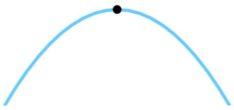
Intervals that are Stationary or Strictly Changing

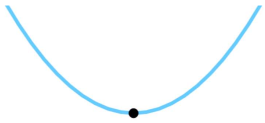
Stationary Points

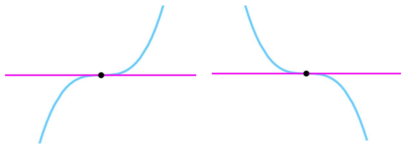
A function is stationary at a point if, at x_1, the tangent is horizontal, that is, has a slope of zero $f'(x_1) = 0$.

Nature of Stationary Points

Local Maximum Turning Point **Local Minimum Turning Point** **Stationary Points of Inflection**







The larger the value of the derivative, regardless of sign, the steeper the slope is at that point.

Methods to identify the nature of the stationary point include:

- use of the symmetry of the function
- identifying the function as an increasing or decreasing function
- use of range
- use of graph
- a sign table.

Local Maximum Turning Point

$f'(x)$	$+$	0	$-$
Slope			

Stationary Point of Inflection

$f'(x)$	$+$	0	$+$
Slope			

Local Minimum Turning Point

$f'(x)$	$-$	0	$+$
Slope			

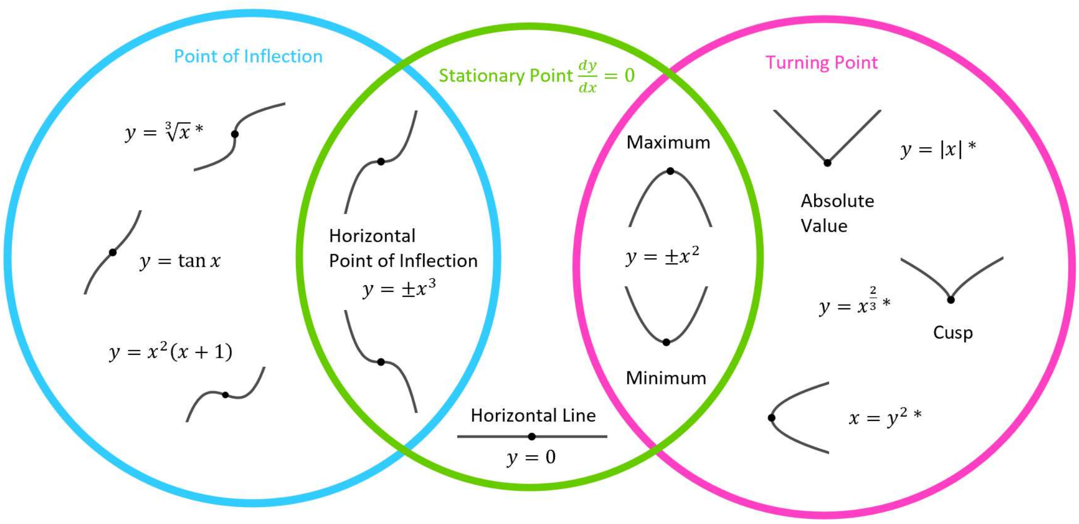
$f'(x)$	$-$	0	$-$
Slope			

Example VCAA 2014 Exam 2 Question 4

Let f be a function with domain R such that $f'(5) = 0$ and $f'(x) < 0$ when $x \neq 5$. At $x = 5$, the graph of f has a stationary point of inflection.

$x < 5$	$x = 5$	$x > 5$

* Not differentiable at this point
$\frac{dy}{dx}$ is undefined at this point



Locating Stationary Points

Stationary points occur when the derivative, that is the gradient, is zero. By determining the derivative and solving it equal to zero you will determine the x-coordinates of any stationary points. If the question asks for the coordinate of the stationary points, make sure to substitute the x-coordinates into the original function to determine the y values for each stationary point.

Example VCAA 2002 Exam 2 Question 3a

An equation in x, the solutions of which give the x-coordinates of the stationary points of the curve whose equation is $y = \frac{1}{2}(2x^4 - x^3 - 5x^2 + 3x)$.

$\frac{dy}{dx} = \frac{1}{2}(8x^3 - 3x^2 - 10x + 3)$, Stationary points when $\frac{dy}{dx} = 0 \therefore \frac{1}{2}(8x^3 - 3x^2 - 10x + 3) = 0$

Example VCAA 2014 Exam 1 Question 5a

Consider the function $f: [-1,3] \to R, f(x) = 3x^2 - x^3$.

The coordinates of the stationary points of the function are

$f'(x) = 6x - 3x^2$. Stationary points when $f'(x) = 0$:

$6x - 3x^2 = 0 \Rightarrow 3x(2 - x) = 0 \Rightarrow x = 0, x = 2 \quad f(0) = 3(0)^2 - (0)^3 = 0, f(2) = 3(2)^2 - (2)^3 = 4$

Therefore, the coordinates of the stationary points are $(0,0)$ and $(2,4)$.

Example VCAA 2017 Exam 2 Question 1a

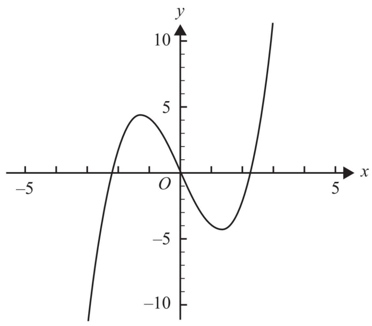
Let $f: R \to R, f(x) = x^3 - 5x$. Part of the graph of f is shown below.

The coordinates of the turning points are

$f'(x) = 3x^2 - 5$. Stationary points when $f'(x) = 0$

$3x^2 - 5 = 0 \Rightarrow x = \pm\sqrt{\frac{5}{3}} = \pm\frac{\sqrt{15}}{3}$

$f\left(\frac{\sqrt{15}}{3}\right) = -\frac{10\sqrt{15}}{9}, \quad f\left(-\frac{\sqrt{15}}{3}\right) = \frac{10\sqrt{15}}{9}$



Therefore, the coordinates of the turning points are $\left(\frac{\sqrt{15}}{3}, -\frac{10\sqrt{15}}{9}\right)$ and $\left(-\frac{\sqrt{15}}{3}, \frac{10\sqrt{15}}{9}\right)$.

Strictly Increasing

A function is strictly increasing over an interval if $f(x_1) < f(x_2)$ for all $x_1 < x_2$

Strictly Decreasing

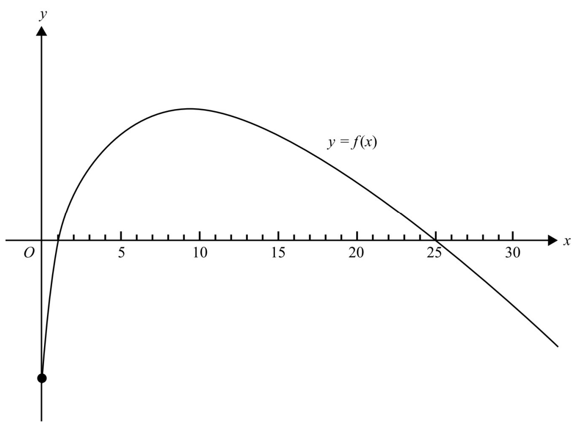
A function is strictly decreasing over an interval if $f(x_1) > f(x_2)$ for all $x_1 < x_2$

If the derivative of a function is strictly positive, $f'(x) > 0$, for all x, then it is a strictly increasing function. If the derivative of a function is strictly negative, $f'(x) < 0$, for all x, then it is a strictly decreasing function.

Example VCAA 2009 Exam 2 Question 1a

Let $f: R^+ \cup \{0\} \to R, f(x) = 6\sqrt{x} - x - 5$.

The graph of $y = f(x)$ is shown below. The interval for which the graph of f is strictly decreasing



$f'(x) = \frac{3}{\sqrt{x}} - 1 = 0$

$\Rightarrow 3 = \sqrt{x}$

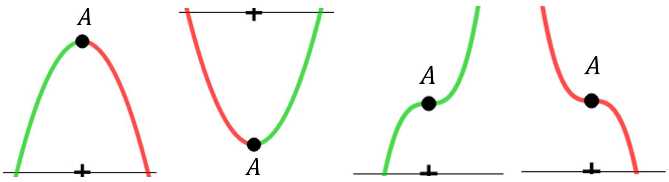
$\Rightarrow x = 9$

Strictly decreasing for $x \in [9, \infty)$ or $x \geq 9$

Be Careful using the Derivative!

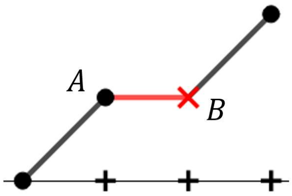
A graph is increasing if its derivative is positive and decreasing if its derivative is negative.
However, we cannot rely on that set of x-values where the derivative is positive or negative for strictly increasing or decreasing as they do not include endpoints of an interval (such as turning points and points of inflection) satisfy the definition, as they are greater/lesser than a smaller x-value, and so should be included but aren't in the derivative set.

The graphs shown right are strictly increasing on the green sections including the stationary point and strictly decreasing on the red sections including the stationary point.

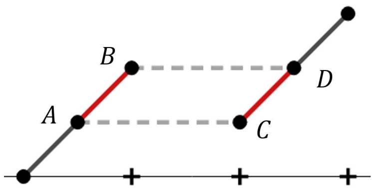


The derivative method will sometimes include unions of intervals that do not satisfy the definition such as

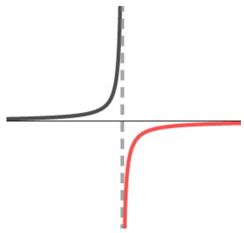
- unions of intervals that repeat y-values, as then $f(x_2) = f(x_1)$ for some $x_2 > x_1$
- unions of intervals where a right-more set of x-values has y-values that are lesser/greater than the y-values from a left-more set of x-values breaking the strictly increasing/decreasing y-values.



Only one of point A or B or a point along the line segment AB can be included since they all share the same y-value.



Line segments on AB and CD must be chosen so that y-values are not repeated.



Both halves are strictly increasing separately, however the right half is less than the left half, so it is not strictly increasing over the domain.