# Conditional Probability - Continuous Random Variables

## Continuous

The probability distribution function for the continuous random variable $X$ is given by $f$ such that

$$\int_{-\infty}^{\infty} f(x)\mathrm{d}x = 1$$

Given that $x$ lies within $a < x < b$, the area in that domain must equal 1,

That is $k\int_a^b f(x)\mathrm{d}x = 1$ where $k = \frac{1}{\int_a^b f(x)\mathrm{d}x}$ such that $g(x) = kf(x)$.

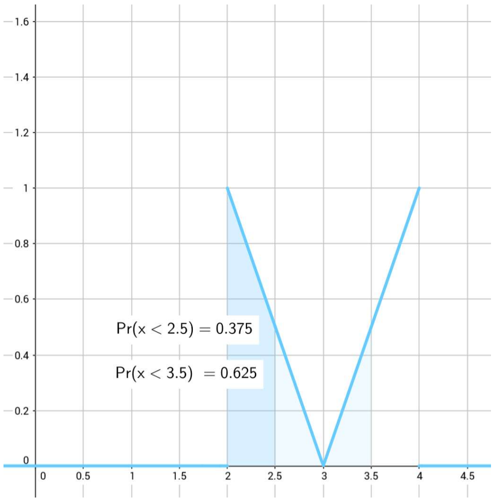
$$\Pr(x_1 < X < x_2 | a < X < b) = \int_{x_1}^{x_2} g(x)\mathrm{d}x = k\int_{x_1}^{x_2} f(x)\mathrm{d}x = \frac{1}{\int_a^b f(x)\mathrm{d}x}\int_{x_1}^{x_2} f(x)\mathrm{d}x$$

Alternatively, $\Pr(x_1 < X < x_2 | a < X < b) = \frac{\int_{x_1}^{x_2} f(x)\mathrm{d}x}{\int_a^b f(x)\mathrm{d}x}, \qquad a < x_1 < x_2 < b$

## Example Modified VCAA 2011 Exam 1 Question 5

The probability distribution function for the continuous random variable $X$ is given by

$$f(x) = \begin{cases} 3 - x, & 2 \le x \le 3 \\ x - 3, & 3 < x \le 4 \\ 0, & \text{elsewhere} \end{cases}$$




$$\Pr(X < 3.5) = \int_2^{3.5} f(x)\mathrm{d}x$$

$$= \int_2^3 (3 - x)\mathrm{d}x + \int_3^{3.5} (x - 3)\mathrm{d}x$$

Using area of triangles:

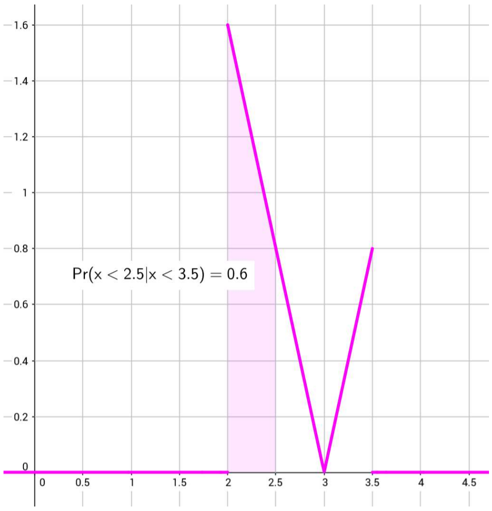
$$= \frac{1}{2}(1 \times 1) + \frac{1}{2}\left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$$

## Given that $x < 3.5$

$$k\int_2^{3.5} f(x)\mathrm{d}x = 1 \;\Rightarrow\; \frac{5k}{8} = 1 \;\Rightarrow\; k = \frac{8}{5}$$

$$g(x) = \frac{8}{5}f(x), \qquad 2 \le x \le 3.5$$

$$g(x) = \frac{8}{5}\begin{cases} 3 - x, & 2 \le x \le 3 \\ x - 3, & 3 < x \le 3.5 \\ 0, & \text{elsewhere} \end{cases}$$




$$\Pr(X < 2.5 | X < 3.5) = \frac{8}{5}\int_2^{2.5} f(x)\mathrm{d}x$$

$$= \frac{8}{5}\int_2^{2.5} (3 - x)\mathrm{d}x = \frac{8}{5} \times \frac{1}{2}\left(\frac{1}{2} + 1\right)\frac{1}{2} = \frac{8}{5} \times \frac{3}{8} = \frac{3}{5}$$

Alternatively

$$\Pr(X < 2.5 | X < 3.5) = \frac{\Pr(X < 2.5)}{\Pr(X < 3.5)} = \frac{\int_2^{2.5} f(x)\mathrm{d}x}{5/8} = \frac{3/8}{5/8} = \frac{3}{5}$$