

Integration and the Fundamental Theorem of Calculus

Integration (The Definite Integral)

The process of finding the area under a curve.

The area between the x -axis and the curve $y = f(x)$,

between the line $x = a$ and the line $x = b$ is $\int_a^b f(x) dx$.

$$\int_{\text{lower bound}}^{\text{upper bound}} \text{function } d(\text{variable})$$

The Fundamental Theorem of Calculus

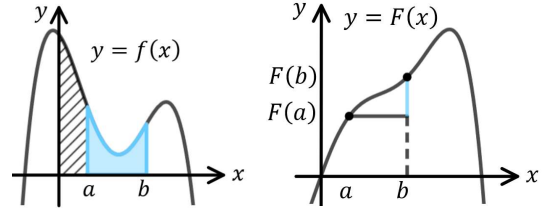
Finding the area under a graph is the converse of finding the gradient of a graph.

That is, integration and differentiation are opposites.

Newton-Leibniz Axiom

The integral (area) is equal to the change in height of the anti-derivative function between the bounds.

$$\int_a^b f(x) dx = F(b) - F(a), \quad \text{where } F'(x) = f(x)$$



Commonly we write the primitive function in square brackets with the bounds on the right side to break up the working so we do not have to substitute immediately into the primitive function.

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$$

The constant of integration can be left out in this calculation as it will always be subtracted.

$$\int_a^b f(x) dx = [F(x) + c]_a^b = (F(b) + c) - (F(a) + c) = F(b) - F(a)$$

Example

$$\int_{-4}^3 e^{2x+1} dx = \left[\frac{1}{2} e^{2x+1} \right]_{-4}^3 = \left(\frac{1}{2} e^{2(3)+1} - \frac{1}{2} e^{2(-4)+1} \right) = \frac{1}{2} (e^7 - e^{-7})$$

Example

$$\int_5^{10} 3 \sin(\pi x) dx = -\frac{3}{\pi} [\cos(\pi x)]_5^{10} = -\frac{3}{\pi} (\cos(10\pi) - \cos(5\pi)) = -\frac{3}{\pi} (1 - (-1)) = -\frac{6}{\pi}$$

Example VCAA 2010 Exam 1 Question 2b

Find p given that $\int_2^3 \frac{1}{1-x} dx = \log_e(p)$.
 Logarithm domain: $1 - x > 0 \Rightarrow x < 1$
 Required domain: $x > 1$, therefore use $-(1 - x) = x - 1$

$$\int_2^3 \frac{1}{1-x} dx = [\log_e(x-1)]_2^3 = \log_e(3-1) - \log_e(2-1) = \log_e(2) - \log_e(1) = \log_e(2) \therefore p = 2$$

The Indefinite Integral

The indefinite integral is another name for the primitive function including the $+c$. It is indefinite because it has no defined bounds to evaluate with so we are looking for the general integral.

$$\int f(x) dx = F(x) + c \quad \text{The indefinite integral of } f(x) \text{ with respect to } x \text{ is the primitive function, } F(x), \text{ plus the constant of integration, } c.$$