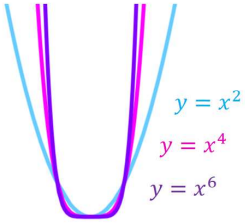
Functions and Graphs Summary

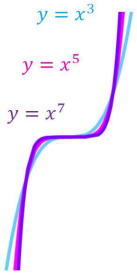
Power Functions

Integer power function $y = x^n$

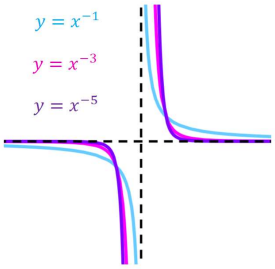
Even $n \geq 1$



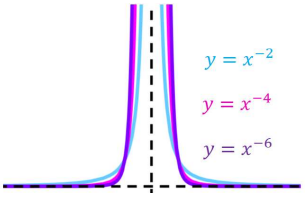
Odd $n \geq 1$



Odd $n \leq -1$

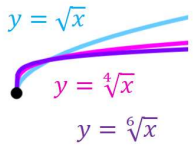


Even $n \leq -1$

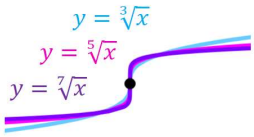


Root functions $y = x^{\frac{1}{n}}$

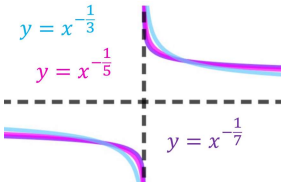
Even $n \geq 1$



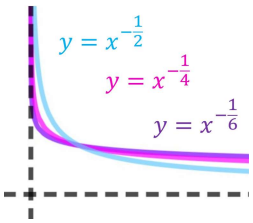
Odd $n \geq 1$



Odd $n \leq -1$

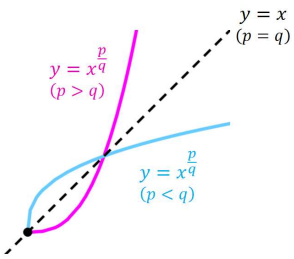


Even $n \leq -1$

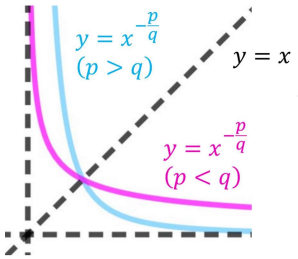


Rational power functions $y = x^{\frac{p}{q}}$

$\frac{p}{q} > 0$

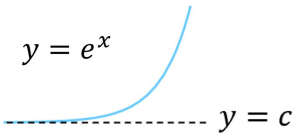


$\frac{p}{q} < 0$

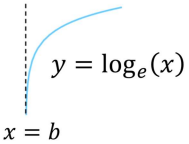


Exponential Functions

Exponential function

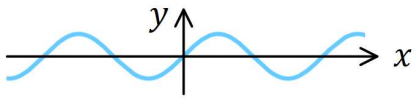


Logarithmic function

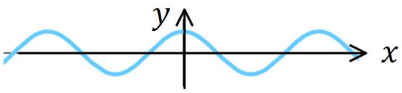


Circular Functions

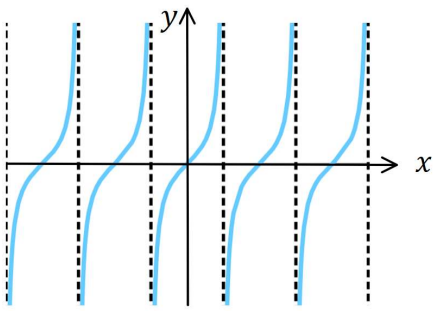
Sine function $y = \sin(x)$



Cosine function $y = \cos(x)$

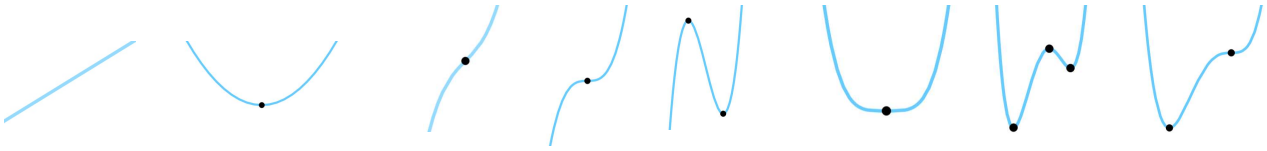


Tangent function $y = \tan(x)$



Polynomials

Linear (x) Quadratic (x^2) Cubic (x^3) Quartic (x^4)



Key features of graphs

Axis intercepts, stationary points, points of inflection, domain (including maximal, implied or natural domain), co-domain and range, asymptotic behaviour and symmetry.

Domain restrictions

$\frac{1}{f(x)}$	$\sqrt{f(x)}$	$\log_e(f(x))$	$\tan(f(x))$
$f(x) \neq 0$	$f(x) \geq 0$	$f(x) > 0$	$f(x) \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

y-intercept: $x = 0$
$y = f(0)$

x-intercept(s): $y = 0$
$0 = f(x)$

Transformations

$$y = \underbrace{f(x)}_{\text{output}} \rightarrow y = A\underbrace{f\big(n(x+b)\big)}_{\text{output}} + c, \qquad A, n, b, c \in \mathbb{R}, \qquad A, n \neq 0$$

(input; dilation/reflection; translation)

	Dilations	Reflections	Translations
y	Dilation by a factor of a from the x-axis ...by a factor of a parallel to the y-axis	A reflection in the x-axis	A translation of d units up A translation of $-d$ units down
x	Dilation by a factor of $\frac{1}{b}$ from the y-axis ...by a factor of $\frac{1}{b}$ parallel to the x-axis	A reflection in the y-axis	A translation of $-c$ units right A translation of c units left

The transformation $T: R^2 \rightarrow R^2$ is defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$.

- Dilation by a factor of a from the x-axis (reflection in y-axis if $a < 0$)
- Dilation by a factor of b from the y-axis (reflection in x-axis if $b < 0$)
- A translation of c units right and d units up

Combinations of functions

Sum function

$(f+g)(x) = f(x) + g(x)$,
$d_{f+g} = d_f \cap d_g$

Difference function

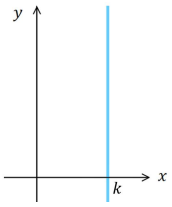
$(f-g)(x) = f(x) - g(x)$,
$d_{f-g} = d_f \cap d_g$

Product function

$(fg)(x) = f(x) \times g(x)$,
$d_{fg} = d_f \cap d_g$

Vertical line

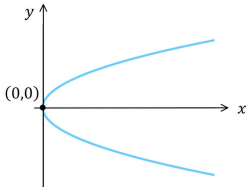
$x = k$



Side parabola

$x = y^2$

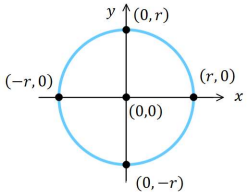
$f_1(x) = \sqrt{x}$
$f_2(x) = -\sqrt{x}$



Circle

$x^2 + y^2 = r^2$

$f_1(x) = \sqrt{r^2 - x^2}$
$f_2(x) = -\sqrt{r^2 - x^2}$



Composite function

$f \circ g(x) = f\big(g(x)\big)$, $\quad d_{f\circ g} = d_g \setminus \{x: g(x) \notin d_f\}$, $\quad r_g \subseteq d_f$

Coordinate geometry

midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $\quad$ distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{\text{run}^2 + \text{rise}^2}$

gradient: $\quad m = \frac{\text{rise}}{\text{run}}$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan(\theta)$ $\quad m_{\parallel} \times m_{\perp} = -1$

equation of a line: $\quad y = m(x-h) + k$ $\quad \frac{1}{x\text{-int}}x + \frac{1}{y\text{-int}}y = 1$