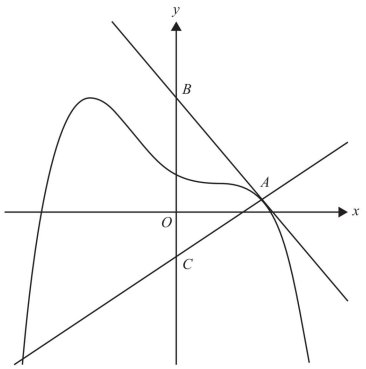
Coordinate Geometry

2016 Exam 2 Question 2

Consider the functions $g(x) = -\frac{x^4}{12} + \frac{x^2}{2} - \frac{2x}{3} + 1$ and $f(x) = g'(x) = -\frac{1}{3}(x+2)(x-1)^2$. The diagram below shows part of the graph of $y = g(x)$, the tangent to the graph at $x = 2$ and a straight line drawn perpendicular to the tangent to the graph at $x = 2$. The equation of the tangent at the point A with coordinates $(2, g(2))$ is $y = 3 - \frac{4x}{3}$. The tangent cuts the $y-$ axis at B.



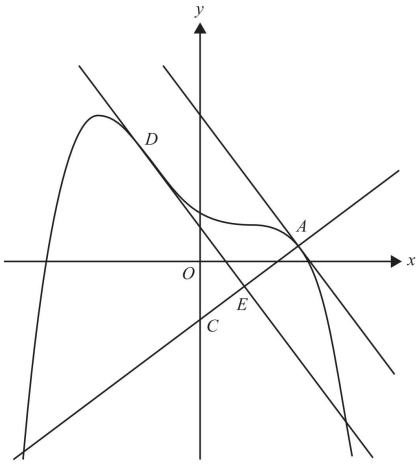
The line perpendicular to the tangent cuts the y-axis at C.

b. i. Find the coordinates of B. 1 mark

ii. Find the equation of the line that passes through A and C and, hence, find the coordinates of C. 2 marks

iii. Find the area of triangle ABC. 2 marks

c. The tangent at $D\left(-1, \frac{25}{12}\right)$ is parallel to the tangent at A. It intersects the line passing through A and C at E.



ii. Find the length of AE. 3 marks

2016 Exam 2 Question 6

Consider the graph of the function defined by $f: [0, 2\pi] \to R, f(x) = \sin(2x)$. The square of the length of the line segment joining the points on the graph for which $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$ is

A. $\frac{\pi^2 + 16}{4}$ **B.** $\pi + 4$ **C.** 4 **D.** $\frac{3\pi^2 + 16\pi}{4}$ **E.** $\frac{10\pi^2}{16}$

2017 NHT Exam 2 Question 1

The gradient of a line perpendicular to the line that passes through $(3, 0)$ and $(0, -6)$ is

A. $-\frac{1}{2}$ **B.** -2 **C.** $\frac{1}{2}$ **D.** 4 **E.** 2

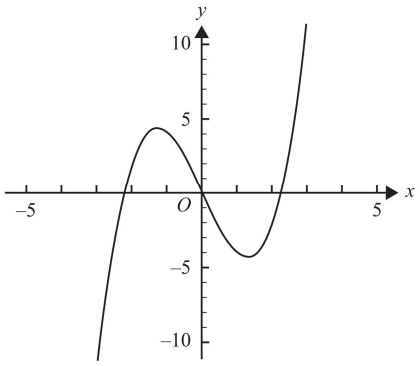
2017 NHT Exam 2 Question 2

Let $f: R \to R$, where $f(x) = (x - 2)^2(x - 5)$.

c. ii. Show that the midpoint of the line segment joining the points on the graph of $y = f(x)$ where $x = 1$ and $x = 5$ also lies on the graph of $y = f(x)$. 2 marks

2017 Exam 2 Question 1

Let $f: R \to R, f(x) = x^3 - 5x$. Part of the graph of f is shown below.



b. $A(-1, f(-1))$ and $B(1, f(1))$ are two points on the graph of f.

i. Find the equation of the straight line through A and B. 2 marks

ii. Find the distance AB. 1 mark

Let $g: R \to R, g(x) = x^3 - kx, k \in R^+$.

c. Let $C(-1, g(-1))$ and $D(1, g(1))$ be two points on the graph of g.

i. Find the distance CD in terms of k. 2 marks

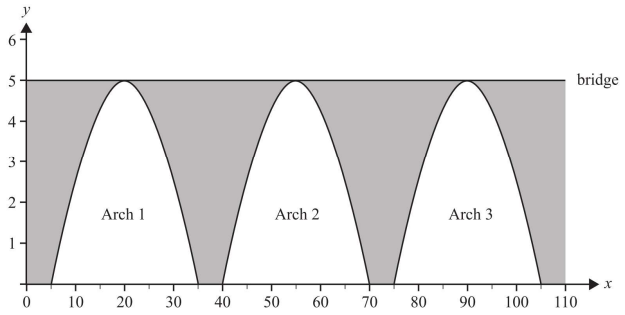
ii. Find the values of k such that the distance CD is equal to $k + 1$. 1 mark

2018 NHT Exam 2 Question 1

c. i. Find the equation of the line through $(0, -8)$ and $(2, 0)$. 1 mark

2018 Exam 2 Question 3

A horizontal bridge positioned 5 m above level ground is 110 m in length. The bridge also touches the top of three arches. Each arch begins and ends at ground level. The arches are 5 m apart at the base, as shown in the diagram below. Let x be the horizontal distance, in metres, from the left side of the bridge and let y be the height, in metres, above ground level.



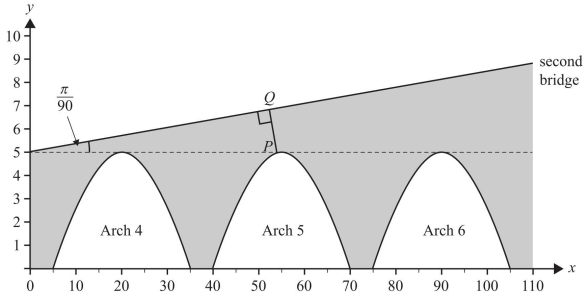
Arch 1 can be modelled by the function $h_1: [5, 35] \to R, h_1(x) = 5\sin\left(\frac{(x-5)\pi}{30}\right)$

Arch 2 can be modelled by the function $h_2: [40, 70] \to R, h_2(x) = 5\sin\left(\frac{(x-40)\pi}{30}\right)$

Arch 3 can be modelled by the function $h_3: [75, 105] \to R, h_3(x) = 5\sin\left(\frac{(x-75)\pi}{30}\right)$

The area above ground level between the arches and the bridge is filled with stone. The stone is represented by the shaded regions shown in the diagram above.

A second bridge has a height of 5 m above the ground at its left-most point and is inclined at a constant angle of elevation of $\frac{\pi}{90}$ radians, as shown in the diagram below.



The second bridge also has three arches below it, which are identical to the arches below the first bridge, and spans a horizontal distance of 110 m.

Let x be the horizontal distance, in metres, from the left side of the second bridge and let y be the height, in metres, above ground level.

The gradient of the second bridge is $\tan\left(\frac{\pi}{90}\right) = 0.035$

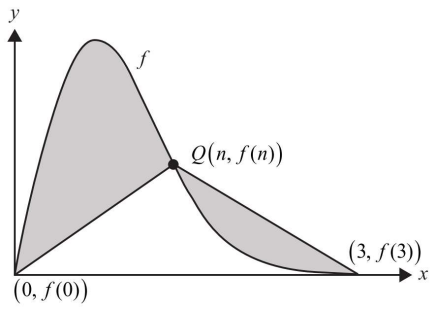
P is a point on Arch 5. The tangent to Arch 5 at point P has the same gradient as the second bridge. The coordinates of P are $(54.36, 4.99)$

f. A supporting rod connects a point Q on the second bridge to point P on Arch 5. The rod follows a straight line and runs perpendicular to the second bridge, as shown in the diagram.
Find the distance PQ, in metres, correct to two decimal places. 3 marks

2020 Exam 2 Question 4

The graph of the function $f(x) = 2xe^{(1-x^2)}$, where $0 \leq x \leq 3$, is shown below.

Two line segments connect the points $(0, f(0))$ and $(3, f(3))$ to a single point $Q(n, f(n))$, where $1 < n < 3$, as shown in the graph below.



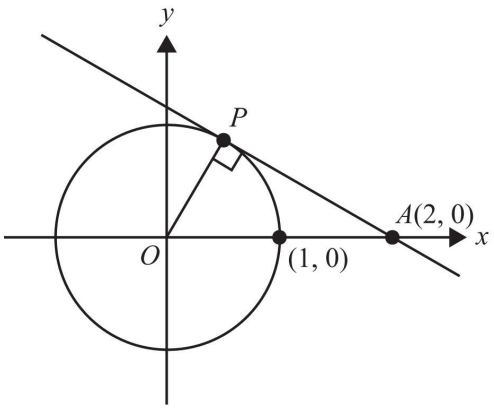
e. i. The first line segment connects the point $(0, f(0))$ and the point $Q(n, f(n))$, where $1 < n < 3$.

Find the equation of this line segment in terms of n. 1 mark

ii. The second line segment connects the point $Q(n, f(n))$ and the point $(3, f(3))$, where $1 < n < 3$.
Find the equation of this line segment in terms of n. 1 mark

2021 Exam 1 Question 9

Consider the unit circle $x^2 + y^2 = 1$ and the tangent to the circle at the point P, shown in the diagram below.



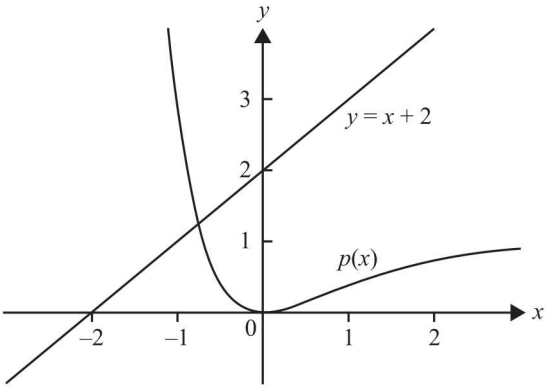
a. Show that the equation of the line that passes through the points A and P is given by $y = -\frac{x}{\sqrt{3}} + \frac{2}{\sqrt{3}}$. 2 marks

2021 Exam 2 Question 3

Let $p(x) = e^{-2x} - 2e^{-x} + 1$. p is not a one-to-one function. The gradient of the tangent to the graph of p at $x = a$ is $-2e^{-2a} + 2e^{-a}$.

The diagram below shows parts of the graph of p and the line $y = x + 2$.

The line $y = x + 2$ and the tangent to the graph of p at $x = a$ intersect with an acute angle of θ between them.



e. Find the value(s) of a for which $\theta = 60°$. Give your answer(s) correct to two decimal places.
3 marks