Bernoulli Distribution

Bernoulli Trials and Distribution

A discrete random variable that has only two possible outcomes: a success or a fail.
It is named after Jacob Bernoulli.

The trials are independent of each other (the outcome of one trial has no influence over the outcome of another).

$\Pr(\text{success}) = p \qquad \Pr(\text{failure}) = 1 - p$

$$\Pr(X = x) = \begin{cases} 1 - p, & x = 0 \quad (0 \text{ successes}) \\ p, & x = 1 \quad (1 \text{ success}) \end{cases}$$

Sampling with replacement has independent events. Without replacement has dependent events.

Example

A fair standard 6-sided die is rolled once.
The probability of getting x 6s is given by

$$\Pr(X = x) = \begin{cases} \frac{5}{6}, & x = 0 \\ \frac{1}{6}, & x = 1 \end{cases}$$

x	0	1
$\Pr(X = x)$	$\frac{5}{6}$	$\frac{1}{6}$

The probability of rolling a 6 is $\Pr(X = 1) = \frac{1}{6}$

The probability of not rolling a 6 is $\Pr(X = 0) = \frac{5}{6}$

Example

A game of chance has a 0.3 probability of winning. One game is played. The probability of getting x wins is given by

$$\Pr(X = x) = \begin{cases} 0.7, & x = 0 \\ 0.3, & x = 1 \end{cases}$$

x	0	1
$\Pr(X = x)$	0.7	0.3

The probability of winning is $\Pr(X = 1) = 0.3$

The probability of not winning is $\Pr(X = 0) = 0.7$

Example

A fair coin is flipped once.
The probability of getting x heads is given by

$$\Pr(X = x) = \begin{cases} 0.5, & x = 0 \\ 0.5, & x = 1 \end{cases}$$

x	0	1
$\Pr(X = x)$	0.5	0.5

The probability of the coin landing on heads is $\Pr(X = 1) = 0.5$

The probability of the coin not landing on heads is $\Pr(X = 0) = 0.5$