Visual Derivatives of Functions

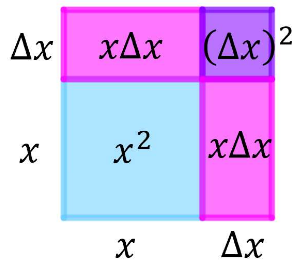
Derivative of x^2

$A = x^2$

$A + \Delta A = (x + \Delta x)^2 = x^2 + 2x\Delta x + (\Delta x)^2$

$\Delta A = 2x\Delta x + (\Delta x)^2$

$\frac{\Delta A}{\Delta x} = 2x + \Delta x, \qquad \lim_{\Delta x \to 0}\left(\frac{\Delta A}{\Delta x}\right) = 2x$



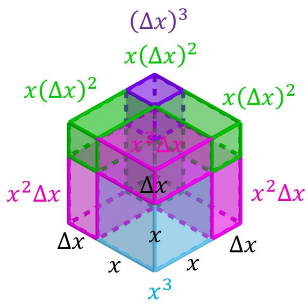
Derivative of x^3

$V = x^3$

$V + \Delta V = (x + \Delta x)^3 = x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + 3(\Delta x)^3$

$\Delta V = 3x^2\Delta x + 3x(\Delta x)^2 + 3(\Delta x)^3$

$\frac{\Delta V}{\Delta x} = 3x^2 + 3x\Delta x + 3(\Delta x)^2, \qquad \lim_{\Delta x \to 0}\left(\frac{\Delta V}{\Delta x}\right) = 3x^2$

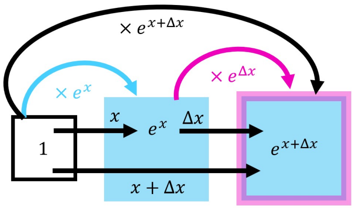


Derivative of Natural Exponential

$A = e^x$

$A + \Delta A = e^{x+\Delta x} = e^x e^{\Delta x}$

$\Delta A = e^x e^{\Delta x} - e^x = e^x(e^{\Delta x} - 1)$



$$\lim_{\Delta x \to 0}\left(\frac{\Delta A}{\Delta x}\right) = e^x \lim_{\Delta x \to 0}\left(\frac{(e^{\Delta x} - 1)}{\Delta x}\right) = e^x \lim_{\Delta x \to 0}\left(\frac{e^{\Delta x} - e^0}{\Delta x}\right) = e^x \times \frac{d}{dx}(e^x)\bigg|_{x=0}$$

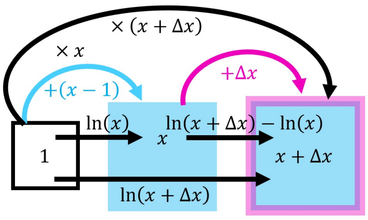
$\frac{d}{dx}(e^x)\bigg|_{x=0}$ = 100% growth of $e^0 = 1$, $\qquad \therefore \lim_{\Delta x \to 0}\left(\frac{\Delta A}{\Delta x}\right) = e^x \times 1 = e^x$

Derivative of the Natural Logarithm

$t = \log_e(x) = \ln(x)$

$t + \Delta t = \ln(x + \Delta x)$

$\Delta t = \ln(x + \Delta x) - \ln(x)$



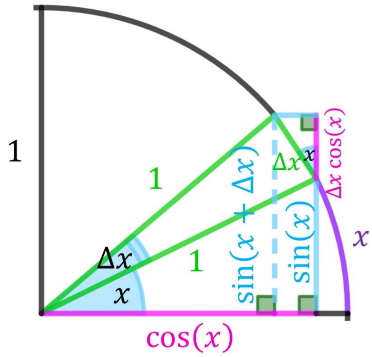
Δt is the time it takes for e^u to increase from x to $(x + \Delta x)$.

Since $\frac{d}{du}(e^u) = e^u$, e^u is increasing at a rate of x when $e^u = x$.

An increase of Δx is the product of the rate, x, and the time: (think distance $=$ speed $\times$ time)

$\Delta x = x\Delta t \;\Rightarrow\; \Delta t = \frac{\Delta x}{x}, \qquad \lim_{\Delta x \to 0}\left(\frac{\Delta t}{\Delta x}\right) = \lim_{\Delta x \to 0}\left(\frac{\Delta x}{x}/\Delta x\right) = \frac{1}{x}$

Derivative of Sine



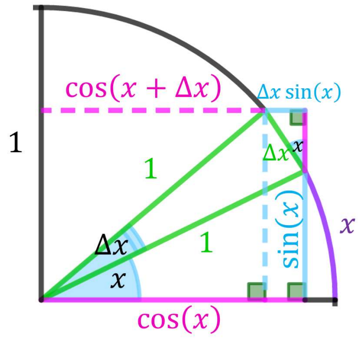
$$h = \sin(x)$$

$$\lim_{\Delta x\to 0}(h + \Delta h) = \lim_{\Delta x\to 0}(\sin(x + \Delta x)) = \sin(x) + \Delta x\cos(x)$$

$$\lim_{\Delta x\to 0}(h + \Delta h) - h = \lim_{\Delta x\to 0}(\Delta h) = \Delta x\cos(x)$$

$$\lim_{\Delta x\to 0}\left(\frac{\Delta h}{\Delta x}\right) = \cos(x)$$

Derivative of Cosine



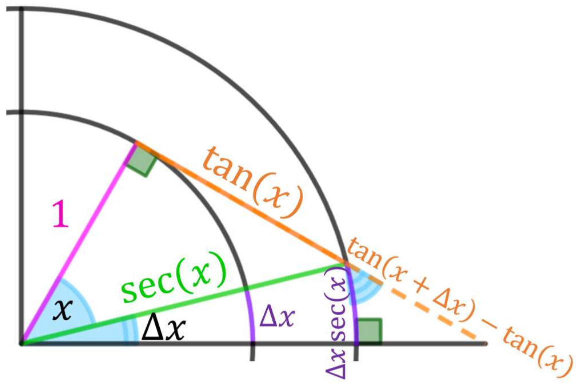
$$w = \cos(x)$$

$$\lim_{\Delta x\to 0}(w + \Delta w) = \lim_{\Delta x\to 0}(\cos(x + \Delta x)) = \cos(x) - \Delta x\sin(x)$$

$$\lim_{\Delta x\to 0}(w + \Delta w) - w = \lim_{\Delta x\to 0}(\Delta w) = -\Delta x\sin(x)$$

$$\lim_{\Delta x\to 0}\left(\frac{\Delta w}{\Delta x}\right) = -\sin(x)$$

Derivative of Tangent



$$d = \tan(x)\,, \qquad \ell = r\theta = \Delta x\sec(x)$$

$$\lim_{\Delta x\to 0}(d + \Delta d) = \lim_{\Delta x\to 0}(\tan(x + \Delta x))$$

$$\lim_{\Delta x\to 0}(\Delta d) = \lim_{\Delta x\to 0}(\tan(x + \Delta x) - \tan(x))$$

$$\lim_{\Delta x\to 0}(\sec(x + \Delta x)) = \frac{\lim_{\Delta x\to 0}(\Delta d)}{\Delta x\sec(x)}$$

$$\frac{\lim_{\Delta x\to 0}(\Delta d)}{\Delta x} = \lim_{\Delta x\to 0}(\sec(x + \Delta x))\sec(x)$$

$$\lim_{\Delta x\to 0}\left(\frac{\Delta d}{\Delta x}\right) = \sec^2(x)$$