Calculus Summary

Derivative notation

For an expression, the derivative is $\frac{\mathrm{d}}{\mathrm{d}x}(\text{expression})$

For y = rule, the derivative is $\frac{\mathrm{d}y}{\mathrm{d}x}$ = derivative of rule

For $f(x)$ = rule, the derivative is $f'(x)$ = derivative of rule

The derivative is equal to the gradient of the function.

Chain rule: derivative of the composition of functions

$$\frac{\mathrm{d}}{\mathrm{d}x}(f[g(x)]) = f'[g(x)]g'(x)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}([f(x)]^n) = n[f(x)]^{n-1}f'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\tan[f(x)]) = \frac{f'(x)}{\cos^2[f(x)]} = f'(x)\sec^2[f(x)]$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\log_e[f(x)]) = \frac{f'(x)}{f(x)}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sin[f(x)]) = f'(x)\cos[f(x)]$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cos[f(x)]) = -f'(x)\sin[f(x)]$$

Sum and difference rules: derivative of sums and differences of functions

$$\frac{\mathrm{d}}{\mathrm{d}x}\big(f(x) \pm g(x)\big) = f'(x) \pm g'(x)$$

Product rule: derivative of products of functions

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)g(x)] = g(x)f'(x) + f(x)g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(uv) = v\frac{du}{dx} + u\frac{dv}{dx} = uv\left(\frac{u'}{u} + \frac{v'}{v}\right)$$

Quotient rule: derivative of quotients of functions

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{u}{v}\right) = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2} = \frac{u}{v}\left(\frac{u'}{u} - \frac{v'}{v}\right)$$

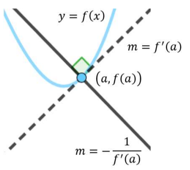
Tangents and perpendicular (normal) lines

$$m_{\text{tangent}} = f'(a)$$

$$y = f'(a)(x-a) + f(a)$$

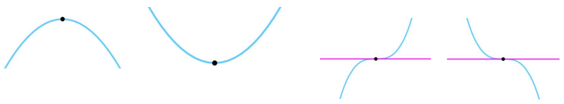
$$m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}} = -\frac{1}{f'(a)}$$

$$y = -\frac{1}{f'(a)}(x-a) + f(a)$$



Stationary points

Stationary points occur when $f'(x) = 0$.



Maxima / minima

Maximums and minimums occur when $f'(x) = 0$ or at the endpoints of the domain.

Strictly increasing

A function is strictly increasing over an interval if for all $x_1 < x_2$, $f(x_1) < f(x_2)$.
That is, if $f'(x) > 0$ for all $x_1 < x_2$.

Strictly decreasing

A function is strictly decreasing over an interval if for all $x_1 < x_2$, $f(x_1) > f(x_2)$.
That is, if $f'(x) < 0$ for all $x_1 < x_2$.

Anti-derivatives

$$\int x^n \mathrm{d}x = \frac{x^{n+1}}{n+1} + c, \qquad n \neq -1$$

$$\int (ax+b)^n \mathrm{d}x = \frac{(ax+b)^{n+1}}{a(n+1)} + c, \qquad n \neq -1$$

$$\int \frac{1}{ax+b} \mathrm{d}x = \frac{1}{a}\log_e(ax+b) + c, x > -\frac{b}{a}$$

$$\int \frac{1}{ax+b} \mathrm{d}x = \frac{1}{a}\log_e\big(-(ax+b)\big) + c, x < -\frac{b}{a}$$

$$\int e^{ax+b} \mathrm{d}x = \frac{1}{a} e^{ax+b} + c$$

$$\int \sin(ax+b)\,\mathrm{d}x = -\frac{1}{a}\cos(ax+b) + c$$

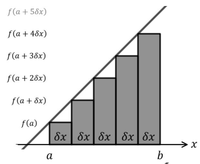
$$\int \cos(ax+b)\,\mathrm{d}x = \frac{1}{a}\sin(ax+b) + c$$

Approximating the area under a curve

The area under $f(x)$ from $x = a$ to $x = b$ approximated using n rectangles of width δx

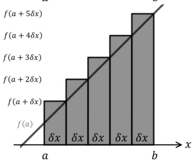
Left-endpoint rectangles

$$f(a)\delta x + f(a+\delta x)\delta x + \cdots + f(a+(n-1)\delta x)\delta x = \sum_{i=0}^{n-1} f(a+\delta x i)\delta x$$



Right-endpoint rectangles

$$f(a+\delta x)\delta x + f(a+2\delta x)\delta x + \cdots + f(a+n\delta x)\delta x = \sum_{i=1}^{n} f(a+\delta x i) \times \delta x$$



Definite integral properties

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a)$$

$$\int_a^b f(x)\mathrm{d}x = \int_a^c f(x)\mathrm{d}x + \int_c^b f(x)\mathrm{d}x$$

$$\int_a^b \big(f(x) \pm g(x)\big)\mathrm{d}x = \int_a^b f(x)\mathrm{d}x \pm \int_a^b g(x)\mathrm{d}x$$

$$\int_a^b f(x)\mathrm{d}x = -\int_b^a f(x)\mathrm{d}x$$

$$\int_a^b kf(x)\mathrm{d}x = k\int_a^b f(x)\mathrm{d}x$$

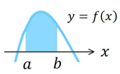
$$\int_a^b \big(f(x) + c\big)\mathrm{d}x = \int_a^b f(x)\mathrm{d}x + \int_a^b c\mathrm{d}x$$

$$\int_{ka}^{kb} f\left(\frac{x}{k}\right)\mathrm{d}x = k\int_a^b f(x)\mathrm{d}x$$

$$\int_{a+c}^{b+c} f(x-c)\mathrm{d}x = \int_a^b f(x)\mathrm{d}x$$

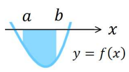
Area

$f(x) \geq 0$



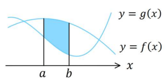
$$A = \int_a^b f(x)\mathrm{d}x \text{ units}^2$$

$f(x) < 0$



$$A = -\int_a^b f(x)\mathrm{d}x = \int_b^a f(x)\mathrm{d}x \text{ units}^2$$

$f(x) > g(x)$



$$A = \int_a^b \big(f(x) - g(x)\big)\mathrm{d}x \text{ units}^2$$

Averages

$$\text{average value} = \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x = \frac{F(b)-F(a)}{b-a}$$

$$\text{average rate of change} = \frac{f(b)-f(a)}{b-a}$$

Kinematics