Graphs of Derivative and Anti-Derivative Functions

Graphs of Derivatives Functions

Since the derivative is the gradient of a function, the graph of the derivative can be sketched by plotting the gradient against the x-coordinate.

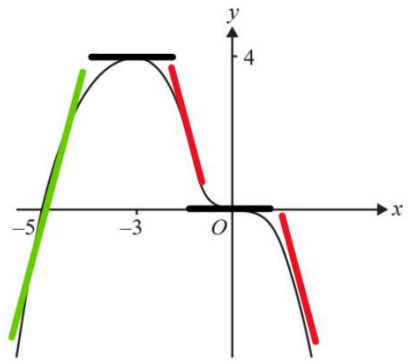
Positive gradients are plotted above the x-axis, negative gradients are plotted below the x-axis.
Stationary points, where the gradient is zero (the tangent is horizontal), are plotted as x-intercepts.
Points where the tangent is vertical have a vertical tangent on derivative.

Local minimums cut the x-axis from the negative to the positive.
Local maximums cut the x-axis from the positive to the negative.
Stationary points of inflection touch the x-axis but do not cross it (a turning point on the x-axis).

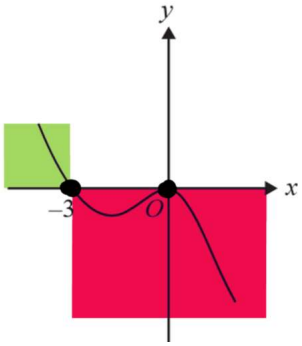
The steeper the gradient, the further away from the x-axis the derivative is plotted.
The flatter the gradient, the closer to the x-axis the derivative is plotted.
If the gradient approaches horizontal, the derivative will approach a horizontal asymptote, $y = 0$.

Example VCAA 2011 Exam 2 Question 9

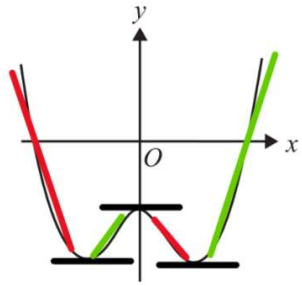
$y = f(x)$

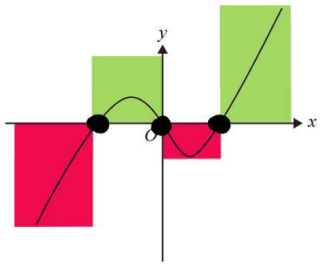


$y = f'(x)$

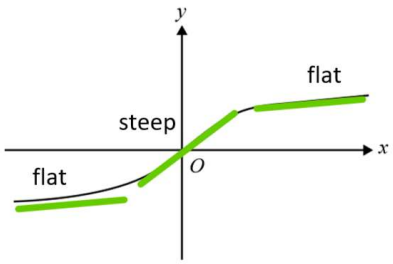


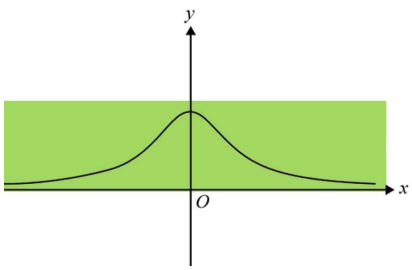
Example Modified VCAA 2010 Exam 2 Question 19



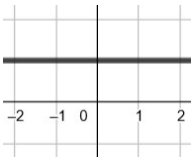
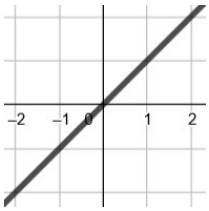
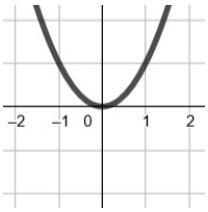
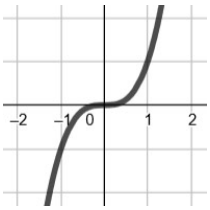
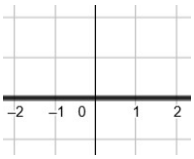
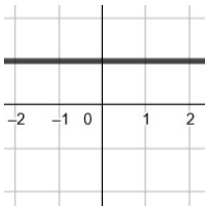
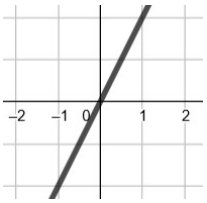
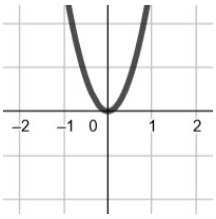


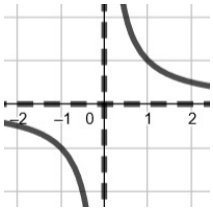
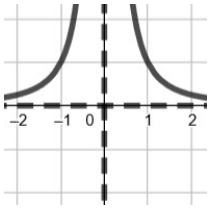
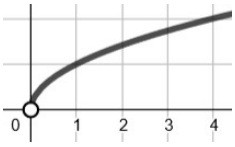
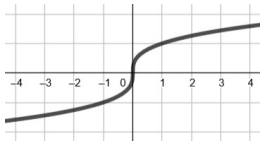
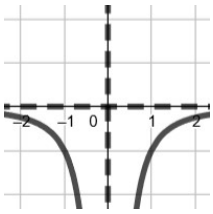
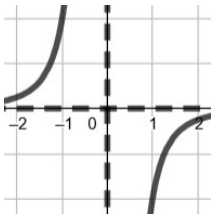
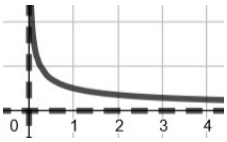
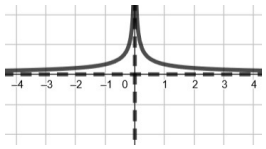
Example Modified VCAA 2008 Exam 2 Question 19

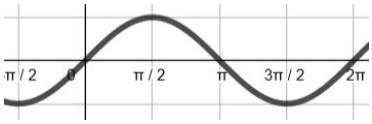
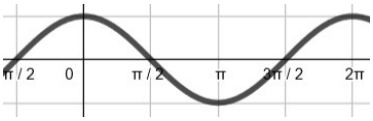
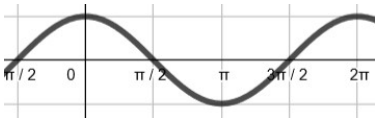
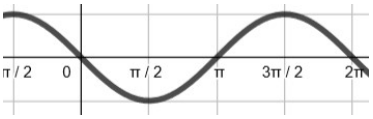


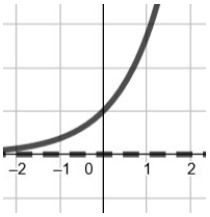
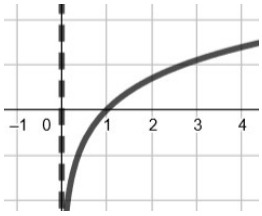
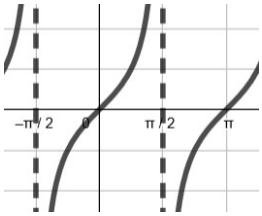
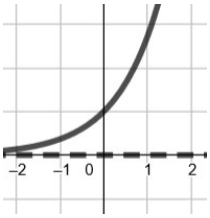
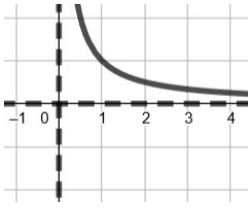
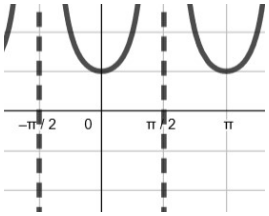


Graphs of the Derivatives of Functions

	$y = c$	$y = x$	$y = x^2$	$y = x^3$
Function				
Derivative				

	$y = \frac{1}{x}$	$y = \frac{1}{x^2}$	$y = \sqrt{x}$	$y = \sqrt[3]{x}$
Function				
Derivative				

	$y = \sin(x)$	$y = \cos(x)$
Function		
Derivative		

	$y = e^x$	$y = \log_e(x)$	$y = \tan(x)$
Function			
Derivative			

Graph of the Anti-Derivative Function

The graph of the anti-derivative can be found in two ways:

- the reverse of sketching the derivative, read of the derivative and use that to sketch the slope
- plot the area under the curve (from the y-axis) up to a point against the x-coordinate of that point (be careful if choosing a graph since any vertical translation is also an anti-derivative)

For using the area:
The area is positive when measuring left to right, and negative when measuring right to left.
The area is positive when it is above the x-axis, and negative when it is below the x-axis.
If the area is below the x-axis and measured right to left, the negative, negative area is positive.

Assuming no vertical translation, the anti-derivative will pass through the origin. If it does not, then the points that describe the area are from that y-intercept.

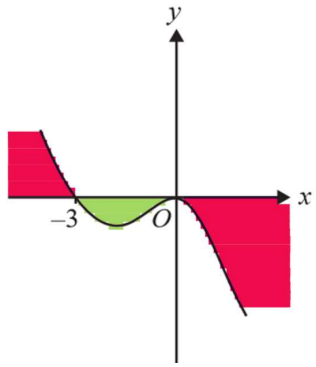
When the area in a section is positive, the graph of the anti-derivative will be decreasing away from the y-axis. When the area in a section is negative, the graph of the anti-derivative will be increasing away from the y-axis.

When the graph has an x-intercept, the graph of the anti-derivative will have a stationary point as the rate of change of the area is zero at that point.

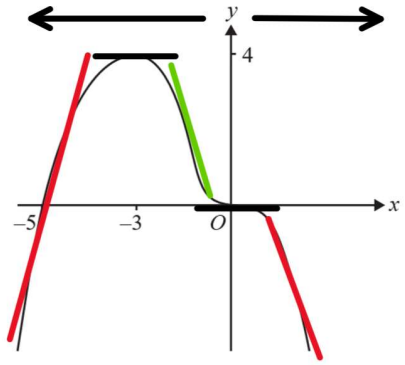
When the total area is positive, the graph of the anti-derivative will be above the y-intercept.
When the total area is negative, the graph of the anti-derivative will be below the y-intercept.

Example Modified VCAA 2011 Exam 2 Question 9

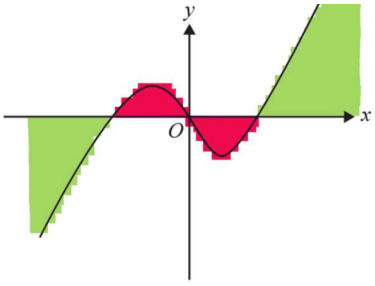
$y = g(x)$

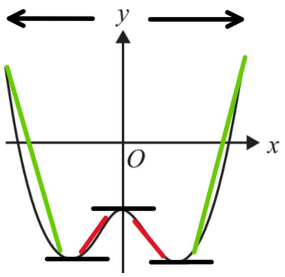


$y = G(x)$



Example VCAA 2010 Exam 2 Question 19





Example VCAA 2008 Exam 2 Question 19