Visual Derivatives of Combinations

Derivatives of Products

$A = fg$

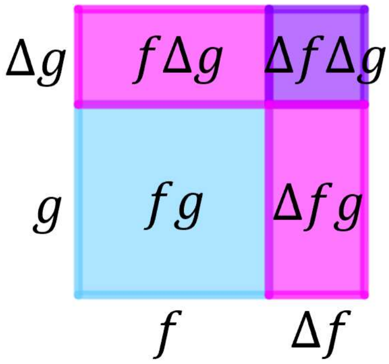
$A + \Delta A = f + \Delta(fg) = (f + \Delta f)(g + \Delta g)$

$f + \Delta(fg) = fg + f\Delta g + \Delta f g + \Delta f \Delta g$

$\Delta(fg) = f\Delta g + \Delta f g + \Delta f \Delta g$

$$\frac{\Delta(fg)}{\Delta x} = f\frac{\Delta g}{\Delta x} + \frac{\Delta f}{\Delta x}g + \frac{\Delta f \Delta g}{\Delta x}$$

$$\lim_{\Delta x \to 0}\left(\frac{\Delta(fg)}{\Delta x}\right) = f\frac{\Delta g}{\Delta x} + \frac{\Delta f}{\Delta x}g = fg' + f'g$$



Derivatives of Quotients

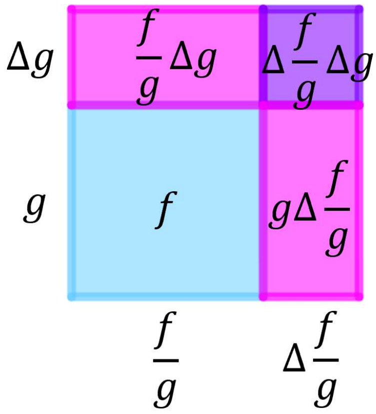
$A = f = \frac{f}{g}g$

$A + \Delta A = f + \Delta f = (g + \Delta g)\left(\frac{f}{g} + \Delta\frac{f}{g}\right)$

$f + \Delta f = \frac{f}{g}g + \frac{f}{g}\Delta g + \Delta\frac{f}{g}(g + \Delta g)$

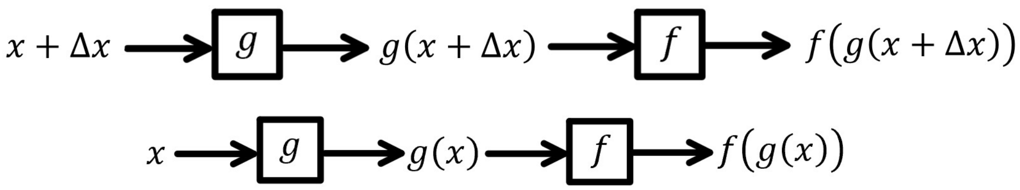
$\Delta f = \frac{f}{g}\Delta g + \Delta\frac{f}{g}(g + \Delta g)$

$\Delta\frac{f}{g}(g + \Delta g) = \Delta f - \frac{f}{g}\Delta g$



$$\Delta\frac{f}{g} = \frac{\Delta f - \frac{f}{g}\Delta g}{g + \Delta g} \Rightarrow \frac{\Delta\frac{f}{g}}{\Delta x} = \frac{\frac{\Delta f}{\Delta x} - \frac{f}{g}\frac{\Delta g}{\Delta x}}{g + \Delta g} \times \frac{g}{g} = \frac{\frac{\Delta f}{\Delta x}g - f\frac{\Delta g}{\Delta x}}{g^2 + g\Delta g} \Rightarrow \lim_{\Delta x \to 0}\left(\frac{\Delta\frac{f}{g}}{\Delta x}\right) = \frac{\frac{\Delta f}{\Delta x}g - f\frac{\Delta g}{\Delta x}}{g^2} = \frac{f'g - fg'}{g^2}$$

Derivatives of Composites



Δx $\quad \Delta g = g(x + \Delta x) - g(x)$

Rate that g is changing with respect to x

$$\frac{\Delta g}{\Delta x} = \frac{g(x + \Delta x) - g(x)}{\Delta x} = g'(x)$$

$\Delta f(g) = f(g(x + \Delta x)) - f(g(x))$

Rate that f is changing with respect to g

$$\frac{\Delta f(g)}{\Delta g} = \frac{f(g(x + \Delta x)) - f(g(x))}{g(x + \Delta x) - g(x)} = f'(g(x))$$

Rate that f is changing with respect to x

$$\frac{\Delta f(g)}{\Delta g} \times \frac{\Delta g}{\Delta x} = \frac{\Delta f(g)}{\Delta x} = f'(g(x))g'(x)$$