Statistics of Binomial Random Variables

Statistics of Bernoulli Distributions

$$\Pr(X = x) = p(x) = \begin{cases} p, & x = 1 \\ 1 - p, & x = 0 \\ 0, & \text{elsewhere} \end{cases}$$

Expected Value / Mean

$$\mathrm{E}(X) = \mu = \sum x \Pr(X = x)$$

$$= 0(1 - p) + 1 \times p = p$$

Variance

$$\mathrm{var}(X) = \sigma^2 = \sum (x - \mu)^2 \Pr(X = x)$$

$$= (0 - p)^2(1 - p) + (1 - p)^2 p$$

$$= p^2(1 - p) + (1 - p)^2 p$$

$$= p(1 - p)\big(p + (1 - p)\big) = p(1 - p)$$

Standard Deviation

$$\mathrm{sd}(X) = \sigma = \sqrt{\mathrm{var}(X)} = \sqrt{p(1 - p)}$$

Statistics of Binomial Distributions

Let $X = B_1 + B_2 + \cdots + B_n = \sum B$, where B_i are independent Bernoulli trials with the same probability.

Expected Value / Mean

$$\mathrm{E}(X) = \mu = \mathrm{E}(B_1 + B_2 + \cdots + B_n)$$

$$= \mathrm{E}(B_1) + \mathrm{E}(B_2) + \cdots + \mathrm{E}(B_n)$$

$$= p + p + \cdots + p = np$$

Variance

$$\mathrm{var}(X) = \sigma^2 = \mathrm{var}(B_1 + B_2 + \cdots + B_n)$$

$$= \mathrm{var}(B_1) + \mathrm{var}(B_2) + \cdots + \mathrm{var}(B_n)$$

$$= p(1 - p) + p(1 - p) + \cdots + p(1 - p)$$

$$= np(1 - p) = \mu(1 - p)$$

Standard Deviation

$$\mathrm{sd}(X) = \sigma = \sqrt{\mathrm{var}(X)} = \sqrt{np(1 - p)}$$

Example Modified VCAA 2001 Exam 2 Question 2dii

The Candlelite Company produces scented candles. The Candlelite Company claims that 90% of all the scented candles, regardless of the scent, will burn for at least 30 hours. Assuming that this claim is correct, the expected number of scented candles in a box of 25 that will burn for more than 30 hours is

$X \sim \mathrm{Bi}(25, 0.9)$, $\mathrm{E}(X) = 25 \times 0.9 = 22.5$

The variance and standard deviation of scented candles in a box of 25 that will burn for more than 30 hours are

$\mathrm{var}(X) = 25 \times 0.9 \times 0.1 = 2.25$, $\mathrm{sd}(X) = \sqrt{2.25} = 1.5$

Example VCAA 2008 Exam 2 Question 5

Let X be a discrete random variable with a binomial distribution.
The mean of X is 1.2 and the variance of X is 0.72.
The values of n (the number of independent trials) and p (the probability of success in each trial) are

$\mu = np = 1.2$
$\sigma^2 = np(1 - p) = 0.72$

$1.2(1 - p) = 0.72$
$\Rightarrow 1 - p = 0.6$
$\Rightarrow p = 0.4$

$0.4n = 1.2$
$\Rightarrow n = 3$