Sampling without Replacement

Combinations

Out of n choices, choose r of them.

$$^{n}\mathrm{C}_{r} = \binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n!}{(n-r)!\,r!} = \binom{n}{n-r} = {}^{n}\mathrm{C}_{n-r}$$

$= n$th row, rth position in Pascal's Triangle

Properties of Combinations

$$^{n}\mathrm{C}_{n} = \binom{n}{n} = 1 \qquad {}^{n}\mathrm{C}_{0} = \binom{n}{0} = 1 \qquad {}^{0}\mathrm{C}_{0} = \binom{0}{0} = 1 \qquad {}^{n}\mathrm{C}_{1} = \binom{n}{1} = n$$

Sampling without Replacement

When selecting objects without replacing them, the probabilities change on each selection as the number of the object that was just selected is reduced and the total number of objects is reduced. If using this method, a tree diagram is helpful for mapping out the changes and each branch that is required.

Sampling without replacement follows a hypergeometric distribution such that for a total N objects where K have property X, the probability of selecting k with property X out of a selection of n is

$$\Pr(X = k) = \frac{{}^{K}\mathrm{C}_{k} \times {}^{N-K}\mathrm{C}_{n-k}}{{}^{N}\mathrm{C}_{n}}$$

If the question cannot be represented as having the property and not having the property, more combinations can be multiplied in the numerator such that the product is the combinations of selecting each individual property and the final is not selecting any of the previous properties (this can be ${}^{y}\mathrm{C}_{0}$).

Example VCAA 2015 Exam 2 Question 12

A box contains five red balls and three blue balls. John selects three balls from the box, without replacing them. The probability that at least one of the balls that John selected is red is

$$\Pr(R \geq 1) = 1 - \Pr(R = 0) = 1 - \left(\frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}\right) = 1 - \frac{1}{56} = \frac{55}{56}$$

$$\Pr(R \geq 1) = 1 - \Pr(R = 0) = 1 - \frac{{}^{5}\mathrm{C}_{0}\,{}^{3}\mathrm{C}_{3}}{{}^{8}\mathrm{C}_{3}} = 1 - \frac{1}{56} = \frac{55}{56}$$

Example VCAA 2017 Exam 2 Question 3

A box contains five red marbles and three yellow marbles. Two marbles are drawn at random from the box without replacement. The probability that the marbles are of different colours is the same as the probability of getting one red marble when selecting two marbles.

$$\Pr(R = 1) = \frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7} = \frac{15}{56} + \frac{15}{56} = \frac{30}{56} = \frac{15}{28}, \qquad \Pr(R = 1) = \frac{{}^{5}\mathrm{C}_{1}\,{}^{3}\mathrm{C}_{1}}{{}^{8}\mathrm{C}_{2}} = \frac{5 \times 3}{28} = \frac{15}{28}$$