Approximation Applications

Distance Travelled in a Straight line

The distance travelled is the product of the speed of the object and the time it travelled for.
$d = s \times t$

If the speed is changing, then the total distance is the sum of the products of the corresponding speeds and times.

$$d = \sum_{i=0}^{n-1} s_i \times t_i$$

Where the speed is changing continuously, the distance can be approximated by evaluating the product of the speed at regular small intervals of time and the small interval of time. The approximation becomes more accurate the smaller each interval of time is.

$$d = \sum_{i=0}^{n-1} s(\delta t i) \times \delta t\,, \qquad n = \frac{\text{total interval of time}}{\delta t}$$

Example

The speed a bike is travelling in metres per second after t seconds is given by the function $f\colon [0,4] \to R, f(t) = t^2$. The distance travelled can be approximated using 1 second intervals. The distance travelled in the first 4 seconds is approximately

$$n = \frac{4}{1} = 4, \qquad d = \sum_{i=0}^{3} (1i)^2 \times 1 = 0^2 + 1^2 + 2^2 + 3^2 = 0 + 1 + 4 + 9 = 14 \text{ metres}$$

The approximation can be improved by taking a smaller interval. Using 0.1 second intervals, the distance travelled in the first 4 seconds is approximately

$$n = \frac{4}{0.1} = 40, \qquad d = \sum_{i=0}^{39} (0.1i)^2 \times 0.1 = 20.54 \text{ metres}$$

Cumulative Effects of Growth

The amount of change is the product of the rate it is changing and the amount of the unit it is changing with respect to.

The amount that the volume increases is the product of the rate it's volume is increasing respect to time or radius or some other length and the elapsed time, or change in radius or length.

Where the rate of change is changing continuously, the amount of change can be approximated by evaluating the product of the rate at regular small intervals of its respective unit and the small interval of the unit. The approximation becomes more accurate the smaller each interval is.

$$\delta A = \sum_{i=0}^{n-1} A'(\delta x i) \times \delta x\,, \qquad n = \frac{\text{total interval of } x}{\delta x}$$

Example

The rate of growth of a population of sheep at a farm after t months is given by

$$f\colon [0,12] \to R, f(t) = e^{0.1t} \sin\left(\frac{\pi t}{4}\right) + 3$$

The number of sheep added to the farm can be approximated using 1 month intervals. The number of sheep added to the farm over the first 5 months is approximately

$$n = \frac{5}{1} = 5, \qquad \delta N = \sum_{i=0}^{4} f(i) \times 1 = 3 + \frac{1}{\sqrt{2}} e^{0.1} + 3 + e^{0.2} + 3 + \frac{1}{\sqrt{2}} e^{0.3} + 3 + 3 \approx 18$$