Area Approximation

Approximating the Area under a Curve

The area under a curve can be approximated by partioning the region into rectangles.
The heights of these rectangles can be determined from the height of the curve at either the left or right side of the rectangle. We keep the widths of the rectangles the same to make the calculations easier. This width can be chosen ahead of time or by evenly dividing up the width of the region.
The area of each rectangle is the given width $\times$ height of the curve on the left end of the rectangle.
For the curve $y = f(x)$ and rectangle widths δx, the area of each rectangle is $f(x)\delta x$ with x values that are δx apart from each other.

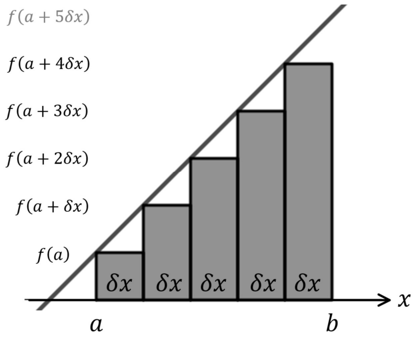
If the rectangle width is not given, then for the interval $[a, b]$ the width for n rectangles is $\delta x = \dfrac{b-a}{n}$.

Left-Endpoint Rectangles

The total area from a to b with n rectangles is
$f(a)\delta x + f(a+\delta x)\delta x + \cdots + f(a+(n-1)\delta x)\delta x$
That is, the sum from $i = 0$ to $i = n-1$ of $f(a + \delta x \times i) \times \delta x$

$$= \sum_{i=0}^{n-1} f(a + \delta x \times i)\delta x$$

The height of the first rectangle is $f(a)$.
The height of the last rectangle is $f(b - \delta x)$.

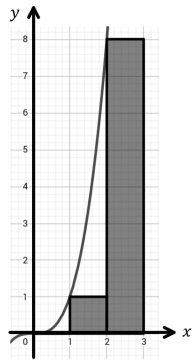


Example VCAA 2001 Exam 1 Question 18

Using the left rectangle approximation with rectangles of width 1, the area of the region bounded by the curve $y = x^3$, the x-axis, and the lines $x = 0$ and $x = 3$ is approximated by

$$\sum_{i=0}^{2}\left((0+1i)^3\right) \times 1 = 0^3 + 1^3 + 2^3$$

$$= 0 + 1 + 8 = 9 \text{ square units}$$

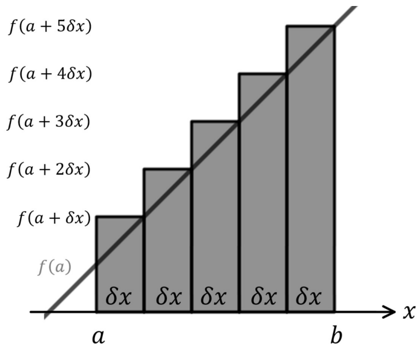


Right-Endpoint Rectangles

The total area from a to b with n rectangles is
$f(a+\delta x)\delta x + f(a+2\delta x)\delta x + \cdots + f(a+n\delta x)\delta x$
That is, the sum from $i = 1$ to $i = n$ of $f(a + \delta x \times i) \times \delta x$

$$= \sum_{i=1}^{n} f(a + \delta x \times i) \times \delta x$$

The height of the first rectangle is $f(a + \delta x)$.
The height of the last rectangle is $f(b)$.



Example VCAA 2002 Exam 1 Question 19

The **right** rectangle approximation with rectangles of width 1, for the area of the region bounded by the x-axis, the y-axis, the line $x = 3$ and the curve $y = \sqrt{(1+x)}$, is

$$\sum_{i=1}^{3}\left(\sqrt{(1+(0+1i))}\right) \times 1 = \sqrt{1+1} + \sqrt{1+2} + \sqrt{1+3}$$

$$= \sqrt{2} + \sqrt{3} + \sqrt{4} = \sqrt{2} + \sqrt{3} + 2 \text{ square units}$$