Transformations and Matrix Transformations

2016 Sample Exam 2 Question 1 / 2014 Exam 2 Question 1
The point $P(4, -3)$ lies on the graph of a function f. The graph of f is translated four units vertically up and then reflected in the y-axis. The coordinates of the final image of P are

A. $(-4, 1)$ **B.** $(-4, 3)$ **C.** $(0, -3)$ **D.** $(4, -6)$ **E.** $(-4, -1)$

2016 Sample Exam 2 Question 10 / 2014 Exam 2 Question 12
The transformation $T: R^2 \to R^2$ with rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ maps the line with equation $x - 2y = 3$ onto the line with equation

A. $x + y = 0$ **B.** $x + 4y = 0$ **C.** $-x - y = 4$ **D.** $x + 4y = -6$ **E.** $x - 2y = 1$

2016 Sample Exam 2 Question 5 / 2014 Exam 2 Question 5
Let $f: R \to R, f(x) = (x-3)(x-1)(x^2+3)$ and $g: R \to R, g(x) = x^4 - 8x = x(x-2)\left((x+1)^2 + 3\right)$.
b. Describe the translation that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$. 2 marks

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c. Find the values of d such that the graph of $y = f(x + d)$ has
i. one positive x-axis intercept 1 mark

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ii. two positive x-axis intercepts. 1 mark

2016 Exam 2 Question 12
The graph of a function f is obtained from the graph of the function g with rule $g(x) = \sqrt{2x - 5}$ reflection in the x-axis followed by a dilation from the y-axis by a factor of $\frac{1}{2}$
Which one of the following is the rule for the function f?

A. $f(x) = \sqrt{5 - 4x}$ **B.** $f(x) = -\sqrt{x - 5}$ **C.** $f(x) = \sqrt{x + 5}$

D. $f(x) = -\sqrt{4x - 5}$ **E.** $f(x) = -\sqrt{4x - 10}$

2016 Exam 2 Question 1

Let $f: [0, 8\pi] \rightarrow R, f(x) = 2\cos\left(\frac{x}{2}\right) + \pi$. The rule for the derivative function f' is $f'(x) = -\sin\left(\frac{x}{2}\right)$

e. The rule of f' can be obtained from the rule of f under a transformation T, such that

$T: R^2 \rightarrow R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\pi \\ b \end{bmatrix}$

Find the value of a and the value of b. 3 marks

f. Find the values of x, $0 \leq x \leq 8\pi$, such that $f(x) = 2f'(x) + \pi$. 2 marks

2017 NHT Exam 1 Question 7

Let $f: R \rightarrow R$, where $f(x) = 2x^3 + 1$, and let $g: R \rightarrow R$, where $g(x) = 4 - 2x$.

The composite functions $g(f(x)) = 2 - 4x^3$ and $f(g(x)) = 1 - 16(x - 2)^3$ are also defined.

b. The transformation $T: R^2 \rightarrow R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b \\ c \end{bmatrix}$, where a, b and c are integers, maps the graph of $y = g(f(x))$ onto the graph of $y = f(g(x))$. Find the values of a, b and c. 3 marks

2017 NHT Exam 2 Question 4

The graph of the function $f: [0, \infty) \to R$, where $f(x) = 4x^{\frac{1}{3}}$, reflected in the x-axis and then translated five units to the right and six units vertically down. Which one of the following is the rule of the transformed graph?

A. $y = 4(x-5)^{\frac{1}{3}} + 6$ **B.** $y = -4(x+5)^{\frac{1}{3}} - 6$ **C.** $y = -4(x+5)^{\frac{1}{3}} + 6$

D. $y = -4(x-5)^{\frac{1}{3}} - 6$ **E.** $y = 4(x-5)^{\frac{1}{3}} + 1$

2017 Exam 2 Question 10

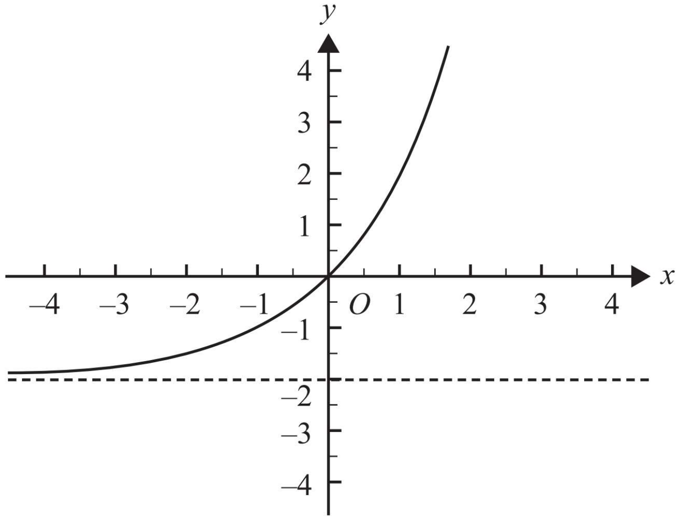
A transformation $T: R^2 \to R^2$ with rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ maps the graph of $y = 3\sin\left(2\left(x + \frac{\pi}{4}\right)\right)$

onto the graph of

A. $y = \sin(x + \pi)$ **B.** $y = \sin\left(x - \frac{\pi}{2}\right)$ **C.** $y = \cos(x + \pi)$ **D.** $y = \cos(x)$ **E.** $y = \cos\left(x - \frac{\pi}{2}\right)$

2017 Exam 2 Question 4

Let $f: R \to R: f(x) = 2^{x+1} - 2$. Part of the graph of f is shown below.



a. The transformation $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$ maps the graph of $y = 2^x$ onto the graph of f. State the values of c and d.

2018 NHT Exam 2 Question 14

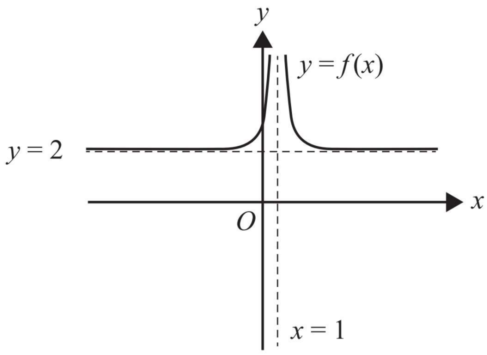
The graph of the function f is obtained from the graph of the function g with rule $g(x) = 3\cos\left(x - \frac{\pi}{6}\right)$ by a dilation of a factor of $\frac{1}{2}$ from the x-axis, a reflection in the y-axis, a translation of $\frac{\pi}{6}$ units in the negative x direction and a translation of 4 units in the negative y direction, in that order. The rule of f is

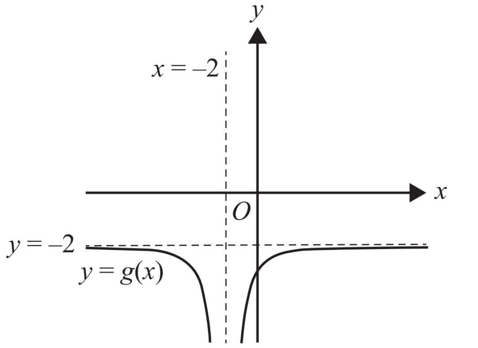
A. $f(x) = \frac{3}{2}\cos\left(-x - \frac{\pi}{3}\right) - 4$ **B.** $f(x) = \frac{3}{2}\cos(-x) - 4$

C. $f(x) = -\frac{3}{2}\cos(x) - 4$ **D.** $f(x) = -3\cos\left(\frac{x}{2} - \frac{\pi}{3}\right) - 4$

E. $f(x) = \frac{3}{2}\cos\left(-x + \frac{\pi}{3}\right) - 4$

2018 NHT Exam 2 Question 18
Consider the graphs of f and g below, which have the same scale.





If T transforms the graph of f onto the graph of g, then

A. $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ -4 \end{bmatrix}$

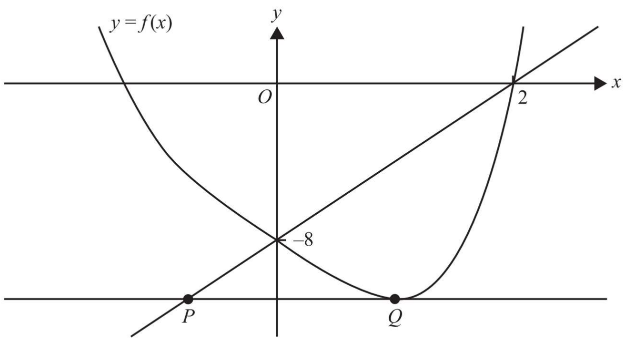
B. $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ -4 \end{bmatrix}$

C. $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \end{bmatrix}$

D. $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

E. $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

2018 NHT Exam 2 Question 1
Let $f: R \to R, f(x) = x^4 - 4x - 8$
$f(x) = (x-2)(x^3 + 2x^2 + 4x + 4)$. A solution to $f(x) = 0$ is in the interval $(m, n) = (-2, -1)$.
The diagram below shows part of the graph of f and a straight line drawn through the points $(0, -8)$ and $(2, 0)$. A second straight line is drawn parallel to the horizontal axis and it touches the graph of f at the point Q. The two straight lines intersect at the point P.



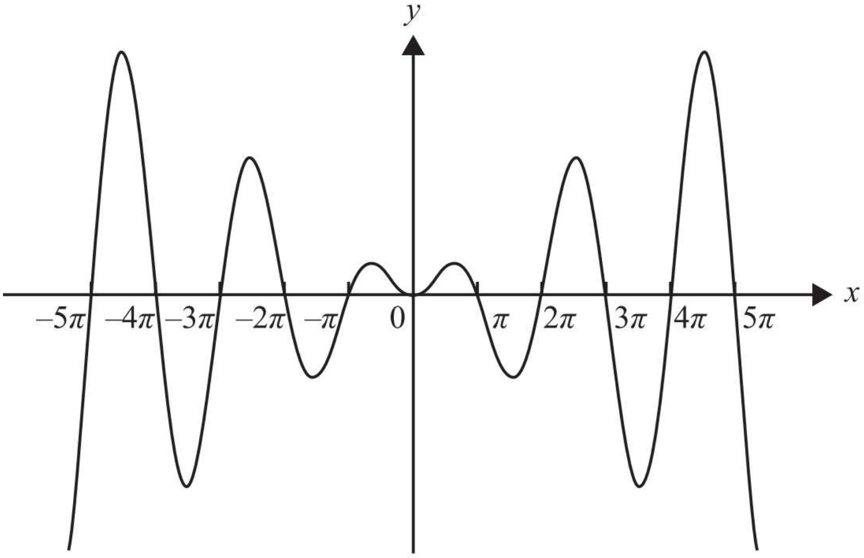
The equation of the line through $(0, -8)$ and $(2, 0)$ is $y = 4x - 8$.
The equation of the line through the points P and Q is $y = -11$.
The coordinates of the points $P\left(-\frac{3}{4}, -11\right)$ and $Q(1, -11)$.

d. A transformation $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d \\ 0 \end{bmatrix}$ is applied to the graph of f.

i. Find the value of d for which P is the image of Q. 1 mark

ii. Let $(m', 0)$ and $(n', 0)$ be the images of $(m, 0)$ and $(n, 0)$ respectively, under the transformation T, where $m = -2$ and $n = -1$. Find the values of m' and n'. 1 mark

2018 Exam 1 Question 9
Consider a part of the graph of $y = x\sin(x)$, as shown below.

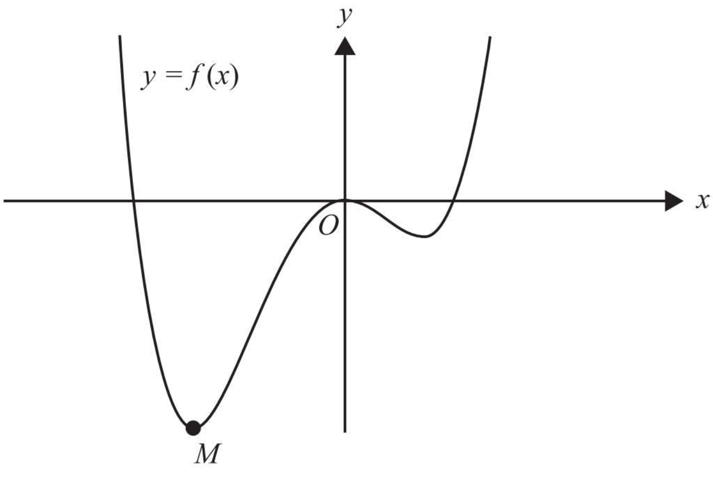


c. The translation T maps the graph of $y = x\sin(x)$ onto the graph of $y = (3\pi - x)\sin(x)$, where $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a \\ 0 \end{bmatrix}$ and a is a real constant. State the value of a. 1 mark

2018 Exam 2 Question 4
The point $A(3, 2)$ lies on the graph of the function f. A transformation maps the graph of f to the graph of g, where $g(x) = \frac{1}{2}f(x-1)$. The same transformation maps the point A to the point P. The coordinates of the point P are

A. $(2, 1)$ **B.** $(2, 4)$ **C.** $(4, 1)$ **D.** $(4, 2)$ **E.** $(4, 4)$

2018 Exam 2 Question 1
Consider the quartic $f: R \to R, f(x) = 3x^4 + 4x^3 - 12x^2$ and part of the graph of $y = f(x)$ below.

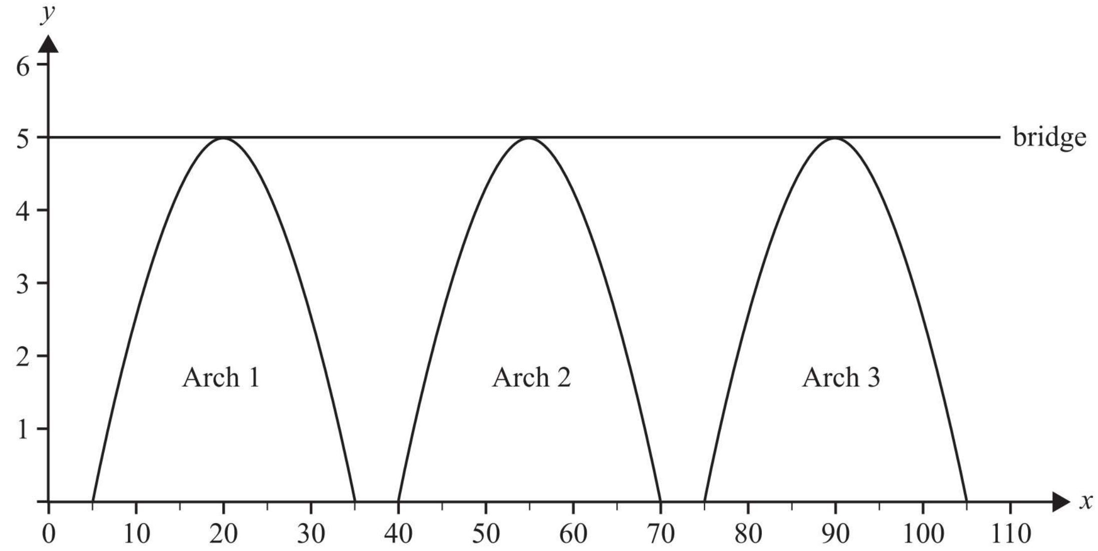


The coordinates of the point M, at which the minimum value of the function f occurs are $(-2, -32)$.

b. State the values of $b \in R$ for which the graph of $y = f(x) + b$ has no x-intercepts. 1 mark

2018 Exam 2 Question 3

A horizontal bridge positioned 5 m above level ground is 110 m in length. The bridge also touches the top of three arches. Each arch begins and ends at ground level. The arches are 5 m apart at the base, as shown in the diagram below. Let x be the horizontal distance, in metres, from the left side of the bridge and let y be the height, in metres, above ground level.



Arch 1 can be modelled by the function $h_1: [5, 35] \to R, h_1(x) = 5\sin\left(\frac{(x-5)\pi}{30}\right)$

Arch 2 can be modelled by the function $h_2: [40, 70] \to R, h_2(x) = 5\sin\left(\frac{(x-40)\pi}{30}\right)$

Arch 3 can be modelled by the function $h_3: [a, 105] \to R, h_3(x) = 5\sin\left(\frac{(x-a)\pi}{30}\right)$

a. State the value of a, where $a \in R$. 1 mark

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b. Describe the transformation that maps the graph of $y = h_2(x)$ to $y = h_3(x)$. 1 mark

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2019 NHT Exam 2 Question 12

The transformation $T: R^2 \to R^2$, which maps the graph of $y = -\sqrt{2x+1} - 3$ onto the graph of $y = \sqrt{x}$, has rule

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ -3 \end{bmatrix}$ **B.** $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ **C.** $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ **E.** $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

2019 NHT Exam 2 Question 17

The graph of the function g is obtained from the graph of the function f with rule $f(x) = \cos(x) - \frac{3}{8}$ by a dilation of factor translation of $\frac{4}{\pi}$ from the y-axis, a dilation of factor $\frac{4}{3}$ from the x-axis, a reflection in the y-axis and a translation of $\frac{3}{2}$ units in the positive y direction, in that order.

The range and the period of g are respectively

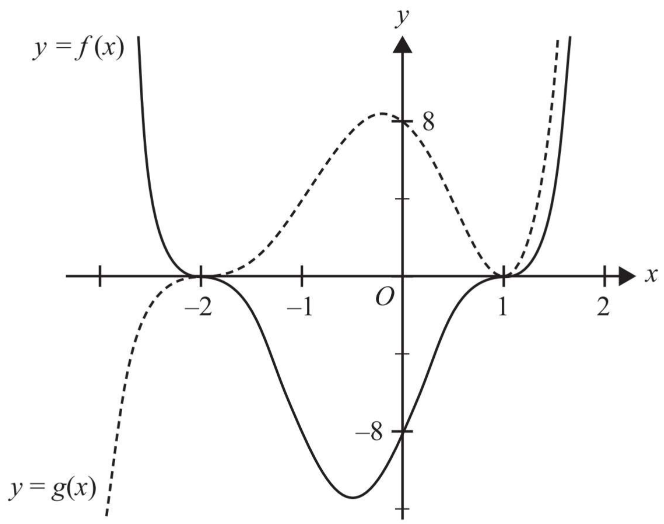
A. $\left[-\frac{1}{3}, \frac{7}{3}\right]$ and 2 **B.** $\left[-\frac{1}{3}, \frac{7}{3}\right]$ and 8 **C.** $\left[-\frac{7}{3}, \frac{1}{3}\right]$ and 2 **D.** $\left[-\frac{7}{3}, \frac{1}{3}\right]$ and 8 **E.** $\left[-\frac{4}{3}, 4\right]$ and $\frac{\pi^2}{2}$

2019 NHT Exam 2 Question 1

Parts of the graphs of $f(x) = (x-1)^3(x+2)^3$ and $g(x) = (x-1)^2(x+2)^3$ are shown on the axes below.

The two graphs intersect at three points, $(-2, 0)$, $(1, 0)$ and (c, d).

The point (c, d) is not shown in the diagram above.



e. Find the values of h such that $g(x+h) = 0$ has exactly one negative solution. 2 marks

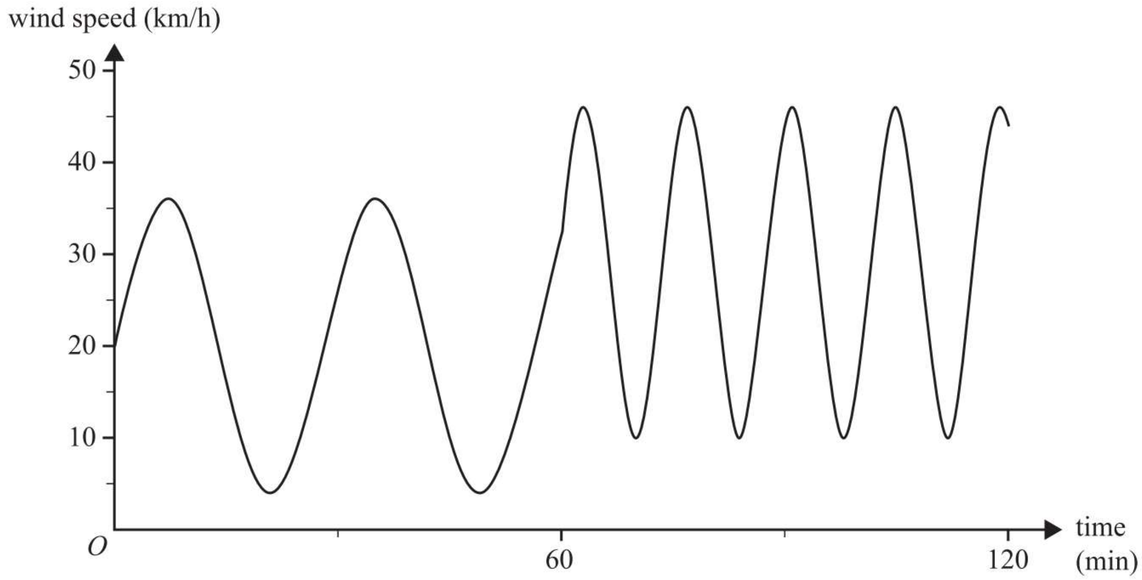
f. Find the values of k such that $f(x) + k = 0$ has no solutions. 1 mark

2019 NHT Exam 2 Question 2

The wind speed at a weather monitoring station varies according to the function $v(t) = 20 + 16\sin\left(\frac{\pi t}{14}\right)$ where v is the speed of the wind, in kilometres per hour (km/h), and t is the time, in minutes, after 9 am. $v(60) = 32.5093$

A sudden wind change occurs at 10 am. From that point in time, the wind speed varies according to the new function $v_1(t) = 28 + 18\sin\left(\frac{\pi(t-k)}{7}\right)$ where v_1 is the speed of the wind, in kilometres per hour, t is the time, in minutes, after 9 am and $k \in R^+$.

The wind speed after 9 am is shown in the diagram below.



e. Find the smallest value of k, correct to four decimal places, such that $v(t)$ and $v_1(t)$ are equal and are both increasing at 10 am. 2 marks

g. Let $f(x) = 20 + 16\sin\left(\frac{\pi x}{14}\right)$ and $g(x) = 28 + 18\sin\left(\frac{\pi(x-k)}{7}\right)$.

The transformation $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$ maps the graph of f onto the graph of g.

State the values of a, b, c and d, in terms of k where appropriate. 3 marks

2019 Exam 1 Question 2

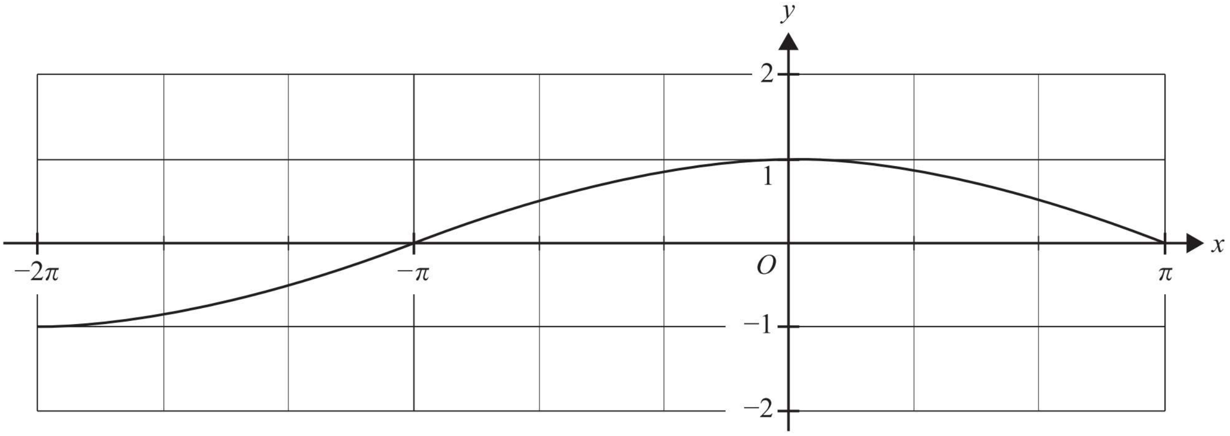
Let $f:\left(\frac{1}{3},\infty\right) \to R, f(x) = \frac{1}{3x-1}$ and $f^{-1}:(0,\infty) \to R, f^{-1}(x) = \frac{1}{3x} + \frac{1}{3}$.

c. Let g be the function obtained by applying the transformation T to the function f, where $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$ and $c, d \in R$. Find the values of c and d given that $g = f^{-1}$. 1 mark

2019 Exam 1 Question 4

The solutions to the equation $1 - \cos\left(\frac{x}{2}\right) = \cos\left(\frac{x}{2}\right)$ for $x \in [-2\pi, \pi]$ are $x = -\frac{2\pi}{3}$ or $x = \frac{2\pi}{3}$.

b. The function $f:[-2\pi,\pi] \to f(x) = \cos\left(\frac{x}{2}\right)$ is shown on the axes below.



Let $g:[-2\pi,\pi] \to R, g(x) = 1 - f(x)$. Sketch the graph of g on the axes above. Label all points of intersection of the graphs of f and g, and the endpoints of g, with their coordinates. 2 marks

2019 Exam 2 Question 13

The graph of the function f passes through the point $(-2, 7)$.

If $h(x) = f\left(\frac{x}{2}\right) + 5$, then the graph of the function h must pass through the point

A. $(-1,-12)$ **B.** $(-1, 19)$ **C.** $(-4, 12)$ **D.** $(-4,-14)$ **E.** $(3, 3.5)$

2019 Exam 2 Question 9

The point (a, b) is transformed by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} \\ -2 \end{bmatrix}$

If the image of (a, b) is $(0, 0)$, then (a, b) is

A. $(1, 1)$ **B.** $(-1, 1)$ **C.** $(-1, 0)$ **D.** $(0, 1)$ **E.** $(1,-1)$

2019 Exam 2 Question 1

Let $f: R \to R,\ f(x) = x^2 e^{-x^2}. f'(x) = (2x - 2x^3)e^{-x^2}$

b. The stationary point on the graph of f at the origin is a minimum.

The maximum value of the function f is $\frac{1}{e}$ and occurs at $x = -1$ and $x = 1$.

iii. Find the values of $d \in R$ for which $f(x) + d$ is always negative. 1 mark

2020 Exam 2 Question 13

The transformation $T: R^2 \to R^2$ that maps the graph of $y = \cos(x)$ onto the graph of $y = \cos(2x + 4)$ is

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -4 \\ 0 \end{bmatrix}\right)$ **B.** $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -4 \\ 0 \end{bmatrix}$ **C.** $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \end{bmatrix}\right)$

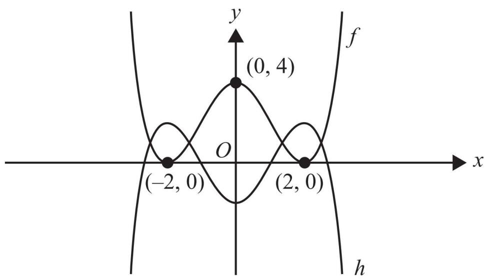
D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}\right)$ **E.** $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

2020 Exam 2 Question 1

Let $f: R \to R, f(x) = \frac{1}{4}(x + 2)^2(x - 2)^2$.

Let $h: R \to R, h(x) = -\frac{1}{4}(x + 2)^2(x - 2)^2 + 2$.

Parts of the graphs of f and h are shown below.



d. Write a sequence of two transformations that map the graph of f onto the graph of h. 1 mark

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2021 NHT Exam 1 Question 4

Let $f: R \to R, f(x) = 2e^x + 1$ and let $g: (-2, \infty) \to R, g(x) = \log_e(x + 2)$.

b. Let $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$ and let the graph of the function h be the transformation of the graph of the function g under T. If $h = f^{-1}$, then find the values of c and d. 3 marks

2021 NHT Exam 2 Question 12

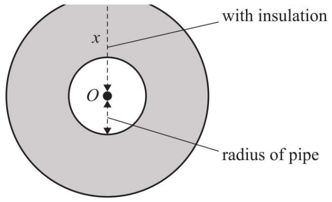
The transformation $T: R^2 \to R^2$ maps the graph of $y = x^3 - x$ onto the graph of $y = 2(x - 1)^3 - 2(x - 1) + 4$. The transformation T could be given by

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ **B.** $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ **C.** $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

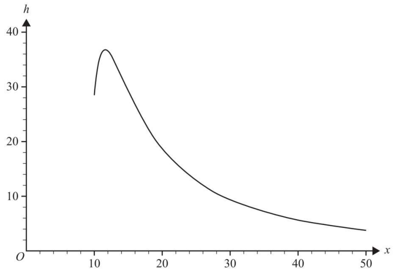
D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ **E.** $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

2021 NHT Exam 2 Question 2

The function $h(x) = \dfrac{3200}{(x-5)^2}\log_e\left(\dfrac{x-5}{4}\right)$, where $x \in [10, 50]$, models the rate at which heat is lost from the water in a hot-water pipe with insulation, where $h(x)$ is the rate at which units of heat are lost from the water and x is the radius of the hot-water pipe with its insulation, in millimetres. The diagram below shows a cross-section of the pipe with its insulation.



The radius of the pipe without its insulation is 10 mm. The graph of the rate of heat lost from the water over the given domain is shown below.



A particular insulated pipe has the same rate of heat lost from the water as a pipe with no insulation.

d. i. If both the radius of the pipe without insulation and the radius of the pipe with insulation, as shown in the diagram, are doubled, show that the rate of heat lost from the water, h_1, is now given by $h_1(x) = \dfrac{12\,800}{(x-10)^2}\log_e\left(\dfrac{x-10}{8}\right)$, and state the domain of h_1. 2 marks

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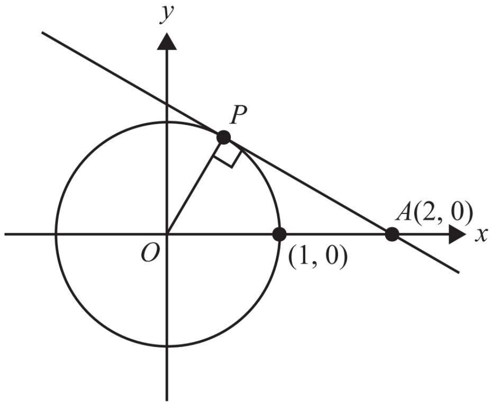
2021 Exam 1 Question 9
Consider the unit circle $x^2 + y^2 = 1$ and the tangent to the circle at the point P, shown in the diagram below.



a. Show that the equation of the line that passes through the points A and P is given by $y = -\frac{x}{\sqrt{3}} + \frac{2}{\sqrt{3}}$. 2 marks

Let $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & q \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, where $q \in R \setminus \{0\}$, and let the graph of the function h be the transformation of the line that passes through the points A and P under T.

b. i. Find the values of q for which the graph of h intersects with the unit circle at least once. 1 mark

ii. Let the graph of h intersect the unit circle twice.
Find the values of q for which the coordinates of the points of intersection have only positive values. 1 mark

ii. Describe the transformation that maps the graph of h to the graph of h_1. 1 mark

2021 Exam 1 Question 5
Let $f: R \to R, f(x) = x^2 - 4$ and $g: R \to R, g(x) = 4(x-1)^2 - 4$.
The graphs of f and g have a common horizontal axis intercept at $(2, 0)$.
The coordinates of the other horizontal axis intercept of the graph of g are $(0, 0)$.

b. Let the graph of h be a transformation of the graph of f where the transformations have been applied in the following order:

- dilation by a factor of $\frac{1}{2}$ from the vertical axis (parallel to the horizontal axis)
- translation by two units to the right (in the direction of the positive horizontal axis)

State the rule of h and the coordinates of the horizontal axis intercepts of the graph of h. 2 marks