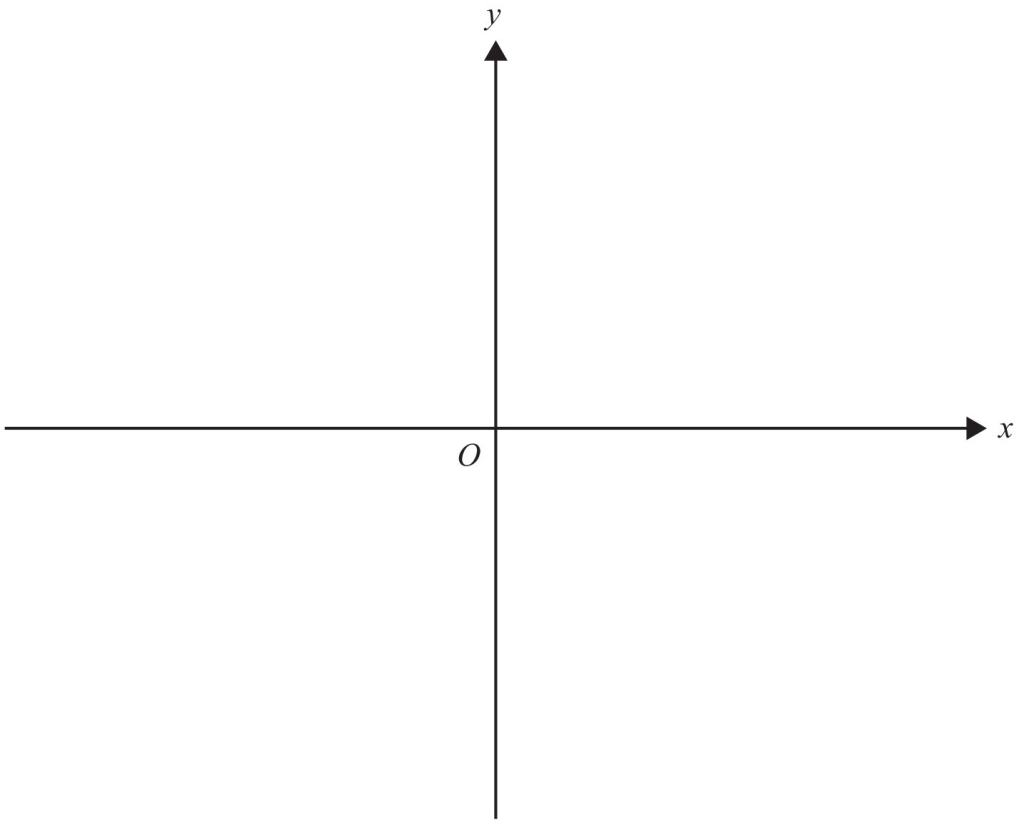
Graphs, Functions, and Relations

2016 Sample Exam 1 Question 2 / 2008 Exam 1 Question 2

On the axes below, sketch the graph of $f: R \setminus \{-1\} \to R, f(x) = 2 - \frac{4}{x+1}$.

Label each axis intercept with its coordinates. Label each asymptote with its equation. 3 marks



2016 Sample Exam 2 Question 1 / 2014 Exam 2 Question 1

The population of wombats in a particular location varies according to the rule $n(t) = 1200 + 400\cos\left(\frac{\pi t}{3}\right)$, where n is the number of wombats and t is the number of months after 1 March 2013. The period and amplitude of the function n are 3 and 400 respectively.

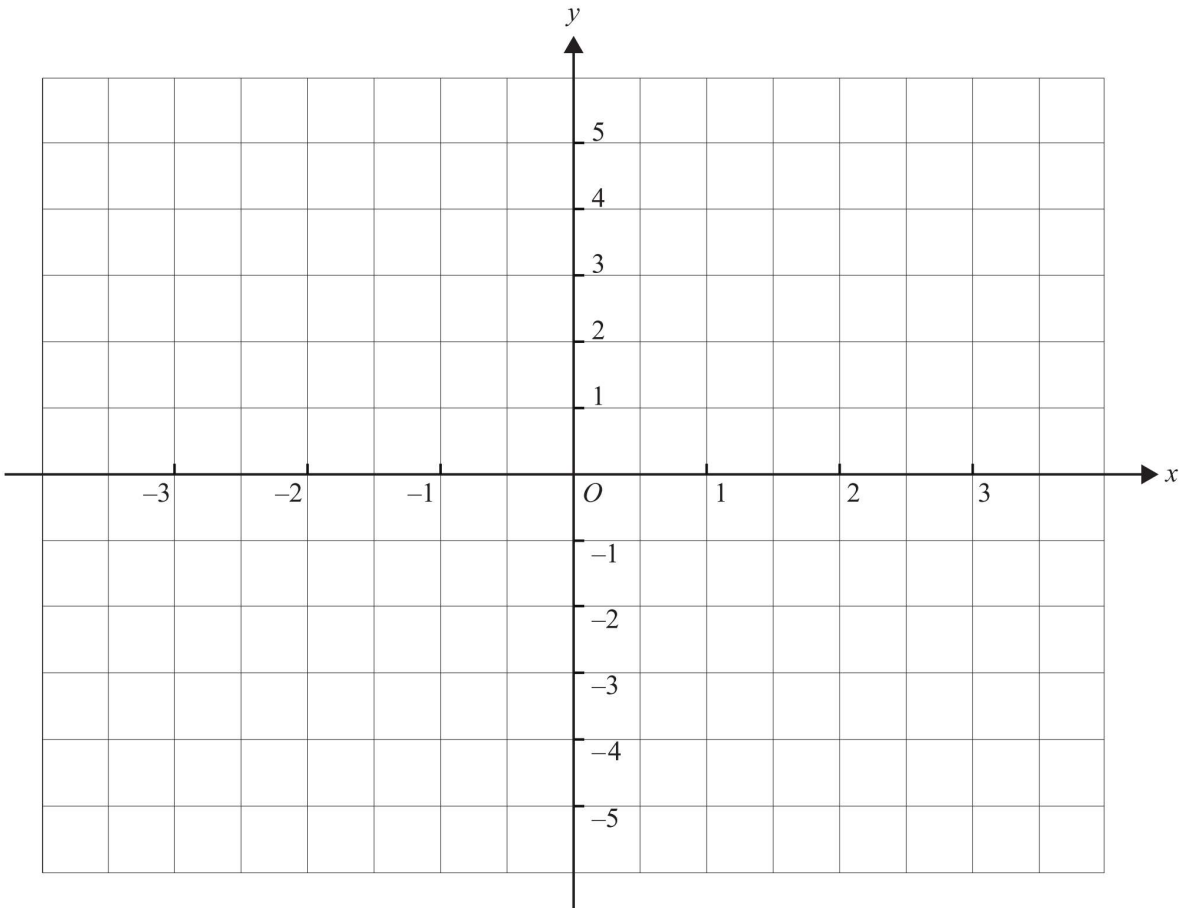
b. Find the maximum and minimum populations of wombats in this location. 2 marks

c. Find $n(10)$. 1 mark

2016 Exam 1 Question 3

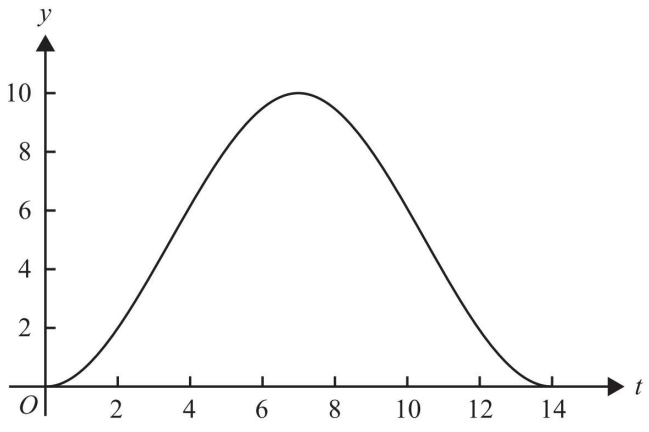
Let $f: R \setminus \{1\} \to R$, where $f(x) = 2 + \dfrac{3}{x-1}$.

a. Sketch the graph of f. Label the axis intercepts with their coordinates and label any asymptotes with the appropriate equation. 3 marks



2016 Exam 2 Question 8

The UV index, y, for a summer day in Melbourne is illustrated in the graph below, where t is the number of hours after 6 am. The graph is most likely to be the graph of



A. $y = 5 + 5\cos\left(\dfrac{\pi t}{7}\right)$

B. $y = 5 - 5\cos\left(\dfrac{\pi t}{7}\right)$

C. $y = 5 + 5\cos\left(\dfrac{\pi t}{14}\right)$

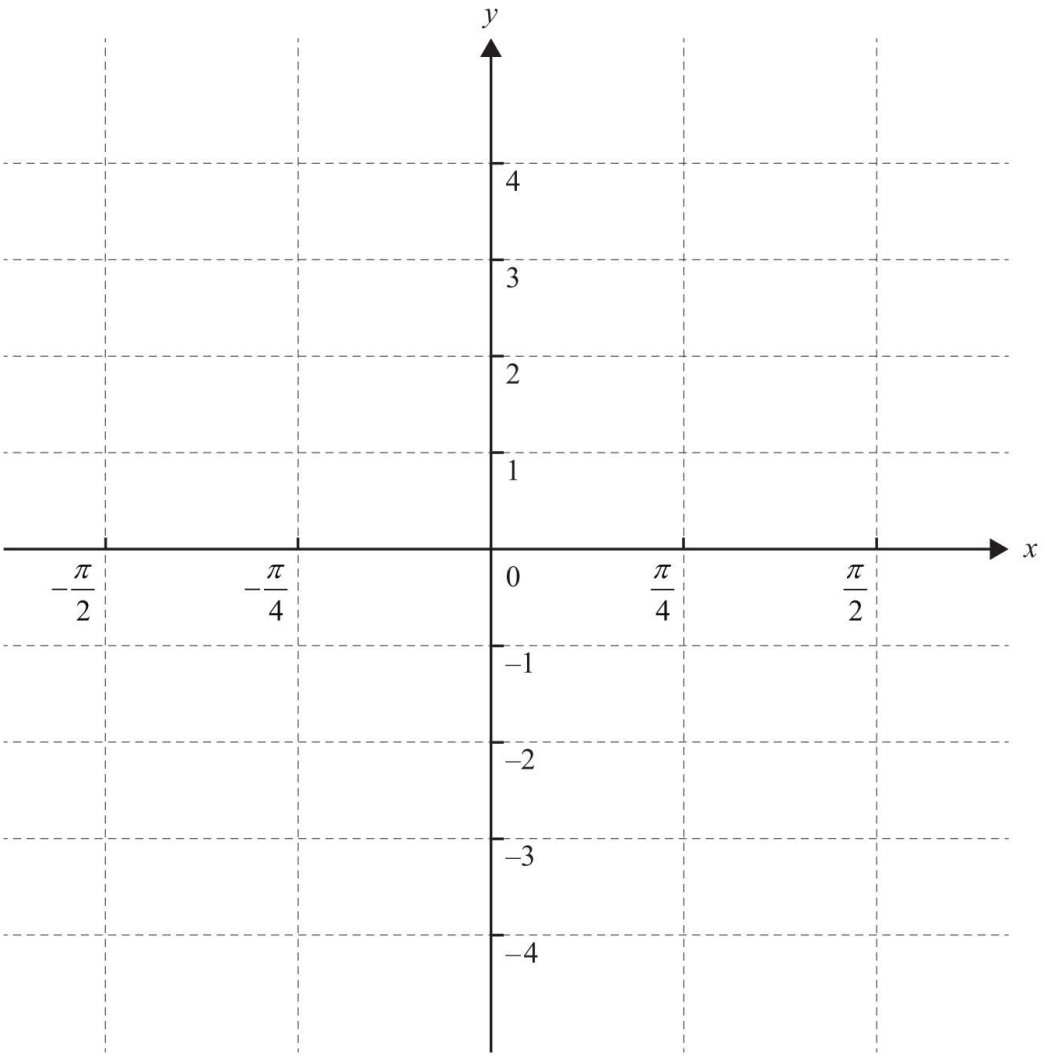
D. $y = 5 - 5\cos\left(\dfrac{\pi t}{14}\right)$

E. $y = 5 + 5\cos\left(\dfrac{\pi t}{14}\right)$

2017 NHT Exam 1 Question 4

Let $f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to R$, where $f(x) = \tan(2x) + 1$.

a. Sketch the graph of f on the axes below. Label any asymptotes with the appropriate equation, and label the end points and the axis intercepts with their coordinates. 4 marks



2017 NHT Exam 2 Question 14

The rule of the function with the graph shown above could be

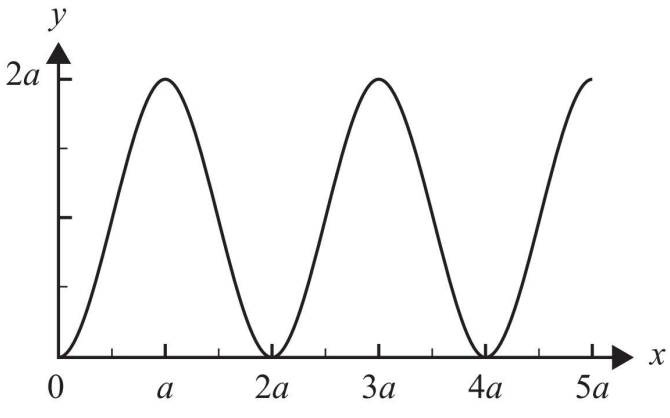
A. $y = a\cos\left(\frac{\pi x}{2a}\right) + a$

B. $y = 2a\sin\left(\frac{\pi x}{a}\right) + a$

C. $y = -a\cos\left(\frac{\pi x}{2a}\right) + a$

D. $y = a\sin\left(\frac{\pi}{a}(x - a)\right) + a$

E. $y = -a\cos\left(\frac{\pi x}{a}\right) + a$



2017 NHT Exam 2 Question 1

The temperature, T °C, in an office is controlled. For a particular weekday, the temperature at time t, where t is the number of hours after midnight, is given by the function

$$T(t) = 19 + 6\sin\left(\frac{\pi}{12}(t-8)\right), 0 \leq t \leq 24$$

a. What are the maximum and minimum temperatures in the office? 2 marks

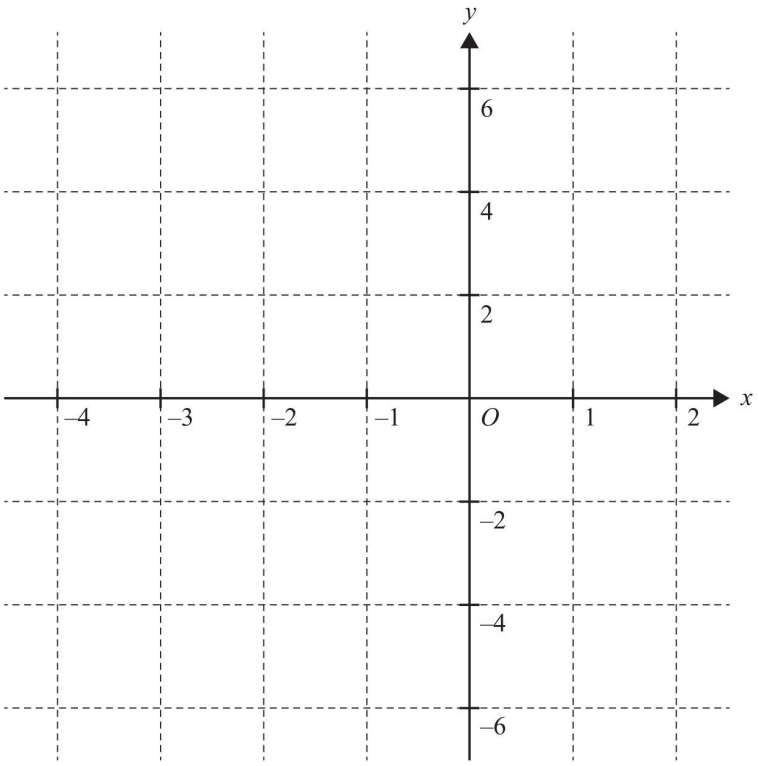
b. What is the temperature in the office at 6.00 am? 1 mark

c. Most of the people working in the office arrive at 8.00 am.
What is the temperature in the office when they arrive? 1 mark

2017 Exam 1 Question 3

Let $f: [-3,0] \to R$,
$f(x) = (x+2)^2(x-1)$
$\quad = x^3 + 3x^2 - 4.$

b. Sketch the graph of f on the axes below. Label the axis intercepts and any stationary points with their coordinates. 3 marks

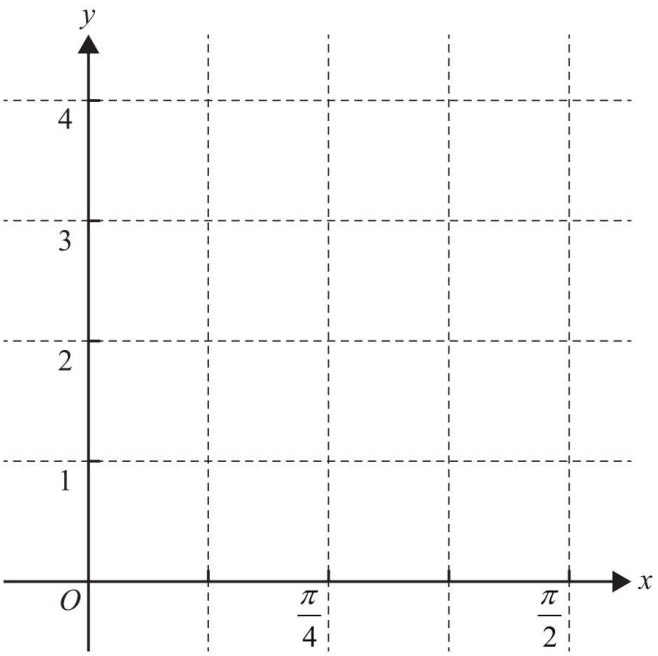


2018 NHT Exam 1 Question 7

Let $f: \left[0, \frac{\pi}{2}\right] \to R, f(x) = 4\cos(x)$

and $g: \left[0, \frac{\pi}{2}\right] \to R, g(x) = 3\sin(x)$.

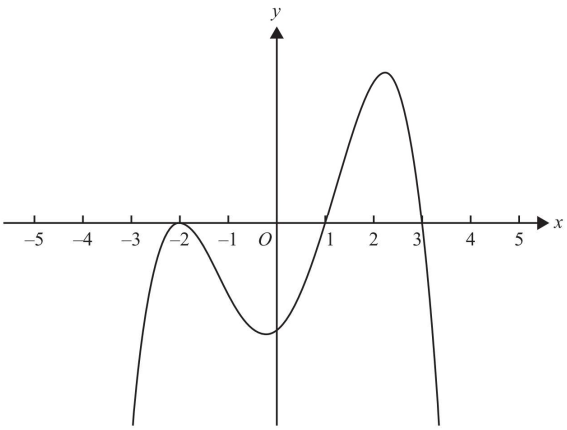
a. Sketch the graph of f and the graph of g on the axes provided below. 2 marks



2018 NHT Exam 2 Question 2

The diagram below shows part of the graph of a polynomial function. A possible rule for this function is

A. $y = (x+2)(x-1)(x-3)$
B. $y = (x+2)^2(x-1)(x-3)$
C. $y = (x+2)^2(x-1)(3-x)$
D. $y = -(x-2)^2(x-1)(3-x)$
E. $y = -(x+2)(x-1)(x-3)$

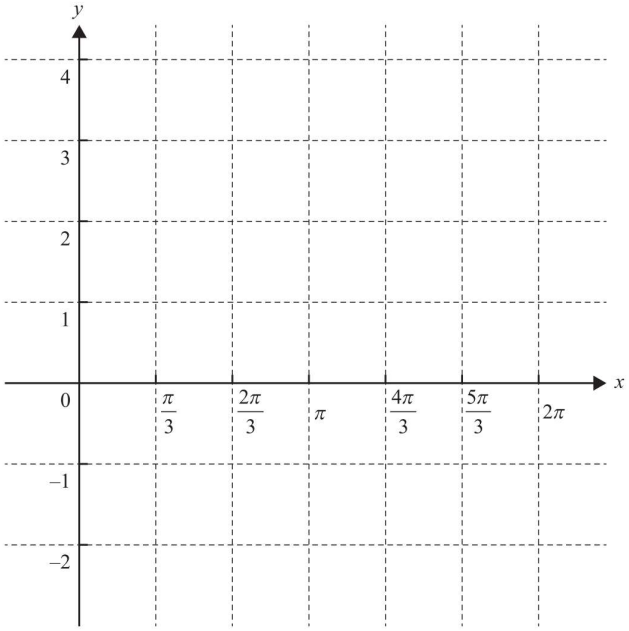


2018 Exam 1 Question 3

Let $f: [0, 2\pi] \to R, f(x) = 2\cos(x) + 1$,

$2\cos(x) + 1 = 0$ for $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$.

b. Sketch the graph of the function f on the axes below. Label the endpoints and local minimum point with their coordinates. 3 marks



2018 Exam 2 Question 11

The graph of $y = \tan(ax)$, where $a \in R^+$, has a vertical asymptote $x = 3\pi$ and has exactly one x-intercept in the region $(0, 3\pi)$. The value of a is

A. $\frac{1}{6}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{2}$ **D.** 1 **E.** 2

2018 Exam 2 Question 18

Consider the functions $f: R^+ \to R, f(x) = x^{\frac{p}{q}}$ and $g: R^+ \to R, g(x) = x^{\frac{m}{n}}$, where p, q, m and n are positive integers, and $\frac{p}{q}$ and $\frac{m}{n}$ are fractions in simplest form.

If $\{x: f(x) > g(x)\} = (0, 1)$ and $\{x: g(x) > f(x)\} = (1, \infty)$, which of the following must be false?

A. $q > n$ and $p = m$

B. $m > p$ and $q = n$

C. $pn < qm$

D. $f'(c) = g'(c)$ for some $c \in (0, 1)$

E. $f'(d) = g'(d)$ for some $d \in (1, \infty)$

2018 Exam 2 Question 5

Consider the function $g: \left[0, \frac{2a}{3}\right] \to R, g(x) = \frac{81x^2(a - x)}{4a^4}$, where a is a positive real number.

d. Evaluate $\frac{2a}{3} \times g\left(\frac{2a}{3}\right)$. 1 mark

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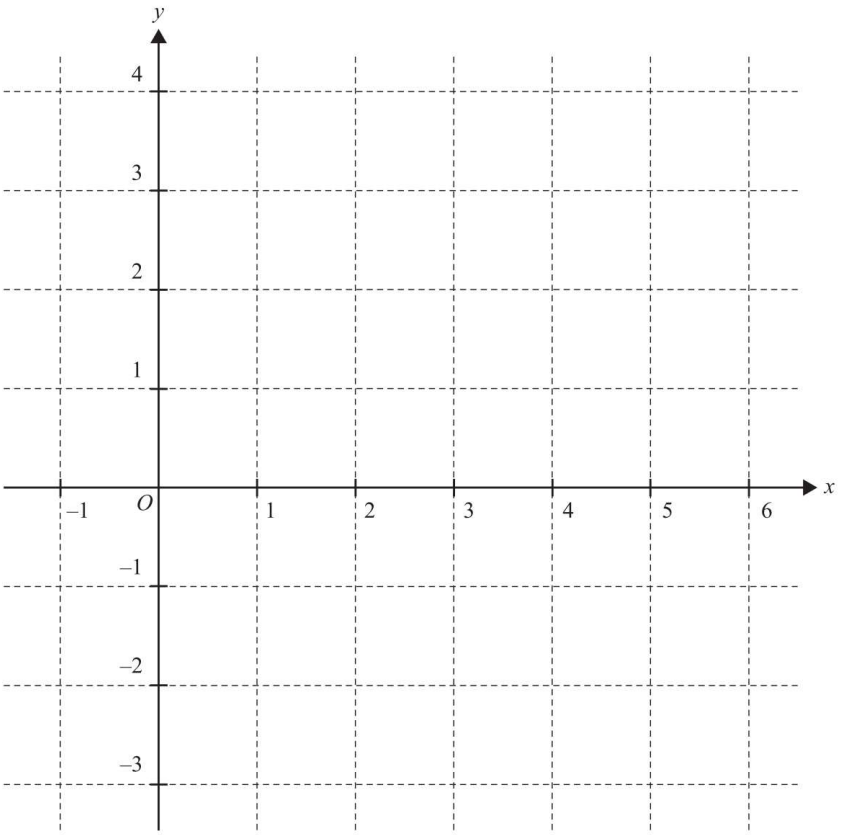
2019 NHT Exam 1 Question 4

A function g has rule $g(x) = \log_e(x - 3) + 2$.

b. The equation of the tangent to the graph of g at $(4, 2)$ is $y = x - 2$.

ii. On the axes below, sketch the graph of the function g, labelling any asymptote with its equation. Also draw the tangent to the graph of g at $(4, 2)$. 4 marks



2019 NHT Exam 2 Question 2

The wind speed at a weather monitoring station varies according to the function $v(t) = 20 + 16\sin\left(\frac{\pi t}{14}\right)$ where v is the speed of the wind, in kilometres per hour (km/h), and t is the time, in minutes, after 9 am.

b. What are the maximum and minimum wind speeds at the weather monitoring station? 1 mark

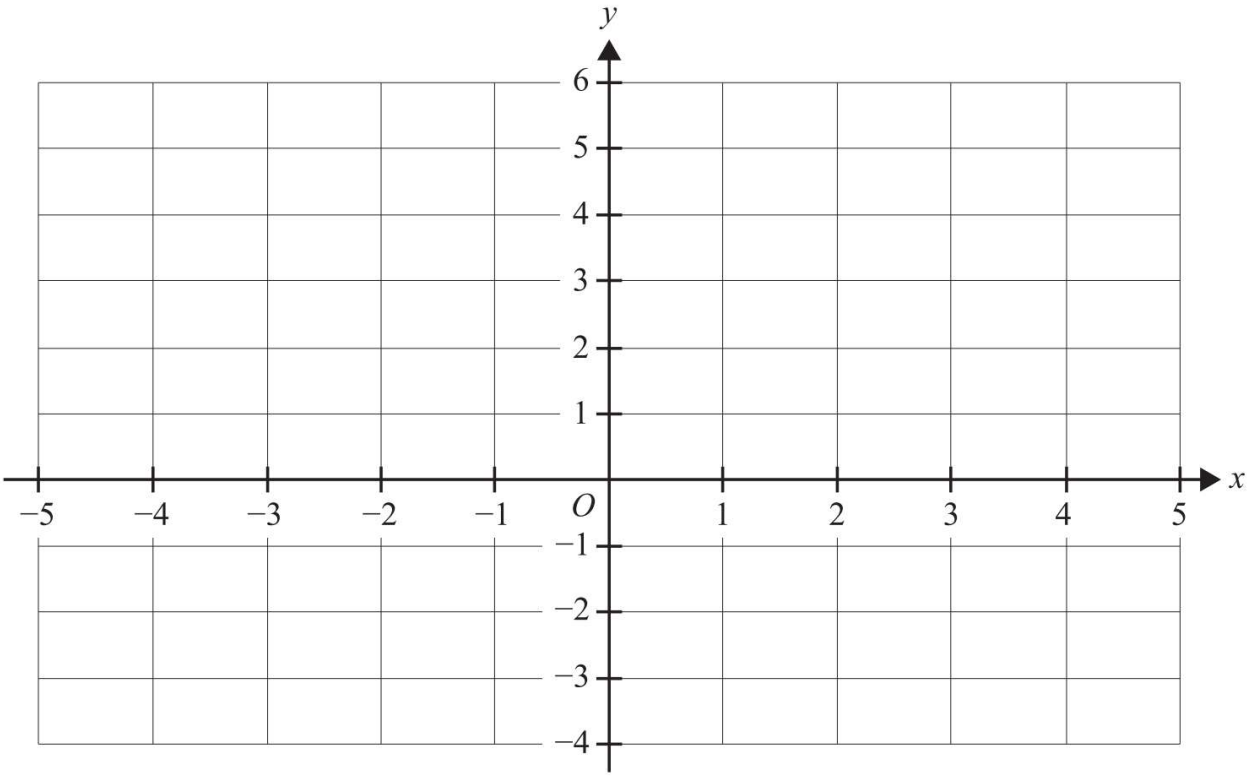
c. Find $v(60)$, correct to four decimal places. 1 mark

2019 Exam 1 Question 5

Let $f: R \setminus \{1\} \to R, f(x) = \dfrac{2}{(x-1)^2} + 1$.

a. i. Evaluate $f(-1)$. 1 mark

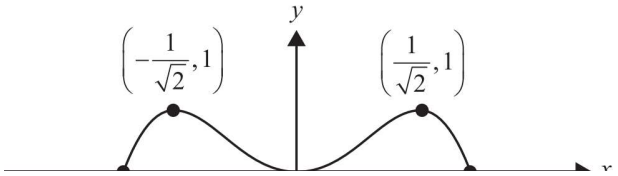
ii. Sketch the graph of f on the axes below, labelling all asymptotes with their equations. 2 marks



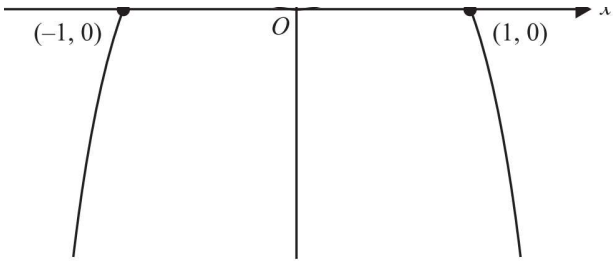
2019 Exam 1 Question 8

The function $f: R \to R, f(x)$ is a polynomial function of degree 4.

Part of the graph of f is shown below.



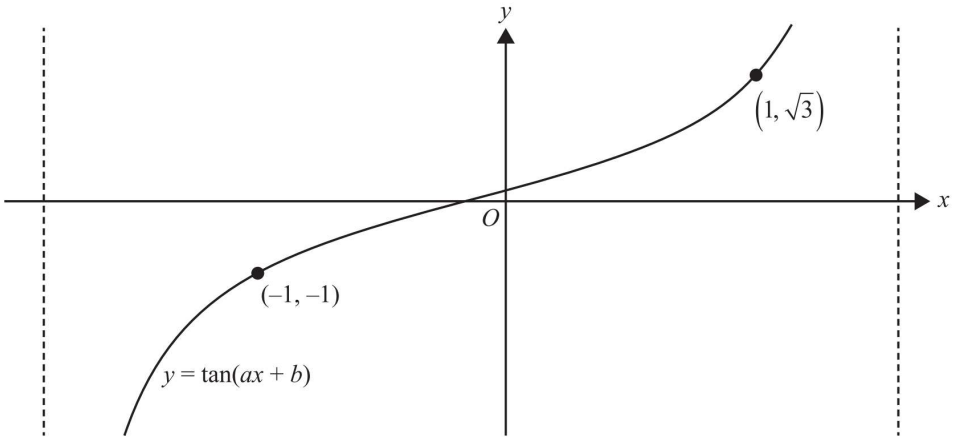
The graph of f touches the x-axis at the origin.



a. Find the rule of f. 1 mark

2020 Exam 1 Question 3

Shown below is part of the graph of a period of the function of the form $y = \tan(ax + b)$.

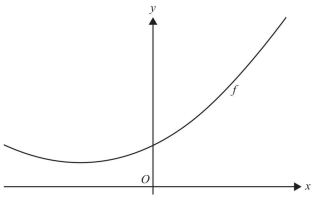


The graph is continuous for $x \in [-1, 1]$.

Find the value of a and the value of b, where $a > 0$ and $0 < b < 1$. 3 marks

2020 Exam 1 Question 7

Consider the function $f(x) = x^2 + 3x + 5$ and the point $P(1, 0)$.
Part of the graph of $y = f(x)$ is shown below.

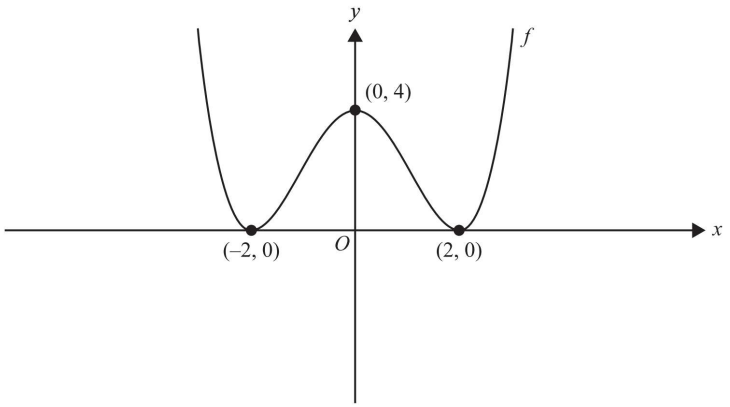


a. Show that point P is not on the graph of $y = f(x)$. 1 mark

2020 Exam 2 Question 1

Let $f: R \to R, f(x) = a(x + 2)^2(x - 2)^2$,
where $a \in R$.

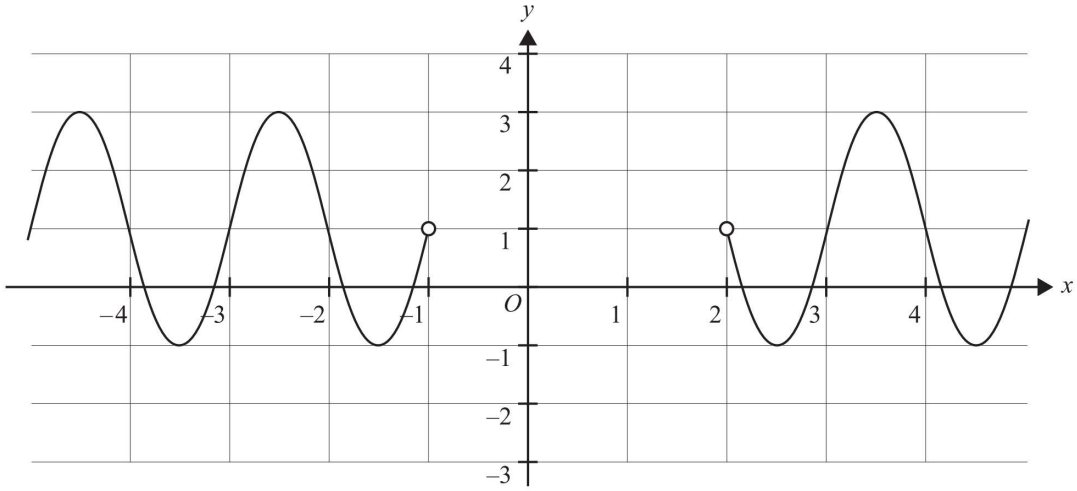
Part of the graph of f is shown below.



a. Show that $a = \frac{1}{4}$. 1 mark

2021 NHT Exam 1 Question 5

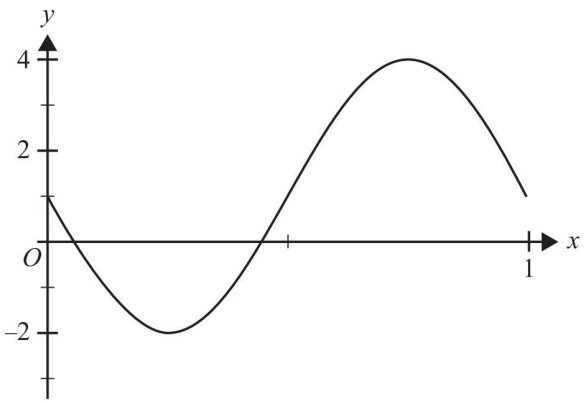
Part of the graph of $f: (-\infty, -1) \cup (2, \infty) \to R, f(x) = -2\sin(\pi x) + 1$ is shown below.



Let $g: [-1, 2] \to R, g(x) = -2\sin(\pi x) + 1$.
a. Sketch the graph of g on the axes provided above. 1 mark

2021 NHT Exam 2 Question 1

The graph below shows one cycle of a circular function. The rule for the function could be



A. $y = 3\sin(x) + 1$

B. $y = -3\sin\left(\frac{x}{2\pi}\right) + 1$

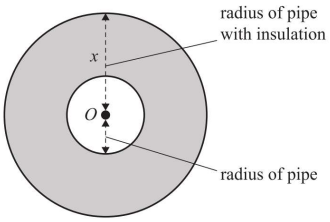
C. $y = -3\cos(2\pi x)$

D. $y = 3\sin(2\pi x) - 1$

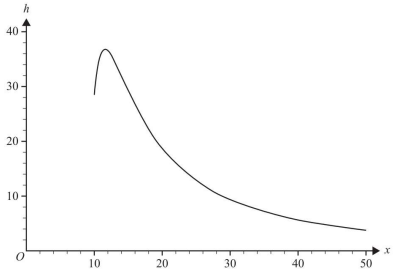
E. $y = -3\sin(2\pi x) + 1$

2021 NHT Exam 2 Question 2

The function $h(x) = \frac{3200}{(x-5)^2}\log_e\left(\frac{x-5}{4}\right)$, where $x \in [10, 50]$, models the rate at which heat is lost from the water in a hot-water pipe with insulation, where $h(x)$ is the rate at which units of heat are lost from the water and x is the radius of the hot-water pipe with its insulation, in millimetres. The diagram below shows a cross-section of the pipe with its insulation.



The radius of the pipe without its insulation is 10 mm. The graph of the rate of heat lost from the water over the given domain is shown below.



a. Find the rate at which heat is lost from the water in a pipe with no insulation, correct to three decimal places. 1 mark

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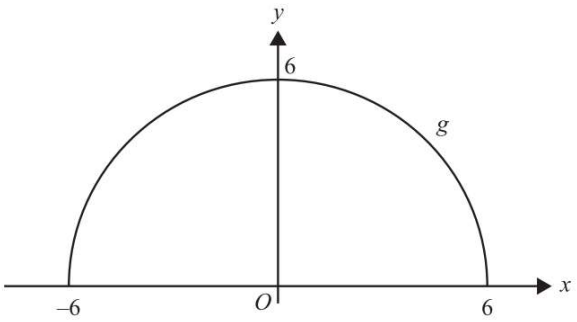
2021 NHT Exam 2 Question 3

A different tunnel has a semicircular arch. This arch can be modelled by the function $g: [-6, 6] \to R, g(x) = \sqrt{r^2 - x^2}$, where $r > 0$. The graph of g is shown below.



c. State the value of r. 1 mark

2021 NHT Exam 2 Question 5

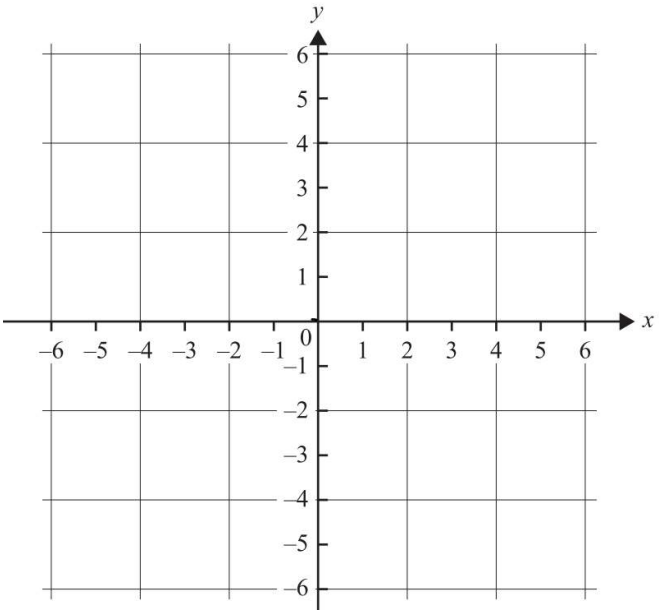
Let $p: R^+ \to R, p(x) = \frac{1}{ax^2} - \frac{a+1}{a}$ and $q: R \to R, q(x) = \frac{(1-a)x}{a}$ for $a > 1$.

d. Find the positive x-intercept of p in terms of a. 1 mark

2021 Exam 1 Question 4

a. Sketch the graph of $y = 1 - \dfrac{2}{x-2}$ on the axes below.

Label asymptotes with their equations and axis intercepts with their coordinates.
3 marks



2021 Exam 1 Question 5

Let $f: R \to R, f(x) = x^2 - 4$ and $g: R \to R, g(x) = 4(x-1)^2 - 4$.

a. The graphs of f and g have a common horizontal axis intercept at $(2, 0)$.
Find the coordinates of the other horizontal axis intercept of the graph of g. 2 marks

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2021 Exam 2 Question 3

Let $p(x) = e^{-2x} - 2e^{-x} + 1$.

c. Explain why p is not a one-to-one function. 1 mark

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