Normal Distribution

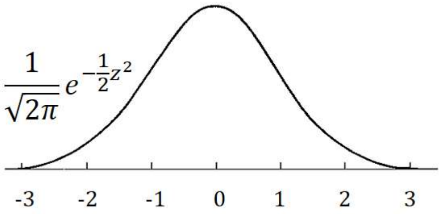
Standard Normal Distribution

A perfectly symmetric continuous random variable defined by the Gaussian Bell Curve.
The area under the entire curve is 1. It is 'normal' in the sense that it is standard model.

$Z \sim \mathrm{N}(0,1)$

Mean / Expected Value	Median	Variance	Standard Deviation
$\mathrm{E}(X) = \mu = 0$	$m = 0$	$\mathrm{var}(X) = \sigma^2 = 1$	$\mathrm{sd}(X) = \sigma = 1$

$$g(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$



$$g(z) = \frac{1}{\sqrt{2\pi e^{z^2}}}$$

Properties of the Standard Normal Distribution

$\Pr(Z < z) + \Pr(Z > z) = 1$

$\Pr(Z > z) = 1 - \Pr(Z < z)$

$\Pr(Z < -z) = \Pr(Z > z)$

$\Pr(Z > 0) = 0.5$

$\Pr(a < Z < b) = \Pr(Z < b) - \Pr(Z < a)$

Transformed Normal Distribution

A transformation of the standard normal distribution with a dilation by a scale factor of $\frac{1}{\sigma}$ parallel to the y-axis, a dilation by a scale factor of σ parallel to the x-axis, a translation of μ to the right.

$X \sim \mathrm{N}(\mu, \sigma^2)$ $\qquad z = \frac{x - \mu}{\sigma}$ $\qquad x = \mu + z\sigma$ $\qquad f(x) = \frac{1}{\sigma} g\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Mean / Expected Value	Median	Variance	Standard Deviation
$\mathrm{E}(X) = \mu$	$m = \mu$	$\mathrm{var}(X) = \sigma^2$	$\mathrm{sd}(X) = \sigma$

Example VCAA 2016 Exam 2 Question 16

The random variable, X, has a normal distribution with mean 12 and standard deviation 0.25
If the random variable, Z, has the standard normal distribution, then the probability that X is greater than 12.5 is equal to

$X \sim \mathrm{N}(12, 0.25^2)$, $\qquad Z \sim \mathrm{N}(0, 1)$, $\qquad \Pr(X > 12.5) = \Pr\left(Z > \frac{12.5 - 12}{0.25}\right) = \Pr(Z > 2)$

Example VCAA 2015 Exam 1 Question 6 a

Let the random variable X be normally distributed with mean 2.5 and standard deviation 0.3.
Let Z be the standard normal random variable, such that $Z \sim \mathrm{N}(0, 1)$.

The value of b such that $\Pr(X > 3.1) = \Pr(Z < b)$ is

$$\Pr(X > 3.1) = \Pr\left(Z > \frac{3.1 - 2.5}{0.3}\right) = \Pr(Z > 2) = \Pr(Z < -2)$$

$\therefore b = -2$

Determining Probabilities with Normal Distribution

Probabilities for the normal distribution are generally determined on the CAS.

Example VCAA 2017 NHT Exam 2 Question 12

The maximum temperature reached by the water heated in a kettle each time it is used is normally distributed with a mean of 95 °C and a standard deviation of 2 °C. When the kettle is used, the proportion of times that the maximum temperature reached by the water is greater than 98 °C is

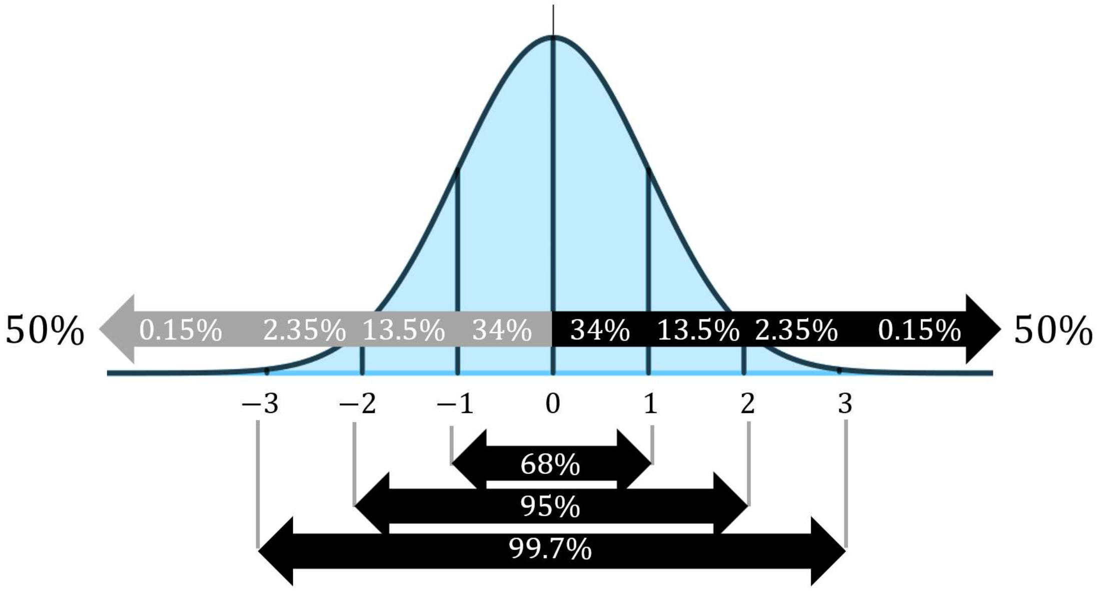
Using $X \sim \mathrm{N}(95, 2^2)$
On CAS use the Normal Cdf function with Lower Bound $= 98$, Upper Bound $= \infty$, $\mu = 95$, $\sigma = 2$
$\Pr(X > 98) = 0.0668$

Using $Z \sim \mathrm{N}(0, 1)$
On CAS use the Normal Cdf function with Lower Bound $= 1.5$, Upper Bound $= \infty$, $\mu = 0$, $\sigma = 1$
$\Pr(Z > 1.5) = 0.0668$

Approximation of the Probabilities of Normally Distributed Variables

We can approximate the probabilities of the normal distribution by using the 68-95-99.7% rule.
All values shown below can be determined by using symmetry and the facts that

- the probability of being within 1 standard deviation of the mean is roughly 0.68
- the probability of being within 2 standard deviations of the mean is roughly 0.95
- the probability of being within 3 standard deviations of the mean is roughly 0.997
- the probability of being more than the mean is 0.5



Example VCAA 2015 Exam 1 Question 6b

Let the random variable X be normally distributed with mean 2.5 and standard deviation 0.3.
Let Z be the standard normal random variable, such that $Z \sim \mathrm{N}(0, 1)$.

Using the fact that, correct to two decimal places, $\Pr(Z < -1) = 0.16$, find $\Pr(X < 2.8 | X > 2.5)$

$\Pr(X < 2.8 | X > 2.5)$, 2.8 is one standard deviation above the mean, and 2.5 is the mean.

$$= \Pr(Z < 1 | Z > 0) = \frac{\Pr(Z < 1 \cap Z > 0)}{\Pr(Z > 0)} = \frac{\Pr(0 < Z < 1)}{\Pr(Z > 0)} = \frac{\Pr(-1 < Z < 0)}{\Pr(Z > 0)}$$

$$= \frac{\Pr(Z < 0) - \Pr(Z < -1)}{\Pr(Z > 0)} = \frac{0.5 - 0.16}{0.5} = \frac{0.34}{0.5} = 0.34 \times 2 = 0.68$$