Kinematics - Straight Line Motion

Displacement $x(t)$

The difference between the final position and the origin of an object.
Unlike distance, displacement can be negative which indicates travelling in a reverse direction.

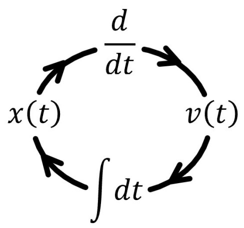
The displacement of an object at any time while it moves can be desribed by a function of time. $x = f(t)$

Velocity $v(t)$

The rate of change of the displacement of an object.
Unlike speed, velocity can be negative which indicates travelling in a reverse direction.

The velocity of an object at any time while it moves can be described by a function of time. $v = f(t)$

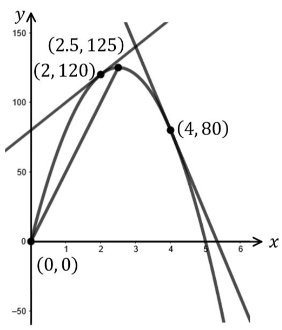
Determining Velocity from Displacement



The average velocity of an object is the rate of the change of the displacement over a change in time: $\dfrac{x_2 - x_1}{t_2 - t_1}$.
As the interval of time approaches 0, the limiting rate of change becomes a derivative. That is, $\dfrac{\mathrm{d}x}{\mathrm{d}t} = x'(t) = v(t)$.

Example

The displacement of a car in kilometres after t hours is given by the function $x(t) = 100t - 20t^2$.



The average velocity of the car in the first 2.5 hours is

$$\frac{x(2.5) - x(0)}{2.5 - 0} = \frac{125}{2.5} = 50 \text{ km/h.}$$

The instantaneous velocity is given by $x'(t) = 100 - 40t$.
The instantaneous velocity at $t = 2$ is $x'(2) = 100 - 40 \times 2 = 20$ km/h.

The velocity at $t = 4$ is $x'(4) = 100 - 40 \times 4 = -60$ km/h.
The speed at $t = 4$ is 60 km/h.

Displacement between Two Times

The approximate displacement travelled is the sum of the products of the velocity and regular intervals of time. $\sum v(\delta t)\delta t$. As the interval of time approaches 0, the limiting sum becomes a definite integral.
That is, the displacement from t_1 to t_2 $\displaystyle\int_{t_1}^{t_2} v(t)\mathrm{d}t = x(t_2) - x(t_1)$.

Example

The speed a bike is travelling in metres per second after t seconds is given by the function $f: [0,4] \to R, f(t) = t^2$. The distance travelled in the first 4 seconds is

$$\int_0^4 t^2 \mathrm{d}t = \frac{1}{3}[t^3]_0^4 = \frac{1}{3}(4^3 - 0^3) = \frac{64}{3} \text{ metres}$$

Distance between Two Times

Since distance is always positive, it is the signed area rather than just the definite integral of the velocity.

Determining the Displacement Function from the Velocity Function

The approximate displacement travelled is the sum of the products of the velocity and regular intervals of time. $\sum v(\delta t)\delta t$. As the interval of time approaches 0, the limiting sum becomes a definite integral.
That is, $\displaystyle\int_0^t v(u)\mathrm{d}u = x(t) + x(0)$.