# Differentiation and the Derivative

## The Derivative / The Gradient Function

The derivative is the rate of change of a function's outputs with respect to its inputs. That is, the rule of the gradient of the graph of a function. Finding the derivative or the gradient of a function is the same thing. By obtaining a gradient function, the gradient at any point in the domain can then be quickly evaluated.

## Notations for Derivatives

For the function $y = f(x)$ the derivative with respect to $x$ can be written as $\frac{\mathrm{d}y}{\mathrm{d}x}, f'(x), \frac{\mathrm{d}}{\mathrm{d}x}\big(f(x)\big)$.

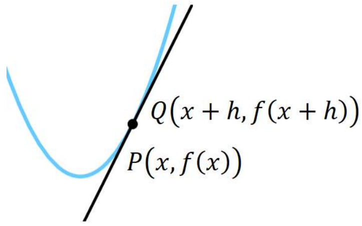
| | | |
|---|---|---|
| Joseph Louis Lagrange | $f'$ or $f^{(1)}$ | ' is called a prime mark, and $f'$ is read as $f$ prime |
| Gottfried Leibniz | $\frac{\mathrm{d}y}{\mathrm{d}x}, \frac{\mathrm{d}f(x)}{\mathrm{d}x}$, or $\frac{\mathrm{d}}{\mathrm{d}x}\big(f(x)\big)$ | $\frac{\mathrm{d}}{\mathrm{d}x}$ is called the differential operator adapted from $\frac{\delta y}{\delta x}$ |

Keep in mind that $\mathrm{d}x$ is one term. It is not $d \times x$ and should not be separated as such.

## Differentiation by First Principles

A method of expressing the derivative function using the limit of the gradient of a secant in terms of $x$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\big(f(x)\big) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$




The limit can only be evaluated when the denominator is no longer $h$.
That is, you must divide it with a factor of $h$ in the numerator.
The limit must be written every time until it is evaluated.

Be careful when using $y =$ or $f(x) =$ that you do not state the derivative is equal to the original function.
Do not write the derivate equals the original function then equate it to its derivative: $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x) = f'(x)$.
Do not write out the function then equate it to its derivate: $y = f(x) = f'(x)$. This includes over a line.

### Example

$y = x^2$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \lim_{h\to 0}\left(\frac{(x+h)^2 - x^2}{h}\right) = \lim_{h\to 0}\left(\frac{x^2+2hx+h^2-x^2}{h}\right) = \lim_{h\to 0}\left(\frac{2hx+h^2}{h}\right) = \lim_{h\to 0}(2x+h) = 2x$$

## Differentiability

A function is not differentiable at $x = a$ if the graph of the function

- is not continuous at $x = a$
- has a cusp or sharp point at $x = a$
- has a vertical tangent at $x = a$

Generally speaking, a function is differentiable when its graph is smooth and continuous.

### Example VCAA 2007 Exam 1 Question 3b

The diagram shows the graph of a function with domain $R$.

The domain of the derivative function is $R \setminus \{0, 3\}$ as the graph has a sharp point at $x = 0$ and a jump discontinuity at $x = 3$.