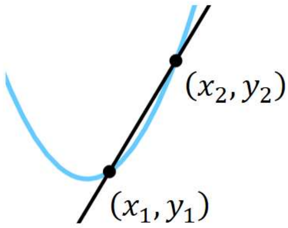
Average and Instantaneous Rates of Change

Secant through a Graph

A line that passes through a graph at two points.



Average Rate of Change

The average rate of change between two points is the gradient of the secant that through the two points.

average rate of change $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$ between $(x_1, f(x_1))$ and $(x_2, f(x_2))$

Example VCAA 2010 Exam 2 Question 2

For $f(x) = x^3 + 2x$, the average rate of change with respect to x for the interval $[1, 5]$ is

$$\text{average rate of change} = \frac{f(5) - f(1)}{5 - 1} = \frac{5^3 + 2(5) - (1^3 + 2(1))}{4} = \frac{125 + 10 - 1 - 2}{4} = \frac{132}{4} = 33$$

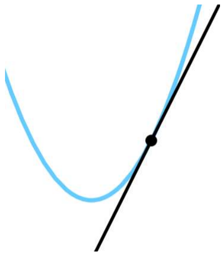
Example VCAA 2007 Exam 2 Question 4

The average rate of change of the function with rule $f(x) = x^3 - \sqrt{x + 1}$ between $x = 0$ and $x = 3$ is

$$\text{average rate of change} = \frac{f(3) - f(0)}{3 - 0} = \frac{3^3 - \sqrt{3 + 1} - (0^3 - \sqrt{0 + 1})}{3} = \frac{27 - 2 + 1}{3} = \frac{26}{3}$$

Tangent to a Graph

A line that just touches a graph. The gradient of a tangent is the same as the gradient (derivative) of the graph at that point. The gradient of the tangent is also the tangent of the angle the line makes with the x-axis. That is, $m = \tan(\theta°)$.



Instantaneous Rate of Change

The instantaneous rate of change is the same as the gradient of the tangent of the relation at that point. That is, the instantaneous rate of change of $f(x)$ at $x = a$ is $f'(a) = \lim\limits_{h \to 0}(\text{average rate of change})$.

Example

The height of a ball, in metres, thrown into the air is given by $h(t) = -4.9t^2 + 5.25t + 1.6$.
The rate at which the height of the ball is increasing after 1 second is

$$h'(t) = -9.8t + 5.25$$

$$h'(1) = -9.8 + 5.25 = -4.55 \text{ m/s}$$

Example VCAA 2001 Exam 1 Question 5a

Let $V\colon [0, \infty) \to R$ be given by $V(t) = -0.5(t + 5)^2 + 2000$.
The value of t for which the instantaneous rate of change of V with respect to t is -10 is

$$V(t) = -0.5(t + 5)^2 + 2000 = -0.5t^2 - 5t - 12.5 + 2000$$

$$V'(t) = -t - 5$$

$$-10 = -t - 5$$

$$t = 5$$