Cumulative Effects of Growth

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The approximate amount of change is the sum of the products of the rate the amount is changing and regular intervals of the unit it is changing with respect to: $\sum A'(\delta xi)\delta x$.
As the interval of the unit approaches 0, the limiting sum becomes a definite integral.
That is, the amount of change from x_1 to x_2 $\int_{x_1}^{x_2} A'(x)\mathrm{d}x = A(x_2) - A(x_1)$.

Some common things that can be integrated to determine the change in amount:

- Velocity is the rate of change of the displacement with respect to the change in time: $\mathrm{d}x/\mathrm{d}t$
  Integrating the velocity gives the amount the displacement has changed from a change in time.

- Population rate is the rate of change of a population with respect to time: $\mathrm{d}P/\mathrm{d}t$
  Integrating the population rate gives the amount the population has changed in that time.

- Flow rates are the rates of change that a liquid is flowing with respect to time $\mathrm{d}L/\mathrm{d}t$
  Integrating the flow rate gives the amount of liquid that has flown in that time.

- Rate of change of area is the rate that an object's area changes with respect to time: $\mathrm{d}A/\mathrm{d}t$
  Integrating the rate of change of area gives the amount the area has changed in that time.

- Time rates: any rate per a change in time.
  Integrating a time rate gives the amount of change in that time

- Cost per mass is the rate of cost of an item with respect to its mass: $\mathrm{d}C/\mathrm{d}m$
  Integrating the cost per mass gives the amount of cost for an amount of mass.

- Density is the rate of change of the mass with respect to the change in volume: $\mathrm{d}m/\mathrm{d}V$
  Integrating the density gives the amount the mass has changed from a change in volume.

- Pressure is the amount of perpendicular force applied to a surface per unit of area: $\mathrm{d}F/\mathrm{d}A$
  Integrating the pressure gives the amount of perpendicular force applied for an amount of area.

- The height of an object as a function of its width: $h(w)$
  Integrating the height gives the amount of area for an particular width interval.

- Pollution level is the function of the pollution at a certain position: $P(x)$
  Integrating the pollution level gives the total amount of pollution in that distance interval.

Example VCAA 2011 Exam 2 Question 1a

Two ships, the Else and the Violet, have collided. Fuel immediately starts leaking from the Elsa into the sea. The captain of the Elsa estimates that at the time of the collision his ship has 6075 litres of fuel on board and he also forecasts that it will leak into the sea at a rate of $\frac{t^2}{5}$ litres per minute, where t is the number of minutes that have elapsed since the collision.

For all the fuel from the Elsa to leak into the sea at this rate, it will take

$$\int_0^a \frac{t^2}{5}\mathrm{d}t = 6075$$

$$\frac{1}{15}[t^3]_0^a = 6075$$

$$a^3 - 0 = 91125$$

$$a = \sqrt[3]{91125} = 45 \text{ minutes}$$