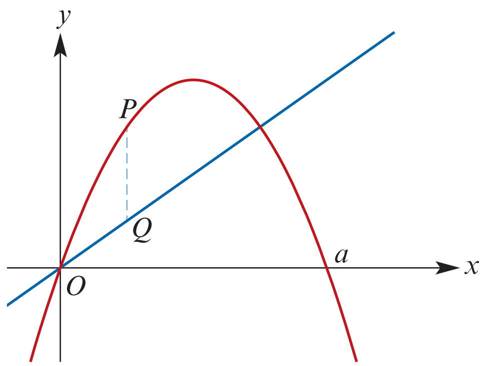
b Find the horizontal position of the pendulum for:

i $t = 0$ **ii** $t = \frac{1}{2}$ **iii** $t = 1$

c Find the first time that the pendulum has horizontal position $x = 2$.

d Find the period of the pendulum, i.e. the time it takes to go from A to B and back to A.

5 The parabola $y = ax - x^2$ and line $y = x$ are shown, where a is a constant with $a > 1$.



Point $P(x, y)$ is on the parabola and point $Q(x, x)$ is on the line, for $0 \le x \le a - 1$.

Let h be the length of line segment PQ. Then h is the 'vertical distance' between the two graphs for a given value of x.

a Find an expression for h in terms of x.

b Find the value of x (expressed in terms of a) for which h is a maximum.

c Find this maximum value of h in terms of a.

d Find the value of a for which the maximum value of h is:

i $\dfrac{1}{4}$ **ii** 1 **iii** 5 **iv** 9 **v** 10

6 Let $f(x) = x^2 - ax$ and $g(x) = bx - x^2$, where a and b are constants with $0 < a < b$.

a Find the coordinates of the points of intersection of the graphs of f and g.

b Sketch the graphs of f and g on the one set of axes.

Consider points $P(x, f(x))$ and $Q(x, g(x))$ for $0 \le x \le \dfrac{a+b}{2}$.

c Find an expression for the distance PQ in terms of x, a and b.

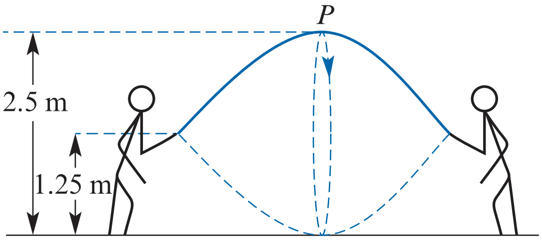
d Hence find the maximum possible distance PQ in terms of a and b.

7 Two people are rotating a skipping rope. The rope is held 1.25 m above the ground. It reaches a height of 2.5 m above the ground, and just touches the ground.

Revision

The vertical position, y m, of the point P on the rope at time t seconds is given by the rule

$$y = -1.25\cos(2\pi t) + 1.25$$



a Find y when:

 i $t = 0$

 ii $t = \frac{1}{2}$

 iii $t = 1$

b How long does it take for one rotation of the rope?

c Sketch the graph of y against t.

d Find the first time that the point P on the rope reaches a height of 2 m above the ground.

8 The population of a country is found to be growing *continuously* at an annual rate of 2.96% after 1 January 1950. The population t years after 1 January 1950 is given by the formula

$$p(t) = (150 \times 10^6)e^{kt}$$

a Find the value of k.

b Find the population on 1 January 1950.

c Find the population on 1 January 2000.

d After how many years would the population be 300×10^6?

9 A large urn was filled with water. It was turned on, and the water was heated until its temperature reached 95°C. This occurred at exactly 2 p.m., at which time the urn was turned off and the water began to cool. The temperature of the room where the urn was located remained constant at 15°C.

Commencing at 2 p.m. and finishing at midnight, Jenny measured the temperature of the water every hour on the hour for the next 10 hours and recorded the results.
At 4 p.m., Jenny recorded the temperature of the water as 55°C. She found that the temperature, T°C, of the water could be described by the equation

$$T = Ae^{-kt} + 15, \quad \text{for } 0 \le t \le 10$$

where t is the number of hours after 2 p.m.

a Find the values of A and k.

b Find the temperature of the water at midnight.

c At what time did Jenny first record a temperature less than 24°C?

d Sketch the graph of T against t.

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10 A football is kicked so that it leaves the player's foot with a velocity of V m/s. The total horizontal distance travelled by the football, x m, is given by

$$x = \frac{V^2 \sin(2\alpha)}{10}$$

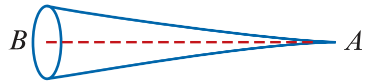
where α is the angle of projection.

a Find the horizontal distance travelled by the ball if $V = 25$ m/s and $\alpha = 45°$.

b For $V = 20$, sketch the graph of x against α for $0° \le \alpha \le 90°$.

c If the ball goes 30 m and the initial velocity is 20 m/s, find the angle of projection.

11 The diagram shows a conical glass fibre. The circular cross-sectional area at end B is 0.02 mm^2.



The cross-sectional area diminishes by a factor of $(0.92)^{\frac{1}{10}}$ per metre length of the fibre. The total length is 5 m.

a Write down a rule for the cross-sectional area of the fibre at a distance x m from B.

b What is the cross-sectional area of the fibre at a point one-third of its length from B?

c The fibre is constructed such that the strength increases in the direction B to A. At a distance of x m from B, the strength is given by the rule $S = (0.92)^{10-3x}$. If the load the fibre will take at each point before breaking is given by

load = strength × cross-sectional area

write down an expression, in terms of x, for the load the fibre will stand at a distance of x m from B.

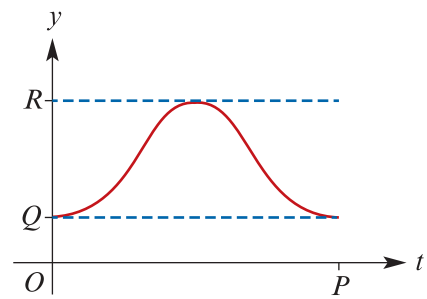
d A piece of glass fibre that will have to carry loads of up to $0.02 \times (0.92)^{2.5}$ units is needed. How much of the 5 m fibre could be used with confidence for this purpose?

12 **a** The graph is of one complete cycle of

$$y = h - k\cos\left(\frac{\pi t}{6}\right)$$

i How many units long is OP?

ii Express OQ and OR in terms of h and k.



b For a certain city in the northern hemisphere, the number of hours of daylight on the 21st day of each month is given by the table:

x	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
y	7.5	8.2	9.9	12.0	14.2	15.8	16.5	15.9	14.3	12.0	9.8	8.1	7.5

Using suitable scales, plot these points and draw a curve through them. Call December month 0, January month 1, etc., and treat all months as of equal length.

c Find the values of h and k so that your graph is approximately that of

$$y = h - k\cos\left(\frac{\pi t}{6}\right)$$

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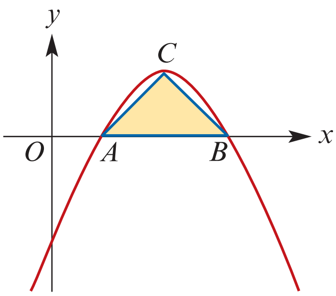
13 On an overnight interstate train, an electrical fault affected the illumination in two carriages, A and B. Before the fault occurred, the illumination in carriage A was I units and that in carriage B was $0.66\,I$ units. Every time the train stopped, the illumination in carriage A reduced by 17% and that in carriage B by 11%.

a Write down exponential expressions for the expected illumination in each carriage after the train had stopped for the nth time.

b At some time after the fault occurred, the illumination in both carriages was approximately the same. At how many stations did the train stop before this occurred?

14 **a** The curve $y = 1 - a(x - 3)^2$ in the figure intersects the x-axis at A and B. Point C is the vertex of the curve and a is a positive constant.



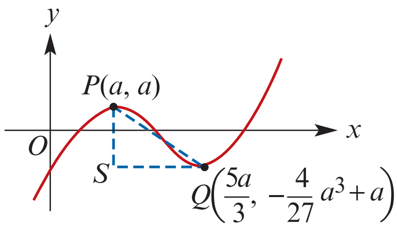
i Find the coordinates of A and B in terms of a.

ii Find the area of triangle ABC in terms of a.

b The graph shown has rule

$$y = (x - a)^2(x - 2a) + a$$

where $a > 0$.



i Use a calculator to sketch the graph for $a = 1, 2, 3$.

ii Find the values of a for which $\frac{-4}{27}a^3 + a = 0$.

iii Find the values of a for which $\frac{-4}{27}a^3 + a < 0$.

iv Find the value of a for which $\frac{-4}{27}a^3 + a = -1$.

v Find the value of a for which $\frac{-4}{27}a^3 + a = 1$.

vi Plot the graphs $y = (x - a)^2(x - 2a) + a$ for the values of a obtained in **iv** and **v**.

c Triangle PSQ is a right-angled triangle.

i Give the coordinates of S.

ii Find the length of PS and SQ in terms of a.

iii Give the area of triangle PSQ in terms of a.

iv Find the value of a for which the area of the triangle is 4.

v Find the value of a for which the area of the triangle is 1500.

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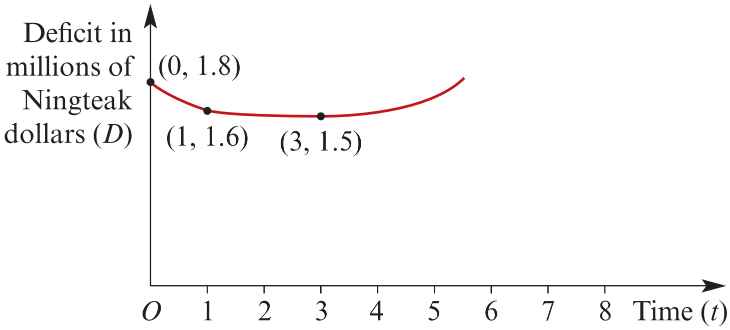
15 The deficit of a government department in Ningteak, a small monarchy east of Africa, is continually assessed over a period of 8 years. The following graph shows the deficit over these 8 years.



The graph is read as follows: The deficit at the beginning of the 8-year period is \$1.8 million. At the end of the third year the deficit is \$1.5 million, and this is the smallest deficit for the period $0 \le t \le 8$.

a Find the rule for D in terms of t, assuming that it is of the form $D = at^2 + bt + c$.

b Use this model to predict the deficit at the end of 8 years.

16 The rate of rainfall, R mm per hour, was recorded during a very rainy day in North Queensland. The recorded data are given in the table.

Assume a quadratic rule of the form

$$R = at^2 + bt + c$$

is applicable for $0 \le t \le 12$, where $t = 0$ is 4 a.m.

Use the quadratic model to predict the rate of rainfall at noon. At what time was the rate of rainfall greatest?

Time	Rainfall
4 a.m.	7.5 mm per hour
8 a.m.	9.0 mm per hour
10 a.m.	8.0 mm per hour

17 A machine in a factory has 20 different power settings. The noise produced by the machine, N dB, depends on the power setting, P, according to a rule of the form

$$N = a\log_{10}(bP) \quad \text{for } P = 1, 2, 3, \ldots, 20$$

where a and b are constants.

a Find the values of a and b, given that the machine produces a noise of 45 dB on power setting 1 and a noise of 90 dB on power setting 10.

b Find the maximum noise level produced by the machine (to the nearest decibel).

c On weekends, the local council imposes a noise-level restriction of 75 dB on the factory. What is the maximum power setting that can be used on the machine if it is being run on the weekend?

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18 A population of insects is determined by a rule of the form

$$n = \frac{c}{1 + ae^{-bt}}, \quad t \geq 0$$

where n is the number of insects alive at time t days.

a Consider the population for $c = 5790$, $a = 4$ and $b = 0.03$.

i Find the equation of the horizontal asymptote by considering values of n as t becomes large.

ii Find n when $t = 0$.

iii Sketch the graph of the function.

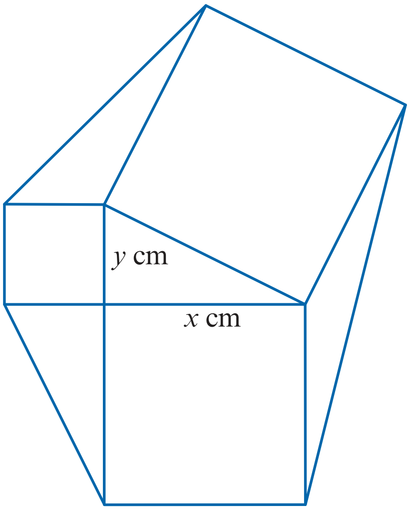
iv Find the exact value of t for which $n = 4000$.

b **i** Use your calculator to find values of a, b and c such that the population growth yields the table on the right.

t	1	10	100
n	1500	2000	5000

ii Sketch the graph for this population.

19 The two shorter sides of a right-angled triangle have lengths x cm and y cm. A square is constructed on each side of the triangle and then a hexagon is constructed as shown.



a Show that the area, A cm^2, of the hexagon is given by $A = 2(x^2 + xy + y^2)$.

b Given that $x + y = 7$, find the minimum area of the hexagon and the values of x and y for which this occurs.

c Given that $x + y = a$, where a is a positive real number, find the minimum area of the hexagon and the values of x and y for which this occurs.

20 For $x \in \mathbb{R}$ the functions f and g are defined by the rules $f(x) = x^2 + 12ax + 6a^2$ and $g(x) = 8x - 4$ where a is a positive constant.

a Find $f(g(x))$.

b Complete the square for the quadratic expression in x, $f(g(x))$.

c Determine the range of $y = f(g(x))$ in terms of a.

d Determine the rule and range of $y = f(g^{-1}(x))$ in terms of a.

e If $f(g(2)) = 630$, find the value of a.

21 Let $f : [-1, \infty) \to \mathbb{R}$ where $f(x) = a - x$. Let $g(-\infty, 2] \to \mathbb{R}$ where $g(x) = x^2 + a$.

a Find the largest set, S, of values of a such that $f(g(x))$ and $g(f(x))$ both exist,

b Find $a \in S$ such that the inverse function of $g(f(x))$ exists and give the rule, domain and range of this inverse function.

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8D Algorithms and pseudocode

Skill-sheet

An introduction to pseudocode is given in Appendix A of this book and the reader is referred to that appendix for explanations of the terms used in this section. You may like to use a device to implement the algorithms in this section; see the coding appendices in the Interactive Textbook for instructions.

Functions

In Chapters 1 to 7 we have discussed functions and in this section we discuss how the concept can be utilised in the context of algorithms and pseudocode.

1 The function shown opposite calculates the sum

$S_n = -1 + 2 - 3 + 4 + \cdot + (-1)^n n.$

If $n = 5$, $S_5 = -1 + 2 - 3 + 4 - 5 = -3$.

```
define f(n):
  sum ← 0
  for i from 1 to n
      sum ← sum + (−1)^i × i
  end for
  return sum
```

a Undertake a desk check for the sum S_6.

b Change the code to describe the algorithm which sums the first n multiples of 3.

c Change the code to describe the algorithm which gives the product of the first n natural numbers.

2 A piecewise defined function is shown below to the left and the associated pseudocode is given to the right.
f: {natural numbers ≤ 50} $\to \mathbb{N}$.

$$f(n) = \begin{cases} n^2 & \text{if } n \leq 10 \\ n^2 + 1 & \text{if } 10 < n \leq 20 \\ n^2 + 2 & \text{if } 20 < n \leq 40 \\ n^2 + 3 & \text{if } 40 < n \leq 50 \end{cases}$$

```
define f(n):
  if n ≤ 10 then
    T ← n^2
  else if n ≤ 20 then
    T ← n^2 + 1
  else if n ≤ 40 then
    T ← n^2 + 2
  else if n ≤ 50
    T ← n^2 + 3
  else:
    T ← "value out of domain"
  end if
  return T
```

a Use the pseudocode to evaluate

i $f(11)$ **ii** $f(40)$ **iii** $f(34)$

b Write the pseudocode that will print out all of the ordered pairs of this function.

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Optimisation

3 Consider a rectangular box open at the top. Let x, y and z be the dimensions of the box measured in metres. Let S m^2 be the surface area. Assume that the volume of the box is 32 m^3 and that the dimensions of the box are integer lengths. Assume that the length and width of the open top and base are x m and y m.

a Show that $S = xy + \frac{64}{x} + \frac{64}{y}$.

b Here is an algorithm described in pseudocode to find the minimum surface area. Evaluate for the following pairs (x, y): $(1, 1), (3, 3), (4, 4), (5, 5)$ using the formula.

c Adjust the code so that print($min, xmin, ymin$) is implemented at every iteration.

d Implement the code on a device to determine the minimum surface area and the values of $xmin$ and $ymin$ that yield this minimum.

e Rewrite the code to determine the minimum surface area, if the volume of the box is 64 m^3.

```
min ← 100
for x from 1 to 32
  for y from 1 to 32
    S ← x × y + 64/x + 64/y
      if S ≤ min then
        min ← S
        xmin ← x
        ymin ← y
      end if
  end for
end for
print min, xmin, ymin
```

f Now, write the rule, in terms of two of the dimensions x and y, to give the surface area, S_1 of a closed box with volume 24 m^3 and write the pseudocode which can be used to determine the minimum surface area. What is this minimum surface area?

Note: We are assuming integer side lengths. For a different volume, the side lengths which maximise the surface area would not necessarily be integers. However, the program will give you valuable information - the maximum surface area for integer side lengths.

4 To fill an order for 100 units of its product, a firm wishes to distribute the production between two plants, Plant 1 and Plant 2. The total cost function is

$$C = x^3 + 100x^2 + y^3 + y^2 + 10\,000.$$

where x and y are the number of units produced at plants 1 and 2 respectively. Describe an algorithm using pseudocode to show how to distribute the production between Plant 1 and Plant 2 to minimize the cost.

5 A chocolate manufacturer makes has two popular products, A and B. The sales of each affect the sales of the other. The cost of production of the two are 70 cents per kilogram and 80 cents per kilogram respectively. The number of kilograms of each that can be sold each week are x kg and y kg respectively and the selling prices are a cents per kg and b cents per kg respectively.

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The variables x and y are determined by

$$x = 240(b - a) \qquad y = 240(150 + a - 2b)$$

a Show that the total profit P is given by

$$P = 240(a - 70)(b - a) + 240(b - 80)(150 + a - 2b)$$

b Describe an algorithm using pseudocode to find the find the maximum profit and the corresponding values of a and b.

6 Let x, y and z be positive integers such that $x + y + z = 48$. The algorithm shown opposite determines the values of x, y and z such that their product P is a maximum, assuming that there is only one triple (x, y, z) that gives the maximum.

```
max ← 1
for x from 1 to 48
  for y from 1 to 48
    P ← xy(48 − x − y)
      if P > max and (x + y) < 48 then
        max ← P
        xmax ← x
        ymax ← y
      end if
  end for
end for
print max, xmax, ymax
```

a Add to the code to check if there are any other triples that give the maximum value of P.

b Let x, y and z be positive integers such that $x + y + z = 64$. Describe the algorithm using pseudocode to find the values of x, y and z such that their product P is a maximum.

c Let x, y and z be positive integers such that $x + y + z = 27$, Describe the algorithm using pseudocode to find the minimum value of $x^2 + y^2 + z^2$

7 A mining company is required to move 200 workers and 36 tonnes of equipment by air. It is able to charter two aircraft: a Hawk, which can accommodate 20 workers and 6 tonnes of equipment; and an Eagle, which can accommodate 40 workers and 4 tonnes of equipment. Let x denote the number of trips made by the Hawk aircraft and let y denote the number of trips made by the Eagle aircraft. The four constraints on the values x and y can take are:

constraint 1: $x \geq 0$ and $y \geq 0$

constraint 2: $20x + 40y \geq 200$

constraint 3: $6x + 4y \geq 36$

Hawk aircraft cost \$3000 per trip while Eagle aircraft cost \$4000 per trip.

a Write down an expression for the cost, \$$C$, of making x trips with a Hawk aircraft and y trips with an Eagle aircraft.

b Describe an algorithm using pseudocode to determine the number of trips that should be made by each of the aircraft to minimise the total cost. Determine this cost.

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Solving equations

8 Let $f(x) = 2^x - 3^x + 2$. Then, $f(1) = 1 > 0$ and $f(2) = -3 < 0$ and it can be shown that $f(x) > 0$ for all $x < 1$ and $f(x) < 0$ for all $x > 2$. Therefore the equation $2^x - 3^x + 2 = 0$ has one real solution, which lies in the interval $[1, 2]$.

Using pseudocode, we write an algorithm to find this solution. The algorithm is known as the bisection method which you met in Mathematical Methods Units 1&2.

We introduce a *count* variable to keep track of the number of iterations in the while loop

```
define f(x):
  return 2^x − 3^x + 2

a ← 1
b ← 2
m ← 1.5
count = 0
while b − a > 2 × 0.0001
    if f(a) × f(m) < 0 then
        b ← m
    else
        a ← m
    end if
    m ← (a + b)/2
    count ← count + 1
end while
print m, count
```

We use the bisection method.

- Define the function $f(x) = 2^x - 3^x + 2$.
- Assign initial values to the variables: the left endpoint a, the right endpoint b and the midpoint m.
- We use a `while` loop, since we don't know how many iterations will be required. We want to continue until $b - a \leq 2 \times 0.01$.
  - Use an `if-then` block to update the value of the left endpoint a or the right endpoint b.
  - Then recalculate the value of the midpoint m.
  - At the end of each pass of the loop, print the values of m, b, $f(a)$, $f(m)$ and $f(b)$.
- After the `while` loop is complete, print the value of m, which is the approximate solution.

a For the given code do a desk check for the first 4 iterations, listing the values of a, b, m and *count*.

b Adapt the pseudocode for the solution of the equation $2^x - 7 = 0$. Note that $2^2 - 7 = -3$ and $2^3 - 7 = 1$. Starting with $a = 2$ and $b = 3$, do a desk check for the first 4 iterations.

c Adapt the pseudocode for the solution of the equation $\sin x - 0.7 = 0$ for $x \in \left[0, \frac{\pi}{2}\right]$.

Note that $\frac{\pi}{6} \approx 0.5236$ and $\frac{\pi}{4} \approx 0.78541$.

Starting with $a = \frac{\pi}{6}$ and $b = \frac{\pi}{4}$, do a desk check for the first 4 iterations.

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